Standards Resource Guide

Algebra I

Overview

This guide is designed to support educators in interpreting and implementing the 2025 Louisiana Student Standards for Mathematics (LSSM). It provides descriptions of each standard to clarify its meaning and application to student learning. The document outlines the intended level of rigor and includes coherence to prerequisite and corequisite standards. Examples are provided for illustrative purposes only and do not represent a comprehensive list.

Additional information on the 2025 Louisiana Student Standards for Mathematics, including guidance on reading standards codes, a listing of standards for each grade or course, and links to additional resources, is available on the Louisiana Math Page. Questions may be directed to math@la.gov.

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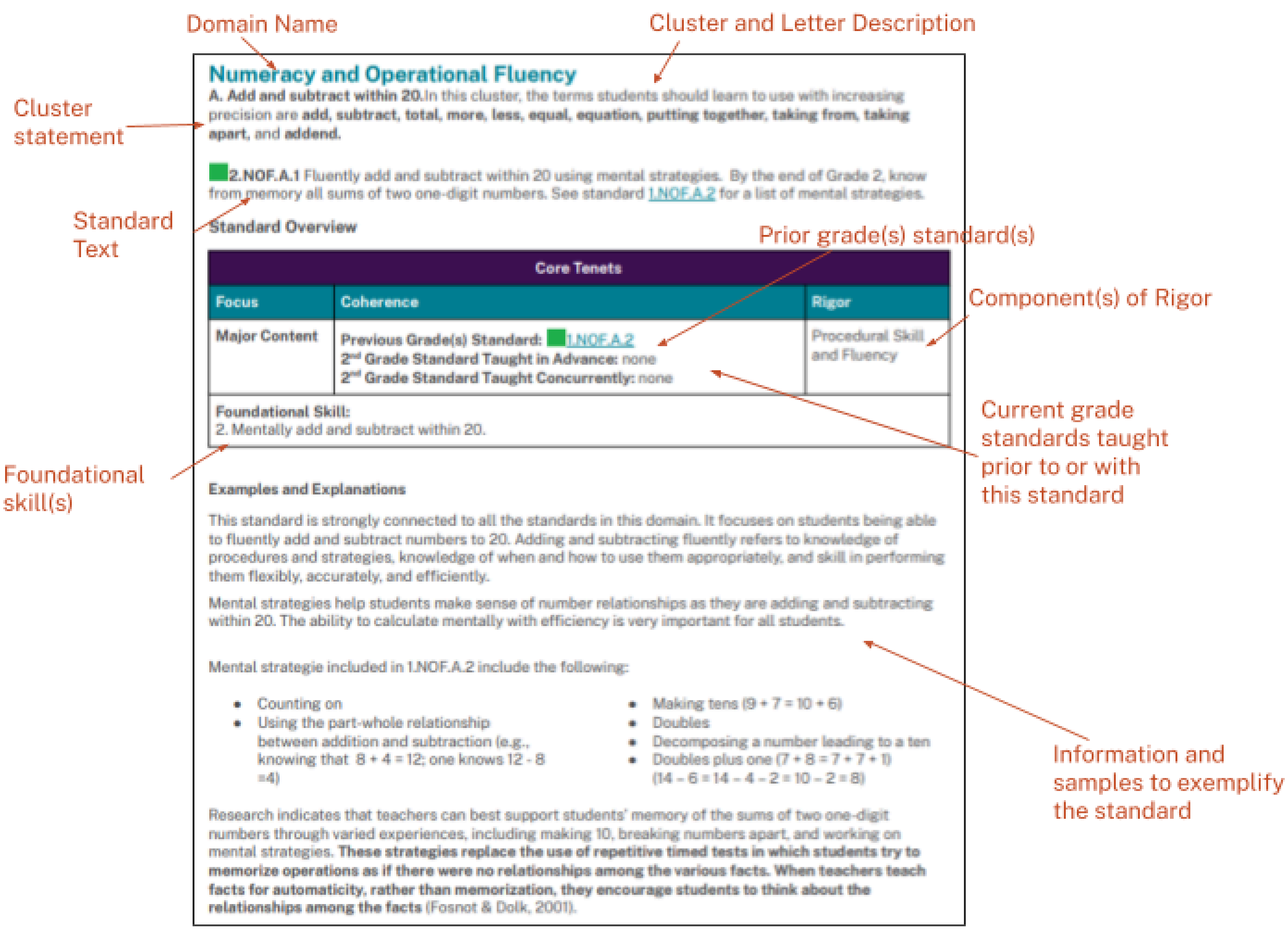
Introduction

How to Read This Guide

The diagram below provides an overview of the information found in all standards resource guides. Definitions and more complete descriptions are provided on the next page.



Text of the standard

Shading of Standard Codes: **Major Work,** **Supporting Work,** **Additional Work**

1. **Domain Name and Abbreviation:** A grouping of standards consisting of related content that is further divided into clusters. Each domain has a unique abbreviation, which is provided in parentheses beside the domain name.
2. **Cluster Letter and Description:** Each cluster within a domain begins with a letter. The description provides a general overview of the focus of the standards in the cluster.
3. **Previous Grade(s) Standards:** One or more standards that students should have mastered in previous grades to prepare them for new learning related to the current grade-level standard. For

students who have unfinished learning, teachers should build in proactive, just-in-time supports to build essential prerequisite knowledge.

4. **Standards Taught in Advance:** These grade-level standards demonstrate within-grade coherence and include concepts on which the target standard is built. Students should have exposure to these standards before instruction in the target standard. As with previous-grade standards, teachers should plan proactive, just-in-time supports to build within-grade prerequisite knowledge ahead of instruction on the target standard.
5. **Standards Taught Concurrently:** These grade-level standards demonstrate within-grade coherence and include standards that should be taught at the same time as the target standard to provide coherence and connectedness in instruction.
6. **Component(s) of Rigor:** See full explanation on components of rigor below.
7. **Sample Problem:** The sample problem provides an example of how a student might meet the requirements of the standard. Multiple examples are provided for some standards. However, sample problems should not be considered an exhaustive list. Explanations, when appropriate, are also included.
8. **Text of Standard:** The complete text of the targeted Louisiana Student Standards of Mathematics is provided.

Classification of Major, Supporting, and Additional Work

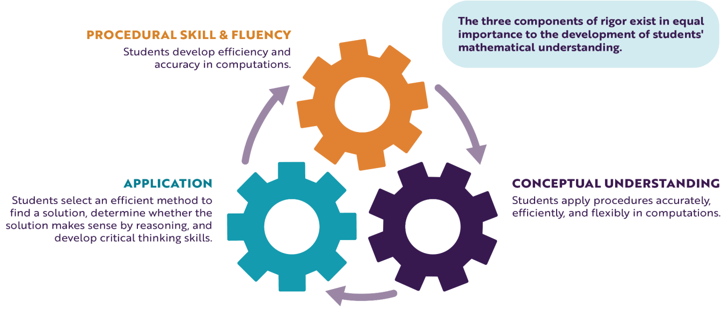
Students should spend the majority of their time on the major work of the grade, supporting work and, where appropriate, additional work engages students in the major work of the grade, often through a change of the **context** within which students are performing mathematics. Each standard is color-coded to quickly and simply determine the focus for the grade. Furthermore, standards from previous grades that provide foundational skills for current grade standards are also color-coded to show whether those standards are classified as major, supporting, or additional in their respective grades.

Core Tenets

The 2016 Louisiana Student Standards for Math called for three instructional shifts. Since then, teachers have used these shifts to ensure deep understanding and consistent application in mathematics classrooms across the state. Now, these shifts are recognized as core tenets that serve as the foundation and guiding principles of Louisiana's mathematics standards.

Coherence in mathematics refers to a coherent body of knowledge composed of interconnected concepts, principles, and procedures. These concepts are logically linked, building upon one another, and work together to deepen understanding, support problem solving, and enable learners to apply mathematical ideas consistently and meaningfully across different contexts.

Rigor in mathematics instruction does not simply mean increasing the difficulty or complexity of practice problems. Incorporating rigor into classroom instruction and student learning requires students to engage deeply with standards and grapple meaningfully with the concepts and ideas embedded within them. The **three** components of rigor, listed



below, exist in equal importance to the development of students' mathematical understanding.

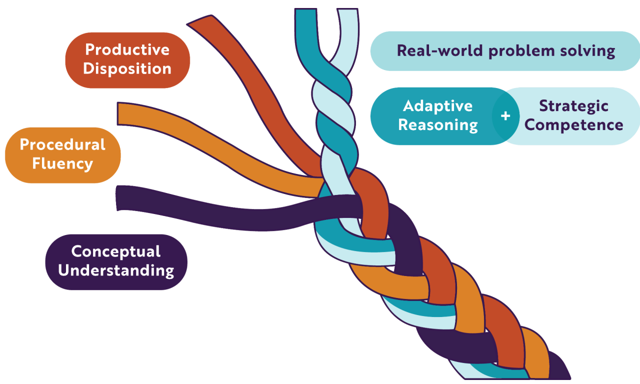
- **Conceptual understanding** refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.
- **Procedural skill and fluency** is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills without dictating the strategy or method chosen. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.
- **Application** provides valuable context for learning and the opportunity to solve problems in a relevant and meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

Focus calls for intentional emphasis on specific topics, skills, and ideas within grade level mathematics instruction. Not all content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Louisiana Standards for Mathematical Practice. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Students should spend the large majority of their time on the major ( ) work of the grade, supporting ( ) work and, where appropriate, additional ( ) work can engage students in the major work of the grade.

The Strands for Mathematical Proficiency

The strands of mathematical proficiency were introduced by the National Research Council (NRC) in their 2001 report, Adding It Up: helping children learn mathematics. These proficiencies provide a framework describing what it means for a learner to be proficient in mathematics.

The strands of mathematical proficiency are intertwined, gradually developed, and deepened over time. When intentionally presented in a connected and balanced way, each strand supports the others, ultimately strengthening the proficiency of learners.



1. **Conceptual Understanding** - Students develop an understanding of key mathematical ideas and relationships and explain why procedures work.
2. **Procedural Fluency** - Students use mathematical procedures with skill and flexibility, guided by a strong grasp of underlying concepts.
3. **Productive Dispositions** - Students view mathematics as coherent, valuable, and meaningful
4. **Real World and Problem Solving** - A combination of both adaptive reasoning and strategic competence, students represent and solve real-world mathematical problems logically, justify solutions, and clearly explain and defend their reasoning.

The Relationship Between Mathematical Proficiency and the Components of Rigor

While there is some overlap between the Strands of Mathematical proficiency and the Core Tenets, there is a distinct difference between the two. The strands are a framework that is focused on learner outcomes and what it means for a student to be proficient in mathematics. The Core Tenets provide a framework for how teachers should design instruction to support learning. The core tenets reflect what should be done instructionally, while the proficiency strands represent the outcome, or what students become, as a result of that instruction.

Mathematical Fluency

Fluency in mathematics is the ability to apply foundational skills and concepts flexibly, accurately, and efficiently, grounded in strong conceptual understanding to solve problems, make connections, and reason mathematically across various contexts. While fluency is an essential component of mathematics instruction, it does not represent the full depth of a mathematical standard; it must be developed alongside conceptual understanding, reasoning, and problem-solving to ensure deep and meaningful learning.

Concrete - Representational - Abstract Framework

The Concrete-Representational-Abstract (CRA) Framework describes the instructional processes that students experience to build conceptual understanding of mathematics. The first stage is where students build a concrete understanding of a concept through the use of manipulatives or other physical objects. During the second phase, learners move into visual representations of concepts. Finally, during the abstract phase, students apply their understanding of the concepts abstractly through the use of numbers and symbols. When students engage in learning that follows this process, it allows them to build a solid understanding of key mathematical concepts and ideas, ensuring comprehension that is transferable and flexibly applied across grade bands.

An example of how this framework is represented across the progression of the Louisiana Student Standards for Mathematics can be seen in the way student learning progresses across a concept, such as linear functions. In fourth grade (4.AR.A.1), students are introduced to multiplicative comparison, where much of the learning is very **concrete**. At this phase, students are using counters and other manipulatives to physically compare quantities to determine the meaning of “times as many.” When they move into the middle grades, this becomes more representational as students apply concrete understanding to recognize and represent proportional relationships. These are represented in various ways, such as ratio tables, double number lines, and graphs (6.PF.A.2, 7.PF.A.2). When students enter high school, the learning becomes **abstract** as they interpret linear functions such as y=kx and come to understand slope as rate of change.

While the example above represents a linear version of this instructional process across multiple grades, it is important to note that the concrete and representational components can occur at different times throughout the progression of learning, and may occur simultaneously across various periods. Additionally, this instructional process may also occur within lessons, units, or within the grade level.

Standards for Mathematical Practice

The Louisiana Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students in grades K-12. Below are a few examples of how these practices may be integrated into tasks that students in high school complete.

Louisiana Standards for Mathematical Practice (MP)	
Louisiana Standard	**Explanations and Examples**
HS.MP.1 Make sense of problems and persevere in solving them.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2 Reason abstractly and quantitatively.	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3 Construct viable arguments and critique the reasoning of others.	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an argument — explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
HS.MP.4 Model with mathematics.	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest

	depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They can identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
HS.MP.5 Use appropriate tools strategically.	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
HS.MP.6 Attend to precision.	High school students try to communicate precisely with others by using clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and label axes to clarify the correspondence between quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.
HS.MP.7 Look for and make use of structure.	By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, compose functions, and transform figures.
HS.MP.8 Look for and express regularity in repeated reasoning.	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas, or make generalizations, high school students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

What is modeling?

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder or whether a two-dimensional disk works well enough for our purposes. Other situations, modeling a delivery route, a production schedule, or a comparison of loan amortizations,need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

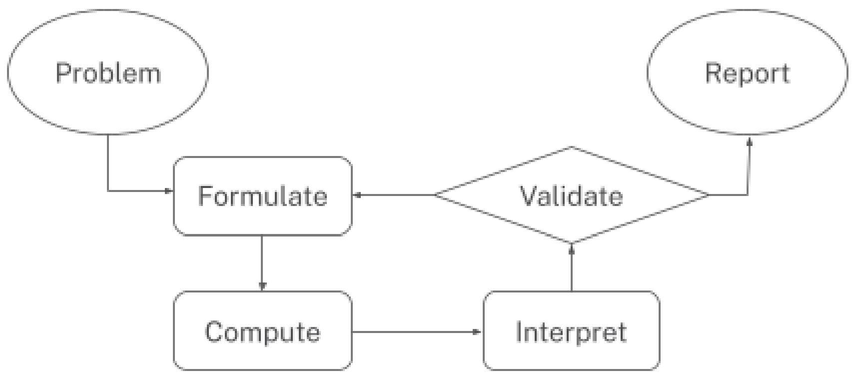
Some examples of such situations might include:

- Estimate how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Plan a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other.
- Design the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyze the stopping distance for a car.
- Model a savings account balance, bacterial colony growth, or investment growth.
- Engage in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyze the risk in situations such as extreme sports, pandemics, and terrorism.
- Relate population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets, and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model — for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data based on deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Number and Quantity: The Real Number System (N-RN)

A. Use properties of rational and irrational numbers. In this cluster, the terms students should learn to use with increasing precision are rational and irrational numbers.

A1: N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Since every difference can be rewritten as a sum and every quotient as a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students' understanding of fractions and negative numbers. Many students will already have the required understanding but have likely not studied it formally, nor been pushed to articulate their reasoning. Rather than directly instructing students of these facts, students should be given opportunities to explore, make conjectures, and be guided to test their conjectures, resulting in a deeper understanding than would be produced from direct instruction.

Operations with Rational and Irrational Numbers[1]

Experiment with sums and products of two numbers from the following list to answer the questions that follow:

$$5, \tfrac{1}{2}, 0\, \sqrt{2}, -\sqrt{2}, \tfrac{1}{\sqrt{2}}, \Pi$$

Based on the above information, conjecture which of the statements is ALWAYS true, which is SOMETIMES true, and which is NEVER true?

a. The sum of a rational number and a rational number is rational.
b. The sum of a rational number and an irrational number is irrational.
c. The sum of an irrational number and an irrational number is irrational.
d. The product of a rational number and a rational number is rational.
e. The product of a rational number and an irrational number is irrational.

[1] https://www.illustrativemathematics.org/content-standards/HSN/RN/B/3/tasks/690

f. The product of an irrational number and an irrational number is irrational.

Solution

a. Always true.
b. Always true.
c. Sometimes true (for instance, the sum of additive inverses like $\sqrt{2}$ and $-\sqrt{2}$ will be 0).
d. Always true.
e. Sometimes true (for instance, the product of 0 and $\sqrt{2}$ is 0).
f. Sometimes true (for instance, the product of multiplicative inverses like $\sqrt{2}$ and $\frac{1}{\sqrt{2}}$ will be 1).

Sums of rational and irrational numbers[2]

a. Explain why the sum and product of two rational numbers are always a rational number.
b. Kaylee says, *I know that* π *is an irrational number, so its decimal never repeats. I also know that* $\frac{1}{7}$ *is a rational number, so its decimal repeats. But I don't know how to add or multiply these decimals, so I am not sure if* $\pi + \frac{1}{7}$ *and* $\pi \times \frac{1}{7}$ *are rational or irrational.*

Using part (a), help Kaylee decide whether or not $\pi + \frac{1}{7}$ and $\pi \times \frac{1}{7}$ are rational or irrational.

Solution

a. Suppose $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions. This means that b and d are non-zero integers and a and c are integers. We can calculate the sum by finding a common denominator:
$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$
Since b and d are not zero, the denominator bd is not zero. Moreover, sums and products of integers are integers, so both the numerator $ad + bc$ and the denominator bd are integers. This means that the sum $\frac{a}{b} + \frac{c}{d}$ is a rational number. The same argument works for the product and is quicker:
$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$
Since b and d are not zero, the denominator bd is not zero. Products of integers are integers, so the numerator ac and the denominator bd are both integers and the product $\frac{a}{b} \times \frac{c}{d}$ is a rational number.
b. The number $x = \pi + \frac{1}{7}$ is either rational or irrational. If x were rational then from part (a) $x - \frac{1}{7}$ would also be rational. But $x - \frac{1}{7} = \pi$ is not rational. Since x is not rational, it must be irrational. A similar argument shows that $x = \pi \times \frac{1}{7}$ is irrational. Again, x is either rational or irrational. If x were rational then $7x$ would also be rational but $7x = \pi$ is irrational. So x must be irrational.

[2] https://www.illustrativemathematics.org/content-standards/HSN/RN/B/3/tasks/1817

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.I.A1.7, LEAP.II.A1.1

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Number Systems LEAP.I.A1.7	Identifies rational and irrational numbers.	Identifies rational and irrational numbers.	Identifies rational and irrational numbers.	Identifies rational and irrational numbers.
	Calculates sums and products of two rational and/or irrational numbers **and determines whether and generalizes when the sums and products are rational or irrational.**	**Calculates sums and products of two rational and/or irrational numbers.**		

Number and Quantity: Quantities (N-Q)

A. Reason quantitatively and use units to solve problems. In this cluster, the terms students should learn to use with increasing precision are interpret units, descriptive modeling, error, tolerance, and level of accuracy.

A1: N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems.

a. Choose and interpret units consistently in formulas.
b. Choose and interpret the scale and the origin in graphs and data displays.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	Previous Grade(s) Standard: 6.PF.A.3 Algebra I Standard Taught in Advance: none Algebra I Standard Taught Concurrently: A1: N-Q.A.2	Conceptual Understanding

Examples and Explanations

Harvesting the Field[3]

A team of farm-workers was assigned the task of harvesting two fields, one twice the size of the other. They worked for the first half of the day on the larger field. Then the team split into two groups of equal size. The first group continued working in the larger field and finished it by evening. The second group harvested the smaller field, but did not finish by evening. The next day, one farm-worker finished the smaller field in a single day's work. How many farm-workers were on the team?

Solution
Method 1: reasoning with rates
First, notice that the larger field would have taken the entire team $\frac{3}{4}$ of a day to harvest. This is because they worked on it for half a day, and then half a team worked on it for another half a day. If the whole team had worked on it for the second half-day, they would have finished it in half the time, that is, $\frac{1}{4}$ day. Adding this to the first half-day, we get $\frac{3}{4}$ day for the whole team.

