Standards Resource Guide

Geometry

Overview

This guide is designed to support educators in interpreting and implementing the 2025 Louisiana Student Standards for Mathematics (LSSM). It provides descriptions of each standard to clarify its meaning and application to student learning. The document outlines the intended level of rigor and includes coherence to prerequisite and corequisite standards. Examples are provided for illustrative purposes only and do not represent a comprehensive list.

Additional information on the 2025 Louisiana Student Standards for Mathematics, including guidance on reading standards codes, a listing of standards for each grade or course, and links to additional resources, is available on the Louisiana Math Page. Questions may be directed to math@la.gov.

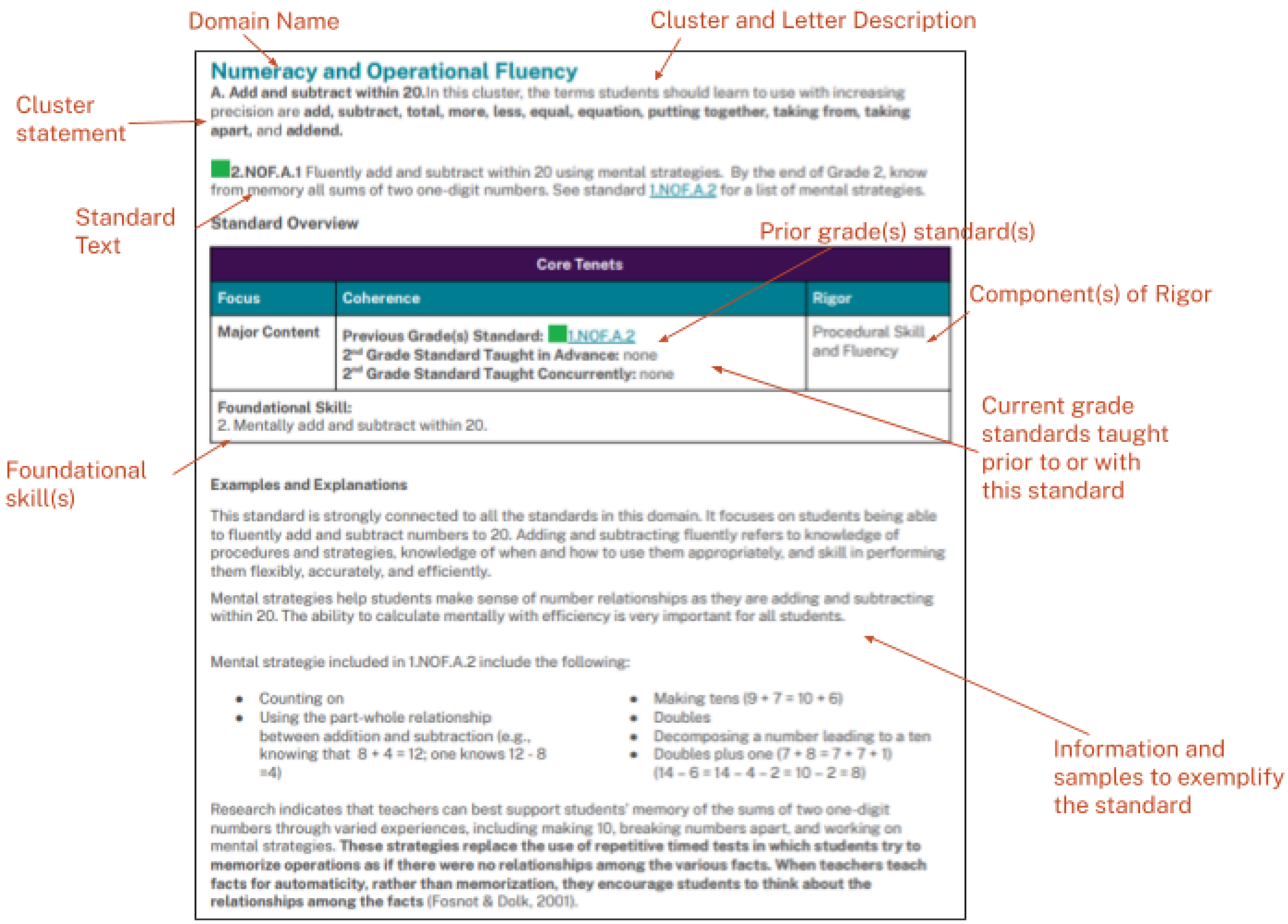
August 2026

Table of Contents

Introduction

How to Read This Guide

The diagram below provides an overview of the information found in all standards resource guides. Definitions and more complete descriptions are provided on the next page.



Text of the standard

Shading of Standard Codes: **Major Work,** **Supporting Work,** **Additional Work**

1. **Domain Name and Abbreviation:** A grouping of standards consisting of related content that is further divided into clusters. Each domain has a unique abbreviation, which is provided in parentheses beside the domain name.
2. **Cluster Letter and Description:** Each cluster within a domain begins with a letter. The description provides a general overview of the focus of the standards in the cluster.

3. **Previous Grade(s) Standards:** One or more standards that students should have mastered in previous grades to prepare them for new learning related to the current grade-level standard. For students who have unfinished learning, teachers should build in proactive, just-in-time supports to build essential prerequisite knowledge.
4. **Standards Taught in Advance:** These grade-level standards demonstrate within-grade coherence and include concepts on which the target standard is built. Students should have exposure to these standards before instruction in the target standard. As with previous-grade standards, teachers should plan proactive, just-in-time supports to build within-grade prerequisite knowledge ahead of instruction on the target standard.
5. **Standards Taught Concurrently:** These grade-level standards demonstrate within-grade coherence and include standards that should be taught at the same time as the target standard to provide coherence and connectedness in instruction.
6. **Component(s) of Rigor:** See full explanation on components of rigor below.
7. **Sample Problem:** The sample problem provides an example of how a student might meet the requirements of the standard. Multiple examples are provided for some standards. However, sample problems should not be considered an exhaustive list. Explanations, when appropriate, are also included.
8. **Text of Standard:** The complete text of the targeted Louisiana Student Standards of Mathematics is provided.

Classification of Major, Supporting, and Additional Work

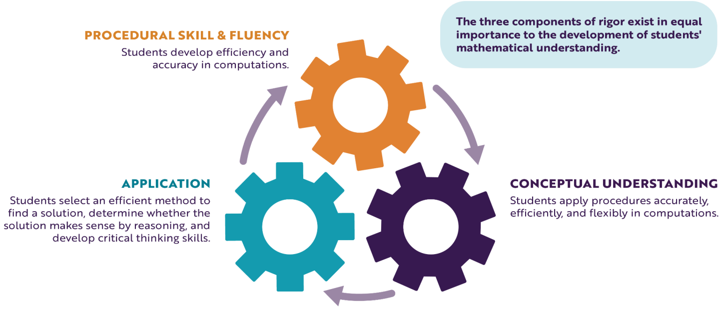
Students should spend the majority of their time on the major work of the grade, supporting work, and, where appropriate, additional work engages students in the major work of the grade, often through a change of the **context** within which students are performing mathematics. Each standard is color-coded to quickly and simply determine the focus for the grade. Furthermore, standards from previous grades that provide foundational skills for current grade standards are also color-coded to show whether those standards are classified as major, supporting, or additional in their respective grades.

Core Tenets

The 2016 Louisiana Student Standards for Math called for three instructional shifts. Since then, teachers have used these shifts to ensure deep understanding and consistent application in mathematics classrooms across the state. Now, these shifts are recognized as core tenets that serve as the foundation and guiding principles of Louisiana's mathematics standards.

Coherence in mathematics refers to a coherent body of knowledge composed of interconnected concepts, principles, and procedures. These concepts are logically linked, building upon one another, and work together to deepen understanding, support problem solving, and enable learners to apply mathematical ideas consistently and meaningfully across different contexts.

Rigor in mathematics instruction does not simply mean increasing the difficulty or complexity of practice problems. Incorporating rigor into classroom instruction and student learning requires students to engage deeply with standards and grapple meaningfully with the concepts and ideas embedded



within them. The **three** components of rigor, listed below, exist in equal importance to the development of students' mathematical understanding.

- **Conceptual understanding** refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.
- **Procedural skill and fluency** is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills without dictating the strategy or method chosen. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.
- **Application** provides valuable context for learning and the opportunity to solve problems in a relevant and meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

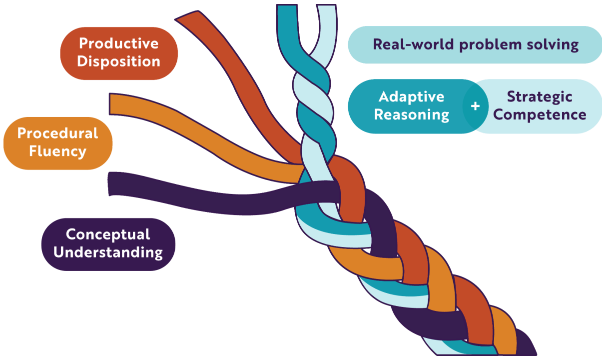
Focus calls for intentional emphasis on specific topics, skills and ideas within grade-level mathematics instruction. Not all content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Louisiana Standards for Mathematical Practice. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Students should spend the majority of their time on the major ( ) work of the grade, supporting ( ) work, and, where appropriate, additional ( ) work can engage students in the major work of the grade.

The Strands for Mathematical Proficiency

The strands of mathematical proficiency were introduced by the National Research Council (NRC) in their 2001 report, Adding It Up: helping children learn mathematics. These proficiencies provide a framework describing what it means for a learner to be proficient in mathematics.

The strands of mathematical proficiency are intertwined, gradually developed, and deepened over time. When intentionally presented in a connected and balanced way, each strand supports the others, ultimately strengthening the proficiency of learners.

1. **Conceptual Understanding** - Students develop an understanding of key mathematical ideas and relationships and explain why procedures work.



2. **Procedural Fluency** - Students use mathematical procedures with skill and flexibility, guided by a strong grasp of underlying concepts.
3. **Productive Dispositions** - Students view mathematics as coherent, valuable, and meaningful
4. **Real World and Problem Solving** - A combination of both adaptive reasoning and strategic competence, students represent and solve real-world mathematical problems logically, justify solutions, and clearly explain and defend their reasoning.

The Relationship Between Mathematical Proficiency and the Components of Rigor

While there is some overlap between the Strands of Mathematical proficiency and the Core Tenets, there is a distinct difference between the two. The strands are a framework that is focused on learner outcomes and what it means for a student to be proficient in mathematics. The Core Tenets provide a framework for how teachers should design instruction to support learning. The core tenets reflect what should be done instructionally, while the proficiency strands represent the outcome, or what students become, as a result of that instruction.

Mathematical Fluency

Fluency in mathematics is the ability to apply foundational skills and concepts flexibly, accurately, and efficiently, grounded in strong conceptual understanding to solve problems, make connections, and reason mathematically across various contexts. While fluency is an essential component of mathematics instruction, it does not represent the full depth of a mathematical standard; it must be developed alongside conceptual understanding, reasoning, and problem-solving to ensure deep and meaningful learning.

Concrete - Representational - Abstract Framework

The Concrete-Representational-Abstract (CRA) Framework describes the instructional processes that students experience to build conceptual understanding of mathematics. The first stage is where students build a concrete understanding of a concept through the use of manipulatives or other physical objects. During the second phase, learners move into visual representations of concepts. Finally, during the abstract phase, students apply their understanding of the concepts abstractly through the use of numbers and symbols. When students engage in learning that follows this process, it allows them to build a solid understanding of key mathematical concepts and ideas, ensuring comprehension that is transferable and flexibly applied across grade bands.

An example of how this framework is represented across the progression of the Louisiana Student Standards for Mathematics can be seen in the way student learning progresses across a concept, such as linear functions. In fourth grade (4.AR.A.1), students are introduced to multiplicative comparison, where much of the learning is very **concrete**. At this phase, students are using counters and other manipulatives to physically compare quantities to determine the meaning of “times as many.” When they move into the middle grades, this becomes more representational as students apply concrete understanding to recognize and represent proportional relationships. These are represented in various ways, such as ratio tables, double number lines, and graphs (6.PF.A.2, 7.PF.A.2). When students enter high school, the learning becomes **abstract** as they interpret linear functions such as y=kx and come to understand slope as rate of change.

While the example above represents a linear version of this instructional process across multiple grades, it is important to note that the concrete and representational components can occur at different times throughout the progression of learning, and may occur simultaneously across various periods. Additionally, this instructional process may also occur within lessons, units, or within the grade level.

Standards for Mathematical Practice

The Louisiana Standards for Mathematical Practice are expected to be integrated into every mathematics lesson for all students in grades K-12. Below are a few examples of how these practices may be integrated into tasks that students in high school complete.

Louisiana Standards for Mathematical Practice (MP)	
Louisiana Standard	**Explanations and Examples**
HS.MP.1 Make sense of problems and persevere in solving them.	High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2 Reason abstractly and quantitatively.	High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3 Construct viable arguments and critique the reasoning of others.	High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

HS.MP.4 Model with mathematics.	High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They can identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
HS.MP.5 Use appropriate tools strategically.	High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
HS.MP.6 Attend to precision.	High school students try to communicate precisely with others by using clear definitions in discussions with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and label axes to clarify the correspondence between quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school, they have learned to examine claims and make explicit use of definitions.
HS.MP.7 Look for and make use of structure.	By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line to solve problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, compose functions, and transform figures.
HS.MP.8 Look for and express regularity in	High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to

repeated reasoning.	the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas, or make generalizations, high school students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

What is modeling?

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder or whether a two-dimensional disk works well enough for our purposes. Other situations, such as modeling a delivery route, a production schedule, or a comparison of loan amortizations, need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

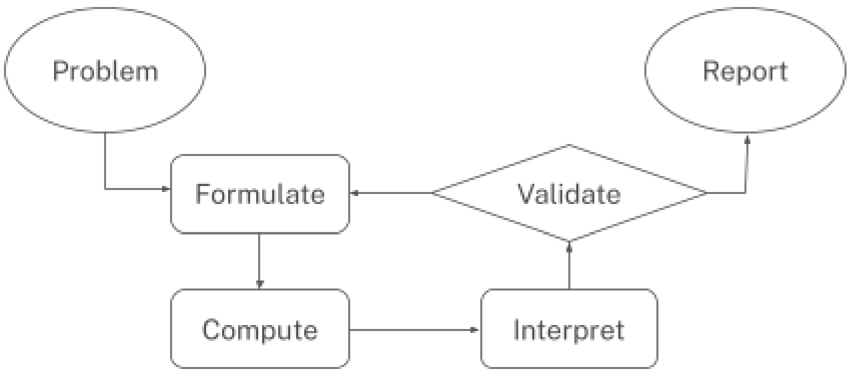
Some examples of such situations might include:

- Estimate how much water and food are needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Plan a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other.
- Design the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyze the stopping distance for a car.
- Model a savings account balance, bacterial colony growth, or investment growth.
- Engage in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyze the risk in situations such as extreme sports, pandemics, and terrorism.
- Relate population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets, and other technologies, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model, for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data based on deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Geometric Reasoning and Logic: Congruence (G-CO)

A. Experiment with transformations in the plane. In this cluster, the terms students should learn to use with increasing precision are definitions for angle, circle, perpendicular line, parallel line, and line segment; transformation, rotation, reflection, translation, and sequence of transformations.

GM: G-CO.A.1 Based on the undefined notions of point, line, distance along a line, and distance around a circular arc, know the precise definitions of

- angle,
- circle,
- perpendicular line,
- parallel line, and
- line segment.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Remediation - Previous Grade(s) Standard:** 4.DM.C.5, 4.GL.A.1, 4.GL.A.2 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Students recognize the importance of having precise definitions and use the vocabulary to accurately describe figures and relationships among figures. Students define angles, circles, perpendicular lines, parallel lines, and line segments precisely using the undefined terms.

Example:

- Draw an example of each of the following and justify how it meets the definition of the term.
    a. Angle
    b. Circle
    c. Perpendicular lines
    d. Parallel lines
    e. Line segment

Example:

- Alex and his friends are studying for a geometry test, and one of the main topics covered is parallel lines in a plane. They each write down what they think it means for two distinct lines in a plane to be parallel:
    a. Rachel writes, Two distinct lines are parallel when they are both perpendicular to a third line."
    b. Alex writes, Two distinct lines are parallel when they do not meet."

c. Briana writes, Two distinct lines are parallel when they have the same slope.''