Since the smaller field is half the size of the larger field, it would have taken the whole team $\frac{3}{8}$ of a day to harvest the smaller field. As it was, only half the team worked on the smaller field, so it would have taken them $\frac{3}{4}$ of a day to finish. They only worked for $\frac{1}{2}$ of a day, so they still had $\frac{1}{4}$ of a day's work left, which would have been $\frac{1}{8}$ of a day's work for the entire team. It took one worker a whole day to finish up, so it

[3] https://www.illustrativemathematics.org/content-standards/HSN/Q/A/1/tasks/83

took one worker a day to do what the entire team could have done in $\frac{1}{8}$ day. Therefore, the team had 8 workers.

Method 2: Setting up an equation

Let x be the number of farm-workers on the team (in units of people per team), and let R be the area one farm-worker harvests in a day (in units of acres per person per day).

The area harvested in that first half day was $\frac{1}{2}day \cdot R\frac{acres}{person{\cdot}day} \times x\frac{people}{team} \cdot 1\,team = \frac{1}{2}xR\,acres$.

In the second half-day, the area harvested was $\frac{1}{2}day \cdot R\frac{acres}{person{\cdot}day} \times x\frac{people}{team} \cdot \frac{1}{2}\,team = \frac{1}{4}xR\,acres$.

So the area of the larger field is $A = \frac{1}{2}xR + \frac{1}{4}xR = \frac{3}{4}xR$.

The area of the smaller field is the area harvested by half the team in half a day, added to the area harvested by one person in 1 day:

$\frac{1}{2}day \cdot R\frac{acres}{person{\cdot}day} \times x\frac{people}{team} \cdot \frac{1}{2}\,team + R\frac{acres}{person{\cdot}day} \cdot 1\,person \cdot 1\,day = \frac{1}{4}xR + R\,acres$.

This is also half of the area of the larger meadow, so $\frac{1}{4}xR + R = \frac{1}{2} \cdot \frac{3}{4}xR$.

We can divide both sides of this equation by R since it is not zero, and solve the resulting equation x:

$$\frac{1}{4}x + 1 = \frac{3}{8}x$$

$$1 = \frac{1}{8}x$$

$$x = 8$$

Method 3: choosing a size for the fields

Let's assume that the smaller field is 1 acre and the larger one is 2 acres. Assuming these particular sizes do not change our final answer, if the sizes are different, then the rate at which the farm-workers can harvest is different, but the total number of farm-workers stays the same. [A more sophisticated way of saying this would be to say that we are working in units of size one small field. In working with this solution, it might make sense to make up a name for this unit.]

Then the larger field would be cleared by the whole team in $\frac{3}{4}$ of a day, which means that working together, they clear $\frac{8}{3}$ of an acre per day. Then half of them clear $\frac{2}{3}$ of an acre in a half day, so for the 1 acre field, this would mean that there is $\frac{1}{3}$ of an acre left for the single farm-worker, so he or she clears $\frac{1}{3}$ of an acre per day. This is $\frac{1}{8}$ of what the entire team can clear, so the entire team is 8 farm-workers.

Assessment Connections

Assessment Item Type(s): Type III
Evidence Statement(s): LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
LEAP.III.A1.1 LEAP.III.A1.2 LEAP.III.A1.3 LEAP.III.A1.4	Using stated assumptions and making assumptions and approximations to simplify a real-world situation (include micro-models).	Using stated assumptions **and making assumptions** and approximations to simplify a real-world situation (**include micro-models**).	Using stated assumptions and approximations to simplify a real-world situation.	Using stated assumptions and approximations to simplify a real-world situation.
	Mapping relationships between important quantities.	**Mapping relationships between important quantities.**	**Illustrating relationships between important quantities.**	Identifying important quantities.
	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically **between quantities** to draw conclusions.	Analyzing relationships mathematically to draw conclusions.
	Interpreting mathematical results in the context of the situation.	Interpreting mathematical results **in the context of the situation**.	Interpreting mathematical results in a simplified context.	
	Reflecting on whether the results make sense.	Reflecting on whether the results make sense.	**Reflecting on whether the results make sense**.	
	Improving the model if it has not served its purpose.	**Improving** the model if it has not served its purpose.	**Modifying the model if it has not served its purpose**.	

	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.
	Applying proportional reasoning and percentages **justifying and defending models which lead to a conclusion**.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.
	Writing and using functions in any form to describe how one quantity of interest depends on another.	Writing and using functions **in any form** to describe how one quantity of interest depends on another.	**Writing and** using functions to describe how one quantity of interest depends on another.	Using functions to describe how one quantity of interest depends on another.
	Using statistics.	Using statistics.	Using statistics.	Using statistics.
	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using **reasonable** estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.
	Analyzing and/or creating constraints, relationships and goals.			

A1: N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** A1: N-Q.A.1	Conceptual Understanding

Examples and Explanations

Giving Raises[4]

A small company wants to give raises to its 5 employees. They have $10,000 available to distribute. Imagine you are in charge of deciding how the raises should be determined.

a. What are some variables you should consider?
b. Describe mathematically different methods to distribute the raises.
c. What information do you need to compute the raises for each employee?
d. Make up the information you need to compute specific raises for 2 different methods and apply them to the situation. Compute the specific dollar amount each employee receives as a raise.
e. Choose one of the methods that you think is most fair and construct an argument that supports your decision.

Solution

a. Some variables to consider are:
    - What is the salary of each employee?
    - How many hours a week do they work? Are they full-time or part-time?
    - What is their education?
    - What is their position? (clerical staff, manager, etc.)
    - How long have they been working at the company?

b. There are many different methods.
    - Method 1: Each person gets the same raise.
    - Method 2: Each person gets the same percent raise based on their salary.
    - Method 3: This is a slight variation on Method 1 - If some employees are part-time and others are full-time, every person gets the same raise per hour of work. For example, if one person works 40 hours and another person works 20 hours, then the first person gets twice as much money as the second person.

[4] https://www.illustrativemathematics.org/content-standards/HSN/Q/A/2/tasks/1850

c. For each method, we need different information.
   - For Method 1, we don't need any additional information. Each person gets a $2000 raise.
   - For Method 2, we need to know the salary of each employee. Then we can figure out what percent increase will result in a total of $10,000.
   - For Method 3, we need to know how much each person is working. Then we can figure out what the raise per hour will be and compute the final raises.

d. We already computed the raises for
   - Method 1: Everybody gets $10,000/5=$2000.

   - Method 3 is a slight variation of this. Assume three employees work 40 hours per week, one works 20 hours per week, and the last one works 15 hours per week. Then the total number of hours per week is $3 \cdot 40 + 20 + 15 = 155$. Assuming that the company gives everybody 4 weeks of vacation, the total number of work hours per year is $155 \cdot 48 = 7440$.

     So the $10,000 available for raises comes to $10,000/7440 hours ≈ $1.34 per hour.

     So if you work 40 hours per week for 48 weeks, your raise is $40 \cdot 48 \cdot \$1.34 = \$2572.8$. If you work 20 hours per week for 48 weeks, your raise is $20 \cdot 48 \cdot \$1.34 = \$1286.4$. And if you work 15 hours per week for 48 weeks, your raise is $15 \cdot 48 \cdot \$1.34 = \$964.8$.

   - Method 2: This is probably the most mathematically sophisticated method. We first need to know everybody's salary. Suppose the salaries are $70,000, $50,000, $50,000, $35,000, $30,000. If we want to give everybody the same percent raise, we are looking for a number r, such that

     $$(70{,}000+50{,}000+50{,}000+35{,}000+30{,}000) \cdot r = 10{,}000.$$

     The sum of all the salaries is $235,000. Therefore, we need to solve $\$235{,}000\, r = \$10{,}000$ for r, which gives $r \approx 0.043$. So everybody will get about a 4.3% raise. The new salaries after the raises will be $73,010, $52,150, $52,150, $36,505, and $31,290.

The answer here depends on your definition of fairness. In method 2, everybody gets the same percent raise. So if you have a high salary, you get a larger pay raise than if you have a lower salary. This is probably the method used most commonly.

If your goal is to support lower-paid employees, then method 3 is designed to accomplish this. Here the raise rewards number of hours worked, and it does not depend on your salary at all. So lower-paid employees receive a larger percentage raise than higher-paid employees.

Method 1 is probably the least fair by most definitions, since no matter how much you work, you receive the same raise. A person working few hours or earning a smaller salary would receive a much higher percentage raise than a person working many hours or earning a higher salary.

Assessment Connections

Assessment Item Type(s): Type III

Evidence Statement(s): LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
LEAP.III.A1.1 LEAP.III.A1.2 LEAP.III.A1.3 LEAP.III.A1.4	Using stated assumptions and making assumptions and approximations to simplify a real-world situation (include micro-models).	Using stated assumptions **and making assumptions** and approximations to simplify a real-world situation (**include micro-models**).	Using stated assumptions and approximations to simplify a real-world situation.	Using stated assumptions and approximations to simplify a real-world situation.
	Mapping relationships between important quantities.	**Mapping relationships between important quantities.**	**Illustrating relationships between important quantities.**	Identifying important quantities.
	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically **between quantities** to draw conclusions.	Analyzing relationships mathematically to draw conclusions.
	Interpreting mathematical results in the context of the situation.	Interpreting mathematical results **in the context of the situation**.	Interpreting mathematical results in a simplified context.	
	Reflecting on whether the results make sense.	Reflecting on whether the results make sense.	**Reflecting on whether the results make sense.**	
	Improving the model if it has not served its purpose.	**Improving** the model if it has not served its purpose.	**Modifying the model if it has not served its purpose.**	

	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.
	Applying proportional reasoning and percentages **justifying and defending models which lead to a conclusion**.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.
	Writing and using functions in any form to describe how one quantity of interest depends on another.	Writing and using functions **in any form** to describe how one quantity of interest depends on another.	**Writing and** using functions to describe how one quantity of interest depends on another.	Using functions to describe how one quantity of interest depends on another.
	Using statistics.	Using statistics.	Using statistics.	Using statistics.
	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using **reasonable** estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.
	Analyzing and/or creating constraints, relationships and goals.			

A1: N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** 8.AR.A.4 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

The margin of error and tolerance limit vary according to the measure, tool used, and context.

Example

Weed Killer[5]

A liquid weed-killer comes in four different bottles, all with the same active ingredient. The accompanying table gives information about the concentration of the active ingredient in the bottles, the size of the bottles, and the price of the bottles. Each bottle's contents is made up of the active ingredient and water.

	Concentration	Amount in Bottle	Price of Bottle
A	1.04%	64 fl oz	$12.99
B	18.00%	32 fl oz	$22.99
C	41.00%	32 fl oz	$39.99
D	1.04%	24 fl oz	$5.99

a. You need to apply a 1% solution of the weed killer to your lawn. Rank the four bottles in order of best to worst buy. How did you decide what made a bottle a better buy than another?
b. The size of your lawn requires a total of 14 fl oz of the active ingredient. Approximately how much would you need to spend if you bought only the A bottles? Only the B bottles? Only the C bottles? Only the D bottles?

5 https://www.illustrativemathematics.org/content-standards/HSN/Q/A/2/tasks/81

Supposing you can only buy one type of bottle, which type should you buy so that the total cost to you is the least for this particular application of weed killer?

Solution

a. All of the bottles have the same active ingredient, and all can be diluted down to a 1% solution, so all that matters in determining value is the cost per fl oz of active ingredient. We estimate this in the following table:

	Amount active in bottle	**Price of bottle**	**Cost per ounce**
A	$1.04\% \times 64 \approx 0.64$ fl oz	\$12.99 ≈ \$13	$\frac{13}{0.64} \approx$ \$20 per fl oz
B	$18.00\% \times 32 \approx 6$ fl oz	\$29.99 ≈ \$23	$\frac{23}{6} \approx$ \$4 per fl oz
C	$41.00\% \times 32 \approx 13$ fl oz	\$39.99 ≈ \$40	$\frac{40}{13} \approx$ \$3 per fl oz
D	$1.04\% \times 24 \approx 0.24$ fl oz	\$5.99 ≈ \$6	$\frac{6}{0.24} \approx$ \$24 per fl oz

If we assume that receiving more of the active ingredient per dollar is a better buy than less of the active ingredient per dollar, the ranking in order of best-to-worst buy is C,B,A,D.

b. The A bottles have about 0.64 fl oz of active ingredient per bottle, so to get 14 fl oz we need $\frac{14\ fl\ oz}{0.64\ fl\ oz/bottle} \approx 22$ bottles. Purchasing 22 A bottles at about \$13 each will cost about \$286.
The B bottles have a little less than 6 fl oz of active ingredient per bottle, so to get 14 fl oz we need 3 bottles. Purchasing 3 B bottles at about \$23 each will cost about \$69.
The C bottles have a little more than 13 fl oz of active ingredient per bottle, so we need 2 bottles. Purchasing 2 C bottles at about \$40 each will cost about \$80.

The D bottles have only 0.24 fl oz of active ingredient per bottle, so to get 14 fl oz we need $\frac{14\ fl\ oz}{0.24\ fl\ oz/bottle} \approx 58$ bottles. Purchasing 58 D bottles at about \$6 each will cost about \$348.

Thus, although the C bottle is the cheapest when measured in dollars/fl oz, the B bottles are the best deal for this job because there is too much unused when you buy C bottles.

[Assessment Connections](#)

Assessment Item Type(s): Type III

Evidence Statement(s): LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
LEAP.III.A1.1 LEAP.III.A1.2 LEAP.III.A1.3 LEAP.III.A1.4	Using stated assumptions and making assumptions and approximations to simplify a real-world situation (include micro-models).	Using stated assumptions **and making assumptions** and approximations to simplify a real-world situation (**include micro-models**).	Using stated assumptions and approximations to simplify a real-world situation.	Using stated assumptions and approximations to simplify a real-world situation.
	Mapping relationships between important quantities.	**Mapping relationships between important quantities.**	**Illustrating relationships between important quantities.**	Identifying important quantities.
	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically **between quantities** to draw conclusions.	Analyzing relationships mathematically to draw conclusions.
	Interpreting mathematical results in the context of the situation.	Interpreting mathematical results **in the context of the situation**.	Interpreting mathematical results in a simplified context.	
	Reflecting on whether the results make sense.	Reflecting on whether the results make sense.	**Reflecting on whether the results make sense.**	

	Improving the model if it has not served its purpose.	**Improving** the model if it has not served its purpose.	**Modifying the model if it has not served its purpose.**	
	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.
	Applying proportional reasoning and percentages **justifying and defending models which lead to a conclusion**.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.
	Writing and using functions in any form to describe how one quantity of interest depends on another.	Writing and using functions **in any form** to describe how one quantity of interest depends on another.	**Writing and** using functions to describe how one quantity of interest depends on another.	Using functions to describe how one quantity of interest depends on another.
	Using statistics.	Using statistics.	Using statistics.	Using statistics.
	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using **reasonable** estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.
	Analyzing and/or creating constraints, relationships and goals.			

Algebra: Seeing Structure in Expressions (A-SSE)

A. Interpret the structure of expressions. In this cluster, the terms students should learn to use with increasing precision are expression, term, factor, coefficient, and rewrite an expression in a different form.

A1: A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 6.AR.A.2, 7.AR.A.2 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context.

Students:

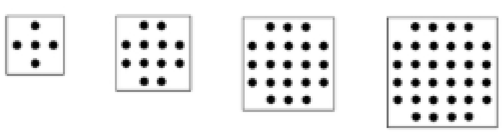
- Use appropriate vocabulary for the parts that make up the whole expression.
- Identify the different parts of the expression and explain their meaning within the context of a problem.
- Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.

Seeing Dots[6]

Consider the algebraic expressions below:

$$(n + 2)^2 - 4 \quad \text{and} \quad n^2 + 4n$$

a. Use the figures below to illustrate why the expressions are equivalent:



[6] https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/21

b. Find some ways to algebraically verify the same result.

Students interpret complicated expressions by viewing one or more of their parts as a single entity.

The Physics Professor[7]

A physics professor says, "Of course, it is easy to see that $L_0\sqrt{1 - \frac{v^2}{c^2} = 0}$ when $v = c$."

a. Give a possible explanation in terms of the structure of the expression on the left why the professor might say that.

b. Assuming that L_0 and c are positive, what is the greatest possible value of the expression on the left? Explain your answer in terms of the structure of the expression.

Solution

When $v = c$, the fraction $\frac{v^2}{c^2}$ is 1 and so $\left(1 - \frac{v^2}{c^2}\right)$ is 0. Since the square root of 0 is 0, the entire expression is 0 when $v = c$.

a. The greatest possible value of the expression is L_0. To see this, we observe that the term $1 - \frac{v^2}{c^2}$ is always at most 1, since the quantity $\frac{v^2}{c^2}$ is always at least zero. Now the square root of a quantity which is at most 1 is again at most 1, and so when multiplied by L_0, we find that the entire expression is at most L_0. Note this value does indeed occur, when (and only when) $v = 0$.

- Given that I, income from a concert is the price, p, of a ticket times each person in attendance, consider the equation $I = 4000p - 250p^2$ that represents income from a concert where p is the price per ticket. (The equivalent factored form, $p(4000 - 250p)$, shows that the income can be interpreted as the price times the number of people in attendance based on the price charged. Students recognize $(4000 - 250p)$ as a single quantity for the number of people in attendance.)
- The expression $10000(1.055)^n$ is the amount of money in an investment account with interest compounded annually for n years. Determine the initial investment and the annual interest rate. (The factor of 1.055 can be rewritten as $(1 + 0.055)$, revealing the growth rate of 5.5% per year.)

[7] https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/23

Assessment Connections
Assessment Item Type(s): Type I
Evidence Statement(s):

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Expressions A1: A-SSE.A.1 A1: A-SSE.A.2 A1: A-APR.A.1	Writes and **analyzes** equivalent numerical and polynomial expressions in one variable, using addition, subtraction, multiplication and factoring, **including multi-step problems.**	Writes equivalent numerical and polynomial expressions in one variable, using addition, subtraction, multiplication, **and factoring**.	Writes equivalent numerical and polynomial expressions in one variable, using addition, subtraction, and multiplication.	Writes equivalent numerical and polynomial expressions in one variable, using addition, subtraction, and multiplication.
	Interprets parts of exponential and quadratic expressions that represent a quantity in terms of its context	**Interprets parts of exponential and quadratic expressions that represent a quantity in terms of its context.**	Identifies components of exponential **and quadratic** expressions.	Identifies components of exponential expressions.

A1: A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it for a specific purpose.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 6.AR.A.3, 7.AR.A.1 **Algebra I Standard Taught in Advance:** A1: A-SSE.A.1 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Explanation:

See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$, or see $2x^2 + 8x$ as $(2x)(x) + 2x(4)$, thus recognizing it as a polynomial whose terms are products of monomials, and the polynomial can be factored as $2x(x + 4)$.

Equivalent Expressions[8]

Find a value for a, a value for k, and a value for n so that $(3x + 2)(2x - 5) = ax^2 + kx + n$.

Solution

No matter what the value of x, the distributive property of multiplication over addition tells us that $(3x + 2)(2x - 5) = (3x + 2)(2x) + (3x + 2)(-5) = 6x^2 + 4x - 15x - 10 = 6x^2 - 11x - 10$.

So, if $a = 6$, $k = -11$, and $n = -10$, then the expression on the left has the same value as the expression on the right for all values of x; that is, the two expressions are equivalent.

[8] https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/2/tasks/87

[Assessment Connections](#)
Assessment Item Type(s): Type I
Evidence Statement(s):

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Expressions A1: A-SSE.A.1 A1: A-SSE.A.2 A1: A-APR.A.1	Writes and **analyzes** equivalent numerical and polynomial expressions in one variable, using addition, subtraction, multiplication and factoring, **including multi-step problems**.	Writes equivalent numerical and polynomial expressions in one variable, using addition, subtraction, multiplication, **and factoring**.	Writes equivalent numerical and polynomial expressions in one variable, using addition, subtraction, and multiplication.	Writes equivalent numerical and polynomial expressions in one variable, using addition, subtraction, and multiplication.
	Interprets parts of exponential and quadratic expressions that represent a quantity in terms of its context	**Interprets parts of exponential and quadratic expressions that represent a quantity in terms of its context.**	Identifies components of exponential **and quadratic** expressions.	Identifies components of exponential expressions.

B. Write expressions in equivalent forms to solve problems. In this cluster, the terms students should learn to use with increasing precision are equivalent forms of an expression; factor and complete the square of a quadratic expression, and properties of exponents.