Analyze each definition, indicating if it is mathematically correct and if it has any drawbacks.[1]

Example:

- Three students have proposed these ways to describe when two lines ℓ and m are perpendicular:
    a. ℓ and m are perpendicular if they meet at one point and one of the angles at their point of intersection is a right angle.
    b. ℓ and m are perpendicular if they meet at one point and all four of the angles at their point of intersection are right angles.
    c. ℓ and m are perpendicular if they meet at one point and reflection about ℓ maps m to itself.

  Explain why each of these definitions is correct. What are some of the advantages and disadvantages with each?[2]

[1] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/1/tasks/1543
[2] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/1/tasks/1544

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.I.GM.3, LEAP.I.GM.4, LEAP.I.GM.5, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Transformations GM: G-CO.A.1 GM: G-CO.A.3 GM: G-CO.A.5	Given a figure and a sequence of transformations, identifies the transformed figure.	Given a figure and a transformation, identifies the transformed figure.	Given a figure and a transformation, identifies the transformed figure.	Given a figure and a transformation, identifies a transformed figure.
	Uses precise geometric terminology to specify a sequence of transformations that will carry a figure onto itself or another.	Specifies a sequence of transformations that will carry a figure onto another.		

GM: G-CO.A.2 Use a plane to:

a. Represent transformations with and without technology,
b. Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
c. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Remediation - Previous Grade(s) Standard:** 8.GL.A.1, 8.GL.A.2, 8.GL.A.3, 8.GL.A.4, A1: F-BF.B.3 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** GM: G-CO.A.3, GM: G-CO.A.5	Conceptual Understanding

Examples and Explanations

Students describe and compare function transformations on a set of points as inputs to produce another set of points as outputs and is an extension of the work started in Grade 8. They distinguish between transformations that are rigid (preserve distance and angle measure: reflections, rotations, translations, or combinations of these) and those that are not (dilations or rigid motions followed by dilations). Transformations produce congruent figures while dilations produce similar figures.

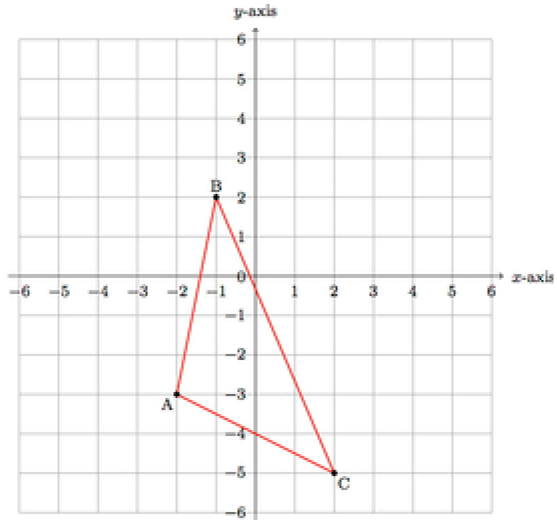
Example:

- A plane figure is translated 3 units right and 2 units down. The translated figure is then dilated with a scale factor of 4, centered at the origin.
    a. Draw a plane figure and represent the described transformation of the figure in the plane.
    b. Explain how the transformation is a function with inputs and outputs.
    c. Determine the relationship between the pre-image and the image after a series of transformations. Provide evidence to support your answer.
- Transform ΔABC with vertices $A\ (1, 1)$, $B\ (6, 3)$ and $C\ (2, 13)$ using the function rule $(x,\ y)\ (-\ y,\ x)$ and describe the transformation as completely as possible.
- Let f be the map which dilates the plane by a factor r>0 with respect to a center O. We will denote the image f(A) of a point A by A′.
    a. Suppose O, P, and Q are collinear. What is the relationship between |P′Q′| and |PQ|? Draw a picture and explain.
    b. Suppose O, P, and Q are not collinear. What is the relationship between |P′Q′| and |PQ|? Draw a picture and explain.[3]

[3] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/2/tasks/1546

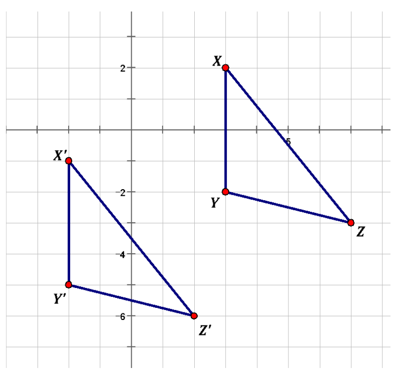
Example:

- Suppose f is the map of the plane that takes each point (x,y) to the point (x−3,y) and g is the map of the plane that takes each (x,y) to (3x,y).
    a. Triangle ABC is shown. Show the image of △ABC after applying each of the maps f and g.
    b. Does f preserve distances and angles? Explain.
    c. Does g preserve distances and angles? Explain.[4]



Example:

- Complete the rule for the transformation below, $(x,\ y)$ (___, ___) and determine if the transformations preserve distance and angle. Justify your answer.



[4] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/2/tasks/1924

Assessment Connections

Assessment Item Type(s): Type II
Evidence Statement(s): LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.GM.1 LEAP.II.GM.2 LEAP.II.GM.3 LEAP.II.GM.4	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions	using an approach based on a conjecture and/or stated assumptions
	providing an efficient and logical progression of steps or chain of reasoning with appropriate justification	providing a logical progression of steps or chain of reasoning with appropriate justification	providing a logical, but incomplete, progression of steps or chain of reasoning	providing an incomplete or illogical progression of steps or chain of reasoning
	performing precise calculations	performing precise calculations	performing minor calculation errors	making an intrusive calculation error
	using correct grade- level vocabulary, symbols and labels	using correct grade-level vocabulary, symbols and labels	using some grade-level vocabulary, symbols and labels	using limited grade-level vocabulary, symbols and labels
	providing a justification of a conclusion	providing a justification of a conclusion	providing a partial justification of a conclusion based on own calculations	providing a partial justification of a conclusion based on own calculations

	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate) – and providing a counter example where applicable.	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate).	evaluating the validity of others' approaches and conclusions	
	determining whether an argument or conclusion is generalizable			

GM: G-CO.A.3 Describe the rotations and reflections that map a preimage onto itself when given a rectangle, parallelogram, trapezoid, or regular polygon.

Standard Overview

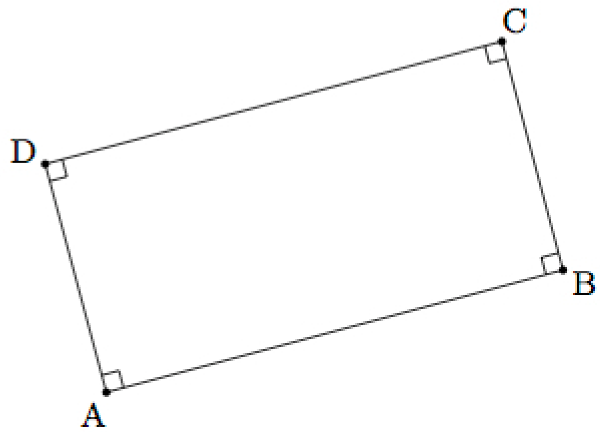
Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Remediation - Previous Grade(s) Standard:** 8.GL.A.2, 8.GL.A.3 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** GM: G-CO.A.2, GM: G-CO.A.4, GM: G-CO.A.5	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students describe and illustrate how a rectangle, parallelogram, isosceles trapezoid, or regular polygon is mapped onto itself using transformations. Students determine the number of lines of reflection symmetry and the degree of rotational symmetry of any regular polygon.

Example:

- Jennifer draws the rectangle ABCD below:

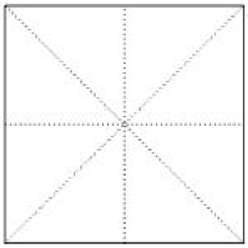


a. Find all rotations and reflections that carry rectangle ABCD onto itself.
b. Lisa draws a different rectangle, and she finds a larger number of symmetries (than Jennifer) for her rectangle. What can you conclude about Lisa's rectangle? Explain.[5]

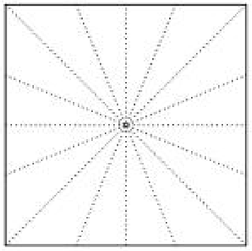
[5] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/3/tasks/1469

Example:

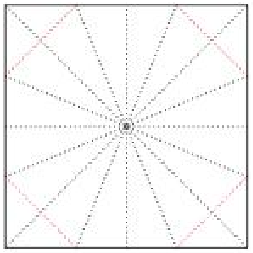
- Lisa makes an octagon by successively folding a square piece of paper as follows. First, she folds the square in half vertically and horizontally and also along both diagonals leaving these creases:



Next, Lisa makes four more folds, identifying each pair of adjacent lines of symmetry for the square used for the folds in the first step:



Finally, Lisa folds the four corners of her shape along the red creases marked below:



Explain why the shape Lisa has made is a regular octagon.[6]

[6] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/3/tasks/1487

Assessment Connections

Assessment Item Type(s): Type I, Type II

Evidence Statement(s): LEAP.I.GM.3, LEAP.I.GM.4, LEAP.I.GM.5, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Transformations GM: G-CO.A.1 GM: G-CO.A.3 GM: G-CO.A.5	Given a figure and a sequence of transformations, identifies the transformed figure.	Given a figure and a transformation, identifies the transformed figure.	Given a figure and a transformation, identifies the transformed figure.	Given a figure and a transformation, identifies a transformed figure.
	Uses precise geometric terminology to specify a sequence of transformations that will carry a figure onto itself or another.	Specifies a sequence of transformations that will carry a figure onto another.		

GM: G-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Remediation - Previous Grade(s) Standard:** [8.GL.A.1](), [8.GL.A.3]() **Geometry Standard Taught in Advance:** [GM: G-CO.A.1]() **Geometry Standard Taught Concurrently:** [GM: G-CO.A.3]()	Conceptual Understanding

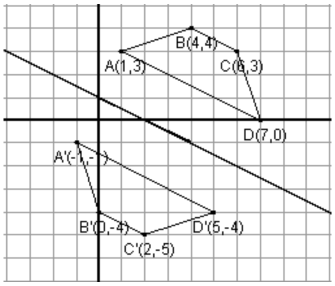
Examples and Explanations

Students develop the definition of each transformation with regard to the characteristics between pre-image and image points.

- **For a *translation*:** connecting any point on the pre-image to its corresponding point on the translated image, and connecting a second point on the pre-image to its corresponding point on the translated image, the two segments are equal in length, translate in the same direction, and are parallel.
- **For a *reflection*:** connecting any point on the pre-image to its corresponding point on the reflected image, the line of reflection is a perpendicular bisector of the line segment.
- **For a *rotation*:** connecting the center of rotation to any point on the pre-image and to its corresponding point on the rotated image, the line segments are equal in length, and the measure of the angle formed is the angle of rotation.

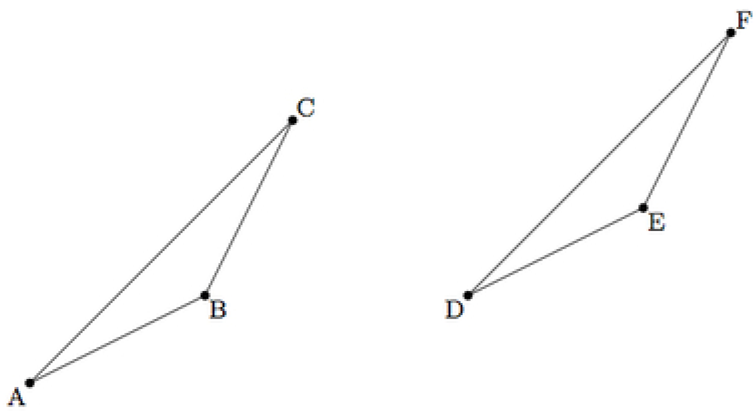
Example:

- Is quadrilateral A'B'C'D' a reflection of quadrilateral ABCD across the given line? Justify your reasoning.



Example:

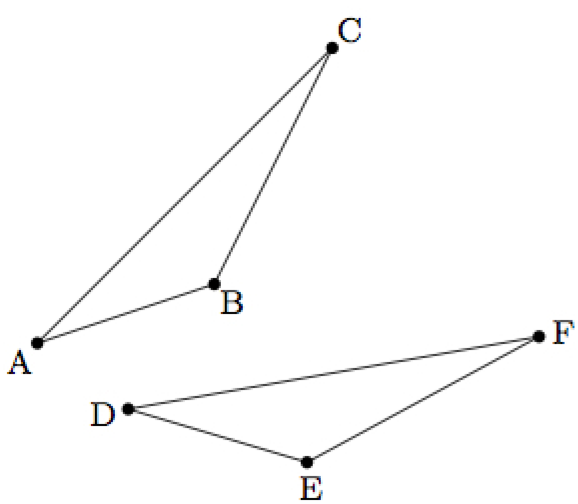
- Suppose △DEF below can be obtained from △ABC by a translation:



a. Explain why ABED, ACFD, and BCFE are parallelograms.
b. Explain why|AD|=|BE|=|CF|.[7]

Example:

- Below is triangle ABC and a rotated image triangle DEF.



a. Explain how to identify the center of rotation.
b. Once you have found the center, how do you find the angle of rotation?[8]

[7] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/4/tasks/1912
[8] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/4/tasks/1913

Assessment Connections

Assessment Item Type(s): Type II
Evidence Statement(s): LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.GM.1 LEAP.II.GM.2 LEAP.II.GM.3 LEAP.II.GM.4	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions	using an approach based on a conjecture and/or stated assumptions
	providing an efficient and logical progression of steps or chain of reasoning with appropriate justification	providing a logical progression of steps or chain of reasoning with appropriate justification	providing a logical, but incomplete, progression of steps or chain of reasoning	providing an incomplete or illogical progression of steps or chain of reasoning
	performing precise calculations	performing precise calculations	performing minor calculation errors	making an intrusive calculation error
	using correct grade- level vocabulary, symbols and labels	using correct grade-level vocabulary, symbols and labels	using some grade-level vocabulary, symbols and labels	using limited grade-level vocabulary, symbols and labels
	providing a justification of a conclusion	providing a justification of a conclusion	providing a partial justification of a conclusion based on own calculations	providing a partial justification of a conclusion based on own calculations

	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate) – and providing a counter example where applicable.	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate).	evaluating the validity of others' approaches and conclusions	
	determining whether an argument or conclusion is generalizable			

GM: G-CO.A.5 Use a geometric figure and a rotation, reflection, translation, or sequence of transformations

a. draw the transformed figure with and without technology.
b. specify a sequence that will map a given figure onto another.

Standard Overview

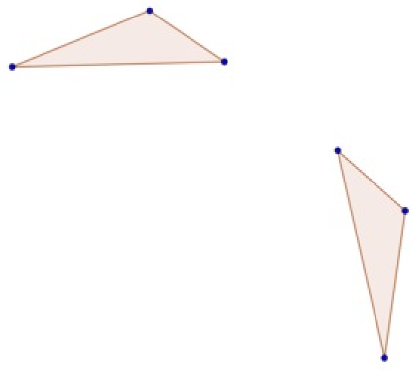
Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Remediation - Previous Grade(s) Standard:** 8.GL.A.2, 8.GL.A.3 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** GM: G-CO.A.2, GM: G-CO.A.3	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students transform a geometric figure given a rotation, reflection, or translation. They create sequences of transformations that map a geometric figure onto itself and another geometric figure. Students predict and verify the sequence of transformations (a composition) that will map a figure onto another.