A1: A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions, emphasizing integer exponents. For example, the growth of bacteria can be modeled by either $f(t) = 3^{(t+2)}$ or $g(t) = 9(3^t)$ because the expression $3^{(t+2)}$ can be rewritten as $(3^t)(3^2) = 9(3^t)$.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** 6.AR.A.3, 7.AR.A.1, 8.AR.A.1 **Algebra I Standard Taught in Advance:** A1: A-SSE.A.1, A1: A-SSE.A.2 **Algebra I Standard Taught Concurrently:** A1: A-REI.B.4	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students factor quadratic expressions and find the zeroes of the quadratic function they represent. Students explain the meaning of the zeroes as they relate to the problem.

Profit of a Company[9]

The profit that a company makes selling an item (in thousands of dollars) depends on the price of the item (in dollars). If p is the price of the item, then three equivalent forms for the profit are:

Standard form: $-2p^2 + 24p - 54$
Factored form: $-2(p-3)(p-9)$
Vertex form: $-2(p-6)^2 + 18$

Which form is the most useful for finding

a. The prices that give a profit of zero dollars?

[9] https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/434

b. The profit when the price is zero?
c. The price that gives the maximum profit?

Solution

a. The factored form gives the values of p that make the profit zero. Since the factored form is $-2(p-3)(p-9)$, the profit is zero when $p=3$ or $p=9$. The company breaks even if the priced charged for the product is $\$3$ or $\$9$.
b. The standard form is the easiest one to use to find the profit when the price is zero. Substituting $p=0$ into the standard form $-2p^2+24p-54$, we see that the profit is -54 (in thousands of dollars) when the price is zero. If the company gives the product away for free, it loses $\$54,000$.
c. The vertex form shows us what price maximizes profit. From the expression $-2(p-6)^2+18$, we see that the maximum profit is 18 thousand dollars, and it occurs when $p=6$. The company should charge a price of $\$6$ for this product.

Students use completing the square to rewrite a quadratic expression in the form $y=a(x-h)^2+k$ to identify the vertex of the parabola (h,k) and explain its meaning in context. This implies a thorough understanding of when it is appropriate to use vertex form; thus, students should use this strategy when looking for the maximum or minimum value.

Profit of a company, assessment variation[10]

The profit, P (in thousands of dollars), that a company makes selling an item is a quadratic function of the price, x (in dollars), that they charge for the item. The following expressions for $P(x)$ are equivalent:

$$P(x)=2x^2+24x-54$$

$$P(x)=2(x-3)(x-9)$$

$$P(x)=2(x-6^2)+18$$

1. Which of the equivalent expressions for $P(x)$ reveals *the price which gives a profit of zero* without changing the form of the expression?

$$P(x)=-2x^2+24x-54$$

$$P(x)=-2(x-3)(x-9)$$

$$P(x)=-2(x-6)^2+18$$

2. Find a price which gives a profit of zero.
3. Which of the equivalent expressions for $P(x)$ reveals *the profit when the price is zero* without changing the form of the expression?

$$P(x)=-2x^2+24x-54$$

[10] https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/1344

$$P(x) =- 2(x - 3)(x - 9)$$

$$P(x) =- 2(x - 6)^2 + 18$$

4. Find the profit when the price is zero.
5. Which of the equivalent expressions for $P(x)$ reveals *the price which produces the highest possible profit* without changing the form of the expression?

$$P(x) =- 2x^2 + 24x - 54$$

$$P(x) =- 2(x - 3)(x - 9)$$

$$P(x) =- 2(x - 6)^2 + 18$$

6. Find the price which gives the highest possible profit.

Students use properties of exponents to transform expressions for exponential functions.

Ice Cream[11]

After a container of ice cream has been sitting in a room for t minutes, its temperature in degrees Fahrenheit is $a - b \cdot 2^{-t} + b$, where a and b are positive constants. Write this expression in a form that

a. Shows that the temperature is always less than $a + b$.
b. Shows that the temperature is never less than a.

Solution

a. To begin, we can first rearrange this expression into

$$(a + b) - b \cdot 2^{-t}.$$

We can now see that we have an $a + b$ together on the left, and our last term is $b \cdot 2^{-t}$, and this term will dictate if the temperature is greater or less than $a + b$. Since b is a positive constant, and since 2^{-t} is positive regardless of the value of t, we know that $b \cdot 2^{-t}$, is positive. Se we have

$$(a + b) - b \cdot 2^{-t} < a + b.$$

b. We can rearrange the expression in the following way:

$$a + b - b \cdot 2^{-t} = a + b\left(1 - \frac{1}{2^t}\right).$$

We see that now the term $b\left(1 - \frac{1}{2^t}\right)$ is going to dictate if the temperature is greater or less than a. Since t is the number of minutes that the ice cream has been sitting in the room, we know that t will always be

[11] https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/551

greater than zero. Therefore, $2^t > 1$, so $\frac{1}{2^t} < 1$, so $\left(1 - \frac{1}{2^t}\right) > 0$. From this we can conclude that since $b > 0$, we have $b\left(1 - \frac{1}{2^t}\right) > 0$. Therefore, $a + b\left(1 - \frac{1}{2^t}\right) > a$.

Assessment Connections

Assessment Item Type(s): Type I, Type III
Evidence Statement(s): LEAP.I.A1.4, LEAP.I.A1.5, LEAP.I.A1.6, LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Equivalent Expressions and Functions A1: A-SSE.B.3 A1: F-IF.C.8a	Determines equivalent forms of quadratic **and exponential (with integer domain)** expressions and functions to reveal and explain **their properties**.	**Determines** equivalent forms of quadratic expressions and functions. Uses equivalent forms **to reveal and explain** zeros, **extreme values** and symmetry.	Identifies equivalent forms of quadratic expressions and functions. **Identifies zeros and symmetry**.	Identifies equivalent forms of quadratic expressions and functions in cases where suitable factorizations are provided.

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)

A. Perform arithmetic operations on polynomials. In this cluster, the terms students should learn to use with increasing precision are polynomial and operations on polynomials.

A1: A-APR.A.1 Understand that polynomials form a system comparable to the integers, as they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 6.AR.A.3, 6.AR.A.4, 7.AR.A.1, 8.AR.A.1 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, polynomial, factor, and term.

- Simplify each of the following:
    a. $(4x + 3) - (2x + 1)$
    b. $(x^2 + 5x - 9) + 2x(4x - 3)$

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.II.A1.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Expressions A1: A-SSE.A.1 A1: A-SSE.A.2 A1: A-APR.A.1	Writes and **analyzes** equivalent numerical and polynomial expressions in one variable, using addition, subtraction, multiplication and factoring, **including multi-step problems**.	Writes equivalent numerical and polynomial expressions in one variable, using addition, subtraction, multiplication, **and factoring**.	Writes equivalent numerical and polynomial expressions in one variable, using addition, subtraction, and multiplication.	Writes equivalent numerical and polynomial expressions in one variable, using addition, subtraction, and multiplication.
	Interprets parts of exponential and quadratic expressions that represent a quantity in terms of its context	**Interprets parts of exponential and quadratic expressions that represent a quantity in terms of its context.**	Identifies components of exponential **and quadratic** expressions.	Identifies components of exponential expressions.

B. Understand the relationship between zeros and factors of polynomials. In this cluster, the terms students should learn to use with increasing precision are zeros and sketching of a quadratic function.

A1: A-APR.B.3 Identify zeros of quadratic functions, and use the zeros to sketch a graph of the function defined by the polynomial.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** 7.AR.A.1 **Algebra I Standard Taught in Advance:** A1: A-SSE.A.2, A1:SSE.B.3 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

This standard calls for a sketch of the graph after zeros are identified. Sketching implies that the graph should be done by hand rather than generated by a graphing calculator.

Examples:

- Given the function $f(x) = (x - 3)(x + 4)$, list the zeroes of the function and sketch the graph.
- Sketch the graph of the function, $f(x) = (x + 5)^2$. How many zeros does this function have? Explain.

Assessment Connections

Assessment Item Type(s): Type I

Evidence Statement(s): LEAP.I.A1.4, LEAP.I.A1.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Graphs of Functions A1: A-APR.B.3 A1: F-IF.C.7	Graphs linear, quadratic, **and piecewise-defined functions, showing key features**.	Graphs linear and quadratic functions, showing key features.	Graphs linear **and quadratic functions**, showing key features.	Graphs linear functions, showing key features.
	Determines a function, given a graph with key features identified.			

Algebra: Creating Equations (A-CED)

A. Create equations that describe numbers or relationships. In this cluster, the terms students should learn to use with increasing precision are equations and inequalities in one variable, equations in two variables, systems of equations/inequalities, exponential equations, and rearrange a formula.

A1: A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential situation functions.

Standard Overview

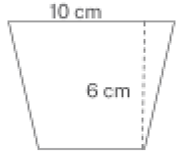
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 7.AR.B.3, 8.AR.C.7 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** A1: A-REI.A.1, A1:A-REI.B.3	Conceptual Understanding, Procedural Skill and Fluency, Application

Examples and Explanations

Equations can represent real-world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.

Examples:

- To be considered a 'fuel-efficient' vehicle, a car must get more than 30 miles per gallon. Consider a test run of 200 miles. How many gallons of fuel can a car use and be considered 'fuel-efficient'?
- Given that the following trapezoid has an area of 54 cm^2, set up an equation to find the length of the base, and solve the equation.



- Lava coming from the eruption of a volcano follows a parabolic path. The height h in feet of lava t seconds after it is ejected from the volcano is given by $h(t) =- t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet?
- A rental agreement locks the monthly rent of an apartment to a constant value for one year. The average rate to rent an apartment is $750 per month and increases at an inflation rate of 8% per year. If inflation continues at the current rate, in which year will the rent be at least $1000 per month?

Assessment Connections

Assessment Item Type(s): Type I, Type III

Evidence Statement(s): LEAP.I.A1.4, LEAP.I.A1.5, LEAP.I.A1.6, LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Solving Algebraically A1: A-REI.B.3 A1: A-REI.B.4 A1: A-CED.A.4 LEAP.I.A1.4 LEAP.I.A1.5 LEAP.I.A1.6	Algebraically solves linear equations, linear inequalities, and quadratics in one variable (at complexity appropriate to the course), including those with coefficients represented by letters.	Algebraically solves linear equations, linear inequalities, and quadratics in one variable (at complexity appropriate to the course), **including those with coefficients represented by letters**.	Algebraically solves linear equations, linear inequalities, and **quadratics** in one variable (at complexity appropriate to the course).	Algebraically solves linear equations and linear inequalities in one variable (at complexity appropriate to the course).
	Utilizes structure and rewriting as strategies for solving.			

A1: A-CED.A.2 Create equations in two variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 8.AR.C.8, 8.PF.A.3, 8.PF.B.4 **Algebra I Standard Taught in Advance:** A1: A-CED.A.1 **Algebra I Standard Taught Concurrently:** A1: A-REI.D.10	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students:

- Create equations in two variables.
- Graph equations on coordinate axes with labels and scales clearly labeling the axes, defining what the values on the axes represent, and the unit of measure.
- Select intervals for the scale that are appropriate for the context and display adequate information about the relationship.
- Choose appropriate minimum and maximum values for a graph.

Linear equations can be written in a multitude of ways; $y = mx + b$ and $ax + by = c$ are commonly used forms (given that x and y are the two variables). Students should be flexible in using multiple forms and recognize from the context, which is appropriate to use in creating the equation. Time spent manipulating linear equations from one form to another is fruitless; moreover, students should be able to graph from either form listed above.

Examples:

- At a fundraising event, the FFA sold hot dogs for $1.50 and drinks for $2.00. The FFA earned $400 at the fundraiser.
    a. Write an equation to calculate the total of $400 based on the hot dog and drink sales.
    b. Graph the relationship between hot dog sales and drink sales.

- A spring with an initial length of 25 cm will compress 0.5 cm for each pound applied.
    a. Write an equation to model the relationship between the amount of weight applied and the length of the spring.
    b. Graph the relationship between pounds and length.
    c. What does the graph reveal about limitation on weight?

Quadratic equations can be written in a multitude of ways; $y = ax^2 + bx + c$ and $y = k(x + m)(k + n)$ are commonly used forms (given that x and y are the two variables). Students should be flexible in using multiple forms and recognize from the context, which is appropriate to use in creating the equation. As with linear equations, time spent manipulating quadratic equations from one form to another is mostly fruitless; moreover, students should be able to graph from either form listed above.

Throwing a Ball[12]

A ball thrown vertically upward at a speed of v ft/sec rises a distance of d feet in t seconds, given by

$$d = 6 + vt - 16t^2.$$

Write an equation whose solution is

a. The time it takes a ball thrown at a speed of 88 ft/sec to rise 20 feet.
b. The speed with which the ball must be thrown to rise 20 feet in 2 seconds.

Solution

We have an equation in three unknowns, and each part specifies two of them, giving an equation in one unknown whose solution gives the desired quantity.

a. We want $d = 20$, and we are given $v = 88$, so the equation is $20 = 6 + 88t - 16t^2$.
b. We want $d = 20$, and we are given $t = 2$, so the equation is $20 = 6 + 2v - 16 \cdot 2^2$. Simplifying the right-hand side, we get $20 =- 58 + 2v$.

The local park is designing a new rectangular sandlot. The sandlot is to be twice as long as the original square sandlot and 3 feet less than its current width. What must be true of the original square lot to justify that the new rectangular lot has more area? Note: Students can construct the equations for each area and graph each equation. Students should select scales for the length of the original square and the area of the lots suitable for the context.

Exponential equations can be written in different ways; $y = ab^x$ and $y = a(1 \pm r)^x$ are the most common forms (given that x and y are variables). Students should be flexible in using all forms and recognize from the context which is appropriate to use in creating the equation.

In a woman's professional tennis tournament, the money a player wins depends on her finishing place in the standings. The first-place finisher wins half of $1,500,000 in total prize money. The second-place finisher wins half of what is left; then the third-place finisher wins half of that, and so on.

a. Write a rule to calculate the actual prize money in dollars won by the player finishing in nth place, for any positive integer n.

[12] https://www.illustrativemathematics.org/content-standards/HSA/CED/A/2/tasks/437

b. Graph the relationship between the first 10 finishers and the prize money in dollars.
c. What pattern is indicated in the graph? What type of relationship exists between the two variables?

Assessment Connections

Assessment Item Type(s): Type III
Evidence Statement(s): LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
LEAP.III.A1.1 LEAP.III.A1.2 LEAP.III.A1.3 LEAP.III.A1.4	Using stated assumptions and making assumptions and approximations to simplify a real-world situation (include micro-models).	Using stated assumptions **and making assumptions** and approximations to simplify a real-world situation (**include micro-models**).	Using stated assumptions and approximations to simplify a real-world situation.	Using stated assumptions and approximations to simplify a real-world situation.
	Mapping relationships between important quantities.	**Mapping relationships between important quantities.**	**Illustrating relationships between important quantities.**	Identifying important quantities.
	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically **between quantities** to draw conclusions.	Analyzing relationships mathematically to draw conclusions.
	Interpreting mathematical results in the context of the situation.	Interpreting mathematical results **in the context of the situation**.	Interpreting mathematical results in a simplified context.	
	Reflecting on whether the results make sense.	Reflecting on whether the results make sense.	**Reflecting on whether the results make sense.**	
	Improving the model if it has not served its purpose.	**Improving** the model if it has not served its purpose.	**Modifying the model if it has not served its purpose.**	

	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.
	Applying proportional reasoning and percentages **justifying and defending models which lead to a conclusion**.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.
	Writing and using functions in any form to describe how one quantity of interest depends on another.	Writing and using functions **in any form** to describe how one quantity of interest depends on another.	**Writing and** using functions to describe how one quantity of interest depends on another.	Using functions to describe how one quantity of interest depends on another.
	Using statistics.	Using statistics.	Using statistics.	Using statistics.
	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using **reasonable** estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.
	Analyzing and/or creating constraints, relationships and goals.			

A1: A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** A1: A-CED.A.1, A1: A-CED.A.2, A1: A-REI.D.10 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Application

Examples and Explanations

Students recognize when a constraint can be modeled with an equation, inequality, or system of equations/inequalities. These constraints may be stated directly or be implied through the context of the given situation.

For example, students can represent inequalities describing nutritional and cost constraints on combinations of different foods.

In Algebra I, focus on linear equations and inequalities. Students should approach optimization problems using a problem-solving process, such as using a table.

A club is selling hats and jackets as a fundraiser. Their budget is $1500, and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5, and each jacket costs $8.

a. Write a system of inequalities to represent the situation.
b. If the club buys 150 hats and 100 jackets, will the conditions be satisfied?

Assessment Connections

Assessment Item Type(s): Type I, Type III

Evidence Statement(s): LEAP.III.A1.2, LEAP.III.A1.3, LEAP.III.A1.4

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Solving Graphically A1: A-CED.A.3 A1: A-REI.D.10 A1: A-REI.D.11 A1: A-REI.D.12	Graphs **and analyzes** the solution sets of equations, linear inequalities, and systems of linear inequalities.	Graphs the solution sets of equations, linear inequalities, **and systems of linear equations and linear inequalities.**	Graphs the solution sets of equations and linear inequalities.	Graphs the solution sets of equations and linear inequalities.
	Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	**Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.**	Given the graph, identify the solutions of a system of two polynomial functions.
	Writes a system of linear inequalities given a context.			

A1: A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** A1: A-CED.A.2 **Algebra I Standard Taught Concurrently:** A1: A-REI.A.1, A1: A-REI.B.3	Procedural Skill and Fluency

Examples and Explanations

Students solve multi-variable formulas for a specific variable.

In Algebra I, limit to formulas that are linear in the variable of interest, or to formulas involving squared or cubed variables. Explicitly connect this to the process of solving equations using inverse operations.

Examples:

- The Pythagorean Theorem expresses the relation between legs a and b of a right triangle and its hypotenuse c with the equation $a^2 + b^2 = c^2$.
    a. Why might the theorem need to be solved for c?
    b. Solve the equation for c and write a problem situation where this form of the equation might be useful.
- Solve $V = \frac{4}{3}\pi r^3$ for radius r.
- Motion can be described by the formula $s = ut + \frac{1}{2}at^2$, where t = time elapsed, u = initial velocity, a = acceleration, and s = distance traveled.
    a. Why might the equation need to be rewritten in terms of a?
    b. Rewrite the equation in terms of a.

Assessment Connections

Assessment Item Type(s): Type I, Type III

Evidence Statement(s): LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Solving Algebraically A1: A-REI.B.3 A1: A-REI.B.4 A1: A-CED.A.4 LEAP.I.A1.4 LEAP.I.A1.5 LEAP.I.A1.6	Algebraically solves linear equations, linear inequalities, and quadratics in one variable (at complexity appropriate to the course), including those with coefficients represented by letters.	Algebraically solves linear equations, linear inequalities, and quadratics in one variable (at complexity appropriate to the course), **including those with coefficients represented by letters**.	Algebraically solves linear equations, linear inequalities, and **quadratics** in one variable (at complexity appropriate to the course).	Algebraically solves linear equations and linear inequalities in one variable (at complexity appropriate to the course).
	Utilizes structure and rewriting as strategies for solving.			

Algebra: Reasoning with Equations and Inequalities★ (A1: A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning. In this cluster, the terms students should learn to use with increasing precision are properties of equations, explain each step in solving an equation, and construct a viable solution.

A1: A-REI.A.1 Use properties of equality to justify and explain each step obtained from the previous step when solving an equation, assuming the original equation has a solution.

a. Construct a viable argument to justify the solution method.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 7.AR.B.3, 8.AR.C.7 **Algebra I Taught in Advance:** none **Algebra I Taught Concurrently:** A1: A-CED.A.1, A1: A-CED.A.4, A1: A-REI.B.3, A1: A-REI.B.4	Conceptual Understanding

Examples and Explanations

Students should focus on solving all applicable equation types presented in Algebra I and be able to extend and apply their reasoning to other types of equations in future courses. When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.

Examples:

- Assuming an equation has a solution, construct a convincing argument that justifies each step in the solution process. Justifications may include the associative, commutative, division and identity properties, combining like terms, etc.

Steps	Justification
$5(x+3)-3x=55$	Given equation
$5x+15-3x=55$	Distributive property
$2x+15=55$	Combined like terms
$2x+15-15=55-15$	Subtraction property of equality
$2x=40$	Result of subtracting
$\frac{2x}{2}=\frac{40}{2}$	Division property of equality
$x=20$	Result of division

- For quadratic examples, see Zero Product Property 4[13]:

The Zero Product Property states that if the product of two numbers is zero, then at least one of the numbers is zero. In symbols, if $ab = 0$, then $a = 0$ or $b = 0$. Sometimes, we can take advantage of this property to help us find solutions to equations. Explain how the property can be used to find both solutions to each of the following equations, and explain each step in your reasoning.

a. $(x - 1)(x - 3) = 0$
b. $2x(x - 1) + 3x - 3 = 0$
c. $x + 4 = x(x + 4)$
d. $x^2 = 6x$
e. $x^2 + 10 = 7x$

Solution

a. The solution is 1 or 3. This equation is written as the product of two expressions. Since it is equal to zero, the ZPP says that at least one of those factors must equal zero. Either $x - 1 = 0$, so 1 is a solution or $x - 3 = 0$, so 3 is a solution.
b. The solution is $-\frac{3}{2}$ or 1. This equation is not of the form "a product of two factors equals zero." Using the structure on the left-hand side, I can apply the distributive property to rewrite the equation as $2x(x - 1) + 3(x - 1) = 0$, and then use the distributive property again to write $(x - 1)(2x + 3) = 0$. Now using ZPP, I can say $x - 1 = 0$ or $2x + 3 = 0$, so $x = 1$ or $x =- \frac{3}{2}$.
c. The solution is -4 or 1. Since I hope to use the ZPP, I need to rewrite the expression as some polynomial of x equals 0. So I'll subtract everything on the right-hand side from both sides and rewrite the equation as $x + 4 - x(x + 4) = 0$. If I view the first $x + 4$ as the product of 1 and $x + 4$, I can apply the distributive property and rewrite the equation as $(x + 4)(x - 1) = 0$. Now using ZPP, I can say $x + 4 = 0$ or $x - 1 = 0$, so $x =- 4$ or $x = 1$.
d. The solution is 0 or 6. This could be figured out by inspecting the operations involved, but if I want to use ZPP, I rewrite the equation as $x^2 - 6x = 0$ and then apply the distributive property to find $x(x - 6) = 0$. Now I can apply ZPP to say $x = 0$ or $x = 6$.
e. The solution is 2 or 5. I first subtract $7x$ from both sides to write $x^2 - 7x + 10 = 0$. Then I factor the left-hand side to get $(x - 2)(x - 5) = 0$. Applying the ZPP to this equation gives $x - 2 = 0$ or $x - 5 = 0$, so $x = 2$ or $x = 5$.

[13] https://www.illustrativemathematics.org/content-standards/tasks/2144

Assessment Connections

Assessment Item Type(s): Type II
Evidence Statement(s): LEAP.II.A1.9

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.A1.1 LEAP.II.A1.2 LEAP.II.A1.3 LEAP.II.A1.4 LEAP.II.A1.5 LEAP.II.A1.6 LEAP.II.A1.7 LEAP.II.A1.8 LEAP.II.A1.9 LEAP.II.A1.10	Using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate).	Using a logical approach based on a conjecture and/or stated assumptions, **utilizing mathematical connections (when appropriate)**.	Using a **logical** approach based on a conjecture and/or stated assumptions.	Using an approach based on a conjecture and/or stated assumptions.
	Providing an **efficient and** logical progression of steps or chain of reasoning with appropriate justification.	Providing a logical progression of steps or chain of reasoning **with appropriate justification**.	Providing a logical, **but incomplete**, progression of steps or chain of reasoning.	Providing an incomplete or illogical progression of steps or chain of reasoning.
	Performing precise calculations.	Performing **precise** calculations.	**Performing minor** calculation errors.	Making an intrusive calculation error.
	Using correct grade-level vocabulary, symbols and labels.	Using **correct** grade-level vocabulary, symbols and labels.	Using **some** grade-level vocabulary, symbols and labels.	Using limited grade-level vocabulary, symbols and labels.
	Providing a justification of a conclusion.	Providing a **justification** of a conclusion.	Providing a partial justification of a conclusion based on own calculations.	Providing a partial justification of a conclusion based on own calculations.

	Evaluating, interpreting and critiquing the validity of others' responses, approaches, and reasoning – using mathematical connections (when appropriate) – and **providing a counter-example where applicable**	**Evaluating, interpreting and critiquing the validity of others' responses, approaches, and reasoning – using mathematical connections(when appropriate).**	**Evaluating the validity of others' approaches and conclusions.**	
	Determining whether an argument or conclusion is generalizable.			

B. Solve equations and inequalities in one variable. In this cluster, the terms students should learn to use with increasing precision are linear equations and inequalities in one variable, solve a quadratic by completing the square, and solve by inspection.

A1: A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 7.AR.B.3, 8.AR.C.7 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** A1: A-CED.A.1, A1:-CED.A.4, A1: A-REI.A.1, A1: A-REI.B.4	Procedural Skill and Fluency

Examples and Explanations

It is important to note that students have not solved linear inequalities since Grade 7 as there are no standards requiring such work in Grade 8. Furthermore, the work in Grade 7 was limited to two-step linear inequalities. Students should have explored the result of multiplying or dividing both sides of an inequality by a negative value but would likely benefit from a refresher on the topic.

Examples:

- Solve $ax + 7 = 12$ for x.
- $\frac{3+x}{7} = \frac{x-9}{4}$
- Solve $(y - y_1) = m(x - x_2)$ for m.

Assessment Connections

Assessment Item Type(s): Type I

Evidence Statement(s):

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Solving Algebraically A1: A-REI.B.3 A1: A-REI.B.4 A1:A-CED.A.4 LEAP.I.A1.4 LEAP.I.A1.5 LEAP.I.A1.6	Algebraically solves linear equations, linear inequalities, and quadratics in one variable (at complexity appropriate to the course), including those with coefficients represented by letters.	Algebraically solves linear equations, linear inequalities, and quadratics in one variable (at complexity appropriate to the course), **including those with coefficients represented by letters**.	Algebraically solves linear equations, linear inequalities, and **quadratics** in one variable (at complexity appropriate to the course).	Algebraically solves linear equations and linear inequalities in one variable (at complexity appropriate to the course).
	Utilizes structure and rewriting as strategies for solving.			

A1: A-REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.
c. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 7.AR.A.1, 8.AR.A.2 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** A1: A-SSE.B.3, A1: A-CED.A.1, A1: A-REI.A.1, A1: A-REI.B.3	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

The zero product property is used to explain why the factors are set equal to zero. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$. Students in Algebra I should recognize when roots are complex and indicate that there are no real solutions rather than writing in $a + bi$ form.

Examples:

- Determine the type of roots for the equation $2x^2 + 5 = 2x$? Explain how you know.
- What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.I.A1.4, LEAP.I.A1.5, LEAP.II.A1.2, LEAP.II.A1.9

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Solving Algebraically A1: A-REI.B.3 A1: A-REI.B.4 A1: A-CED.A.4 LEAP.I.A1.4 LEAP.I.A1.5 LEAP.I.A1.6	Algebraically solves linear equations, linear inequalities, and quadratics in one variable (at complexity appropriate to the course), including those with coefficients represented by letters.	Algebraically solves linear equations, linear inequalities, and quadratics in one variable (at complexity appropriate to the course), **including those with coefficients represented by letters**.	Algebraically solves linear equations, linear inequalities, and **quadratics** in one variable (at complexity appropriate to the course).	Algebraically solves linear equations and linear inequalities in one variable (at complexity appropriate to the course).
	Utilizes structure and rewriting as strategies for solving.			

C. Write and solve systems of equations. In this cluster, the terms students should learn to use with increasing precision are multiple of an equation, sum of two equations, and solve a system of equations exactly and approximately.

A1: A-REI.C.5 Write and solve systems of linear equations in two variables.

a. Use methods such as substitution, elimination, and graphing to solve.
b. Justify a method for solving such systems.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Previous Grade(s) Standard:** 8.AR.C.8 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Procedural Skill and Fluency

Examples and Explanations

The solution methods can include, but are not limited to, graphical, elimination/linear combination, and substitution. While this standard simply calls for procedural skill and fluency in solving systems of linear equations, it is appropriate to provide students with experiences connecting this work to the A-CED domain. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations.

Example:

- Solve the system of equations. Use a second method to verify your answer.

$$\begin{cases} x + y = 11 \\ 3x - y = 5 \end{cases}$$

Assessment Connections

Assessment Item Type(s): Type I, Type II, Type III
Evidence Statement(s): LEAP.II.A1.3, LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Multiple Representations of Functions A1: A-REI.C.6 A1: F-LE.A.2 A1: F-IF.C.9	Writes **and analyzes** systems of linear equations in multi-step contextual problems.	Writes systems of linear equations in multi-step contextual problems.	Writes systems of linear equations in **multi-step** contextual problems.	Writes systems of linear equations in simple contextual problems.
	Represents linear and exponential (with domain in the integers) functions symbolically, graphically, **with a verbal description, as a sequence**, and with input- output pairs to solve mathematical problems.	**Represents** linear and exponential (with domain in the integers) functions symbolically, graphically, and with input-output pairs to solve mathematical problems.	Given a symbolic representation, graph, verbal description, sequence, or input-output pairs for linear **and exponential** functions **(with domains in the integers)**, solves mathematical problems.	Given a symbolic representation, graph, verbal description, sequence, or input-output pairs for linear functions, solves mathematical problems.
	Compares the properties of two functions represented in different ways, limited to linear, quadratic, exponential (with domains in the integers), **absolute value, and piecewise**.	Compares the properties of two functions represented in different ways, limited to linear, quadratic, and **exponential (with domains in the integers)**.	Compares the properties of two functions represented in different ways, limited to linear **and quadratic**.	Compares the properties of two linear functions represented in different ways.

D. Represent and solve equations and inequalities graphically. In this cluster, the terms students should learn to use with increasing precision are solution of an equation, solution of a system of equations, rational function, piecewise function, absolute value function, exponential function, graph the solution to an inequality in two variables in a half plane, and set notation for a system of inequalities.

A1: A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Standard Overview

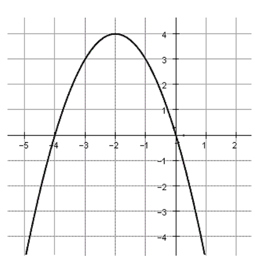
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 8.AR.B.5 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** A1: A-CED.A.2	Conceptual Understanding

Examples and Explanations

In Algebra I, students focus on linear, quadratic, and exponential equations and are able to adapt and apply that learning to other types of equations in future courses. Students can explain and verify that every point (x, y) on the graph of an equation represents all values for x and y that make the equation true.

Examples:

- Which of the following points are on the graph of the equation $-5x + 2y = 20$? How many points are on this graph? Explain.
    a. $(4, 0)$
    b. $(0, 10)$
    c. $(-1, 7.5)$
    d. $(2.3, 5)$
- Verify that $(-1, 60)$ is a solution to the equation $y = 15(\frac{1}{4})^x$. Explain what this means for the graph of the function.
- Given the graph of $y = g(x)$, provide at least three solutions to $g(x) = y$.



Assessment Connections

Assessment Item Type(s): Type I, Type II

Evidence Statement(s): LEAP.II.A1.4

Achievement Level Descriptor				
Content	Advanced	Mastery	Basic	Approaching Basic
Solving Graphically A1: A-CED.A.3 A1: A-REI.D.10 A1: A-REI.D.11 A1: A-REI.D.12	Graphs **and analyzes** the solution sets of equations, linear inequalities, and systems of linear inequalities.	Graphs the solution sets of equations, linear inequalities, **and systems of linear equations and linear inequalities.**	Graphs the solution sets of equations and linear inequalities.	Graphs the solution sets of equations and linear inequalities.
	Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	**Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.**	Given the graph, identify the solutions of a system of two polynomial functions.
	Writes a system of linear inequalities given a context.			

A1: A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, piecewise linear (to include absolute value), and exponential functions.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 8.AR.C.8 **Algebra I Standard Taught in Advance:** A1: A-REI.C.5, A1: A-REI.D.10 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Because A1:REI.D.11 permits the use of technology to graph functions, the standard allows students to solve equations that, algebraically, would be too advanced. Teachers should view this as a system of equations standard that applies to linear, polynomial, rational, piecewise linear (to include absolute value), and exponential functions, with students understanding that intersection points of graphs are solutions to both equations. Students would benefit from having access to a calculator or an application with graphing capabilities.

Example:

- Find the solution to the following system of equations. Solution $(x \approx 5.5,\ y \approx 5.2)$

$$\begin{cases} y = x^2 - 4x - 3 \\ y = x^3 - 4x^2 - 6x - 7 \end{cases}$$

Assessment Connections

Assessment Item Type(s): Type I, Type II, Type III

Evidence Statement(s): LEAP.II.A1.2, LEAP.II.A1.4, LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Solving Graphically A1: A-CED.A.3 A1: A-REI.D.10 A1: A-REI.D.11 A1: A-REI.D.12	Graphs **and analyzes** the solution sets of equations, linear inequalities, and systems of linear inequalities.	Graphs the solution sets of equations, linear inequalities, **and systems of linear equations and linear inequalities**.	Graphs the solution sets of equations and linear inequalities.	Graphs the solution sets of equations and linear inequalities.
	Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	**Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.**	Given the graph, identify the solutions of a system of two polynomial functions.
	Writes a system of linear inequalities given a context.			

A1: A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Standard Overview

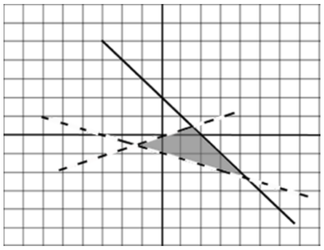
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** A1: A-REI.C.5, A1: REI.D.10 **Algebra I Standard Taught Concurrently:** none	Procedural Skill and Fluency

Examples and Explanations

Examples:

- Graph the solution: $y \leq 2x + 3$.
- Graph the system of linear inequalities below and determine if $(3, 2)$ is a solution to the system.

$$\begin{cases} x - 3y > 0 \\ x + y \leq 2 \\ x + 3y > -3 \end{cases}$$

Solution: 

Assessment Connections

Assessment Item Type(s): Type I, Type II, Type III
Evidence Statement(s): LEAP.II.A1.4, LEAP.III.A1.2

Achievement Level Descriptor				
Content	Advanced	Mastery	Basic	Approaching Basic
Solving Graphically A1: A-CED.A.3 A1: A-REI.D.10 A1: A-REI.D.11 A1: A-REI.D.12	Graphs **and analyzes** the solution sets of equations, linear inequalities, and systems of linear inequalities.	Graphs the solution sets of equations, linear inequalities, **and systems of linear equations and linear inequalities**.	Graphs the solution sets of equations and linear inequalities.	Graphs the solution sets of equations and linear inequalities.
	Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.	**Finds the solutions to two polynomial functions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.**	Given the graph, identify the solutions of a system of two polynomial functions.
	Writes a system of linear inequalities given a context.			

Functions: Interpreting Functions (A1: F-IF)

A. Understand the concept of a function and use function notation. In this cluster, the terms students should learn to use with increasing precision are domain, range, function notation, input, output, evaluate a function, arithmetic sequence, and geometric sequence.

A1: F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 8.PF.A.1, 8.PF.A.2, 8.PF.A.3 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

The foundation for this standard is the standards in 8.F.A; however, this is students' first opportunity to work with function notation, as it is explicitly left out of the Grade 8 standards.

Example:

- Determine which of the following tables represent a function and explain why.

Table A	
x	y
0	1
1	2
2	2
3	4

Table B	
r	T
0	0
1	2
1	3
4	5

Given the function, f, students explain that input values are guaranteed to produce unique output values and use the function rule to generate a table or graph. They identify x as an element of the domain, the input, and $f(x)$ as an element in the range, the output. Students recognize that the graph of the function, f, is the graph of the equation $y = f(x)$ and that $(x, f(x))$ is a point on the graph of f.

Example:

- A pack of pencils cost \$0.75. If n is the number of packs purchased, then the total purchase price is represented by the function $t(n) = 0.75n$.
  a. Explain why t is a function.
  b. What is a reasonable domain and range for the function t?
  c. Graph function t.

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.1, LEAP.I.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
	Evaluates with, uses, and **interprets** with function notation within a context.	Evaluates with and uses function notation **within a context.**	Evaluates with and uses function notation.	Evaluates with and uses function notation.
	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems.**	For linear and quadratic functions that model relationships, determines key features **and graphs the function.**	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
	Determines the domain and relates it to the quantitative relationship it describes for linear, quadratic, exponential (limited to domains in the	Determines the domain **and relates it to the quantitative relationship it describes for** linear, quadratic, **and exponential (limited to**	**Determines the domain of linear and quadratic functions.**	

	integers), **piece-wise, and absolute value** functions.	**domains in the integers)** functions.		

A1: F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 6.AR.A.2 **Algebra I Standard Taught in Advance:** A1.F-IF.A.1 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Examples:

- Evaluate $f(2)$ for the function $f(x) = 5(x - 3) + 17$, both graphically and algebraically.
- Evaluate $f(2)$ for the function $f(x) = 1200(1 + .04)^x$, both graphically and algebraically.
- You placed a yam in the oven and, after 45 minutes, you take it out. Let f be the function that assigns to each minute after you place the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language.
    a. $f(0) = 65$
    b. $f(5) < f(10)$
    c. $f(40) = f(45)$
    d. $f(45) > f(60)$
- The rule $f(x) = 50(0.85)^x$ represents the amount of a drug in milligrams, $f(x)$, which remains in the bloodstream after x hours. Evaluate and interpret each of the following:
    a. $f(0)$
    b. $f(x) < 6$
- If $h(t) = -16t^2 + 5t + 7$ models the path of an object projected into the air where t is time in seconds and $h(t)$ is the vertical height in feet.
    a. Find $h(5)$ and explain the meaning of the solution.
    b. Find t when $h(t) = 0$. Explain the meaning of the solution.

Assessment Connections

Assessment Item Type(s): Type I, Type III

Evidence Statement(s): LEAP.I.A1.1, LEAP.I.A1.2, LEAP.III.A1.3, LEAP.III.A1.4

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
	Evaluates with, uses, and **interprets** with function notation within a context.	Evaluates with and uses function notation **within a context**.	Evaluates with and uses function notation.	Evaluates with and uses function notation.
	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems**.	For linear and quadratic functions that model relationships, determines key features **and graphs the function**.	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
	Determines the domain and relates it to the quantitative relationship it describes for linear, quadratic, exponential (limited to domains in the	Determines the domain **and relates it to the quantitative relationship it describes for** linear, quadratic, **and exponential (limited to**	**Determines the domain of linear and quadratic functions.**	

	integers), **piece-wise, and absolute value** functions.	**domains in the integers)** functions.		

B. Interpret functions that arise in applications in terms of the context. In this cluster, the terms students should learn to use with increasing precision are linear, piecewise linear, absolute value, quadratic and exponential functions; domain, intercepts, intervals, maximum, minimum, symmetry, and end behavior; and average rate of change for listed functions.

A1: F-IF.B.4 For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities,

a. interpret key features of graphs and tables in terms of the quantities, and
b. sketch graphs showing key features given a verbal description of the relationship.
c. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries and end behavior.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 8.PF.B.5 **Algebra I Standard Taught in Advance:** A1: F-IF.A.1, A1: N-Q.A.1 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Students interpret the key features of the different functions listed in the standard. When given a table or graph of a function that models a real-life situation, explain the meaning of the characteristics of the table or graph in the context of the problem.

Key features

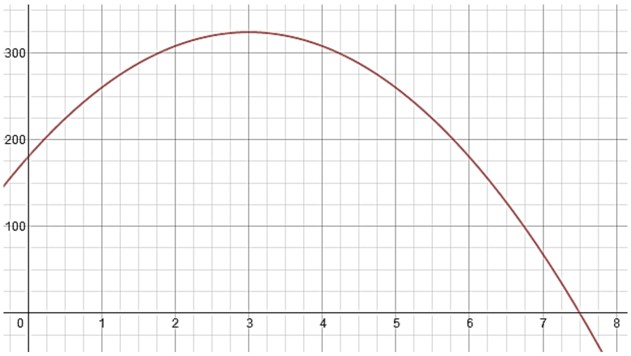
- of a linear function are slope and intercepts
- of a quadratic function are intervals of increase/decrease, positive/negative, maximum/minimum, symmetry, and intercepts
- of an exponential function include y-intercept and increasing/decreasing intervals
- of an absolute value include y-intercept, minimum or maximum, increasing or decreasing intervals, and symmetry

Examples:

- The local newspaper charges for advertisements in its community section. A customer has called to ask about the charges. The newspaper gives the first 50 words for free and then charges a fee per word. Use the table at the right to describe how the newspaper charges for the ads. Include all important information.