Example:

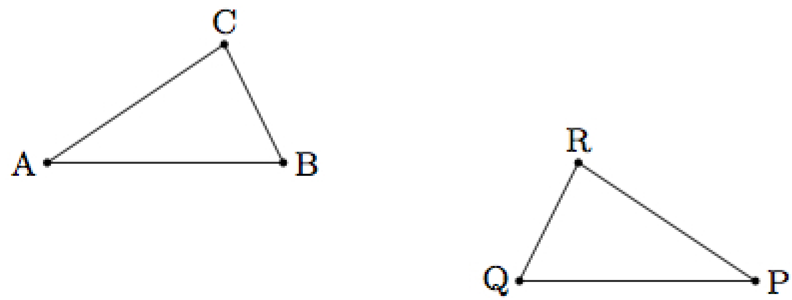
- The triangle in the upper left of the figure below has been reflected across a line into the triangle in the lower right of the figure. Use a straightedge and compass to construct the line across which the triangle was reflected.[9]



[9] ttps://www.illustrativemathematics.org/content-standards/HSG/CO/A/5/tasks/31

Example:

- Triangles ABC and PQR, pictured below, are congruent:

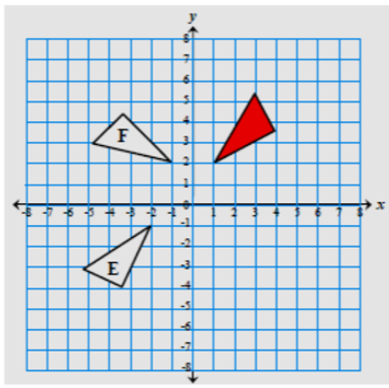


  a. Show the congruence using rigid motions of the plane.
  b. Can the congruence be shown with a single translation, rotation, or reflection? Explain.
  c. Is it possible to show the congruence using only translations? Explain.
  d. Is it possible to show the congruence using only rotations? Explain.
  e. Is it possible to show the congruence using only reflections? Explain[10]

Example:

- Suppose $\triangle ABC$ and $\triangle PQR$ are distinct, congruent triangles. Using the steps below, show that a congruence can always be shown with one, two, or three rigid motions:
  a. A translation taking A to P (if necessary).
  b. A rotation taking B, or the image of B after translation, to Q (if necessary).
  c. A reflection about $\overleftrightarrow{PQ}$ which takes C, or its image after (a) and (b), to R (if necessary).[11]

Example:



- Part 1
  Draw the shaded triangle after:
  a. It has been translated −7 units horizontally and +1 units vertically. Label your answer A.
  b. It has been reflected over the *x*-axis. Label your answer B.
  c. It has been rotated 90° clockwise about the origin. Label your answer C.
  d. It has been reflected over the line *y* = *x*. Label your answer D.

  Part 2
  Describe fully the single transformation that:
  a. Takes the shaded triangle onto the triangle labeled F.
  b. Takes the shaded triangle onto the triangle labeled E.

---

[10] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/5/tasks/1547
[11] https://www.illustrativemathematics.org/content-standards/HSG/CO/A/5/tasks/1549

Assessment Connections

Assessment Item Type(s): Type I, Type II

Evidence Statement(s): LEAP.I.GM.3, LEAP.I.GM.4, LEAP.I.GM.5, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Transformations GM: G-CO.A.1 GM: G-CO.A.3 GM: G-CO.A.5	Given a figure and a sequence of transformations, identifies the transformed figure.	Given a figure and a transformation, identifies the transformed figure.	Given a figure and a transformation, identifies the transformed figure.	Given a figure and a transformation, identifies a transformed figure.
	Uses precise geometric terminology to specify a sequence of transformations that will carry a figure onto itself or another.	Specifies a sequence of transformations that will carry a figure onto another.		

B. Understand congruence in terms of rigid motions. In this cluster, the terms students should learn to use with increasing precision are rigid motion, congruent, corresponding sides, corresponding angles, criteria for triangle congruence (ASA, SSS, and SAS).

GM: G-CO.B.6 Use geometric descriptions of rigid motions to transform figures.

- Predict the effect of a given rigid motion on a given figure.
- Given two figures, use the definition of congruence in terms of rigid motions to determine if they are congruent.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.A.2 **Geometry Standard Taught in Advance:** GM: G-CO.A.5 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane. Students recognize rigid transformations preserve size and shape or distance and angle and develop the definition of congruent. Students determine if two figures are congruent by determining if rigid motions will turn one figure into the other.

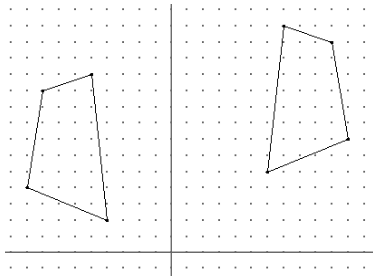
Example:

- Consider parallelogram ABCD with coordinates A(2,-2), B(4,4), C(12,4) and D(10,-2). Perform the following transformations. Make predictions about how the lengths, perimeter, area, and angle measures will change under each transformation.
  a. A reflection over the x-axis.
  b. A rotation of 270° counter-clockwise about the origin.
  c. A dilation of scale factor 3 about the origin.
  d. A translation to the right 5 and down 3.

  Verify your predictions. Compare and contrast which transformations preserved the size and/or shape with those that did not preserve size and/or shape. Generalize, how could you determine if a transformation maintains congruency from the pre-image to the image?

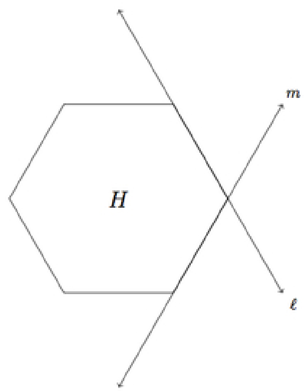
Example:

- Determine if the figures below are congruent. If so, tell what rigid motions were used.

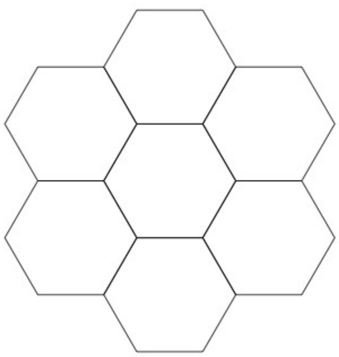


Example:

- Below is a picture of a regular hexagon, which we denote by H, and two lines denoted ℓ and m, each containing one side of the hexagon:



a. Draw rℓ(H), the reflection of the hexagon about ℓ.
b. Draw rm(H), the reflection of the hexagon about line m, together with H and rℓ(H).
c. Show that H and its reflections about the six lines containing its sides make the following pattern:[12]



[12] https://www.illustrativemathematics.org/content-standards/HSG/CO/B/6/tasks/1338

Assessment Connections

Assessment Item Type(s): Type 1, Type II
Evidence Statement(s): LEAP.I.GM.1, LEAP.I.GM.2, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Congruence Transformations GM: G-CO.B.6 LEAP.I.GM.1	Determines and uses appropriate geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve problems and prove statements about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems and prove statements about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems and reason about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems.

GM: G-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Standard Overview

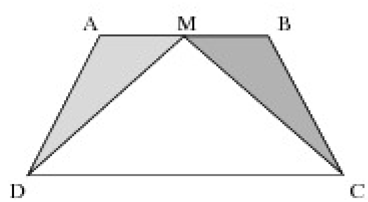
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.A.2 **Geometry Standard Taught in Advance:** GM: G-CO.B.6 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

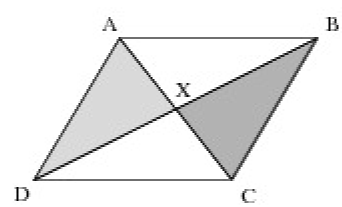
A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to *preserve distances and angle measures*. Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.

Students identify corresponding sides and corresponding angles of congruent triangles. Explain that in a pair of congruent triangles, corresponding sides are congruent (distance is preserved), and corresponding angles are congruent (angles measure is preserved). They demonstrate that when distance is preserved (corresponding sides are congruent) and angle measure is preserved (corresponding angles are congruent), the triangles must also be congruent.

Example:
- In each of the following diagrams, two triangles are shaded. Based on the information given about each diagram, decide whether there is enough information to prove that the two triangles are congruent.
  a. In the figure below, ABCD is a trapezoid with AB parallel to CD, and M is the midpoint of side AB.

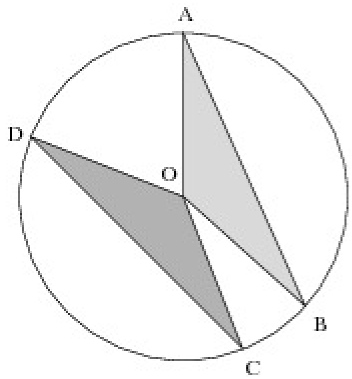


Example:
- In the figure below, ABCD is a parallelogram.



Example:

- In the figure below, the circle is centered at point O, and the arc AB is congruent to the arc CD.[13]



[13] https://tasks.illustrativemathematics.org/content-standards/HSG/CO/B/tasks/33

Assessment Connections

Assessment Item Type(s): Type II
Evidence Statement(s): LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.GM.1 LEAP.II.GM.2 LEAP.II.GM.3 LEAP.II.GM.4	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions	using an approach based on a conjecture and/or stated assumptions
	providing an efficient and logical progression of steps or chain of reasoning with appropriate justification	providing a logical progression of steps or chain of reasoning with appropriate justification	providing a logical, but incomplete, progression of steps or chain of reasoning	providing an incomplete or illogical progression of steps or chain of reasoning
	performing precise calculations	performing precise calculations	performing minor calculation errors	making an intrusive calculation error
	using correct grade- level vocabulary, symbols and labels	using correct grade-level vocabulary, symbols and labels	using some grade-level vocabulary, symbols and labels	using limited grade-level vocabulary, symbols and labels
	providing a justification of a conclusion	providing a justification of a conclusion	providing a partial justification of a conclusion based on own calculations	providing a partial justification of a conclusion based on own calculations

	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate) – and providing a counter example where applicable.	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate).	evaluating the validity of others' approaches and conclusions	
	determining whether an argument or conclusion is generalizable			

GM: G-CO.B.8 Explain how the criteria for triangle congruence ASA, SAS, and SSS follow from the definition of congruence in terms of rigid motions.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.A.2 **Geometry Standard Taught in Advance:** GM: G-CO.B.7 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

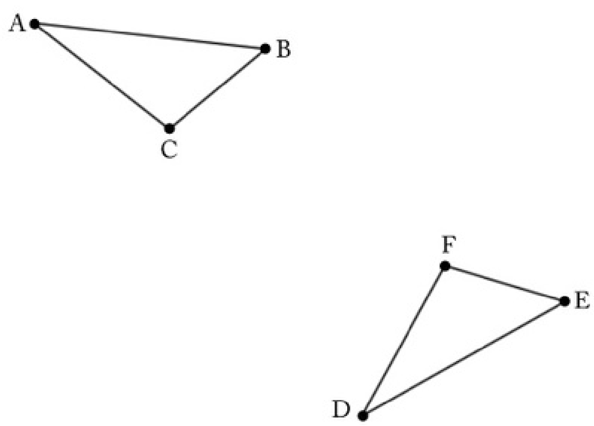
In this cluster, the terms students should learn to use with increasing precision are rigid motion, congruent, corresponding sides, corresponding angles, criteria for triangle congruence (ASA, SSS, and SAS).

Examples and Explanations

Students list the sufficient conditions to prove triangles are congruent: ASA, SAS, and SSS. They map a triangle with one of the sufficient conditions (e.g., SSS) onto the original triangle and show that corresponding sides and corresponding angles are congruent.

Example:

- In the two triangles below, angle A is congruent to angle D, side AC is congruent to side DF and side AB is congruent to side DE:



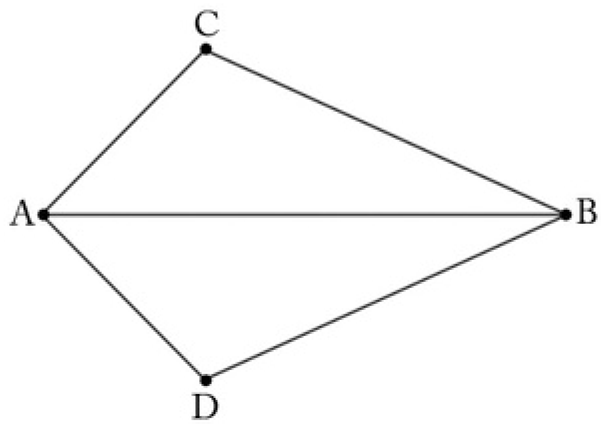
Sally reasons as follows: "If angle A is congruent to angle D, then I can move point A to point D so that side AB lies on top of side DE and side AC lies on top of side DF. Since AB and DE are congruent, as are AC and DF, the two triangles match up exactly, and so they are congruent.''

Explain Sally's reasoning for why triangle ABC is congruent to triangle DEF using the language of reflections:

a. Construct a reflection which maps point A to point D. Call B′ and C′ the images of B and C, respectively, under this reflection.
b. Construct a reflection which does not move D but which sends B′ to E. Call C″ the image of C′ under this reflection.
c. Construct a reflection which does not move D or E but which sends C″ to F.[14]

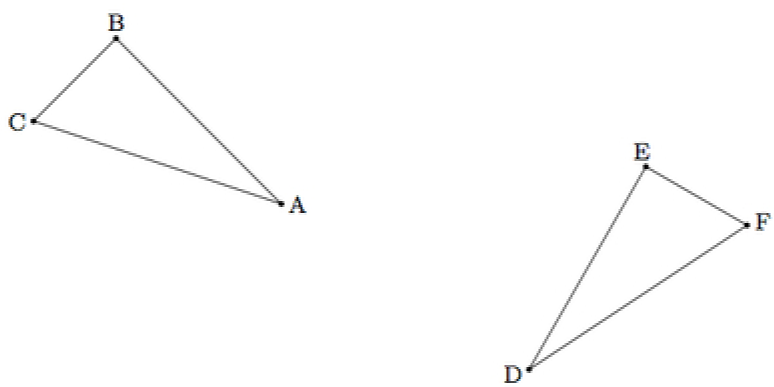
Example:

- In triangles ABC and ABD below, we are given that angle BAC is congruent to angle BAD, and angle ABC is congruent to angle ABD. Show that the reflection of the plane about line AB maps triangle ABD to triangle ABC.[15]



Example:

- In the picture below, segment AB is congruent to segment DE, segment AC is congruent to segment DF, and segment BC is congruent to segment EF:



[14] http://www.illustrativemathematics.org/illustrations/109
[15] https://tasks.illustrativemathematics.org/content-standards/HSG/CO/B/8/tasks/339

Show that the two triangles ABC and DEF are congruent via the following steps, which produce a rigid transformation of the plane sending △ABC to △DEF.

a. Show that there is a translation of the plane which maps A to D. Call B′ and C′ the images of B and C under this transformation.
b. Show that there is a rotation of the plane which does not move D and which maps B′ to E. Call C″ the image of C′ under this transformation.
c. Show that there is a reflection of the plane which does not move D or E and which maps C″ to F.[16]

[16] https://tasks.illustrativemathematics.org/content-standards/HSG/CO/B/8/tasks/110

Assessment Connections

Assessment Item Type(s): Type II
Evidence Statement(s): LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.GM.1 LEAP.II.GM.2 LEAP.II.GM.3 LEAP.II.GM.4	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions	using an approach based on a conjecture and/or stated assumptions
	providing an efficient and logical progression of steps or chain of reasoning with appropriate justification	providing a logical progression of steps or chain of reasoning with appropriate justification	providing a logical, but incomplete, progression of steps or chain of reasoning	providing an incomplete or illogical progression of steps or chain of reasoning
	performing precise calculations	performing precise calculations	performing minor calculation errors	making an intrusive calculation error
	using correct grade- level vocabulary, symbols and labels	using correct grade-level vocabulary, symbols and labels	using some grade-level vocabulary, symbols and labels	using limited grade-level vocabulary, symbols and labels
	providing a justification of a conclusion	providing a justification of a conclusion	providing a partial justification of a conclusion based on own calculations	providing a partial justification of a conclusion based on own calculations

	evaluating, interpreting, and critiquing the validity of others’ responses, approaches and reasoning – using mathematical connections (when appropriate) – and providing a counter example where applicable.	evaluating, interpreting, and critiquing the validity of others’ responses, approaches and reasoning – using mathematical connections (when appropriate).	evaluating the validity of others’ approaches and conclusions	
	determining whether an argument or conclusion is generalizable			

C. Prove and apply geometric theorems. In this cluster, the terms students should learn to use with increasing precision are proof and theorem.

GM: G-CO.C.9 Prove and apply theorems about lines and angles.

Theorems include:
- *vertical angles are congruent;*
- *when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent;*
- *points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

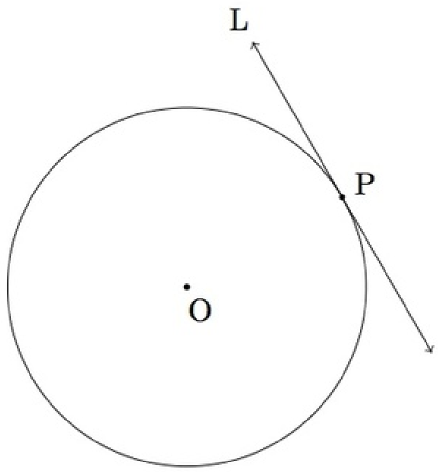
Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 4.DM.C.7, 7.GL.B.5, 8.GL.A.5 **Geometry Standard Taught in Advance:** GM.G-CO.A.1 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Encourage multiple ways of writing proofs, such as narrative paragraphs, using flow diagrams, and two-column format. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between and within geometric objects should be central to any geometric study and certainly to proof. The use of dynamic geometry software, such as GeoGebra, can be an important tool for helping students conceptually understand important geometric concepts. GeoGebra is a free app for tablets, phones, and desktops. Click here to download GeoGebra.

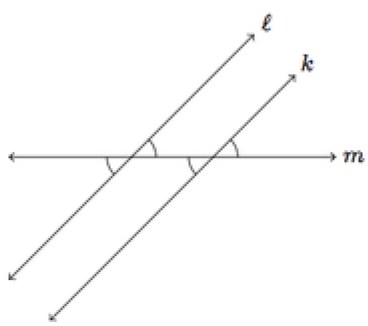
Example:
- Consider a circle with center O, and let P be a point on the circle. Suppose L is a tangent line to the circle at P, that is, L meets the circle only at P.



Show that line segment OP is perpendicular to L.[17]

Example:

- In the picture below ℓ and k are parallel:



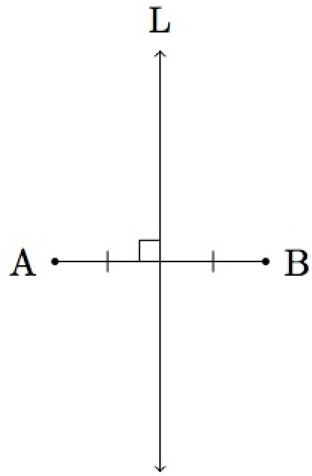
Show that the four angles marked in the picture are congruent.[18]

Example:

- Suppose A and B are two distinct points in the plane, and L is the perpendicular bisector of segment AB as pictured below:

[17] https://www.illustrativemathematics.org/content-standards/HSG/CO/C/9/tasks/963

[18] https://www.illustrativemathematics.org/content-standards/HSG/CO/C/9/tasks/1922



a. If C is a point on L, show that C is equidistant from A and B, that is, show that AC and B are congruent.
b. Conversely, show that if P is a point which is equidistant from A and B, then P is on L.
c. Conclude that the perpendicular bisector of AB is exactly the set of points which are equidistant from A and B.[19]

[19] https://www.illustrativemathematics.org/content-standards/HSG/CO/C/9/tasks/967

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.I.GM.1, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Congruence Transformations GM: G-CO.B.6 LEAP.I.GM.1	Determines and uses appropriate geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve problems and prove statements about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems and prove statements about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems and reason about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems.

GM: G-CO.C.10 Prove and apply theorems about triangles.Theorems include

- measures of interior angles of a triangle sum to 180°;
- base angles of isosceles triangles are congruent;
- the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point

Standard Overview

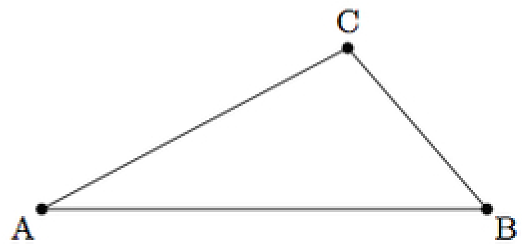
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 7.GL.A.2, 8.GL.A.5 **Geometry Standard Taught in Advance:** GM: G-CO.B.8, GM: G-CO.C.9 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Encourage multiple ways of writing proofs, such as narrative paragraphs, using flow diagrams, and two-column format. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between and within geometric objects should be central to any geometric study and certainly to proof. The use of dynamic geometry software, such as GeoGebra, can be an important tool for helping students conceptually understand important geometric concepts. GeoGebra is a free app for tablets, phones, and desktops. Click here to download GeoGebra.

Example:

- Suppose ABC is a triangle in the plane as pictured below:

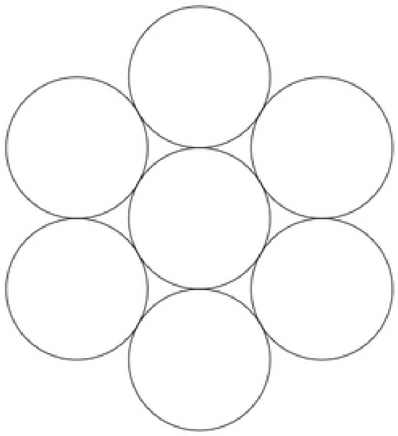


Suppose M is the midpoint of segment AC and N is the midpoint of segment BC.

a. Draw the rotation of $\triangle ABC$ by 180 degrees about M, labeling the image of B as B′.
b. Draw the rotation of $\triangle ABC$ by 180 degrees about N, labeling the image of A as A′.
c. Explain why B′, C, and A′ are collinear.
d. Deduce that $m(\angle A)+m(\angle B)+m(\angle C)=180$.[20]

Example:

- Liz places seven pennies in the arrangement shown below:



She notices that each of the six outer pennies touches the one in the center and its two outer neighbors. Liz wonders whether or not this is true if the pennies are replaced with seven nickels or quarters or, more generally, any set of seven equally sized circles.

What is the answer to Liz's question, that is, is it possible to place six circles around a central circle so that each outer circle touches the one in the center as well as its two outside neighbors?[21]

[20] https://tasks.illustrativemathematics.org/content-standards/HSG/CO/C/10/tasks/1923
[21] https://tasks.illustrativemathematics.org/content-standards/HSG/CO/C/10/tasks/707

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.I.GM.1, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Congruence Transformations GM: G-CO.B.6 LEAP.I.GM.1	Determines and uses appropriate geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve problems and prove statements about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems and prove statements about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems and reason about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems.

GM: G-CO.C.11 Prove and apply theorems about parallelograms. Theorems include

- opposite sides are congruent,
- opposite angles are congruent,
- the diagonals of a parallelogram bisect each other, and the converse of this theorem;
- rectangles are parallelograms with congruent diagonals and the converse of this theorem.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 5.GL.B.3 **Geometry Standard Taught in Advance:** GM: G-CO.B.8, GM: G-CO.C.9 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

In this cluster, the terms students should learn to use with increasing precision are proof and theorem.

Examples and Explanations

Encourage multiple ways of writing proofs, such as *narrative paragraphs*, using *flow diagrams*, and *two-column format.* Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between and within geometric objects should be central to any geometric study and certainly to proof. The use of dynamic geometry software, such as GeoGebra, can be an important tool for helping students conceptually understand important geometric concepts. GeoGebra is a free app for tablets, phones, and desktops. Click here to download GeoGebra.

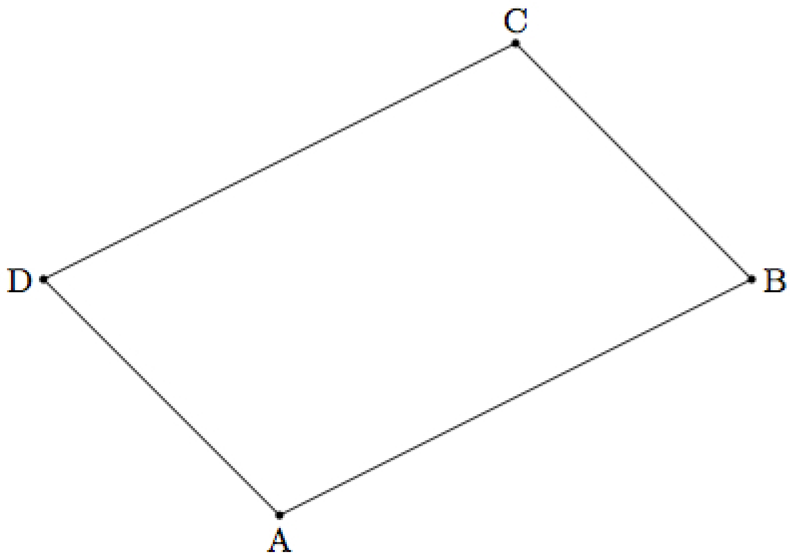
Example:

- Rhianna has learned the SSS and SAS congruence tests for triangles and she wonders if these tests might work for parallelograms.
    a. Suppose ABCD and EFGH are two parallelograms all of whose corresponding sides are congruent, that is $|AB|=|EF|,|BC|=|FG|,|CD|=|GH|$, and $|DA|=|HE|$. Is it always true that ABCD is congruent to EFGH?
    b. Suppose ABCD and EFGH are two parallelograms with a pair of congruent corresponding sides, $|AB|=|EF|$ and $|BC|=|FG|$. Suppose also that the included angles are congruent, $m(\angle ABC)=m(\angle EFG)$. Are ABCD and EFGH congruent?[22]

Example:

[22] https://www.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/1517

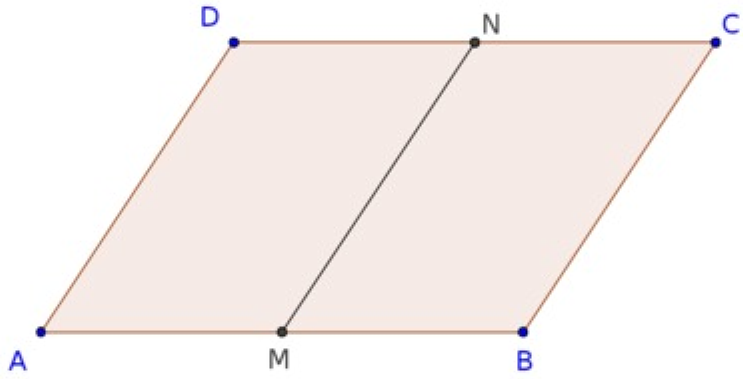
- In quadrilateral $ABCD$ pictured below, $\overline{AB}$ is congruent to $\overline{CD}$ and $\overline{BC}$ is congruent to $\overline{AD}$.



From the given information, can we deduce that ABCD is a parallelogram? Explain. [23]

Example:

- Suppose that ABCD is a parallelogram, and that M and N are the midpoints of segment AB and segment CD, respectively. Prove that MN=AD, and that the line MN is parallel to line AD.[24]

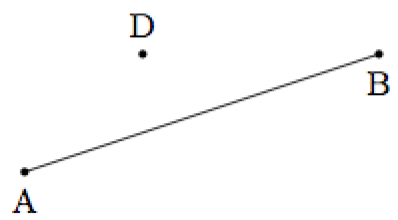


[23] https://www.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/1321

[24] https://www.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/35

Example:

- Suppose AB is a line segment and D is a point not on AB as pictured below:



Let C be the point so that |CD|=|AB|, CD is parallel to AB, and ABCD is a quadrilateral.

Draw a picture of this situation and show that ABCD is a parallelogram. [25]

[25] https://www.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/1511

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.1

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Congruence Transformations GM: G-CO.B.6 LEAP.I.GM.1	Determines and uses appropriate geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve problems and prove statements about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems and prove statements about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems and reason about angle measurement, triangles, distance, line properties, and congruence.	Uses given geometric theorems and properties of rigid motions, lines, angles, triangles, and parallelograms to solve routine problems.

D. Make geometric constructions. In this cluster, the terms students should learn to use with increasing precision are formal geometric construction, bisect, perpendicular bisector, regular polygon, and inscribed.

GM: G-CO.D.12 Make formal geometric constructions

- with a variety of tools and methods, with or without technology
- of an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Supporting Content	**Remediation - Previous Grade(s) Standard:** 7.GL.A.2 **Geometry Standard Taught in Advance:** GM: G-CO.C.9 **Geometry Standard Taught Concurrently:** none	Procedural Skill and Fluency

Examples and Explanations

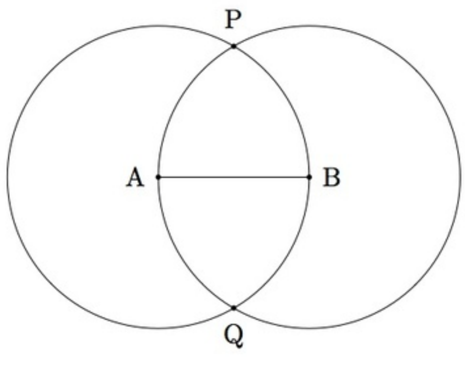
Free resources for this standard include:

- Animated Geometric Constructions (compass/straight edge)[26]

Example:

- Let A and B be two distinct points in the plane and AB the segment joining them. The goal of this problem is to construct the perpendicular bisector of segment AB.

  Draw circles with radius |AB| centered at A and B, respectively, as pictured below:

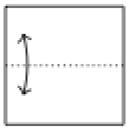


[26] http://www.mathsisfun.com/geometry/constructions.htm

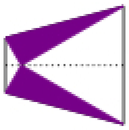
The two points of intersection of these circles are labelled P and Q. Show that line PQ is the perpendicular bisector of AB.[27]

Example:

- Jessica is working to construct an equilateral triangle with origami paper and uses the following steps.
    a. First, she folds the paper in half and then unfolds it, leaving the crease along the dotted line:



    b. Next, Jessica folds two corners over to the center crease as shown in the picture below:



Explain why the two purple triangles meet in a point, as the picture suggests, and why the large white triangle is an equilateral triangle.[28]

[27] https://www.illustrativemathematics.org/content-standards/HSG/CO/D/12/tasks/966
[28] https://www.illustrativemathematics.org/content-standards/HSG/CO/D/12/tasks/1486

Assessment Connections

Assessment Item Type(s): Type I, Type II

Evidence Statement(s): LEAP.I.GM.3, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Geometric Constructions LEAP.I.GM.3	Understands geometric constructions: copying a segment, copying an angle, bisecting an angle, bisecting a segment, including the perpendicular bisector of a line segment.	Understands geometric constructions: copying a segment, copying an angle, bisecting an angle, bisecting a segment, including the perpendicular bisector of a line segment.	Understands basic geometric constructions: copying a segment, copying an angle, bisecting an angle, bisecting a segment, including the perpendicular bisector of a line segment.	Understands basic geometric constructions: copying a segment, and copying an angle
	Given a line and a point not on the line, uses a variety of tools and methods to construct perpendicular and parallel lines.	Given a line and a point not on the line, constructs perpendicular and parallel lines.		
	Uses a variety of tools and methods to construct equilateral triangles, squares, and hexagons inscribed in circles.			

Geometric Reasoning and Logic: Similarity, Right Triangles, and Trigonometry (G-SRT)

A. Understand similarity in terms of similarity transformations. In this cluster, the terms students should learn to use with increasing precision are verify, dilation, similar figures, similarity transformation, and proportionality of sides.

GM: G-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor.

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.A.4 **Geometry Standard Taught in Advance:** GM: G-CO.A.2 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Students should understand that a dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Students perform a dilation with a given center and scale factor on a figure in the coordinate plane.

Example:

- Given ΔABC with $A\,(-2,\ -4)$, $B\,(1, 2)$ and $C\,(4, -3)$ apply the rule $(x,\ y)(3x,\ 3y)$

Students verify that when a side passes through the center of dilation, the side and its image lie on the same line and the remaining corresponding sides of the pre-image and images are parallel.

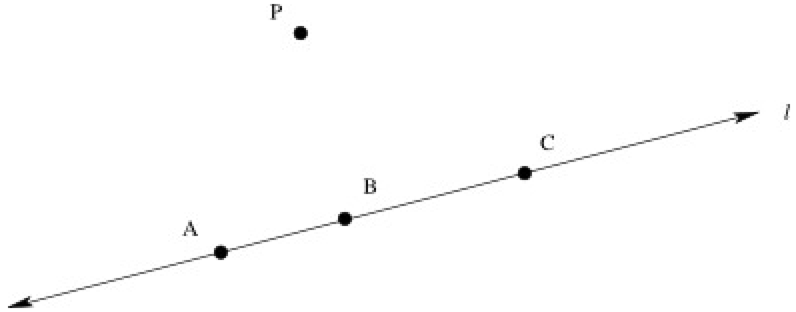
Example:

- Using ΔABC and its image $\Delta A'B'C'$ from the previous example, connect the corresponding pre-image and image points. Describe how the corresponding sides are related. Determine the center of dilation.

Students verify that a side length of the image is equal to the scale factor multiplied by the corresponding side length of the pre-image.

Example:

- Suppose we apply a dilation by a factor of 2, centered at the point P, to the figure below.



a. In the picture, locate the images A', B', and C' of the points A, B, and C under this dilation.
b. Based on your picture in part (a), what do you think happens to the line l when we perform the dilation?
c. Based on your picture in part (a), what appears to be the relationship between the distance A'B' and the distance AB? How about the distances B'C' and BC?
d. Can you prove your observations in part (c)?[29]

[29] https://www.illustrativemathematics.org/content-standards/HSG/SRT/A/1/tasks/602

Assessment Connections

Assessment Item Type(s): Type I, Type II

Evidence Statement(s): LEAP.I.GM.1, LEAP.I.GM.2, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Similarity GM: G-SRT.A.1 GM: G-SRT.A.2 GM: G-SRT.B.5	Uses transformations and congruence and similarity criteria for triangles to prove relationships among geometric figures and to solve problems.	Uses transformations to determine relationships among simple geometric figures and to solve problems.	Identifies transformation relationships in simple geometric figures.	Identifies transformation relationships in simple geometric figures in cases where an image is provided.