# of words	50	60	70	80	90	100
Cost ($)	0.00	0.50	1.00	1.50	2.00	2.50

- The graph of $h(t) = -5t^2 + 10t + 3$, a function giving the height of a diver above the water (in meters), t seconds after the diver leaves the springboard, is shown to the right.
    a. How high above the water is the springboard? Justify your answer.
    b. When does the diver hit the water? Justify your answer.
    c. At what time on the diver's descent toward the water is the diver again at the same height as the springboard? Justify your answer.
    d. When does the diver reach the peak of the dive? Justify your answer.
- The graph represents the height (in feet) of a rocket as a function of the time (in seconds) since it was launched. Use the graph to answer the following:



    a. What is the appropriate domain for t in this context? Why?
    b. What is the height of the rocket two seconds after it was launched?
    c. What is the maximum value of the function and what does it mean in context?
    d. When is the rocket 100 feet above the ground?
    e. When is the rocket 250 feet above the ground?
    f. Why are there two answers to part e but only one practical answer for part d?
    g. What are the intercepts of this function? What do they mean in the context of this problem?
    h. What are the intervals of increase and decrease on the practical domain? What do they mean in the context of the problem?
- Jack planted a mysterious bean just outside his kitchen window. Jack kept a table (shown below) of the plant's growth. He measured the height at 8:00 am each day.

Day	0	1	2	3	4
Height (cm)	2.56	6.4	16	40	100

    a. What was the initial height of Jack's plant?
    b. How is the height changing each day?
    c. If this pattern continues, how tall should Jack's plant be after 8 days?
- The front of a camping tent can be modeled by the function $y = -1.4|x - 2.5| + 3.5$ where x and y are measured in feet and the x-axis represents the ground.

a. What does the point $(2.5, 3.5)$ represent in the context of the situation?
b. What are the height and width of the tent?
c. What are the intervals of increase or decrease? What do they mean in the context of the problem?

[Assessment Connections]

Assessment Item Type(s): Type I, Type III
Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.4, LEAP.I.A1.5, LEAP.I.A1.6, LEAP.III.A1.2, LEAP.III.A1.3

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
	Evaluates with, uses, and **interprets** with function notation within a context.	Evaluates with and uses function notation **within a context**.	Evaluates with and uses function notation.	Evaluates with and uses function notation.
	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems**.	For linear and quadratic functions that model relationships, determines key features **and graphs the function.**	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
	Determines the domain and relates it to the quantitative relationship it describes for linear, quadratic, exponential (limited to domains in the	Determines the domain **and relates it to the quantitative relationship it describes for** linear, quadratic, **and exponential (limited to**	**Determines the domain of linear and quadratic functions.**	

	integers), **piece-wise, and absolute value** functions.	**domains in the integers)** functions.		

A1: F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

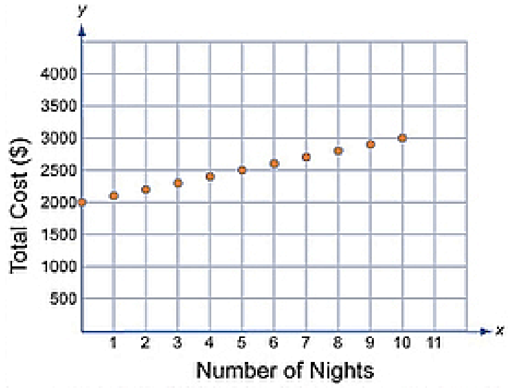
Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** A1: F-IF.A.1, A1: F-IF.B.4 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Examples:

- An all-inclusive resort in Los Cabos, Mexico, provides everything for its customers during their stay, including food, lodging, and transportation. Use the graph below to describe the domain of the total cost function.



- Maggie tosses a coin off of a bridge into a stream below. The distance the coin is above the water is modeled by the equation $y = -16x^2 + 96x + 112$, where x represents time in seconds. What is a reasonable domain for the function?

Oakland Coliseum Problem[14]

Oakland Coliseum, home of the Oakland Raiders, can seat 63,026 fans. For each game, the amount of money that the Raiders' organization brings in as revenue is a function of the number of people, n, in attendance. If each ticket costs $30.00, find the domain and range of this function.

[14] https://www.illustrativemathematics.org/content-standards/HSF/IF/B/5/tasks/631

Solution

Let r represent the revenue that the Raiders's organization makes, so that $r = f(n)$. Since n represents a number of people, it must be a nonnegative whole number. Therefore, since 63,026 is the maximum number of people who can attend a game, we can describe the domain of f as follows:

Domain = {n: $0 \leq n \leq 63{,}026$ and n is an integer}.

The range of the function consists of all possible amounts of revenue that could be earned. To explore this question, note that $r = 0$ if nobody comes to the game, $r = 30$ if one person comes to the game, $r = 60$ if two people come to the game, etc. Therefore, r must be a multiple of 30 and cannot exceed $30 \cdot 63{,}026 = 1{,}890{,}780$, so we see that

Range = {r : $0 \leq r \leq 1{,}890{,}780$ and r is an integer multiple of 30}.

Note that the representations used above are just sample ways of writing down the domain and range, using set-builder notation. Other options for writing down descriptions of the same sets abound.

Assessment Connections

Assessment Item Type(s): Type I, Type III

Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.6, LEAP.III.A1.2, LEAP.III.A1.3

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
	Evaluates with, uses, and **interprets** with function notation within a context.	Evaluates with and uses function notation **within a context**.	Evaluates with and uses function notation.	Evaluates with and uses function notation.
	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems**.	For linear and quadratic functions that model relationships, determines key features **and graphs the function**.	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
	Determines the domain and relates it to the quantitative relationship it describes for linear, quadratic, exponential (limited to domains in the	Determines the domain **and relates it to the quantitative relationship it describes for** linear, quadratic, **and exponential (limited to**	**Determines the domain of linear and quadratic functions.**	

	integers), **piece-wise, and absolute value** functions.	**domains in the integers)** functions.		

A1: F-IF.B.6 Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 8.PF.B.4 **Algebra I Standard Taught in Advance:** A1: F-IF.A.2 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

The average rate of change of a function $y = f(x)$ over an interval $[a, b]$ is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$

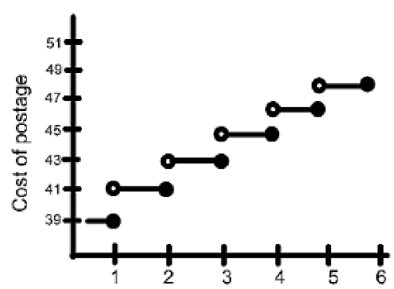
In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change over various intervals.

Examples:

- The plug is pulled in a small hot tub. The table gives the volume of water in the tub from the moment the plug is pulled until it is empty. What is the average rate of change between:
  a. 60 seconds and 100 seconds?
  b. 0 seconds and 120 seconds?
  c. 70 seconds and 110 seconds?

Draining Water from a Hot Tub	
Time (s)	**Volume (L)**
0	1600
10	1344
20	1111
30	900
40	711
50	544
60	400
70	278
80	178
90	100
100	44
110	11
120	0

- Compare the rate of change for the cost of postage for a letter over the interval $1 \leq x \leq 2$ to the interval $3 \leq x \leq 6$.



High School Gym Problem[15]

The school assembly is being held over the lunch hour in the school gym. All the teachers and students are there by noon, and the assembly begins. About 45 minutes after the assembly begins, the temperature within the gym remains a steady 77 degrees Fahrenheit for a few minutes. As the students leave after the assembly ends at the end of the hour, the gym begins to slowly cool down.
Let T denote the temperature of the gym in degrees Fahrenheit and M denote the time, in minutes, since noon.

a. Is M a function of T? Explain why or why not.
b. Explain why T is a function of M, and consider the function $T = g(M)$. Interpret the meaning of $g(0)$ in the context of the problem.
c. Becky says, "The temperature increased 5 degrees in the first half hour after the assembly began." Which of the following equations best represents this statement? Explain your choice.
    i. $g(30) = 5$
    ii. $\frac{g(30)-g(0)}{30} = 5$
    iii. $g(30) - g(0) = 5$
    iv. $T = g(30) - 5$
d. Which of these choices below represents the most reasonable value for the quantity $\frac{g(75)-g(60)}{15}$? Explain your choice:
    i. 4
    ii. 0.3
    iii. 0
    iv. -0.2
    v. -5

Solution

a. No, M is not a function of T. The problem states that the temperature, T, remains a steady 77 degrees Fahrenheit for a few minutes, M. Thus, we know that there is more than one value of M, or more than one minute, associated with the temperature $T = 77$. This means that if M were a function of T there would be more than one output M associated with the single input value of $T = 77$. By definition of a function, each input must be associated with exactly one output. Therefore, M cannot be a function of T because there would be more than one output associated with the input $T = 77$.
b. In contrast the previous part, it is the case that for each number of minutes M after noon, there is one and only one temperature T in the room, so T is a function of M. The quantity $g(0)$ is the value of T when $M = 0$. From the context given in the problem, we know that M denotes the time in minutes since noon. Thus, $M = 0$ is exactly noon, so $g(0)$ is the temperature in the high school gym in degrees Fahrenheit at exactly noon.
c. Let's look at the meaning of each equation:
    i. $g(30) = 5$ says the temperature at 12:30 p.m. is 5 degrees Fahrenheit, which is not what Becky said.

[15] https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/577

ii. $\frac{g(30)-g(0)}{30} = 5$ says that the average rate of change of the temperature of the gym between noon and 12:30 p.m. is 5 degrees per minute, which is not what Becky said.

iii. $g(0) - g(30) = 5$ says that the difference in the temperature of the gym at 12:30 and the temperature of the gym at noon is 5 degrees, which is the same thing as saying the temperature increased by 5 degrees Fahrenheit.

iv. $T = g(30) - 5$ says that the temperature equals the temperature at 12:30 minus five degrees, which is not what Becky said.

d. As in the previous part, we recognize the quantity $g(75) - g(60)$ as the change in the temperature in the room in the 15 minutes following the students' departure from the gym. We are told that the temperature decreases as the students begin to leave, so we know this quantity must be negative.

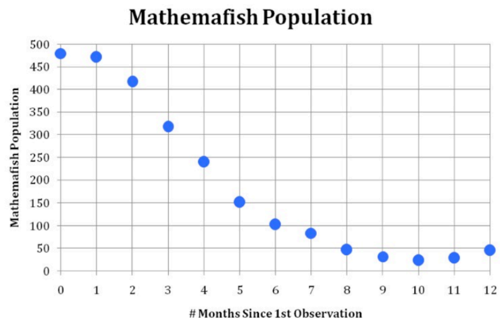
Next, we note that the quantity in question, namely $\frac{g(75)-g(60)}{15}$, is precisely the average rate of change of the temperature over these first fifteen minutes, and so is also negative. This eliminates the first three choices for the value of this quantity immediately.

Finally, we note that the last option corresponds to an average decrease of 5 degrees per minute, which would combine for a 75-degree drop in this 15-minutes span. Since this would leave the gym in a frozen state, we conclude that this was an implausibly fast rate of temperature decrease. This leaves only the value $-\ 0.2$, corresponding to a significantly more plausible temperature drop of 3 degrees over these 15 minutes.

Mathemafish Population[16]

You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

# months since 1st measurement	0	1	2	3	4	5	6	7	8	9	10	11	12
Mathemafish population	480	472	417	318	240	152	103	84	47	32	24	29	46



[16] https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/686

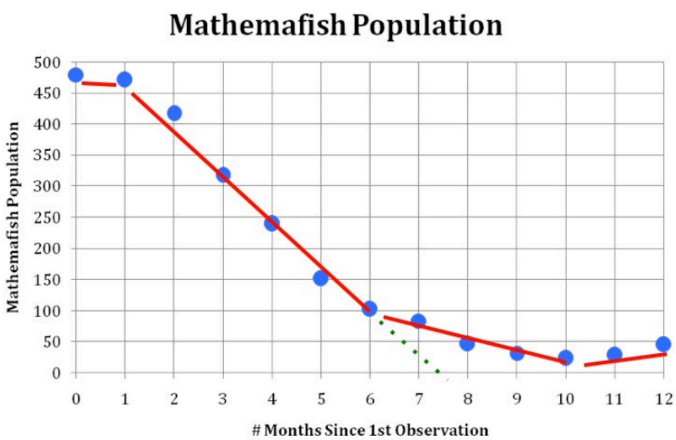
Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project.
What to include in your summary report:

- Calculate the average rate of change of the mathemafish population over specific intervals. Indicate how and why you chose the intervals you chose.
- When was the population decreasing the fastest?
- During what month did you notice the largest effects of the EPA intervention?
- Explain the overall effects of the intervention.
- Remember to justify all your conclusions using supporting evidence.

Solution

Solutions are asked to be put in the format of a report. We include below a sample collection of the most relevant pieces of information -- alternatives abound.

Looking at the graph, it seems that there were four intervals in which the rate of population change was significantly different; that is, where the slope of the line appeared to change. These intervals are: (0,1), (1,6), (6,10), and (10,12).



$$m_{(0.1)} = \frac{\Delta y}{\Delta x} = \frac{472-480}{1-0} = \frac{-8\,fish}{1\,month} = -\ 8\frac{fish}{month}$$

$$m_{(1.6)} = \frac{\Delta y}{\Delta x} = \frac{103-472}{6-1} = \frac{-369\,fish}{5\,month} = -\ 73.8\frac{fish}{month}$$

$$m_{(6.10)} = \frac{\Delta y}{\Delta x} = \frac{24-103}{10-6} = \frac{-79\,fish}{4\,month} = -\ 19.75\frac{fish}{month}$$

$$m_{(10.12)} = \frac{\Delta y}{\Delta x} = \frac{46-24}{12-10} = \frac{22\,fish}{2\,month} = +\ 11\frac{fish}{month}$$

The mathemafish population decreased along three of the intervals, but decreased the fastest between 1 and 6 months with a rate of change of –73.8 fish each month. We begin to see the effects of the EPA intervention around the 6th month when the rate of *decrease* drops from approximately 74 fish per month to 20 fish per month. If not for the intervention, the fluted dropout shark would have decimated the mathemafish population before month 8.

Population in 6th month: 103 fish

Current rate of change: –73.8 fish/month

$$y = 103 - 73.8x$$
$$0 = 103\,fish - (73.8\frac{fish}{month})(x\,months)$$
$$(73.8\frac{fish}{month})(x\,months) = 103\,fish$$
$$x \approx 1.39\,months$$
$$6 + 1.39 = 7.39\,months$$

Additionally, because of the intervention, by month 10, we even begin to see the population *increase* with a rate of change of 11 fish/month. If the EPA intervention remains effective, we can expect to see continued growth in the mathemafish population.

Temperature Change[17]

The table below shows the temperature, T, in Tucson, Arizona t hours after midnight. When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

t **(hours after midnight)**	0	3	4
T **(temp. in °F)**	85	76	70

Solution
Even though the temperature drops more between midnight and 3 a.m. than it does between 3 a.m. and 4 a.m., the temperature decreases fastest between 3 a.m. and 4 a.m., as the calculations below demonstrate:

Interval	ΔT	Δt	$\frac{\Delta T}{\Delta t}$
$0 \leq t \leq 3$	-9°F	3 hours	$\frac{-9}{3} = -\ 3°F$ per hour
$3 \leq t \leq 4$	-6°F	1 hour	$\frac{-6}{1} = -\ 6°F$ per hour

Thus, the temperature decreases at an average rate of only 3°F per hour between midnight and 3 a.m., but it decreases at an average rate of 6°F per hour between 3 and 4 a.m.

17 https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1500

Assessment Connections

Assessment Item Type(s): Type I, Type III

Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.4, LEAP.I.A1.5, LEAP.I.A1.6, LEAP.III.A1.2, LEAP.III.A1.3, LEAP.III.A1.4

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Rate of Change A1: F-IF.B.6	Calculates and interprets the average rate of change of linear, exponential, quadratic, **and piecewise-defined functions (presented symbolically or as a table)** over a specified interval, and estimates the rate of change from a graph.	Calculates the average rate of change of linear, exponential, and quadratic functions (presented symbolically or as a table) over a specified interval **and estimate the rate of change from a graph**.	Calculates the average rate of change of linear, exponential, and quadratic functions (presented **symbolically** or as a table) over a specified interval.	Calculates the average rate of change of linear, exponential, and quadratic functions (presented as a table) over a specified interval.
	Compares rates of change associated with different intervals.			

C. Analyze functions using different representations. In this cluster, the terms students should learn to use with increasing precision are linear, piecewise linear, absolute value, quadratic and exponential functions; symbolically expressed functions; factor and complete the square of a quadratic function to show intercepts, maxima,, minima, zeros, symmetry, extreme values.

A1: F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph piecewise linear (to include absolute value) and exponential functions.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** 8.AR.B.5, 8.PF.A.3 **Algebra I Standard Taught in Advance:** A1: F-IF.A.1 **Algebra I Standard Taught Concurrently:** A1: F-IF.C.8, A1: F-BF.B.3	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Key characteristics include, but are not limited to, maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.

Examples:

- Graph the function $f(x) = |x - 3| + 5$. Identify the key characteristics of the graph.
- Graph and identify the key characteristics of the function described below.

$$f(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ x + 5 \text{ for } x < -1 \end{cases}$$

- Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph.
- Without using the graphing capabilities of a calculator, sketch the graph of $f(x) = x^2 + 7x + 10$ and identify the x-intercepts, y-intercept, and the maximum or minimum point.

Assessment Connections

Assessment Item Type(s): Type I, Type III

Evidence Statement(s): LEAP.I.A1.2. LEAP.I.A1.4, LEAP.I.A1.5, LEAP.I.A1.6, LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Graphs of Functions A1: A-APR.B.3 A1: F-IF.C.7	Graphs linear, quadratic, **and piecewise-defined functions, showing key features.**	Graphs linear and quadratic functions, showing key features.	Graphs linear **and quadratic functions**, showing key features.	Graphs linear functions, showing key features.
	Determines a function, given a graph with key features identified.			

A1: F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** 7.AR.A.1 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** A1: F-IF.C.7	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Focus on factoring as a process to show zeroes, extreme values, and symmetry of the graph. Students should be prepared to factor quadratics in which the coefficient of the quadratic term is an integer that may or may not be the GCF of the expression. Students must use the factors to reveal and explain properties of the function, interpreting them in context. Factoring just to factor does not fully address this standard.

Springboard Dive[18]

Suppose $h(t) = -5t^2 + 10t + 3$ is the height of a diver above the water (in meters), t seconds after the diver leaves the springboard.

a. How high above the water is the springboard? Explain how you know.
b. When does the diver hit the water?
c. At what time on the diver's descent toward the water is the diver again at the same height as the springboard?
d. When does the diver reach the peak of the dive?

Solution

Solution 1:

a. The height of the springboard is found by evaluating h at 0, the instant the diver leaves the springboard:

$$h(0) = 3$$

so the springboard is 3 meters above the water.

[18] https://www.illustrativemathematics.org/content-standards/HSF/IF/C/8/tasks/375

b. The diver will hit the water when $h(t) = 0$. Using the quadratic formula, the solutions to $h(t) = 0$ are $t = \frac{-10 \pm \sqrt{160}}{-10}$.

One of these values of t, namely $t = \frac{-10 \pm \sqrt{160}}{-10}$ is negative and so has no significance relative to the dive. So the diver hits the water after $t = \frac{-10 + \sqrt{160}}{-10}$ seconds or about $2\frac{1}{4}$ seconds.

c. Since we have already determined that the springboard is three meters above the water, we are looking for a second time t for which height above the water when $h(t) = 3$.

Substituting in the definition of $h(t)$ and re-writing, we are left trying to solve $-5t^2 + 10t = 0$. Factoring this gives us $5t(-t + 2) = 0$, which occurs only when $t = 0$ and when $t = 2$. The value $t = 0$ is the moment the dive begins, and so it is two seconds after the dive, $t = 2$, that the diver returns again to the level of the springboard.

d. The graph of $h(t)$ is a parabola. Parabolas are symmetric about the vertical line through the vertex of the parabola. Since $h(0) = h(2) = 3$ the vertex must be at $t = 1$ as this is the only vertical reflection which will interchange the points $(0, 3)$ and $(2, 3)$ on the graph of $h(t)$.