GM: G-SRT.A.2 Using similarity transformations

- Determine if two figures are similar using the definition of similarity transformations.
- Explain the meaning of similarity for triangles as the equality of all corresponding pairs of angles and proportionality of all corresponding sides.

Standard Overview

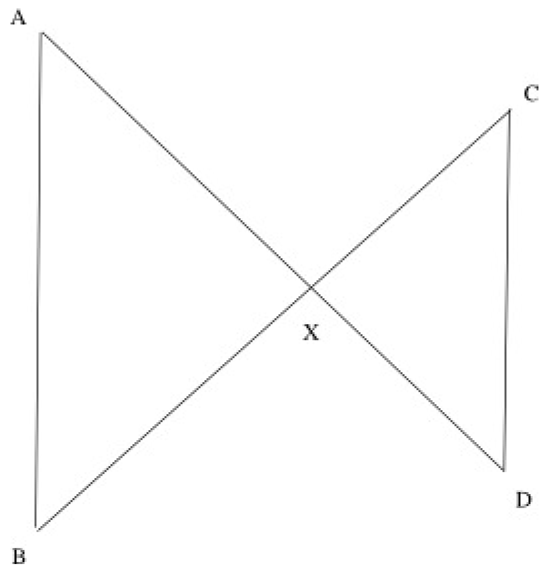
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.A.4 **Geometry Standard Taught in Advance:** GM: G-SRT.A.1 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students use the idea of dilation transformations to develop the definition of similarity. They understand that a similarity transformation is a rigid motion followed by a dilation. Students demonstrate that in a pair of similar triangles, corresponding angles are congruent (angle measure is preserved) and corresponding sides are proportional. They determine that two figures are similar by verifying that angle measure is preserved and corresponding sides are proportional.

Example:

- In the picture given below, line segments AD and BC intersect at X. Line segments AB and CD are drawn, forming two triangles AXB and CXD.



In each part (a)-(d) below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar, and if so, what the correct correspondence of vertices is. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one triangle to the other. If not, explain why not.

a. The lengths AX and XD satisfy the equation 2AX=3XD.
b. The lengths AX, BX, CX, and DX satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$
c. Lines AB and CD are parallel.
d. Angle XAB is congruent to angle XCD.[30]

Example:

- Quadrilaterals ABCD and EFGH have three corresponding congruent angles:
  - m(∠A)=m(∠E)
  - m(∠B)=m(∠F)
  - m(∠C)=m(∠G)

  a. Must it be true that m(∠D)=m(∠H)? Explain.
  b. If ABCD and EFGH are isosceles trapezoids, are they always similar?
  c. If ABCD and EFGH are rhombuses, are they always similar?[31]

Example:

- Alicia has two triangles, ABC and PQR, whose corresponding sides are proportional as pictured below:

  Alicia says:
  I wonder if they are similar because I don't have any information about the angles?

  What is the answer to Alicia's question? Explain.[32]

[30] https://www.illustrativemathematics.org/content-standards/HSG/SRT/A/2/tasks/603
[31] https://www.illustrativemathematics.org/content-standards/HSG/SRT/A/2/tasks/1858
[32] https://www.illustrativemathematics.org/content-standards/HSG/SRT/A/2/tasks/1890

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.I.GM.1, LEAP.I.GM.2, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Similarity GM: G-SRT.A.1 GM: G-SRT.A.2 GM: G-SRT.B.5	Uses transformations and congruence and similarity criteria for triangles to prove relationships among geometric figures and to solve problems.	Uses transformations to determine relationships among simple geometric figures and to solve problems.	Identifies transformation relationships in simple geometric figures.	Identifies transformation relationships in simple geometric figures in cases where an image is provided.

GM: G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Standard Overview

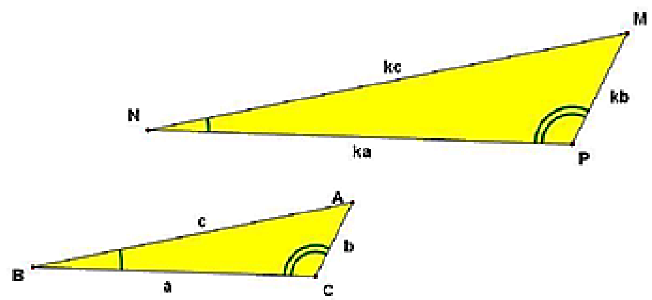
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.A.4, 8.GL.A.5 **Geometry Standard Taught in Advance:** GM: G-SRT.A.2 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

In this cluster, the terms students should learn to use with increasing precision are verify, dilation, similar figures, similarity transformation, and proportionality of sides.

Examples and Explanations

Students can use the theorem that the angle sum of a triangle is 180° and verify that the AA criterion is equivalent to the AAA criterion.

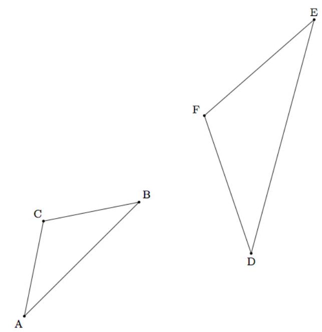
Given two triangles for which AA holds, students use rigid motions to map a vertex of one triangle onto the corresponding vertex of the other in such a way that their corresponding sides are in line. Then show that dilation will complete the mapping of one triangle onto the other.



Example:
- Given that ΔMNP is a dilation of ΔABC with scale factor k, use properties of dilations to show that the AA criterion is sufficient to prove similarity.

Example:

- In the two triangles pictured below m(∠A)=m(∠D) and m(∠B)=m(∠E):



Using a sequence of translations, rotations, reflections, and/or dilations, show that △ABC is similar to △DEF.[33]

[33] https://www.illustrativemathematics.org/content-standards/HSG/SRT/A/3/tasks/1422

Assessment Connections

Assessment Item Type(s): Type II

Evidence Statement(s): LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.GM.1 LEAP.II.GM.2 LEAP.II.GM.3 LEAP.II.GM.4	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions	using an approach based on a conjecture and/or stated assumptions
	providing an efficient and logical progression of steps or chain of reasoning with appropriate justification	providing a logical progression of steps or chain of reasoning with appropriate justification	providing a logical, but incomplete, progression of steps or chain of reasoning	providing an incomplete or illogical progression of steps or chain of reasoning
	performing precise calculations	performing precise calculations	performing minor calculation errors	making an intrusive calculation error
	using correct grade- level vocabulary, symbols and labels	using correct grade-level vocabulary, symbols and labels	using some grade-level vocabulary, symbols and labels	using limited grade-level vocabulary, symbols and labels
	providing a justification of a conclusion	providing a justification of a conclusion	providing a partial justification of a conclusion based on own calculations	providing a partial justification of a conclusion based on own calculations

	evaluating, interpreting, and critiquing the validity of others’ responses, approaches and reasoning – using mathematical connections (when appropriate) – and providing a counter example where applicable.	evaluating, interpreting, and critiquing the validity of others’ responses, approaches and reasoning – using mathematical connections (when appropriate).	evaluating the validity of others’ approaches and conclusions	
	determining whether an argument or conclusion is generalizable			

B. Prove and apply theorems involving similarity. In this cluster, the terms students should learn to use with increasing precision are proof, theorem, converse, and triangle congruence and similarity criteria.

GM: G-SRT.B.4 Prove and apply theorems about triangles including

- a line parallel to one side of a triangle divides the other two proportionally, and the converse of this theorem;
- the Pythagorean Theorem proved using triangle similarity;
- SAS similarity criteria;
- SSS similarity criteria;
- AA similarity criteria.

Standard Overview

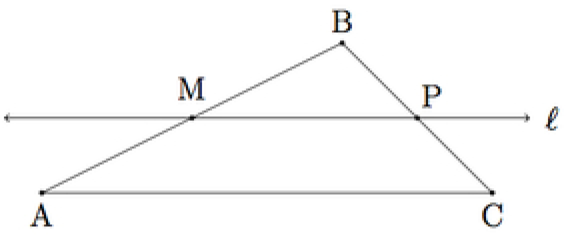
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.B.6 **Geometry Standard Taught in Advance:** GM: G-SRT.A.3 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Use AA, SAS, and SSS similarity theorems to prove triangles are similar. Use triangle similarity to prove other theorems about triangles.

Example:

- Suppose ABC is a triangle. Let M be the midpoint of AB and ℓ the line through M parallel to AC:

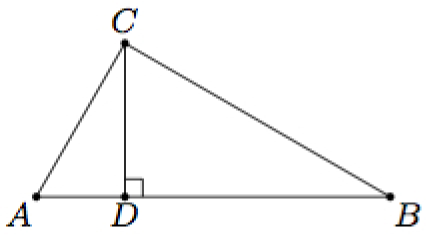


   a. Show that angle CAB is congruent to angle PMB and that angle BPM is congruent to angle BCA. Conclude that triangle MBP is similar to triangle ABC.
   b. Use part (a) to show that P is the midpoint of BC.[34]

[34] https://www.illustrativemathematics.org/content-standards/HSG/SRT/B/4/tasks/1095

Example:

- Below is a picture of a right triangle ABC with right angle C, along with the point D so that CD is perpendicular to AB.



a. Show that △ACB is similar to △ADC and to △CDB.
b. Use part (a) to conclude that $|AC|^2+|BC|^2=|AB|^2$.[35]

[35] https://www.illustrativemathematics.org/content-standards/HSG/SRT/B/4/tasks/1568

Assessment Connections

Assessment Item Type(s): Type II
Evidence Statement(s): LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.GM.1 LEAP.II.GM.2 LEAP.II.GM.3 LEAP.II.GM.4	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions	using an approach based on a conjecture and/or stated assumptions
	providing an efficient and logical progression of steps or chain of reasoning with appropriate justification	providing a logical progression of steps or chain of reasoning with appropriate justification	providing a logical, but incomplete, progression of steps or chain of reasoning	providing an incomplete or illogical progression of steps or chain of reasoning
	performing precise calculations	performing precise calculations	performing minor calculation errors	making an intrusive calculation error
	using correct grade- level vocabulary, symbols and labels	using correct grade-level vocabulary, symbols and labels	using some grade-level vocabulary, symbols and labels	using limited grade-level vocabulary, symbols and labels
	providing a justification of a conclusion	providing a justification of a conclusion	providing a partial justification of a conclusion based on own calculations	providing a partial justification of a conclusion based on own calculations

	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate) – and providing a counter example where applicable.	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate).	evaluating the validity of others' approaches and conclusions	
	determining whether an argument or conclusion is generalizable			

GM: G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Standard Overview

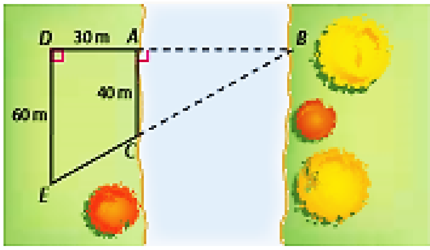
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: G-CO.B.8, GM:G-SRT.A.3 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency, Application

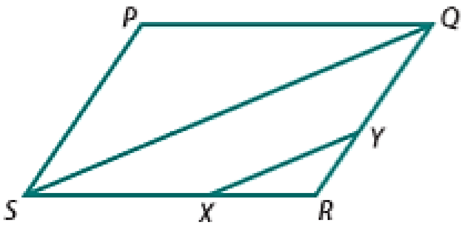
Examples and Explanations

The similarity postulates include SSS, SAS, and AA. The congruence postulates include SSS, SAS, ASA, AAS, and H-L. Students apply triangle congruence and triangle similarity to solve problem situations (e.g., indirect measurement, missing side(s)/angle measure(s), side splitting).

Example:

- Calculate the distance across the river, *AB*.





- In the diagram, quadrilateral *PQRS* is a parallelogram, *SQ* is a diagonal, and *SQ* || *XY*.
  - a. Prove that $\Delta XYR \sim \Delta SQR$.
  - b. Prove that $\Delta XYR \sim \Delta QSP$.

Assessment Connections

Assessment Item Type(s): Type I, Type II

Evidence Statement(s): LEAP.I.GM.1, LEAP.I.GM.2, LEAP.II.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Similarity GM: G-SRT.A.1 GM: G-SRT.A.2 GM: G-SRT.B.5	Uses transformations and congruence and similarity criteria for triangles to prove relationships among geometric figures and to solve problems.	Uses transformations to determine relationships among simple geometric figures and to solve problems.	Identifies transformation relationships in simple geometric figures.	Identifies transformation relationships in simple geometric figures in cases where an image is provided.

C. Define trigonometric ratios and solve problems involving right triangles. In this cluster, the terms students should learn to use with increasing precision are right triangle, side ratios, special right triangles (30-30-90 and 45-45-90), acute angle, trigonometric ratios (sine, cosine, tangent), complementary angles, and "solve a right triangle."

GM: G-SRT.C.6 Understand that by similarity, side ratios in right triangles, including special right triangles (30-60-90 and 45-45-90), are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Standard Overview

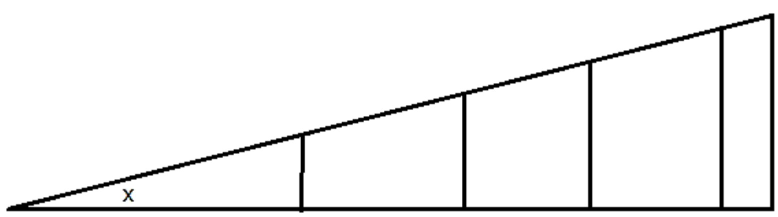
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** [GM: G-SRT.A.2] **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

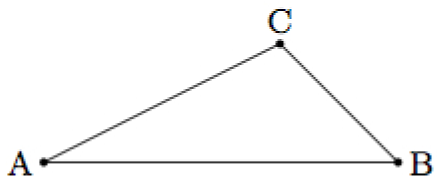
Students establish that the side ratios of a right triangle are equivalent to the corresponding side ratios of similar right triangles and are a function of the acute angle(s).

Example:
- Find the sine, cosine, and tangent of *x*.

Example:
- Explain why the sine of *x* is the same regardless of which triangle is used to find it in the figure to the right.



Example:
- Below is a picture of △ABC:



a. Draw a triangle DEF that is similar (but not congruent) to △ABC.
b. How do $\left|\frac{DE}{DF}\right|$ and $\left|\frac{AB}{AC}\right|$ and compare? Explain.
c. When ∠B is a right angle, the ratio |AB|:|AC| is called the cosine of ∠A, while the ratio |BC|:|AC| is called the sine of ∠A. Why do these ratios depend only on ∠A?
d. The ratios in part (c) make sense whether or not ∠B is a right angle, but they are only given names (sine and cosine) in this special case. What is special about the case where B is a right angle?[36]

[36] https://www.illustrativemathematics.org/content-standards/HSG/SRT/C/6/tasks/1635

Assessment Connections

Assessment Item Type(s): Type I

Evidence Statement(s): LEAP.I.GM.1, LEAP.I.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Similarity in Trigonometry GM: G-SRT.C.6 GM: G-SRT.C.7 GM: G-SRT.C.8	Uses trigonometric ratios, the Pythagorean Theorem, and the relationship between sine and cosine to solve right triangles in applied problems.	Uses trigonometric ratios, the Pythagorean Theorem, and the relationship between sine and cosine to solve right triangles in applied problems.	Uses trigonometric ratios and the Pythagorean Theorem to determine the unknown side lengths and angle measurements of a right triangle.	Uses trigonometric ratios and the Pythagorean Theorem to determine the unknown side lengths of a right triangle.
	Uses similarity transformations with right triangles to define trigonometric ratios for acute angles.			

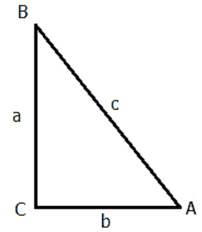
GM: G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 7.GL.B.5 **Geometry Standard Taught in Advance:** GM: G-SRT.C.6 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students can explain why the sine of an acute angle in a right triangle is the cosine of a complementary angle in the same right triangle. Students use the relationship to solve problems.



Example:

- Using the diagram at the right, provide an argument justifying why $sin\ A = cos\ B$.