Solution 2: Finding when the diver reaches the peak by completing the square

The last part of the problem can be solved by rewriting the function $h(t)$ and manipulating as follows:

$$\begin{aligned} h(t) &= -5t^2 + 10t + 3 \\ &= -5(t^2 - 2t - \tfrac{3}{5}) \\ &= -5[(t-1)^2 - \tfrac{8}{5}] \\ &= 8 - 5(t-1)^2 \end{aligned}$$

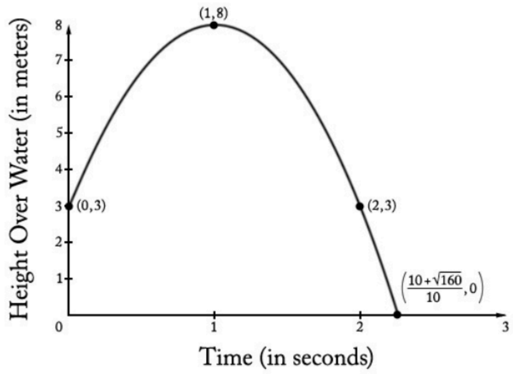
The structure of the expression $8 - (t - 1)^2$ shows that $h(t)$ is largest when $t = 1$ and that the maximal value is 8. Thinking back to the first solution, the expression $8 - 5(t - 1)^2$ for $h(t)$ also makes it clear that the graph of $y = h(t)$ will be symmetric about $t = 1$, that is $h(1 + d)$ will take the same value as $h(1 - d)$ namely $8 - 5d^2$.

Solution 3: Graphing Technology

A calculator can be used either to generate a graph of $h(t)$ to help visualize the situation, or to find and label the important points on the graph:

- The point $(0, 3)$ when the dive begins
- The point $(1, 8)$ when the diver reaches the highest elevation
- The point $(2, 3)$ when the diver returns to the level of the springboard
- The point $\left(\frac{10+\sqrt{160}}{10}, 0\right)$ when the diver reaches the water

Below is a graph of $h(t) = -5t^2 + 10x + 3$ with the appropriate points labelled:


(1,8)
(0,3)
(2,3)
$\left(\frac{10+\sqrt{160}}{10}, 0\right)$
Height Over Water (in meters)
Time (in seconds)
0
1
2
3
4
5
6
7
8

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.4, LEAP.I.A1.5, LEAP.II.A1.8

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Equivalent Expressions and Functions A1: A-SSE.B.3 A1: F-IF.C.8a	Determines equivalent forms of quadratic **and exponential (with integer domain)** expressions and functions to reveal and explain **their properties.**	**Determines** equivalent forms of quadratic expressions and functions. Uses equivalent forms **to reveal and explain** zeros, **extreme values** and symmetry.	Identifies equivalent forms of quadratic expressions and functions. **Identifies zeros and symmetry.**	Identifies equivalent forms of quadratic expressions and functions in cases where suitable factorizations are provided.

A1: F-IF.C.9 Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential), each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.

Standard Overview

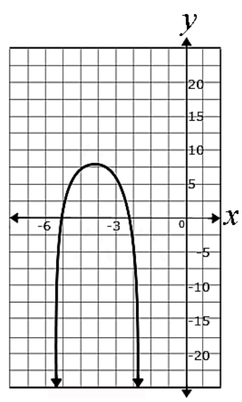
Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** A1: F-IF.B.4, A1: F-IF.C.8 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Examples:

- Examine the functions below.
  Which function has the larger maximum?
  How do you know?

$$f(x) = -2x^2 - 8x + 20$$



- Examine the two functions represented below. Compare the x-intercepts and find the difference between the minimum values.

$$f(x) = x^2 + 8x + 15$$

x	-7	-6	-5	-4	-3	-2	-1
$g(x)$	4	1.5	0	-0.5	0	1.5	4

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.4, LEAP.I.A1.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Multiple Representations of Functions A1: A-REI.C.5 A1: F-LE.A.2 A1: F-IF.C.9	Writes **and analyzes** systems of linear equations in multi-step contextual problems.	Writes systems of linear equations in multi-step contextual problems.	Writes systems of linear equations in **multi-step** contextual problems.	Writes systems of linear equations in simple contextual problems.
	Represents linear and exponential (with domain in the integers) functions symbolically, graphically, **with a verbal description, as a sequence**, and with input- output pairs to solve mathematical problems.	**Represents** linear and exponential (with domain in the integers) functions symbolically, graphically, and with input-output pairs to solve mathematical problems.	Given a symbolic representation, graph, verbal description, sequence, or input-output pairs for linear **and exponential** functions **(with domains in the integers)**, solves mathematical problems.	Given a symbolic representation, graph, verbal description, sequence, or input-output pairs for linear functions, solves mathematical problems.
	Compares the properties of two functions represented in different ways, limited to linear, quadratic, exponential (with domains in the integers), **absolute value, and piecewise.**	Compares the properties of two functions represented in different ways, limited to linear, quadratic, and **exponential (with domains in the integers).**	Compares the properties of two functions represented in different ways, limited to linear **and quadratic.**	Compares the properties of two linear functions represented in different ways.

Functions: Building Functions (A1: F-BF)

A. Build a function that models a relationship between two quantities. In this cluster, the terms students should learn to use with increasing precision are explicit expression, recursive process, function notation, and understand the effect of k on fx where f(x) is linear, piecewise linear, absolute value, quadratic, or an exponential function.

A1: F-BF.A.1 Write a linear, quadratic, or exponential function that describes a relationship between two quantities. ★

a. Determine an explicit expression or steps for calculation from a context.

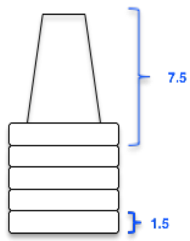
Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** [8.PF.B.4](8.PF.B.4) **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

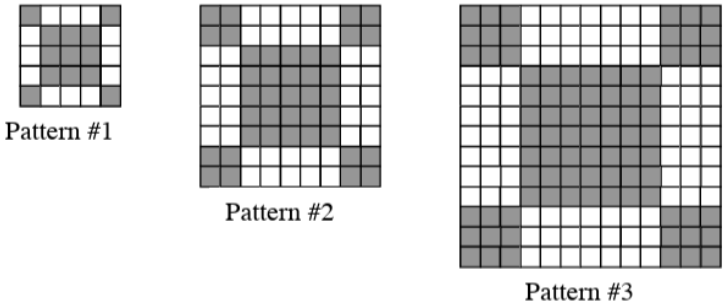
Examples:

- The height of a stack of cups is a function of the number of cups in the stack. If a 7.5" cup with a 1.5" lip is stacked vertically, determine a function that would provide you with the height based on any number of cups.



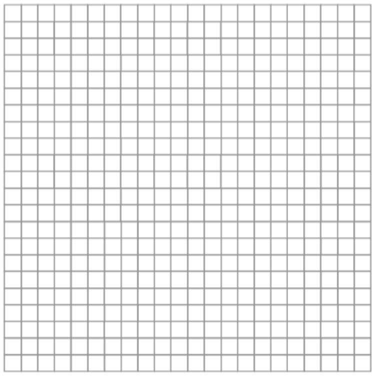
Sidewalk Patterns[19]

In Prague, some sidewalks are made of small square blocks of stone. The blocks are in different shades to make patterns that are in various sizes.



[19] http://map.mathshell.org/tasks.php?unit=HA06&collection=9&redir=1

Draw the next pattern in this series.



Pattern #4

1. Complete the table below.

Pattern number, *b*	1	2	3	4
Number of white blocks	12	40		
Number of gray blocks	13			
Total number of blocks	25			

2. What do you notice about the number of white blocks and the number of gray blocks?
3. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.

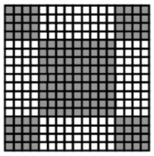
   a. Fill in the blank spaces in this list.
   $25 = 5^2$ $81 =$ _____ $169 =$ _____ $289 = 17^2$

   b. How many blocks will pattern #5 need?
   c. How many blocks will pattern #n need?

4. A. If you know the total number of blocks in a pattern, you can work out the number of white blocks in it. Explain how you can do this.
   B. Pattern #6 has a total of 625 blocks. How many white blocks are needed for pattern #6? Show how you figured this out.

Solution

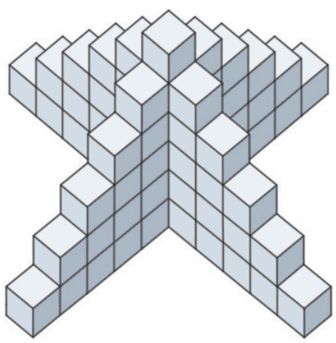
correct pattern:



Pattern number, n	1	2	3	4
Number of white blocks	12	40	84	144
Number of gray blocks	13	41	85	145
Total number of blocks	25	81	169	289

- There is one more gray block than white blocks.
- a. $81 = 9^2 \qquad 169 = 13^2$
- 21^2 or 441
- $(4n + 1)^2$ or equivalent expression
- a. Subtract 1 from the total and divide by 2.
- b. 312 because $\frac{(625-1)}{2} = 312$.

Skeleton Tower [20]



a. How many cubes are needed to build this tower?
b. How many cubes are needed to build a tower like this, but 12 cubes high? Justify your reasoning.
c. How would you calculate the number of cubes needed for a tower n cubes high?

Solution

Arithmetic Sequence, Variation 1

In this solution, we count the cubes in rows (or layers):

a. The top layer has a single cube. The layer below has one cube beneath the top cube, plus 4 new ones, making a total of 5. The third layer below has cubes below these 5, plus 4 new ones to make 9. Continuing to add four each time gives a total of $1 + 5 + 9 + 13 + 17 + 21 = 66$ cubes in the skeleton tower with six layers.

[20] https://tasks.illustrativemathematics.org/content-standards/tasks/75

b. Building upon the reasoning established in a), the number of cubes in the bottom (12th) layer will be $1 + 4 \times 11$, since it is 11 layers below the top. So for this total, we need to add $1 + 5 + 9 + ... + 45$.

One way to do this would be to simply add the numbers. A quicker method, which will work for any number of layers is the so-called Gauss method (which the great mathematician reportedly used to sum a large list of whole numbers when he was very young). The idea is to rewrite the sum backward as $45 + 41 + 37 + ... + 1$.

Now, if this list is placed below the previous list, we can see that each column sums to 46. There are 12 columns so the answer to this problem is half of $12 \times 46 = 552$ or 276.

c. Let $f(n)$ be the number of cubes in the nth layer, counting down from the top. Then $f(1) = 1$ is the number of cubes in the top layer, $f(2) = 5$ the number of cubes in the second layer from the top and so on. In this case, the values of the function $f(1),\ f(2),\ f(3)$,... form an arithmetic sequence, where each term is obtained from the previous one by adding 4. In general, we have $f(n) = 4(n - 1) + 1$. The method used to solve problem b) provides a method both to solve the skeleton tower problem and to add a general arithmetic sequence. For the skeleton tower problem, with n layers in the tower, the sum giving the total number of cubes will be

$$1 + 5 + 9 + ... + f(n) = 1 + 5 + 9 + ... + (4(n - 1) + 1).$$

If we use the method of problem b) here, twice this sum will be equal to $\frac{n(4(n-1)+2)}{2} = n(2n - 1)$.

Arithmetic Sequence, Variation 2

In this solution, we count the cubes in columns.

a. The top layer has a single cube, and there are five layers of cubes below this one, making 6 total cubes in the column with the top one. The next layer introduces 4 new cubes, and there are four layers below, so this gives a total of 20 cubes in these 4 columns. Similarly, each successive layer introduces 4 new columns, so we calculate the total to be

$$6 + 4 \times 5 + 4 \times 4 + 4 \times 3 + 4 \times 2 + 4 \times 1 = 66.$$

b. Imitating this method used in the first problem, we find that the column under the cube on top has 12 cubes. Then there are four columns which are 11 cubes high, four columns which are 10 cubes high, and so on down to four columns which are one cube high. Adding this all up gives

$$12 + 44 + 40 + ... + 4.$$

We can simplify this to $12 + 4(11 + 10 + ... + 1)$.

The sum of the first eleven positive integers is found to be 66 (either by hand or by the method indicated in problem c) below. Thus, there are $12 + 4 \times 66 = 276$ cubes in this tower.

c. Using the reasoning developed in a) and b), the general skeleton tower problem with n layers has a total of $n + 4((n - 1) + (n - 2) + ... + 1)$ cubes. In order to evaluate the sum $((n - 1) + (n - 2) + ... + 1)$, we list the same sum backward, that is $1 + 2 + ... + (n - 1))$, beneath it. When we add the columns created by the two sums, each adds to n. Since there are

$n - 1$ columns, the total of the two sums is $n(n - 1)$. Thus, the sum $((n - 1) + (n - 2) + ... + 1)$ will be half this or $\frac{n(n-1)}{2}$. Thus the total number of cubes in the skeleton tower with n layers is $n + 4 \times \frac{n(n-1)}{2} = 2n^2 - n$.

Assessment Connections

Assessment Item Type(s): Type I, Type III

Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.4, LEAP.I.A1.5, LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
LEAP.III.A1.1 LEAP.III.A1.2 LEAP.III.A1.3 LEAP.III.A1.4	Using stated assumptions and making assumptions and approximations to simplify a real-world situation (include micro-models).	Using stated assumptions **and making assumptions** and approximations to simplify a real-world situation **(include micro-models)**.	Using stated assumptions and approximations to simplify a real-world situation.	Using stated assumptions and approximations to simplify a real-world situation.
	Mapping relationships between important quantities.	**Mapping relationships between important quantities.**	**Illustrating relationships between important quantities.**	Identifying important quantities.
	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically between quantities to draw conclusions.	Analyzing relationships mathematically **between quantities** to draw conclusions.	Analyzing relationships mathematically to draw conclusions.
	Interpreting mathematical results in the context of the situation.	Interpreting mathematical results **in the context of the situation**.	Interpreting mathematical results in a simplified context.	
	Reflecting on whether the results make sense.	Reflecting on whether the results make sense.	**Reflecting on whether the results make sense.**	
	Improving the model if it has not served its purpose.	**Improving** the model if it has not served its purpose.	**Modifying the model if it has not served its purpose.**	

	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.	Writing an algebraic expression or equation to describe a situation.
	Applying proportional reasoning and percentages **justifying and defending models which lead to a conclusion**.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.	Applying proportional reasoning and percentages.
	Writing and using functions in any form to describe how one quantity of interest depends on another.	Writing and using functions **in any form** to describe how one quantity of interest depends on another.	**Writing and** using functions to describe how one quantity of interest depends on another.	Using functions to describe how one quantity of interest depends on another.
	Using statistics.	Using statistics.	Using statistics.	Using statistics.
	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using **reasonable** estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.	Using estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.
	Analyzing and/or creating constraints, relationships and goals.			

B. Build new functions from existing functions. In this cluster, the terms students should learn to use with increasing precision are explicit expression, function notation, understand the effect of k on $f(x)$ where $f(x)$ is linear, piecewise linear, absolute value, quadratic, or an exponential function.

A1: F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, and $f(x + k)$ for specific values of k (both positive and negative). Without technology, find the value of k given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where $f(x)$ is a linear, quadratic, piecewise linear (to include absolute value), or exponential function.

Standard Overview

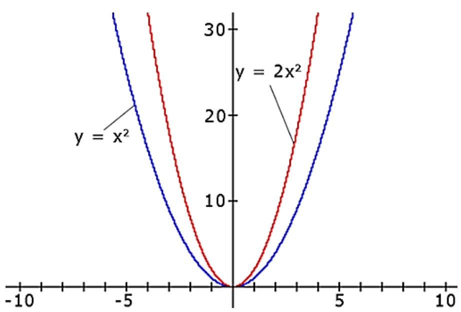
Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** A1: F-IF.C.7	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Although this standard is considered additional work, it should be leveraged to help students see how all of the functions studied in Algebra I behave similarly in terms of transformations on the coordinate plane (a major topic of study in Grade 8).

Examples:

- Compare the shape and position of the graphs of $f(x) = x^2$ and $f(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions.



- Describe the effect of varying the parameters a, h, and k have on the shape and position of the graph of $f(x) = a(x - h)^2 + k$.
- Describe how the graph of $f(x) + k$ compares to $f(x)$ if k positive and if k is negative.

Assessment Connections

Assessment Item Type(s): Type I, Type II, Type III

Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.4, LEAP.I.A1.5, LEAP.II.A1.6, LEAP.III.A1.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Function Transformations A1: F-BF.B.3	Identifies the effects of **multiple** transformations on graphs of linear and quadratic functions and finds the value of k given a transformed graph.	Identifies the effects of a single transformation on graphs of linear and quadratic functions, including $f(x)+k$, $kf(x)$, **$f(kx)$ and $f(x+k)$, and finds the value of k given a transformed graph.**	Identifies the effects of a single transformation on graphs of linear and quadratic functions, limited to $f(x)+k$ **and $kf(x)$.**	Identifies the effects of a single transformation on graphs of linear and quadratic functions, limited to $f(x)+k$.
	Experiments with cases using technology.			
	Given the equation of a transformed linear or quadratic function, creates an appropriate graph.			

Functions: Linear, Quadratic, and Exponential Models★ (A1: F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems. In this cluster, the terms students should learn to use with increasing precision are equal differences, equal intervals, equal factors, constant rate, growth and decay by a constant rate, construct a function, and increase exponentially, linearly, and quadratically.

A1: F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals.
b. Prove that exponential functions grow by equal factors over equal intervals.
c. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
d. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** 8.PF.A.3, 8. PF.B.4 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions.

Students distinguish between a constant rate of change and a constant percent rate of change.

Examples:

- Town A adds 10 people per year to its population, and town B grows by 10% each year. In 2006, each town had 145 residents. For each town, determine whether the population growth is linear or exponential. Explain.
- A couple wants to buy a house in five years. They need to save a down payment of $8,000. They deposit $1,000 in a bank account earning 3.25% interest, compounded quarterly. Would the total amount saved for an unknown period of time be modeled better by a linear or exponential function? Explain.
- Carbon 14 is a common form of carbon which decays exponentially over time. The half-life of Carbon 14, that is the amount of time it takes for half of any amount of Carbon 14 to decay, is approximately 5730 years. Suppose we have a plant fossil and that the plant, at the time it died, contained 10 micrograms of Carbon 14 (one microgram is equal to one millionth of a gram). Would

the total amount of Carbon 14 for an unknown period of time be modeled better by a linear or exponential function? Explain.

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.6, LEAP.II.A1.7

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.A1.1 LEAP.II.A1.2 LEAP.II.A1.3 LEAP.II.A1.4 LEAP.II.A1.5 LEAP.II.A1.6 LEAP.II.A1.7 LEAP.II.A1.8 LEAP.II.A1.9 LEAP.II.A1.10	Using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate).	Using a logical approach based on a conjecture and/or stated assumptions, **utilizing mathematical connections (when appropriate)**.	Using a **logical** approach based on a conjecture and/or stated assumptions.	Using an approach based on a conjecture and/or stated assumptions.
	Providing an **efficient and** logical progression of steps or chain of reasoning with appropriate justification.	Providing a logical progression of steps or chain of reasoning **with appropriate justification**.	Providing a logical, **but incomplete**, progression of steps or chain of reasoning.	Providing an incomplete or illogical progression of steps or chain of reasoning.
	Performing precise calculations.	Performing **precise** calculations.	**Performing minor** calculation errors.	Making an intrusive calculation error.
	Using correct grade-level vocabulary, symbols and labels.	Using **correct** grade-level vocabulary, symbols and labels.	Using **some** grade-level vocabulary, symbols and labels.	Using limited grade-level vocabulary, symbols and labels.
	Providing a justification of a conclusion.	Providing a **justification** of a conclusion.	Providing a partial justification of a conclusion based on own calculations.	Providing a partial justification of a conclusion based on own calculations.

	Evaluating, interpreting and critiquing the validity of others' responses, approaches, and reasoning – using mathematical connections (when appropriate) – and **providing a counter-example where applicable**.	**Evaluating, interpreting and critiquing the validity of others' responses, approaches, and reasoning – using mathematical connections(when appropriate).**	**Evaluating the validity of others' approaches and conclusions.**	
	Determining whether an argument or conclusion is generalizable.			

A1: F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** 8. PF.B.4 **Algebra I Standard Taught in Advance:** A1: F-LE.A.1 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Example:

- Albuquerque boasts one of the longest aerial trams in the world. The tram transports people up to Sandia Peak. The table shows the elevation of the tram at various times during a particular ride.

Minutes into the ride	2	5	9	14
Elevation in feet	7069	7834	8854	10,129

  Write an equation for a function that models the relationship between the elevation of the tram and the number of minutes into the ride.

- After a record-setting winter storm, there are 10 inches of snow on the ground! Now that the sun is finally out, the snow is melting. At 7 a.m., there were 10 inches, and at 12 p.m,. there were 6 inches of snow.
    a. Construct a linear function rule to model the amount of snow.
    b. Construct an exponential function rule to model the amount of snow.
    c. Which model best describes the amount of snow? Provide reasoning for your choice

Assessment Connections

Assessment Item Type(s): Type I, Type III

Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.6, LEAP.III.A1.3, LEAP.III.A1.4

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Multiple Representations of Functions A1: A-REI.C.5 A1: F-LE.A.2 A1: F-IF.C.9	Writes **and analyzes** systems of linear equations in multi-step contextual problems.	Writes systems of linear equations in multi-step contextual problems.	Writes systems of linear equations in **multi-step** contextual problems.	Writes systems of linear equations in simple contextual problems.
	Represents linear and exponential (with domain in the integers) functions symbolically, graphically, **with a verbal description, as a sequence**, and with input-output pairs to solve mathematical problems.	**Represents** linear and exponential (with domain in the integers) functions symbolically, graphically, and with input-output pairs to solve mathematical problems.	Given a symbolic representation, graph, verbal description, sequence, or input-output pairs for linear **and exponential** functions **(with domains in the integers)**, solves mathematical problems.	Given a symbolic representation, graph, verbal description, sequence, or input-output pairs for linear functions, solves mathematical problems.
	Compares the properties of two functions represented in different ways, limited to linear, quadratic, exponential (with domains in the integers), **absolute value, and piecewise**.	Compares the properties of two functions represented in different ways, limited to linear, quadratic, and **exponential (with domains in the integers)**.	Compares the properties of two functions represented in different ways, limited to linear **and quadratic**.	Compares the properties of two linear functions represented in different ways.

A1: F-LE.A.3 Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically, with and without technology.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** A1: F-LE.A.1 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Examples:

- Compare the values of the functions: $f(x) = 2x$, $f(x) = 2^x$, and $f(x) = x^2$ for $x \geq 0$.
- Kevin and Joseph each decide to invest $100. Kevin decides to invest in an account that will earn $5 every month. Joseph decided to invest in an account that will earn 3% interest every month.
    a. Whose account will have more money in it after two years?
    b. After how many months will the accounts have the same amount of money in them?
    c. Describe what happens as the money is left in the accounts for longer periods of time.

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.4

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
	Evaluates with, uses, and **interprets** with function notation within a context.	Evaluates with and uses function notation **within a context.**	Evaluates with and uses function notation.	Evaluates with and uses function notation.
	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems.**	For linear and quadratic functions that model relationships, determines key features **and graphs the function.**	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
	Determines the domain and relates it to the quantitative relationship it describes for linear, quadratic, exponential (limited to domains in the integers),	Determines the domain a**nd relates it to the quantitative relationship it describes for** linear, quadratic, and **exponential (limited to**	**Determines the domain of linear and quadratic functions.**	

	piece-wise, and absolute value functions.	**domains in the integers)** functions.		

B. Interpret expressions for functions in terms of the situation they model. In this cluster, the terms students should learn to use with increasing precision are parameters in terms of context.

A1: F-LE.B.5 Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** A1: F-BF.B.3, A1: F-LE.A.2 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Use real-world situations to help students understand how the parameters of linear and exponential functions depend on the context.

- The total costs for a plumber who charges $50 for a house call and $85 per hour can be expressed as the function $y = 85x + 50$. If the rate were raised to $90 per hour, how would the function change?

- Lauren keeps records of the distances she travels in a taxi and what it costs:

Distance, d (miles)	Fare, f (dollars)
3	8.25
5	12.75
11	26.25

  a. If you graph the ordered pairs (d, f) from the table, they lie on a line. How can this be determined without graphing them?
  b. Show that the equation for Part a is $f = 2.25d + 1.5$.
  c. What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides?

- The equation $y = 8,000(1.04)^x$ models the rising population of a city with 8,000 residents when the annual growth rate is 4%.
  a. What would be the effect on the equation if the city's population were 12,000 instead of 8,000?
  b. What would happen to the population over 25 years if the growth rate were 6% instead of 4%?

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.2, LEAP.I.A1.4, LEAP.I.A1.5, LEAP.I.A1.6

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
	Evaluates with, uses, and **interprets** with function notation within a context.	Evaluates with and uses function notation **within a context**.	Evaluates with and uses function notation.	Evaluates with and uses function notation.
	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems**.	For linear and quadratic functions that model relationships, determines key features **and graphs the function**.	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
	Determines the domain and relates it to the quantitative relationship it describes for linear, quadratic, exponential (limited to domains in the integers),	Determines the domain **and relates it to the quantitative relationship it describes for** linear, quadratic, **and exponential (limited to**	**Determines the domain of linear and quadratic functions.**	

	piece-wise, and absolute value functions.	**domains in the integers)** functions.		

Statistics and Probability: Interpreting Categorical and Quantitative Data (A1: S-ID) ★

A. Summarize, represent, and interpret data on a single count or measurement variable. In this cluster, the terms students should learn to use with increasing precision are data distribution, center, spread, shape, and outliers.

A1: S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Previous Grade(s) Standard:** 6.DA.A.2, 6.DA.B.5 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** A1: S-ID.A.3	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Given one or more sets of data or two graphs, students:

- Identify the similarities and differences in shape, center, and spread.
- Compare data sets and summarize the similarities and differences between the shape, and measures of center and spreads of the data sets.
- Use the correct measure of center and spread to describe a distribution that is symmetric or skewed.
- Identify outliers and their effects on data sets.

The mean and standard deviation are most commonly used to describe sets of data. However, if the distribution is extremely skewed and/or has outliers, it is best to use the median and the interquartile range to describe the distribution since these measures are not sensitive to outliers.

Haircut Costs[21]

Seventy-five female college students and 24 male college students reported the cost (in dollars) of his or her most recent haircut. The resulting data are summarized in the following table.

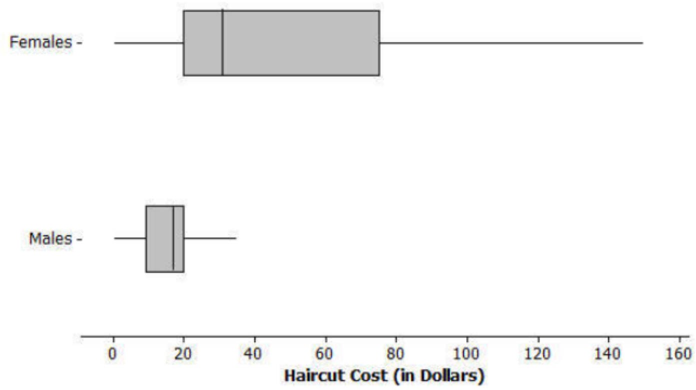
	Females	**Males**
Number of Observations	75	24
Minimum	0	0
Maximum	150	35
1st Quartile	20	9.25
Median	31	17
3rd Quartile	75	20
Mean	52.53	20.13

a. Using the minimum, maximum, quartiles,and median, sketch two side-by-side box plots to compare the haircut costs between males and females in this student's school.
b. How would you describe the difference in haircut costs between males and females? Be sure you discuss differences/similarities in shape, center, and spread.
c. Why is the mean greater than the median both for males and for females? Explain your reasoning.
d. Is the median or mean a more appropriate choice for describing the "centers" of these two distributions?

Solution

a. Students can sketch out a basic box plot with whiskers extending to the min and max, a box extending from the first quartile to the third quartile, and a line at the median, as shown below. In order to compare the haircut costs of males and females, the two boxplots should be plotted side by side on the same scale.

[21] https://www.illustrativemathematics.org/content-standards/HSS/ID/A/2/tasks/942



b. Both boxplots show distributions that are skewed to the right. It makes sense that most haircuts will not cost too much, but a few students will spend a large amount. Since the cost will always be a positive number, the minimum cannot be less than 0, and there is a long right tail. The centers and spreads are quite different. The median cost for females is about twice that of males, and there is much more variability in the haircut costs for women. The interquartile range (IQR) for women is $55, while for men it is $10.75.
c. We should not be surprised that the mean is larger than the median because the distribution appears to be skewed to the right. The mean averages all the values in the data, so is “pulled” toward the high ones. The median is the 50th percentile and is resistant to the extreme values.
d. Since the median gives a better description of the center, or a “typical” haircut cost, it is more appropriate. Note that the mean for males is about equal to the 3rd quartile, indicating that 75% of the males paid less than the mean haircut cost for males. For women, the median is $31, indicating that half of women spent $31 or less, but the mean haircut cost for women is $52.53. The mean doesn’t give us a good idea of what we could expect for a typical student's haircut cost. It is best to only use the mean when the data distribution is reasonably symmetric.

Speed Trap[22]

A statistically-minded state trooper wondered if the speed distributions are similar for cars traveling northbound and for cars traveling southbound on an isolated stretch of interstate highway. He uses a radar gun to measure the speed of all northbound cars and all southbound cars passing a particular location during a fifteen-minute period. Here are his results:

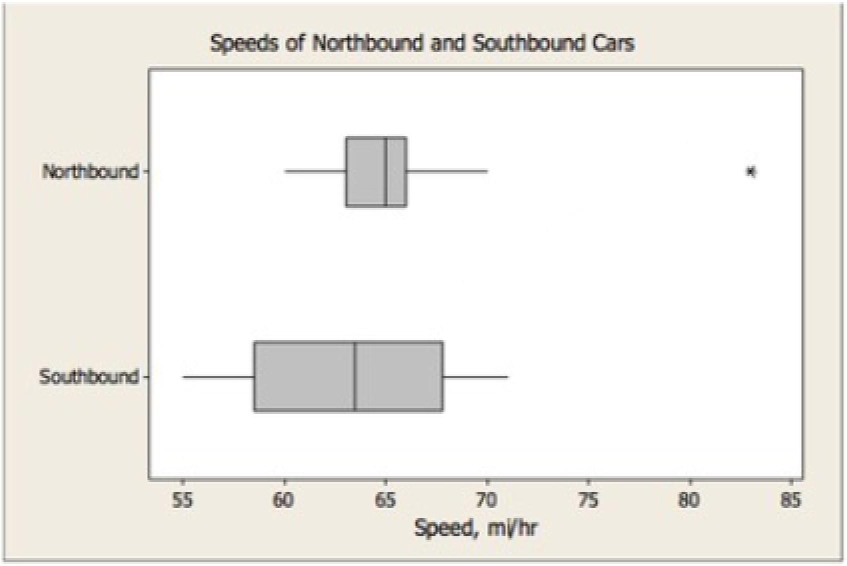
Northbound Cars				
60	62	62	63	63
63	64	64	64	65

[22] https://www.illustrativemathematics.org/content-standards/HSS/ID/A/2/tasks/1027

65	65	65	66	66
67	68	70	83	
Southbound Cars				
55	56	57	57	58
60	61	61	62	63
64	65	65	67	67
68	68	68	68	71

Draw box plots of these two data sets, and then use the plots and appropriate numerical summaries of the data to write a few sentences comparing the speeds of northbound cars and southbound cars at this location during the fifteen-minute time period.

Solution



Using the median to describe typical speed, we would say that typical speed is about the same (median of 65 mph for northbound and 63.5 mph for southbound) for northbound cars and southbound cars. One noticeable difference between the two speed distributions is that the southbound speeds are more variable than the northbound speeds. This means that the northbound speeds tended to be more consistent than the southbound speeds, which tended to differ more from one car to another. Other than the outlier in the northbound speeds, both speed distributions appear to be approximately symmetric.

Delia wanted to find the best type of fertilizer for her tomato plants. She purchased three types of fertilizer and used each on a set of seedlings. After 10 days, she measured the heights (cm) of each set of seedlings. The data she collected is shown on the next page.

a. Construct box plots to analyze the data.
b. Write a brief description comparing the three types of fertilizer.
c. Which fertilizer do you recommend that Delia use? Explain your answer.

Fertilizer A		
7.1	6.3	1.0
5.0	4.5	5.2
3.2	4.6	2.4
5.5	3.8	1.5
6.2	6.9	2.6

Fertilizer B		
11.0	9.2	5.6
8.4	7.2	12.1
10.5	14.0	15.3
6.3	8.7	11.3
17.0	13.5	14.2

Fertilizer C		
10.5	11.8	15.5
14.7	11.0	10.8
13.9	12.7	9.9
10.3	10.1	15.8
9.5	13.2	9.7

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.3

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
	Evaluates with, uses, and **interprets** with function notation within a context.	Evaluates with and uses function notation **within a context**.	Evaluates with and uses function notation.	Evaluates with and uses function notation.
	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems**.	For linear and quadratic functions that model relationships, determines key features **and graphs the function**.	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
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	piece-wise, and absolute value functions.	**domains in the integers)** functions.		

A1: S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Previous Grade(s) Standard:** 6.DA.B.5 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** A1: S-ID.A.2	Conceptual Understanding

Examples and Explanations

Students understand and use the context of the data to explain why its distribution takes on a particular shape (e.g., Is the data skewed? Are there outliers?)

Examples:

- Why does the shape of the distribution of incomes for professional athletes tend to be skewed to the right?
- Why does the shape of the distribution of test scores on a really easy test tend to be skewed to the left?
- Why does the shape of the distribution of heights of the students at your school tend to be symmetrical?

Students understand that the higher the value of a measure of variability, the more spread out the data set is. Measures of variability are range (100% of data), standard deviation (68-95-99.7% of data), and interquartile range (50% of data).

Example:

- On last week's math test, Mrs. Smith's class had an average of 83 points with a standard deviation of 8 points. Mr. Tucker's class had an average of 78 points with a standard deviation of 4 points. Which class was more consistent with their test scores? How do you know?

Students explain the effect of any outliers on the shape, center, and spread of the data sets.

Example:

- The heights of a group of women basketball players are: 5 ft. 9 in., 5 ft. 4 in., 5 ft. 7 in., 5 ft. 6 in., 5 ft. 5 in., 5 ft. 3 in., and 5 ft. 7 in. A new player joins the basketball team. Her height is 6 ft. 10 in.
    a. What is the mean height of the team before the new player transfers in? What is the median height?
    b. What is the mean height after the new player transfers? What is the median height?
    c. What effect does her height have on the team's height distribution and stats (center and spread)?

d. How many players are taller than the new mean team height?
e. Which measure of center most accurately describes the team's average height? Explain.

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.3

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
	Evaluates with, uses, and **interprets** with function notation within a context.	Evaluates with and uses function notation **within a context**.	Evaluates with and uses function notation.	Evaluates with and uses function notation.
	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems**.	For linear and quadratic functions that model relationships, determines key features **and graphs the function**.	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
	Determines the domain and relates it to the quantitative relationship it describes for linear, quadratic, exponential (limited to domains in the integers),	Determines the domain **and relates it to the quantitative relationship it describes for** linear, quadratic, and **exponential (limited to**	**Determines the domain of linear and quadratic functions.**	

	piece-wise, and absolute value functions.	**domains in the integers)** functions.		

B. Summarize, represent, and interpret data on two categorical and quantitative variables. In this cluster, the terms students should learn to use with increasing precision are categorical data, two-way frequency tables, relative frequencies, trends in data, scatter plot, fit a function to data, and residuals.

A1: S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** 8.DA.A.4 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

When students are proficient with analyzing two-way frequency tables, build upon their understanding to develop the vocabulary.

- The table entries are the joint frequencies of how many subjects displayed the respective cross-classified values.
- Row totals and column totals constitute the marginal frequencies.
- Dividing joint or marginal frequencies by the total number of subjects defines relative frequencies, respectively.

Favorite Subject by Grade					
Grade	**English**	**History**	**Math/Science**	**Other**	**Totals**
7th Grade	38	36	28	14	116
8th Grade	47	45	72	18	182
Totals	85	81	100	32	298

Conditional relative frequencies are determined by focusing on a specific row or column of the table and are particularly useful in determining any associations between the two variables.

Students are flexible in identifying and interpreting the information from a two-way frequency table. They complete calculations to determine frequencies and use those frequencies to describe and compare.

Example:

- At the NC Zoo, 23 interns were asked about their preferred location for work. There were three choices: African Region, Aviary, or North American Region. There were 13 who preferred the African Region, 5 of whom were male. There were 6 who preferred the Aviary, 2 males and 4 females. A total of 4 preferred the North American Region, and only 1 of them was female.
    a. Using the information on the NC Zoo Internship to create a two-way frequency table.
    b. Using the two-way frequency table from the NC Zoo Internship, calculate the percentage of males who prefer the African Region
    c. How does the percentage of males who prefer the African Region compare to the percentage of females who prefer the African Region?
    d. 15% of the paid employees are male and work in the Aviary. How does that compare to the interns who are male and prefer to work in the Aviary? Explain how you made your comparison.

[Assessment Connections]

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.3

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Summarizing, Representing, and Interpreting Data A1: S-ID.B.5	Determines appropriate representations of categorical and quantitative data, summarizing and interpreting the data and characteristics of the representations.	**Determines appropriate** representations of categorical and quantitative data, summarizing the data and characteristics of the representations.	Given representations of categorical and quantitative data, **summarizes the data** and characteristics of the representations.	Given representations of categorical and quantitative data, describes the characteristics of the representations.
	Describes and interprets possible associations and trends in the data.			

A1: S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and quadratic models.
b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Previous Grade(s) Standard:** 8.DA.A.1, 8.DA.A.2, 8.DA.A.3 **Algebra I Standard Taught in Advance:** none **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency, Application

Examples and Explanations

A *residual* is the difference between the actual y-value and the predicted y-value ($y - \hat{y}$), which is a measure of the error in prediction. (Note: $\hat{y}$ is the symbol for the predicted y-value for a given *x*-value.) A residual is represented on the graph of the data by the vertical distance between a data point and the graph of the function.

A *residual plot* is a graph that shows the residuals on the vertical axis and the independent variable on the horizontal axis. If the points in a residual plot are randomly dispersed around the horizontal axis, a linear regression model is appropriate for the data; otherwise, a non-linear model is more appropriate.

Example:

The table displays the annual tuition rates of a state college in the U.S. between 1990 and 2000, inclusively. The linear function $R(t) = 326x + 6440$ has been suggested as a good fit for the data. Use a residual plot to determine the goodness of fit of the function for the data provided in the table.

Year (0=1990)	Tuition Rate
0	6546
1	6996
2	6996
3	7350
4	7500
5	7978
6	8377
7	8710
8	9110
9	9411
10	9800

Examples:

- The data in the chart gives the number of miles driven and advertised price for 11 used models of a particular car from 2002 to 2006.

Miles driven (In thousands)	Price (In dollars)
22	17,998
29	16,450
35	14,998
39	13,998
45	14,599
49	14,988
55	13,599
56	14,599
69	11,998
70	14,450
86	10,998

  a. Use your calculator to make a scatter plot of the data.
  b. Use your calculator to find the correlation coefficient for the data above. Describe what the correlation means in regards to the data.
  c. Use your calculator to find an appropriate linear function to model the relationship between miles driven and price for these cars.
  d. How do you know that this is the best-fit model?

Students fit a quadratic function for a scatter plot that suggests a quadratic association.

- A study was done to compare the speed x (in miles per hour) with the mileage y (in miles per gallon) of an automobile. The results are shown in the table.
  (Source: Federal Highway Administration)

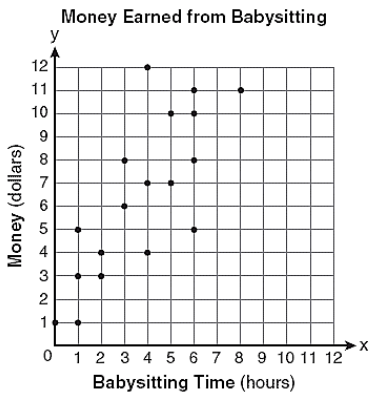
Speed, x (mph)	Mileage, y (mpg)
15	22.3
20	25.5
25	27.5
30	29.0
35	28.8
40	30.0
45	29.9
50	30.2
55	30.4
60	28.8
65	27.4
70	25.3
75	23.3

  a. Use your calculator to make a scatter plot of the data.
  b. Use the regression feature to find a model that best fits the data.
  c. Approximate the speed at which the mileage is the greatest.