Example:

- Complete the following statement:

  If $sin\ 30° = \frac{1}{2}$, then cos ____ $= \frac{1}{2}$

Example:

- Given: Angle *F* and angle *G* are complementary. As the measure of angle *F* varies from a value of *x* to a value of *y*, $sin\ (F)$ increases by 0.2. How does $cos\ (G)$ change as *F* varies from x to y?

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.1, LEAP.I.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Similarity in Trigonometry GM: G-SRT.C.6 GM: G-SRT.C.7 GM: G-SRT.C.8	Uses trigonometric ratios, the Pythagorean Theorem, and the relationship between sine and cosine to solve right triangles in applied problems.	Uses trigonometric ratios, the Pythagorean Theorem, and the relationship between sine and cosine to solve right triangles in applied problems.	Uses trigonometric ratios and the Pythagorean Theorem to determine the unknown side lengths and angle measurements of a right triangle.	Uses trigonometric ratios and the Pythagorean Theorem to determine the unknown side lengths of a right triangle.
	Uses similarity transformations with right triangles to define trigonometric ratios for acute angles.			
Modeling and Applying GM: G-SRT.C.7 GM: G-SRT.C.8 GM: G-GPE.B.6 LEAP.I.GM.2	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.
	Applies geometric concepts and trigonometric ratios to describe, model and	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem,	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean	Applies geometric concepts to describe, model and solve applied problems related to geometric

	solve applied problems (including design problems) related to the Pythagorean Theorem, density, geometric shapes, measures, and properties.	geometric shapes, measures, and properties.	Theorem, geometric shapes, measures, and properties.	shapes, measures, and properties.

GM: G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Standard Overview

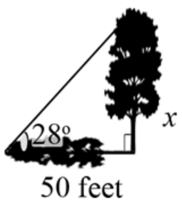
Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.B.7 **Geometry Standard Taught in Advance:** GM: G-SRT.B.4 **Geometry Standard Taught Concurrently:** none	Application

Examples and Explanations

This is a modeling standard, which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Example:

- Find the height of a tree to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the tree is 50 ft.



Example:

- A new house is 32 feet wide. The rafters will rise at a 36° angle and meet above the centerline of the house. Each rafter also needs to overhang the side of the house by 2 feet. How long should the carpenter make each rafter?

Assessment Connections

Assessment Item Type(s): Type I, Type II, Type III

Evidence Statement(s): LEAP.I.GM.1, LEAP.I.GM.2, LEAP.II.GM.3, LEAP.III.GM.3, LEAP.III.GM.4

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Similarity in Trigonometry GM: G-SRT.C.6 GM: G-SRT.C.7 GM: G-SRT.C.8	Uses trigonometric ratios, the Pythagorean Theorem, and the relationship between sine and cosine to solve right triangles in applied problems.	Uses trigonometric ratios, the Pythagorean Theorem, and the relationship between sine and cosine to solve right triangles in applied problems.	Uses trigonometric ratios and the Pythagorean Theorem to determine the unknown side lengths and angle measurements of a right triangle.	Uses trigonometric ratios and the Pythagorean Theorem to determine the unknown side lengths of a right triangle.
	Uses similarity transformations with right triangles to define trigonometric ratios for acute angles.			
Modeling and Applying GM: G-SRT.C.7 GM: G-SRT.C.8 GM: G-GPE.B.6 LEAP.I.GM.2	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.
	Applies geometric concepts and trigonometric ratios to describe, model and	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem,	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean	Applies geometric concepts to describe, model and solve applied problems related to geometric

	solve applied problems (including design problems) related to the Pythagorean Theorem, density, geometric shapes, measures, and properties.	geometric shapes, measures, and properties.	Theorem, geometric shapes, measures, and properties.	shapes, measures, and properties.

Geometric Reasoning and Logic: Circles (G-C)

A. Understand and apply theorems about circles. In this cluster, the terms students should learn to use with increasing precision are: proof, theorem, circle, central angle, radii, chords, tangent segment/line, inscribed and circumscribed (angles and figures).

GM: G-C.A.1 Understand that all circles are similar.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: G-SRT.A.2 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Students use the fact that the ratio of diameter to circumference is the same for all circles to prove that all circles are similar. Students use any two circles in a plane and show that they are related by dilation.

Example:

- For this problem, (a,b) is a point in the x-y coordinate plane, and r is a positive number.
    a. Using a translation and a dilation, show how to transform the circle with radius r centered at (a,b) into the circle of radius 1 centered at (0,0).
    b. Explain how to use your work in part (a) to show that any two circles are similar.[37]

[37] https://www.illustrativemathematics.org/content-standards/HSG/C/A/1/tasks/1368

Assessment Connections

Assessment Item Type(s):

Evidence Statement(s):

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Geometric Constructions LEAP.I.GM.3	Understands geometric constructions: copying a segment, copying an angle, bisecting an angle, bisecting a segment, including the perpendicular bisector of a line segment.	Understands geometric constructions: copying a segment, copying an angle, bisecting an angle, bisecting a segment, including the perpendicular bisector of a line segment.	Understands basic geometric constructions: copying a segment, copying an angle, bisecting an angle, bisecting a segment, including the perpendicular bisector of a line segment.	Understands basic geometric constructions: copying a segment, and copying an angle
	Given a line and a point not on the line, uses a variety of tools and methods to construct perpendicular and parallel lines.	Given a line and a point not on the line, constructs perpendicular and parallel lines.		
	Uses a variety of tools and methods to construct equilateral triangles, squares, and hexagons inscribed in circles.			

GM: G-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords, including

- the relationship that exists between central, inscribed, and circumscribed angles;
- inscribed angles on a diameter are right angles; and
- a radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: G-CO.C.10 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

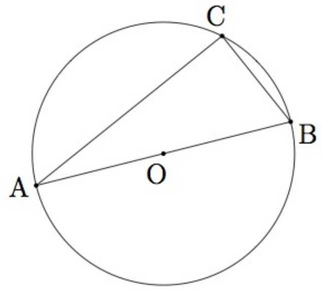
Examples and Explanations

Students can:

- Identify central angles, inscribed angles, circumscribed angles, diameters, radii, chords, and tangents.
- Describe the relationship between a central angle and the arc it intercepts.
- Describe the relationship between an inscribed angle and the arc it intercepts.
- Describe the relationship between a circumscribed angle and the arcs it intercepts.
- Recognize that an inscribed angle whose sides intersect the endpoints of the diameter of a circle is a right angle.
- Recognize that the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Example:

- Suppose AB is a diameter of a circle and C is a point on the circle different from A and B, as in the picture below:

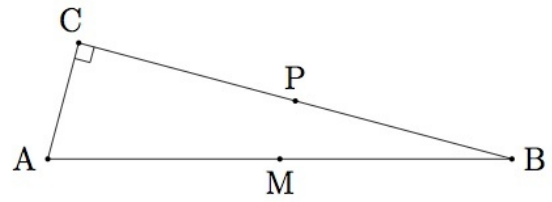


a. Show that triangles COB and COA are both isosceles triangles.

b. Use part (a) and the fact that the sum of the angles in triangle ABC is 180 degrees to show that angle C is a right angle.[38]

Example:

- Suppose ABC is a triangle with angle ACB a right angle. Suppose M is the midpoint of segment AB and P the midpoint of segment BCas pictured below:



a. Show that angle MPB is a right angle.
b. Show that triangle MPB is congruent to triangle MPC.
c. Conclude from (b) that triangle ABC is inscribed in the circle with diameter ABand radius |MA|.[39]

[38] https://www.illustrativemathematics.org/content-standards/HSG/C/A/2/tasks/1091

[39] https://www.illustrativemathematics.org/content-standards/HSG/C/A/2/tasks/1093

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.3, LEAP.I.GM.4, LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Applying Geometric Properties and Theorems GM: G-C.A.2 GM: G-GPE.A.1 LEAP.I.GM.4	Applies properties and theorems of angles, segments and arcs in circles to solve problems and model relationships.	Applies properties and theorems of angles, segments and arcs in circles to solve problems.	Applies properties and theorems of angles, segments and arcs in circles to solve problems.	Applies properties and theorems of angles and segments to solve problems
	Completes the square to find the center and radius of a circle given by an equation.	Completes the square to find the center and radius of a circle given by an equation.		

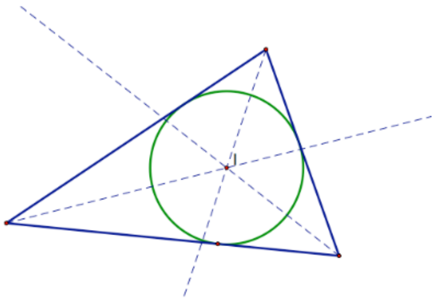
GM: G-C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Standard Overview

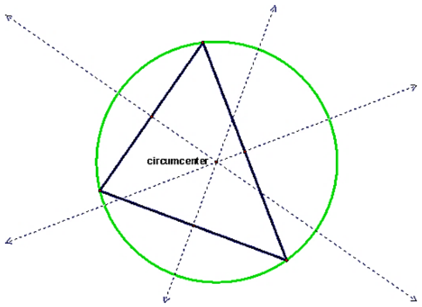
Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: G-C.A.2 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

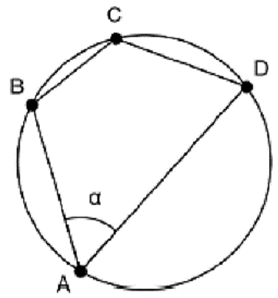
Examples and Explanations

Students construct the inscribed circle whose center is the point of intersection of the angle bisectors (the center).



Students construct the circumscribed circle whose center is the point of intersection of the perpendicular bisectors of each side of the triangle (the circumcenter).





Students prove properties of angles for a quadrilateral inscribed in a circle.

Example:

- Given the inscribed quadrilateral to the right, prove that m∠ B is supplementary to m∠D.

Assessment Connections

Assessment Item Type(s):

Evidence Statement(s):

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Geometric Constructions LEAP.I.GM.3	Understands geometric constructions: copying a segment, copying an angle, bisecting an angle, bisecting a segment, including the perpendicular bisector of a line segment.	Understands geometric constructions: copying a segment, copying an angle, bisecting an angle, bisecting a segment, including the perpendicular bisector of a line segment.	Understands basic geometric constructions: copying a segment, copying an angle, bisecting an angle, bisecting a segment, including the perpendicular bisector of a line segment.	Understands basic geometric constructions: copying a segment, and copying an angle
	Given a line and a point not on the line, uses a variety of tools and methods to construct perpendicular and parallel lines.	Given a line and a point not on the line, constructs perpendicular and parallel lines.		
	Uses a variety of tools and methods to construct equilateral triangles, squares, and hexagons inscribed in circles.			

B. Find arc lengths and areas of sectors of circles. In this cluster, the terms students should learn to use with increasing precision are: arc length, intercepted arc, radian, and area of a sector.

GM: G-C.B.5 Derive and apply the formula for finding area of a sector and arc length.

Standard Overview

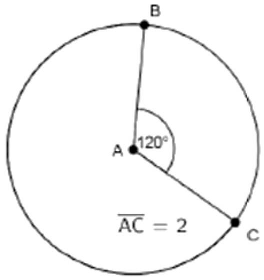
Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: G-CO.B.8, GM: G-C.A.2 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

All circles are similar (G-C.A.1). Sectors with the same central angle have arc lengths that are proportional to the radius. The radian measure of the angle is the constant of proportionality.

Example:

- Find the area of the sectors. What general formula can you develop based on this information?
- Find the area of a sector with an arc length of 40 cm and a radius of 12 cm.



Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.4

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Applying Geometric Properties and Theorems GM: G-C.A.2 GM: G-GPE.A.1 LEAP.I.GM.4	Applies properties and theorems of angles, segments and arcs in circles to solve problems and model relationships.	Applies properties and theorems of angles, segments and arcs in circles to solve problems.	Applies properties and theorems of angles, segments and arcs in circles to solve problems.	Applies properties and theorems of angles and segments to solve problems
	Completes the square to find the center and radius of a circle given by an equation.	Completes the square to find the center and radius of a circle given by an equation.		

Geometric Reasoning and Logic: Expressing Geometric Properties with Equations (G-GPE)

A. Translate between the geometric description and the equation for a conic section. In this cluster, the terms students should learn to use with increasing precision are definition of circle on a coordinate plane, equation of a circle, x-coordinate, y-coordinate, center, and radius

GM: G-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** 8.GL.B.8, A1: A-REI.B.4 **Geometry Standard Taught in Advance:** GM: G-SRT.B.4, GM: G-SRT.C.8 **Geometry Standard Taught Concurrently:** none	Procedural Skill and Fluency

Examples and Explanations

Students define a circle as the set of points whose distance from a fixed point is constant. Given a point on the circle and the fixed point, they identify that the difference in the x-coordinates represent the horizontal distance and the difference in the y-coordinates represent the vertical distance. Students apply the Pythagorean Theorem to calculate the distance between the two points. Generalizing this process, students derive the equation of a circle. Students connect the derivation of the equation of a circle to the distance formula.

Example:

- Write the equation of a circle that is centered at $(-1, 3)$ with a radius of 5 units.
- Write an equation for a circle given that the endpoints of the diameter are $(-2, 7)$ and $(4, -8)$

Assessment Connections

Assessment Item Type(s): Type I

Evidence Statement(s): LEAP.I.GM.3, LEAP.I.GM.4, LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Applying Geometric Properties and Theorems GM: G-C.A.2 GM: G-GPE.A.1 LEAP.I.GM.4	Applies properties and theorems of angles, segments and arcs in circles to solve problems and model relationships.	Applies properties and theorems of angles, segments and arcs in circles to solve problems.	Applies properties and theorems of angles, segments and arcs in circles to solve problems.	Applies properties and theorems of angles and segments to solve problems
	Completes the square to find the center and radius of a circle given by an equation.	Completes the square to find the center and radius of a circle given by an equation.		

B. Use coordinates to prove simple geometric theorems algebraically. In this cluster, the terms students should learn to use with increasing precision are **coordinates**, **coordinate plane**, **theorem**, **prove**, **disprove**, **slope criteria**, **parallel and perpendicular lines**, **directed line segment**, **partition**, **ratio**, **perimeter**, **area**, and **distance formula**.

GM: G-GPE.B.4 Use coordinates to prove geometric theorems algebraically.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.B.8 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** none	Procedural Skill and Fluency

Examples and Explanations

Example:

- Draw a quadrilateral ABCD. Try to draw your quadrilateral so that no two sides are congruent, no two angles are congruent, and no two sides are parallel.

    a. Let P, Q, R, and S be the midpoints of sides AB, BC, CD, and DA, respectively. Use a ruler to locate these points as precisely as you can, and join them to form a new quadrilateral PQRS. What do you notice about the quadrilateral PQRS?
    b. Suppose your quadrilateral ABCD lies in the coordinate plane. Let (x1,y1) be the coordinates of vertex A, (x2,y2) the coordinates of B, (x3,y3) the coordinates of C, and (x4,y4) the coordinates of D. Use coordinates to prove the observation you made in part (a).[40]

[40] https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/4/tasks/605

Assessment Connections

Assessment Item Type(s): Type II
Evidence Statement(s): LEAP.II.GM.1

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.GM.1 LEAP.II.GM.2 LEAP.II.GM.3 LEAP.II.GM.4	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions	using an approach based on a conjecture and/or stated assumptions
	providing an efficient and logical progression of steps or chain of reasoning with appropriate justification	providing a logical progression of steps or chain of reasoning with appropriate justification	providing a logical, but incomplete, progression of steps or chain of reasoning	providing an incomplete or illogical progression of steps or chain of reasoning
	performing precise calculations	performing precise calculations	performing minor calculation errors	making an intrusive calculation error
	using correct grade- level vocabulary, symbols and labels	using correct grade-level vocabulary, symbols and labels	using some grade-level vocabulary, symbols and labels	using limited grade-level vocabulary, symbols and labels
	providing a justification of a conclusion	providing a justification of a conclusion	providing a partial justification of a conclusion based on own calculations	providing a partial justification of a conclusion based on own calculations

	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate) – and providing a counter example where applicable.	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate).	evaluating the validity of others' approaches and conclusions	
	determining whether an argument or conclusion is generalizable			

GM: G-GPE.B.5 Determine the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.AR.B.6, 8.PF.A.3 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students use the formula for the slope of a line to determine whether two lines are parallel or perpendicular. Two lines are parallel if they have the same slope, and two lines are perpendicular if their slopes are opposite reciprocals of each other. In other words, the product of the slopes of lines that are perpendicular is
(-1). Additionally, students find the equations of lines that are parallel or perpendicular, given certain criteria.

Example:

- Suppose a line *k* in a coordinate plane has slope $\frac{c}{d}$.
  a. What is the slope of a line parallel to *k*? Why must this be the case?
  b. What is the slope of a line perpendicular to *k*? Why does this seem reasonable?

Example:

- Two points, $A(0, \ -4)$ $B(2, \ -1)$ determine a line, $\overleftrightarrow{AB}$.
  a. What is the equation of the line AB?
  b. What is the equation of the line perpendicular to $\overleftrightarrow{AB}$. passing through the point $(2, -1)$?

Assessment Connections

Assessment Item Type(s): Type II
Evidence Statement(s): LEAP.II.GM.1

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Reasoning LEAP.II.GM.1 LEAP.II.GM.2 LEAP.II.GM.3 LEAP.II.GM.4	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions, utilizing mathematical connections (when appropriate)	using a logical approach based on a conjecture and/or stated assumptions	using an approach based on a conjecture and/or stated assumptions
	providing an efficient and logical progression of steps or chain of reasoning with appropriate justification	providing a logical progression of steps or chain of reasoning with appropriate justification	providing a logical, but incomplete, progression of steps or chain of reasoning	providing an incomplete or illogical progression of steps or chain of reasoning
	performing precise calculations	performing precise calculations	performing minor calculation errors	making an intrusive calculation error
	using correct grade- level vocabulary, symbols and labels	using correct grade-level vocabulary, symbols and labels	using some grade-level vocabulary, symbols and labels	using limited grade-level vocabulary, symbols and labels
	providing a justification of a conclusion	providing a justification of a conclusion	providing a partial justification of a conclusion based on own calculations	providing a partial justification of a conclusion based on own calculations

	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate) – and providing a counter example where applicable.	evaluating, interpreting, and critiquing the validity of others' responses, approaches and reasoning – using mathematical connections (when appropriate).	evaluating the validity of others' approaches and conclusions	
	determining whether an argument or conclusion is generalizable			

GM: G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

- Apply ratio thinking to find the midpoint of the given line segment.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: G-CO.C.9, GM: G-SRT.A.2 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students' work with ratios began in Grade 6, and their understanding of a ratio will directly impact their ability to master this standard. If students (incorrectly) think a ratio is a fraction, they will likely miss every problem designed to assess this standard. Students must understand a ratio as a part-to-part relationship, not a part-to-whole relationship, as is suggested by a fraction. If a line segment is partitioned into a ratio of
a: b, then the line segment is made up of $a + b$ parts, not b parts. If a student thinks a ratio is a fraction, he/she will also think the latter is true.

Example:

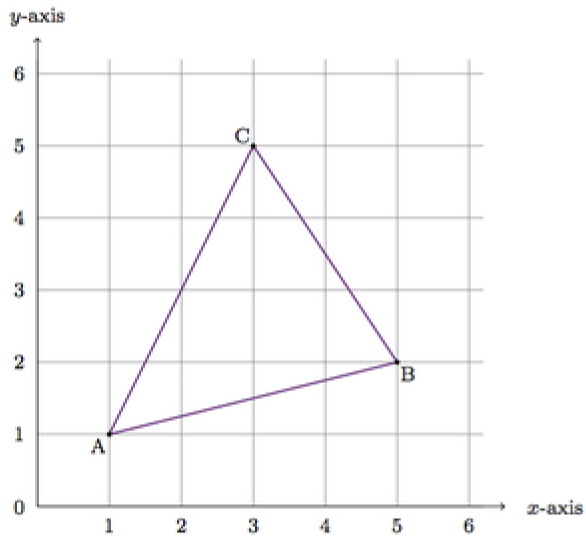
- Given $A(3,\ 2)$ and $B(6,\ 11)$,
  a. Find the point that divides the line segment *AB* two-thirds of the way from *A* to *B*.
  b. Find the midpoint of line segment *AB*.

Example:

- Given directed line segment AB with $A(-\ 1, 2)$ and $B(7, 14)$, find point *P* that partitions the segment into a ratio of 1:3.

Example:

- Below is a picture of △ABC with vertices lying on grid points:



a. Draw the image of △ABC when it is scaled with a scale factor of $\frac{1}{3}$ about the vertex A: label this triangle A′B′C′ and find the coordinates of the points A′, B′, and C′.
b. Draw the image of △ABC when it is scaled with a scale factor of $\frac{2}{3}$ about the vertex B: label this triangle A″B″C″ and find the coordinates of the points A″, B″, and C″.
c. How does A″ compare to B′? Why?[41]

[41] https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/6/tasks/1867

Assessment Connections

Assessment Item Type(s): Type I, Type II, Type III

Evidence Statement(s): LEAP.I.GM.1, LEAP.I.GM.2, LEAP.II.GM.1, LEAP.II.GM.4, LEAP.III.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Modeling and Applying GM: G-SRT.C.7 GM: G-SRT.C.8 GM: G-GPE.B.6 LEAP.I.GM.2	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.
	Applies geometric concepts and trigonometric ratios to describe, model and solve applied problems (including design problems) related to the Pythagorean Theorem, density, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to geometric shapes, measures, and properties.

GM: G-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

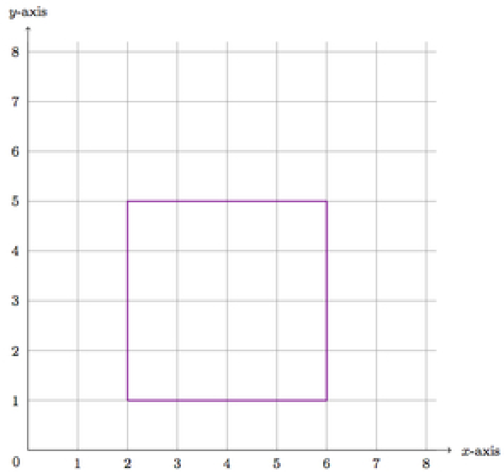
Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 8.GL.B.8 **Geometry Standard Taught in Advance:** GM: G-SRT.B.4 **Geometry Standard Taught Concurrently:** none	Procedural Skill and Fluency

Examples and Explanations
This is a modeling standard, which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential. This standard provides practice with the distance formula and its connection with the Pythagorean Theorem. Students use the coordinates of the vertices of a polygon graphed in the coordinate plane and the distance formula to compute the perimeter and to find the lengths necessary to compute the area.

Example:

- In the picture below, a square is outlined whose vertices lie on the coordinate grid points:

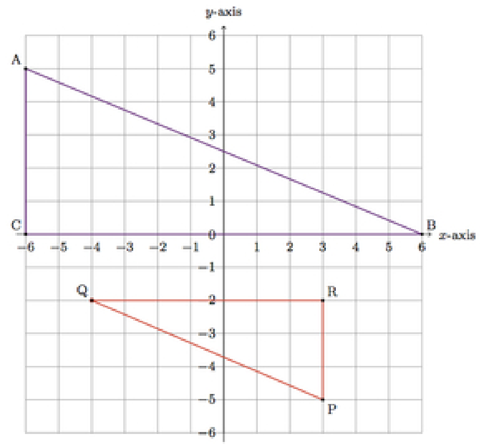


The area of this particular square is 16 square units. For each whole number n between 1 and 10, find a square with vertices on the coordinate grid whose area is n square units or show that there is no such square.[42]

[42] .https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/7/tasks/1684

Example:

- Below is a picture of two triangles with vertices on coordinate grid points:



a. What are the perimeters of △ABC and △PQR?
b. What is the smallest perimeter possible for a triangle with vertices on grid points and with whole-number side lengths? Explain.[43]

43 https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/7/tasks/1816

Assessment Connections

Assessment Item Type(s): Type I, Type II
Evidence Statement(s): LEAP.I.GM.2, LEAP.II.GM.1

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Modeling and Applying GM: G-SRT.C.7 GM: G-SRT.C.8 GM: G-GPE.B.6 LEAP.I.GM.2	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.
	Applies geometric concepts and trigonometric ratios to describe, model and solve applied problems (including design problems) related to the Pythagorean Theorem, density, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to geometric shapes, measures, and properties.

Geometric Reasoning and Logic: Geometric Measurement and Dimension (G-GMD)

A. Explain volume formulas and use them to solve problems. In this cluster, the terms students should learn to use with increasing precision are **informal argument**.

GM: G-GMD.A.1 Give an informal argument, e.g., dissection arguments, Cavalieri's principle, or informal limit arguments for the formulas of:

- circumference of a circle;
- area of a circle; and
- volume of a cylinder, pyramid, and cone.

Standard Overview

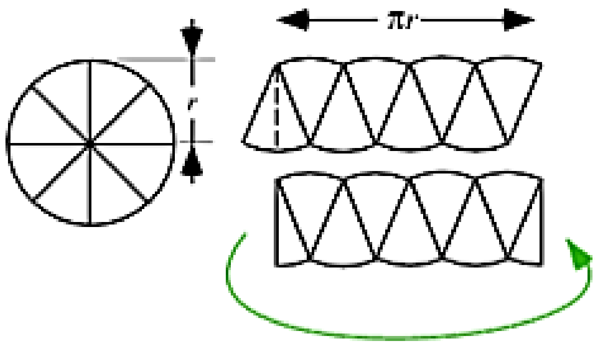
Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** 7.GL.B.4 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Circumference of a circle:
Students begin with the measure of the diameter of the circle or the radius of the circle. They can use string or pipe cleaner to represent the measurement. Next, students measure the distance around the circle using the measure of the diameter. They discover that there are 3 diameters around the circumference with a small gap remaining. Through discussion, students conjecture that the circumference is the length of the diameter π times. Therefore, the circumference can be written as $C = \pi d$. When measuring the circle using the radius, students discover there are 6 radii around the circumference with a small gap remaining. Students conjecture that the circumference is the length of the radius 2π times. Therefore, the circumference of the circle can also be expressed using $C = 2\pi r$.

Area of a circle:

Students may use dissection arguments for the area of a circle. Dissect portions of the circle like pieces of a pie and arrange the pieces into a figure resembling a parallelogram as indicated below. Reason that the base is half of the circumference and the height is the radius. Students use the formula for the area of a parallelogram to derive the area of the circle.



A_{rect} = Base x Height
Area = $\frac{1}{2}(2\pi r) \times r$
Area = $\pi r \times r$
Area = πr^2

Volume of a cylinder:

Students develop the formula for the volume of a cylinder based on the area of a circle stacked over and over again until the cylinder has the given height. Therefore, the formula for the volume of a cylinder is $V = Bh$. This approach is similar to Cavalieri's principle. In Cavalieri's principle, the cross-sections of the cylinder are circles of equal area, which stack to a specific height.

Volume of a pyramid or cone:

For pyramids and cones, the factor $\frac{1}{3}$ will need some explanation. An informal demonstration can be done using a volume relationship set of plastic shapes that permit one to pour liquid or sand from one shape into another. Another way to do this for pyramids is with Geoblocks. The set includes three pyramids with equal bases and altitudes that will stack to form a cube. An algebraic approach involves the formula for the sum of squares $(1^2 + 2^2 + ... + n^2)$. After the coefficient $\frac{1}{3}$ has been justified for the formula of the volume of the pyramid $(A = \frac{1}{3}Bh)$, one can argue that it must also apply to the formula of the volume of the cone by considering a cone to be a pyramid that has a base with infinitely many sides.

Assessment Connections

Assessment Item Type(s): Type I

Evidence Statement(s): LEAP.I.GM.3, LEAP.I.GM.4, LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Geometric Formulas GM: G-GMD.A.1 GM: G-GMD.A.3 GM: G-GMD.B.4	Uses volume formulas to solve mathematical and contextual problems that involve cylinders, pyramids, cones, and spheres.	Using formulas, determines the volume of cylinders, pyramids, cones, and spheres.	Using formulas, determines the volume of cylinders, pyramids, cones, and spheres.	Using formulas, determines the volume of cylinders, pyramids, cones, and spheres.
	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects and identifies three-dimensional objects generated by rotations of two-dimensional objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects, when cross sections are parallel or perpendicular to a base/face.
	Uses dissection arguments, Cavalieri's principle, and informal limit arguments to support the formula for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	Gives an informal argument for the formula for the circumference of a circle and area of a circle, including dissection arguments.		

GM: G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Standard Overview

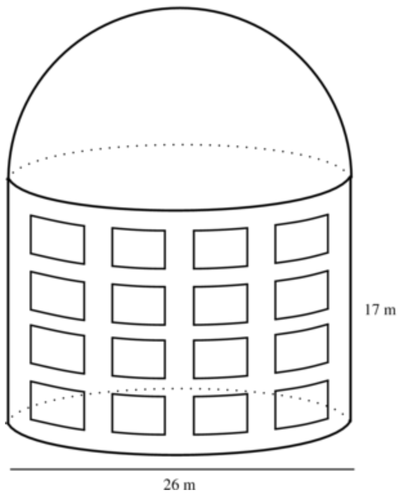
Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** 8.GL.C.9 **Geometry Standard Taught in Advance:** GM: G-GMD.A.1 **Geometry Standard Taught Concurrently:** none	Procedural Skill and Fluency, Application

Examples and Explanations

This is a modeling standard, which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Example:

- The Southern African Large Telescope (SALT) is housed in a cylindrical building with a domed roof in the shape of a hemisphere. The height of the building wall is 17 m, and the diameter is 26 m. To program the ventilation system for heat, air conditioning, and dehumidification, the engineers need the amount of space in the building. What is the volume, in cubic meters, of space in the building?



Assessment Connections

Assessment Item Type(s): Type I, Type II, Type III

Evidence Statement(s): LEAP.I.GM.3, LEAP.I.GM.4, LEAP.I.GM.5, LEAP.II.GM.3, LEAP.II.GM.4, LEAP.III.GM.4, LEAP.III.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Geometric Formulas GM: G-GMD.A.1 GM: G-GMD.A.3 GM: G-GMD.B.4	Uses volume formulas to solve mathematical and contextual problems that involve cylinders, pyramids, cones, and spheres.	Using formulas, determines the volume of cylinders, pyramids, cones, and spheres.	Using formulas, determines the volume of cylinders, pyramids, cones, and spheres.	Using formulas, determines the volume of cylinders, pyramids, cones, and spheres.
	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects and identifies three-dimensional objects generated by rotations of two-dimensional objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects, when cross sections are parallel or perpendicular to a base/face.
	Uses dissection arguments, Cavalieri's principle, and informal limit arguments to support the formula for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	Gives an informal argument for the formula for the circumference of a circle and area of a circle, including dissection arguments.		

B. Visualize relationships between two-dimensional and three-dimensional objects. In this cluster, the terms students should learn to use with increasing precision are **cross section**, **two dimensional**, **three dimensional**, **generate**, and **rotation**.

GM: G-GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

- Apply properties of two-dimensional figures identified to solve problems.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** 7.GL.A.3 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

In this cluster, the terms students should learn to use with increasing precision are cross-section, two-dimensional, three-dimensional, generate, and rotation.

Examples and Explanations

Students identify shapes of two-dimensional cross-sections of three-dimensional objects. The Cross Section Flyer at http://www.shodor.org/interactivate/activities/CrossSectionFlyer/ can be used to allow students to predict and verify the cross-section of different three-dimensional objects.

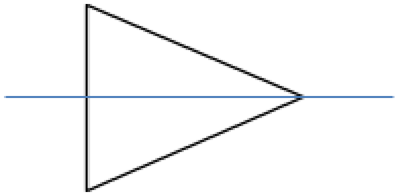
Example:

- Identify two-dimensional cross sections of a rectangular prism.

Students identify three-dimensional objects generated by rotations of two-dimensional objects. The 3D Transmographer at http://www.shodor.org/interactivate/activities/3DTransmographer/ can be used to allow students to predict and verify three-dimensional objects generated by rotations of two-dimensional objects.

Example:

- Identify the object generated when the following object is rotated about the indicated line.



Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.3, LEAP.I.GM.4, LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Geometric Formulas GM: G-GMD.A.1 GM: G-GMD.A.3 GM: G-GMD.B.4	Uses volume formulas to solve mathematical and contextual problems that involve cylinders, pyramids, cones, and spheres.	Using formulas, determines the volume of cylinders, pyramids, cones, and spheres.	Using formulas, determines the volume of cylinders, pyramids, cones, and spheres.	Using formulas, determines the volume of cylinders, pyramids, cones, and spheres.
	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects and identifies three-dimensional objects generated by rotations of two-dimensional objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects.	Identifies the shapes of two-dimensional cross-sections of three-dimensional objects, when cross sections are parallel or perpendicular to a base/face.
	Uses dissection arguments, Cavalieri's principle, and informal limit arguments to support the formula for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	Gives an informal argument for the formula for the circumference of a circle and area of a circle, including dissection arguments.		