Students fit a linear function for a scatter plot that suggests a linear association.

- Which of the following equations best models the (babysitting time, money earned) data?

$y = x$ $\qquad y = \frac{6}{5}x + 2$ $\qquad y = \frac{3}{2}x + 4$ $\qquad y = \frac{1}{4}x + 4$



Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.3

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
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	piece-wise, and absolute value functions.	**domains in the integers)** functions.		

C. Interpret linear models. In this cluster, the terms students should learn to use with increasing precision are rate of change, intercept, correlation coefficient of a linear fit, and correlation versus causation.

A1: S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** 8.DA.A.3 **Algebra I Standard Taught in Advance:** A1: S-ID.B.6 **Algebra I Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Students may use graphing calculators or software to create representations of data sets, create linear models, and to assist students in interpreting the data.

Example:

- Data were collected of the weight of a male white laboratory rat for the first 25 weeks after its birth. A scatterplot of the rat's weight (in grams) and the time since birth (in weeks) shows a fairly strong, positive linear relationship. The linear regression equation $W = 100 + 40t$ (where W = weight in grams and t = number of weeks since birth) models the data fairly well.
    a. What is the slope of the linear regression equation? Explain what it means in context.
    b. What is the y-intercept of the linear regression equation? Explain what it means in context.

Assessment Connections
Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.3

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
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	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems**.	For linear and quadratic functions that model relationships, determines key features **and graphs the function**.	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
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	piece-wise, and absolute value functions.	**domains in the integers)** functions.		

A1: S-ID.C.8 Compute and interpret the correlation coefficient of a linear fit.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** A1: S-ID.B.6 **Algebra I Standard Taught Concurrently:** A1: S-ID.C.9	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students may use graphing calculators or software to create representations of data sets and create linear models and interpret them. The correlation coefficient, r, is a measure of the strength and direction of a linear relationship between two quantities in a set of data. The magnitude (absolute value) of r indicates how closely the data points fit a linear pattern. If $r = 1$, the points all fall on a line. The closer $|r|$ is to 1, the stronger the correlation. The closer $|r|$ is to zero, the weaker the correlation. The sign of r indicates the direction of the relationship, positive or negative.

Example:

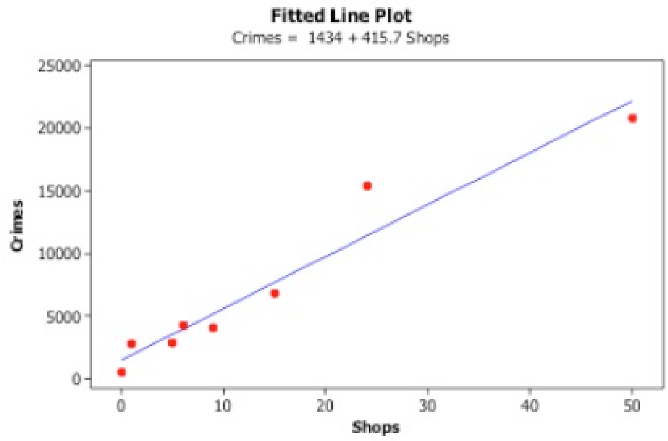
Coffee and Crime[23]

Many counties in the United States are governed by a county council. At public county council meetings, county residents are usually allowed to bring up issues of concern. At a recent public County Council meeting, one resident expressed concern that 3 new coffee shops from a popular coffee shop chain were planning to open in the county, and the resident believed that this would create an increase in property crimes in the county. (Property crimes include burglary, larceny-theft, motor vehicle theft, and arson -- From http://www.fbi.gov/about-us/cjis/ucr/crime-in-the-u.s/2010/crime-in-the-u.s.-2010/property-crime accessed on December 5, 2012.)

[23] https://www.illustrativemathematics.org/content-standards/HSS/ID/C/8/tasks/1307

To support this claim, the resident presented the following data and scatterplot (with the least-squares line shown) for 8 counties in the state:

Country	Shops	Crimes
A	9	4000
B	1	2700
C	0	500
D	6	4200
E	15	6800
F	50	20800
G	5	2800
H	24	15400



The scatterplot shows a positive linear relationship between "Shops" (the number of coffee shops of this coffee shop chain in the county) and "Crimes" (the number of annual property crimes for the county). In other words, counties with more of these coffee shops tend to have more property crimes annually.

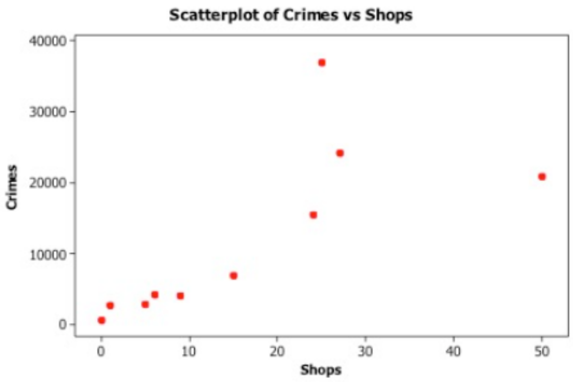
a. Does the relationship between Shops and Crimes appear to be linear? Would you consider the relationship between Shops and Crimes to be strong, moderate, or weak?
b. Compute the correlation coefficient. Does the value of the correlation coefficient support your choice in part (a)? Explain.
c. The equation of the least-squares line for these data is: Predicted Crimes = 1434 + 415.7(Shops). Based on this line, what is the estimated number of additional annual property crimes for a given county that has 3 more coffee shops than another county?

d. Do these data support the claim that building 3 additional coffee shops will necessarily *cause* an increase in property crimes? What other variables might explain the positive relationship between the number of coffee shops for this coffee shop chain and the number of annual property crimes for these counties?
e. If the following two counties were added to the data set, would you still consider using a line to model the relationship? If not, what other types (forms) of model would you consider?

County	Shops	Crimes
I	25	36,900
J	27	24,100

Solution

a. The relationship does appear to be linear. The relationship would be considered a *strong* and positive given how closely the points adhere to a line with positive slope.
b. $r = 0.968$. Since the pattern shown is one of very strong, positive, linear association, a correlation coefficient value near +1 is plausible.
c. $415.7 \cdot 3 = 1247.1$. According to the model, the predicted increase in the number of annual property crimes for a county with 3 additional coffee shops would be 1247.
d. Association (no matter how strong) does not necessarily imply causation. It is unlikely that building a new coffee shop would cause crime rates to increase, for such logic would imply that coffee drinkers engage in more criminal behavior than non-coffee drinkers, the coffee shop attracts criminals to the county, etc. From a perspective of context, students should consider other variables that may be responsible for the association (e.g., counties with higher populations or higher population density may have both more coffee shops and more property crimes). As stated in the "commentary" above, depending upon student knowledge of experiments and observational studies, a discussion can occur reinforcing the risk associated with stating/implying causation based on data from an observational study.
e. With the addition of the two observations, the scatterplot now displays a curved relationship with one outlier at (50, 20800). The scatterplot still shows a positive relationship between "Shops" (the number of coffee shops of this coffee shop chain in the county) and "Crimes" (the number of annual property crimes for the county in the previous year) – but the relationship no longer appears to be linear (or does not appear as linear as before). When only a few observations are used to assess a trend, sometimes just adding one or two points can change the appearance significantly. The new plot is shown below. This relationship might be modeled using a quadratic or an exponential curve.


Scatterplot of Crimes vs Shops
40000
30000
20000
10000
0
Crimes
0
10
20
30
40
50
Shops

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.3

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
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	piece-wise, and absolute value functions.	**domains in the integers)** functions.		

A1: S-ID.C.9 Distinguish between correlation and causation.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Previous Grade(s) Standard:** none **Algebra I Standard Taught in Advance:** A1: S-ID.B.6 **Algebra I Standard Taught Concurrently:** A1: S-ID.C.8	Conceptual Understanding

Examples and Explanations

Some data lead observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment.

Examples:

- A study found a strong, positive correlation between the number of cars owned and the length of one's life. Larry concludes that owning more cars means you will live longer. Does this seem reasonable? Explain your answer.

High blood pressure[24]

In a study of college freshmen, researchers found that students who watched TV for an hour or more on weeknights were significantly more likely to have high blood pressure, compared to those students who watched less than an hour of TV on weeknights. Does this mean that watching more TV raises one's blood pressure? Explain your reasoning.

Solution

This does not mean that watching more TV raises blood pressure. Whether or not we can conclude that "watching more TV raises one's blood pressure" depends on the design of the study. If the researchers had conducted a randomized experiment where some of the participants were randomly assigned to watch less than an hour of TV and others were assigned to watch more than hour of TV, and if we found a statistically significant difference in the average blood pressure of the two groups, we could conclude *causation*. If the study is simply an observational study (which is the case here), we can only conclude that there is an association between time spent watching TV and blood pressure.

[24] https://www.illustrativemathematics.org/content-standards/HSS/ID/C/9/tasks/1100

Math test grades[25]

Ari has collected data on how much her classmates study each week and how well they did on their recent math test. Here is part of the data, showing the percentage of each group (divided according to hours spent studying) who got different grades on the test:

Weekly Hours *w* Studying	C or below on test	B on test	A on test
w < 3	50%	20%	30%
3 ≤w < 6	60%	25%	15%
w ≥ 6	60%	20%	20%

a. What can you conclude about the percentage of Ari's classmates who got B's? What about the percent who got A's?
b. Looking at the table, Ari decides that in order to do well in her math class, she should not study more than 3 hours a week: she has a 30 percent chance of getting an A as long as she does not study more than 3 hours per week. Is Ari's reasoning valid?

Solution

a. In order to accurately calculate the percent of Ari's classmates who got A's or B's we would need to know how many classmates she has and how many of these received an A (or B) on the exam. This information is not available. We can still, however, make some important deductions. Note first that each of Ari's classmates is in one of the three groups listed in the table because the categories are exhaustive. That is, every student must have studied either less than 3 hours per week, between 3 and 6 hours per week, or more than 6 hours per week. Not only are the groups exhaustive but they are also mutually exclusive. It is impossible, for example, to study less than 3 hours per week while also studying at least 6 hours per week.
Suppose we let x denote the number of students who studied less than 3 hours per week, y the number who studied between 3 and 6 hours per week (including exactly 3 hours per week), and z the number who studied at least 6 hours per week. Then, as noted in the previous paragraph, the total number of Ari's classmates is $x + y + z$.
Looking at the given information, the number of Ari's classmates who got a B is $\frac{1}{5}x + \frac{1}{4}y + \frac{1}{5}z$.
Since $\frac{1}{5} < \frac{1}{4}$ we have $\frac{1}{5}(x + y + z) < \frac{1}{5}x + \frac{1}{5}z < \frac{1}{4}(x + y + z)$.

Note that in order to have inequalities above, it is important to know that $x,\ y,\ z$ are non-zero which is true because the percents in the table are all non-zero. Putting all of the information together, the fraction of Ari's classmates who got B's on the exam is $\frac{\frac{1}{5}x\frac{1}{4}y+\frac{1}{5}z}{x+y+z}$ and, according to the

[25] https://www.illustrativemathematics.org/content-standards/HSS/ID/C/9/tasks/1585

inequalities, this is greater than $\frac{\frac{1}{5}x\frac{1}{4}y+\frac{1}{5}z}{x+y+z}=\frac{1}{5}$ and less than $\frac{\frac{1}{4}x+\frac{1}{4}y+\frac{1}{4}z}{x+y+a}=\frac{1}{4}$. Without further information, we can not say more. So the percentage of Ari's classmates who got B's is more than 20 percent but less than 25 percent. For the percentage of Ari's classmates who got A's, similar reasoning applies, telling us that it is more than 15 but less than 30.

b. For Ari's classmates, the largest percentage of A's came from those who studied less than 3 hours a week. But this does not mean that Ari's best chance of getting an A is to study less than 3 hours a week. In the first place, there is no reason to believe that these students received A's on the test *because* they studied fewer than three hours a week. On the contrary, a more plausible explanation would be that they were very proficient with the material and therefore did not need to study much in order to do well on the exam. In other words, there is a ''hidden'' cause of both the study habits and the grades.

   We can rephrase this argument in more concrete terms. In order to get an A on the exam, Ari's best chances come from learning the material well. So her best strategy is to study as much as she needs to in order to feel confident with the material. This might be less than three hours a week or it might be much more. Even if it turns out that Ari only needs to study two hours a week to get an A, the source of the A is her mastery of the material, not the fact that she studied for less than three hours a week.

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.A1.3

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Interpreting Functions A1: F-IF.A.1 A1: F-IF.A.2 A1: F-IF.B.4 A1: F-IF.B.5 LEAP.I.A1.1 LEAP.I.A1.2 LEAP.I.A1.3	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.	Determines if a given relation is a function.
	Evaluates with, uses, and **interprets** with function notation within a context.	Evaluates with and uses function notation **within a context**.	Evaluates with and uses function notation.	Evaluates with and uses function notation.
	Given a context, writes **and analyzes** a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.	Given a context, writes a linear function.
	For linear and quadratic functions that model **contextual** relationships, determines **and interprets** key features, graphs the function, **and solves problems**.	For linear and quadratic functions that model relationships, determines key features **and graphs the function**.	For linear **and quadratic** functions that model relationships, determines key features.	For linear functions that model relationships, determines key features.
	Determines the domain and relates it to the quantitative relationship it describes for linear, quadratic, exponential (limited to domains in the integers),	Determines the domain **and relates it to the quantitative relationship it describes for** linear, quadratic, and **exponential (limited to**	**Determines the domain of linear and quadratic functions.**	

	piece-wise, and absolute value functions.	**domains in the integers)** functions.		

Cross Content Connections

Mathematics supports understanding and skills across all disciplines. The ability to apply mathematics learning across disciplines allows students to transform abstract concepts into practical tools, deepen engagement, think critically and apply mathematics to the real world.

English Language Arts

The connection between math and English language arts (ELA) begins in disciplinary literacy, which requires literacy skills in technical subjects, including the ability to decode complex word problems as narrative structures. Students must master the language of math by using ELA skills like reading comprehension and argumentative writing to explain their reasoning, prove theorems, and articulate the logic behind their numerical solutions.

Science

K-12 math and science are deeply integrated, with mathematics serving as the essential language that allows students to quantify, model, and analyze real-world phenomena they observe in science. As students progress, they use mathematics and computational thinking,ranging from simple data graphing to complex algebraic modeling,to turn raw scientific observations into evidence-based conclusions and predictable patterns. Explicit connections between the LSSM and Louisiana Student Standards for Science (LSSS) can be found on the Connections to K-12 ELA and Math document within the LSSS files.

Computer Science

The integration of mathematics and computer science is codified through state standards that treat computational thinking as a practical application of algebraic logic and problem-solving. Students use mathematical foundations, such as functions and data analysis, to build algorithms and write code that solves real-world challenges in Louisiana's growing technology and energy sectors. The Louisiana CS Frameworks make explicit connections and embed technology tools such as artificial intelligence to support students as they engage in math and computer science learning. Students can strategically use artificial intelligence tools to explore math ideas, test conjectures, analyze patterns and trends, and refine their thinking. The appropriate and skillful use of artificial intelligence in math requires students to use their mathematical reasoning to validate and critique artificial intelligence outputs in addition to naming how the use of artificial intelligence supports and extends their thinking, understanding, and reasoning about concepts, ensuring the student maintains ownership of the mathematics, and leverages artificial intelligence as a resource.

Social Studies

The intersection of math and social studies is most evident in the analysis of historical data and economic trends, allowing students to quantify the impact of events such as the Great Flood of 1927 or the evolution of Louisiana's energy industry. By applying statistical tools to demographic shifts and geographic mapping, students move beyond historical content standards to demonstrate understanding of the skills and practices standards for social studies. Identifying and interpreting mathematical patterns strengthens students' ability to recognize trends that have shaped the geographical, economic, political, and cultural landscape of our state, nation, and world.

■6.PF.A.3 Use and apply ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what unit rate were lawns being mowed?*
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units within and between the U.S. customary and metric systems; manipulate and transform units appropriately when multiplying or dividing quantities.

Return to ◘A1: N-Q.A.1

■6.AR.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.*
b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6\,s^2$ *to find the volume and surface area of a cube with sides of length* $s = 1/2$.

Return to ■ A1: A-SSE.A.1, ■ A1: F-IF.A.2

■6.AR.A.3 Apply the properties of operations to generate equivalent expressions, identify when two expressions are equivalent, and explain why two expressions are equivalent. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* *Return to* ■A1: A-SSE.A.2, ◘A1: A-SSE.B.3, ■A1: A-APR.A.1

■6.AR.A.4 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
Return to ■A1: A-APR.A.1

○6.DA.A.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
Return to ○A1: S-ID.A.2

○6.DA.B.5 Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Return to ○ A1: S-ID.A.2, ○ A1: S-ID.A.3

Grade 7 Standards

■7.AR.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with positive and negative rational coefficients to include multiple grouping symbols (e.g., parentheses, brackets, and braces). *Return to* ■A1: A-SSE.A.2, □A1: A-SSE.B.3, ■A1: A-APR.A.1, □A1: A-APR.B.3, ■A1: A-REI.B.4, □A1: F-IF.C.8

■7.AR.A.2 Understand that rewriting an expression in different forms can help reveal relationships between quantities in the context of a problem. *For example, a + 0.05a is equivalent to 1.05a, which means to "increase by 5%" is the same as "multiply by 1.05."* *Return to* ■A1: A-SSE.A.1

■7.AR.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
Return to ■A1: A-CED.A.1, ■A1: A-REI.A.1, ■A1: A-REI.B.3

Grade 8 Standards

■8.NOF.A.1 Understand the real number system.

a. Distinguish between rational and irrational numbers (e.g., know that $\sqrt{2}$ is irrational.)
b. Understand informally that every number has a decimal expansion.
c. For rational numbers, show that the decimal expansion repeats eventually.
d. Convert a decimal expansion that repeats eventually into a rational number by analyzing repeating patterns.

Return to ○A1: N-RN.B.3

■8.AR.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Return to ■A1: A-SSE.B.3, ■ A1: A-APR.A.1

■8.AR.A.2 Represent solutions to equations, in the form $x^2 = p$ and $x^3 = p$, using the square root and cube root symbols, and determine if the solution is rational or irrational. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. *Return to* ■A1: A-REI.B.4

■8.AR.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. *Return to* ■A1: N-Q.A.3

■8.AR.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

Return to ■A1: A-REI.D.10, ■A1: F-IF-C.7

■8.AR.C.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Return to ■A1: A-CED.A.1, ■A1: A-REI.A.1, ■A1: A-REI.B.3

■8.AR.C.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Return to ■A1: A-CED.A.2, ○A1: A-REI.C.5, ○A1: A-REI.C.5, ■A1: A-REI.D.11

■8.PF.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in this grade level.) *Return to* ■A1: F-IF.A.1

■8.PF.A.2 Compare properties of two functions where each function is represented in a different way (algebraically, graphically, numerically, or through verbal descriptions). For example, given a linear function represented by a table of values (numerically) and a linear function represented by an algebraic expression (algebraically), determine which function has the greater rate of change. *Return to* ■A1: F-IF.A.1

■8.PF.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; categorize functions as linear or nonlinear when given equations, graphs, or tables. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.* *Return to* ■A1: A-CED.A.2, ■A1: F-IF.A.1, □A1: F-IF.C.7, □A1: F-LE.A.1

□8.PF.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values. *Return to* ■A1: A-CED.A.2, ■A1: F-IF.B.6, □A1: F-BF.A.1, □A1: F-LE.A.1, □A1: F-LE.A.2

□8.PF.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. *Return to* ■A1: F-IF.B.4

□8.DA.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. *Return to* □A1: S-ID.B.6

□8.DA.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. *Return to* □A1: S-ID.B.6

8.DA.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. *Return to* A1: S-ID.B.6, A1: S-ID.C.7

8.DA.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
b. Use relative frequencies calculated for rows or columns to describe the possible association between the two variables. For example, collect data from students in your class on whether they have a curfew on school nights and whether they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Return to A1: S-ID.B.5