Geometric Reasoning and Logic: Modeling with Geometry (G-MG)

A. Apply geometric concepts in modeling situations. In this cluster, the terms students should learn to use with increasing precision are **mathematical model**, **density**, and **design problem**.

GM: G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 6.GL.A.4, 7.GL.B.6 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Students recognize situations that require relating two- and three- dimensional objects. They estimate measures (circumference, area, perimeter, volume) of real-world objects using comparable geometric shapes or three-dimensional objects. Students apply the properties of geometric figures to comparable real-world objects (e.g., The spokes of a wheel of a bicycle are equal lengths because they represent the radii of a circle).

Example:

- Describe each of the following as a simple geometric shape or combination of shapes. Illustrate with a sketch and label dimensions important to describing the shape.
    a. Soup can label
    b. A bale of hay
    c. Paperclip
    d. Strawberry

Assessment Connections

Assessment Item Type(s): Type 1
Evidence Statement(s): LEAP.I.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Modeling and Applying GM: G-SRT.C.7 GM: G-SRT.C.8 GM: G-GPE.B.6 LEAP.I.GM.2	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.
	Applies geometric concepts and trigonometric ratios to describe, model and solve applied problems (including design problems) related to the Pythagorean Theorem, density, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to geometric shapes, measures, and properties.

GM: G-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 7.GL.B.6, 8.GL.C.9 **Geometry Standard Taught in Advance:** GM: G-MG.A.1, GM: G-MG.A.3 **Geometry Standard Taught Concurrently:** none	Application

In this cluster, the terms students should learn to use with increasing precision are mathematical model, density, and design problem.

Examples and Explanations

Example:

- An antique waterbed has the following dimensions: 72 in. x 84 in. x 9.5in. It takes 240.7 gallons of water to fill it, which would weigh 2071 pounds. What is the weight of a cubic foot of water?

Example:

- Wichita, Kansas has 344,234 people within 165.9 square miles. What is Wichita's population density?

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Modeling and Applying GM: G-SRT.C.7 GM: G-SRT.C.8 GM: G-GPE.B.6 LEAP.I.GM.2	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.
	Applies geometric concepts and trigonometric ratios to describe, model and solve applied problems (including design problems) related to the Pythagorean Theorem, density, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to geometric shapes, measures, and properties.

GM: G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Major Content	**Remediation - Previous Grade(s) Standard:** 7.GL.A.1, 7.GL.B.6, 8.GL.C.9 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** none	Application

Examples and Explanations

Example:

- You have been hired by the owner of a local ice cream parlor to assist in his company's new venture. The company will soon sell its ice cream cones in the freezer section of local grocery stores. The manufacturing process requires that the ice cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disc covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones. Use a real ice cream cone or the dimensions of a real ice cream cone to complete the following tasks.

  a. Sketch a wrapper like the one described above, using the actual size of your cone. Ignore any overlap required for assembly.
  b. Use your sketch to help you develop an equation the owner can use to calculate the surface area of a wrapper (including the lid) for another cone given its base had a radius of length, r, and a slant height, s.
  c. Using measurements of the radius of the base and slant height of your cone, and your equation from the previous step, find the surface area of your cone.
  d. The company has a large rectangular piece of paper that measures 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit your cone that could be cut from this one piece of paper. Explain your estimate.[44]

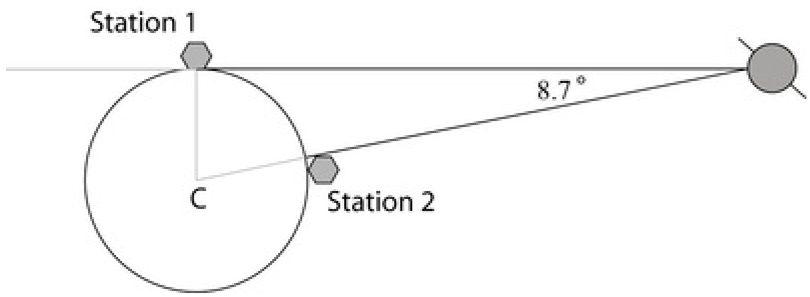
Example:

- A satellite orbiting the Earth uses radar to communicate with two control stations on the Earth's surface. The satellite is in a geostationary orbit. That means that the satellite is always on the line through the center of the Earth and control station 2.

[44] https://www.illustrativemathematics.org/content-standards/HSG/MG/A/3/tasks/414

From the perspective of Station 1, the satellite is on the horizon, and from the perspective of Station 2, the satellite is always directly overhead, as in the following diagram. The angle between the lines from the satellite to the stations is 8.7◦.

Assuming the Earth is a sphere with a radius of 3963 miles, answer the following questions. Round all answers to the nearest whole number.



a. How many miles will a signal sent from Station 1 to the satellite and then to Station 2 have to travel? Explain your answer.
b. A satellite technician is flying from one station to the other in a direct path along the Earth's surface. What is the distance she will have to fly? If she travels an average of 300 mph, how long will the trip take? Explain your answer.
c. If a signal could travel through the earth's surface from one station to the other, what is the shortest distance the signal could travel to get from Station 1 to Station 2? Explain your answer.[45]

[45] https://www.illustrativemathematics.org/content-standards/HSG/MG/A/3/tasks/416

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.2

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Modeling and Applying GM: G-SRT.C.7 GM: G-SRT.C.8 GM: G-GPE.B.6 LEAP.I.GM.2	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses geometric relationships in the coordinate plane to solve problems involving area, perimeter, and ratios of lengths.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.	Uses provided geometric relationships in the coordinate plane to solve problems involving area and perimeter.
	Applies geometric concepts and trigonometric ratios to describe, model and solve applied problems (including design problems) related to the Pythagorean Theorem, density, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to the Pythagorean Theorem, geometric shapes, measures, and properties.	Applies geometric concepts to describe, model and solve applied problems related to geometric shapes, measures, and properties.

Statistics & Probability: Conditional Probability and the Rules of Probability (S-CP)

A. Understand independence and conditional probability and use them to interpret data. In this cluster, the terms students should learn to use with increasing precision are **sample space**, **event**, **outcome**, **union**, **intersection**, **complement of an event**, **independent event**, **probability of an event**, **conditional probability**, **frequency**, and **two-way frequency table**.

GM: S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics or categories of the outcomes, or as unions, intersections, or complements of other events "or," "and," "not".

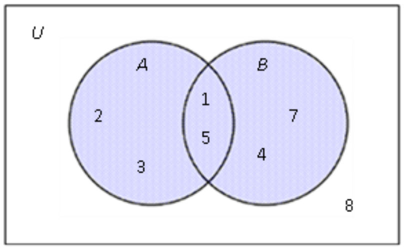
Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** 7.DA.C.8 **Geometry Standard Taught in Advance:** none **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Intersection: The intersection of two sets *A* and *B* is the set of elements that are common to both set *A* and set *B*. It is denoted by $A \cap B$ and is read '*A* intersection *B*.'

- $A \cap B$ in the diagram is {1, 5}
- this means: BOTH/AND



Union: The union of two sets *A* and *B* is the set of elements, which are in *A* or in *B* or in both. It is denoted by $A \cup B$ and is read '*A* union *B*.'

$A \cup B$ in the diagram is {1, 2, 3, 4, 5, 7}

- this means: EITHER/OR/ANY
- *could* be both

Complement: The complement of the set $A \cup B$ is the set of elements that are members of the universal set U but are not in $A \cup B$. It is denoted by $(A \cup B)'$.

$(A \cup B)'$ in the diagram is {8}.

Students define a sample space and events within the sample space. The sample space is the set of all possible outcomes of an experiment. Students describe sample spaces using a variety of different representations.

Example:

- Describe the sample space for rolling two number cubes. *Note: This may be modeled well with a 6x6 table with the rows labeled for the first event and the columns labeled for the second event.*

Example:

- Describe the sample space for picking a colored marble from a bag with red and black marbles. *Note: This may be modeled with set notation.*

Example:

- Andrea is shopping for a new cellphone. She is either going to contract with Company A (60% chance) or with Company B (40% chance). She must choose between phone Q (25% chance) or phone R (75% chance). Describe the sample space. *Note: This may be modeled well with an area model.*

Example:

- The 4 aces are removed from a deck of cards. A coin is tossed, and one of the aces is chosen. Describe the sample space. *Note: This may be modeled well with a tree diagram.*

Students establish events as subsets of a sample space. An event is a subset of a sample space.

Example:

- Describe the event of rolling two number cubes and getting evens.

Example:

- Describe the event of pulling two marbles from a bag of red/black marbles.

Example:

- Describe the event that the sum of two rolled number cubes is larger than 7 and even, and contrast it with the event that the sum is larger than 7 or even.

Assessment Connections

Assessment Item Type(s): Type I

Evidence Statement(s): LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Probability LEAP.I.GM.5	Recognizes, determines and uses conditional probability and independence in multi-step contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes, determines and uses conditional probability and independence in contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes and determines conditional probability and independence in contextual problems.	Recognizes and determines independence in contextual problems.
	Applies the Addition Rule of probability.			

GM: S-CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: S-CP.A.1 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency

Examples and Explanations

Students understand that two events A and B are independent when the probability that one event occurs in no way affects the probability of the other event occurring. In other words, the probability of A is the same even if event B has occurred. If events are independent, then the $P(A \cap B) = P(A) \cdot P(B)$

Example:

- Determine if the events are independent or not. Explain your reasoning.
    a. Flipping a coin and getting heads and rolling a number cube and getting a 4.
    b. When rolling a pair of number cubes, consider the events: getting a sum of 7 and getting doubles.
    c. From a standard deck of cards, consider the events: draw a diamond and draw an ace.

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Probability LEAP.I.GM.5	Recognizes, determines and uses conditional probability and independence in multi-step contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes, determines and uses conditional probability and independence in contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes and determines conditional probability and independence in contextual problems.	Recognizes and determines independence in contextual problems.
	Applies the Addition Rule of probability.			

GM: S-CP.A.3 Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B.*

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: S-CP.A.1, GM: S-CP.A.2 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Students understand conditional probability as the probability of A occurring given that B has occurred.

Example:

- What is the probability that the sum of two rolled number cubes is 6, given that you rolled doubles?

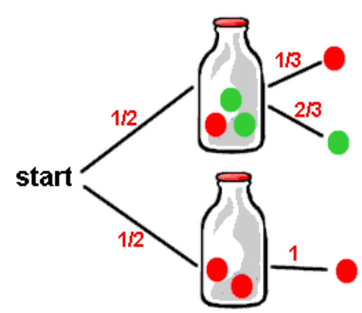
		Curfew		
		Yes	No	Total
Chores	Yes	51	24	75
	No	30	12	42
	Total	81	36	117

Example:

- Each student in the junior class was asked if they had to complete chores at home and if they had a curfew. The table represents the data.
  a. What is the probability that a student who has chores also has a curfew?
  b. What is the probability that a student who has a curfew also has chores?
  c. Are the two events, having chores and having a curfew, independent? Explain.

Example:

- There are two identical bottles. A bottle is selected at random, and a single ball is drawn. Use the tree diagram at the right to determine each of the following:
  a. $P\ (red|bottle\ 1)$
  b. $P\ (red|bottle\ 2)$



Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Probability LEAP.I.GM.5	Recognizes, determines and uses conditional probability and independence in multi-step contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes, determines and uses conditional probability and independence in contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes and determines conditional probability and independence in contextual problems.	Recognizes and determines independence in contextual problems.
	Applies the Addition Rule of probability.			

GM: S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** A1: S-ID.B.5 **Geometry Standard Taught in Advance:** GM: S-CP.A.2, GM: S-CP.A.3 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency, Application

Examples and Explanations

Students:
- Determine when a two-way frequency table is an appropriate display for a set of data.
- Collect data from a random sample.
- Construct a two-way frequency table for the data using the appropriate categories for each variable.
- Calculate probabilities from the table.
- Use probabilities from the table to evaluate the independence of two variables.

Example:

- Each student in a random sample of seniors at a local high school participated in a survey. These students were asked to indicate their gender and their eye color. The following table summarizes the results of the survey.

		Eye Color			
		Brown	**Blue**	**Green**	**Total**
Gender	**Male**	50	40	20	110
	Female	40	40	10	90
	Total	90	80	30	200

a. Suppose that one of these seniors is randomly selected. What is the probability that the selected student is a male?

b. Suppose that one of these seniors is randomly selected. What is the probability that the selected student has blue eyes?
c. Suppose that one of these seniors is randomly selected. What is the probability that the selected student is a male and has blue eyes?
d. Suppose that one of these seniors is randomly selected. What is the probability that the selected student is a male or has blue eyes?
e. Suppose that one of these seniors is randomly selected. What is the probability that the selected student has blue eyes, given that the student is male?
f. Suppose that one of these seniors is randomly selected. What is the probability that the selected student is a male, given that the student has blue eyes?[46]

[46] https://www.illustrativemathematics.org/content-standards/HSS/CP/A/4/tasks/2045

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Probability LEAP.I.GM.5	Recognizes, determines and uses conditional probability and independence in multi-step contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes, determines and uses conditional probability and independence in contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes and determines conditional probability and independence in contextual problems.	Recognizes and determines independence in contextual problems.
	Applies the Addition Rule of probability.			

GM: S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: S-CP.A.3 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding

Examples and Explanations

Example:

- On school days, Janelle sometimes eats breakfast and sometimes does not. After studying probability for a few days, Janelle says, "The events 'I eat breakfast' and 'I am late for school' are independent." Explain what this means in terms of the relationship between Janelle eating breakfast and her probability of being late for school in language that someone who hasn't taken statistics would understand.[47]

[47] https://www.illustrativemathematics.org/content-standards/HSS/CP/A/5/tasks/1019

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Probability LEAP.I.GM.5	Recognizes, determines and uses conditional probability and independence in multi-step contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes, determines and uses conditional probability and independence in contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes and determines conditional probability and independence in contextual problems.	Recognizes and determines independence in contextual problems.
	Applies the Addition Rule of probability.			

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model. In this cluster, the terms students should learn to use with increasing precision are **conditional probability**, **Addition Rule**, **mutually exclusive**, and **disjoint**.

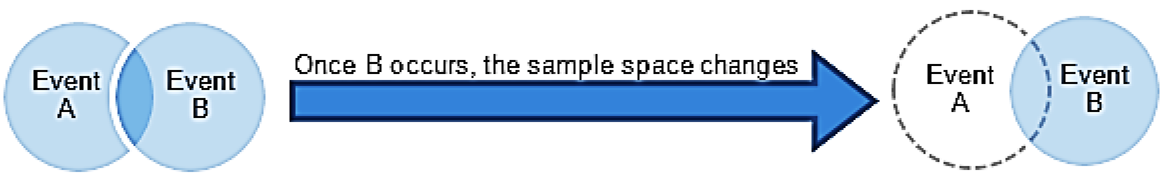
GM: S-CP.B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: S-CP.A.3 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency, Application

Examples and Explanations

The sample space of an experiment can be modeled with a Venn diagram such as:



So, the $P(B) = \frac{P(A \text{ and } B)}{P(B)}$

Example:

- Peter has a bag of marbles. In the bag are 4 white marbles, 2 blue marbles, and 6 green marbles. Peter randomly draws one marble, sets it aside, and then randomly draws another marble. What is the probability of Peter drawing out two green marbles? *Note: Students must recognize that this is a conditional probability, P(green | green).*

Example:

- A teacher gave her class two quizzes. 30% of the class passed both quizzes, and 60% of the class passed the first quiz. What percent of those who passed the first quiz also passed the second quiz?

Example:

- If a balanced tetrahedron with faces 1, 2, 3, 4 is rolled twice, what is the probability that the sum is prime (A) of those that show a 3 on at least one roll (B)?

Assessment Connections

Assessment Item Type(s): Type I
Evidence Statement(s): LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Probability LEAP.I.GM.5	Recognizes, determines and uses conditional probability and independence in multi-step contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes, determines and uses conditional probability and independence in contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes and determines conditional probability and independence in contextual problems.	Recognizes and determines independence in contextual problems.
	Applies the Addition Rule of probability.			

GM: S-CP.B.7 Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B),* and interpret the answer in terms of the model.

Standard Overview

Core Tenets		
Focus	**Coherence**	**Rigor**
Additional Content	**Remediation - Previous Grade(s) Standard:** none **Geometry Standard Taught in Advance:** GM: S-CP.A.1, GM: S-CP.A.3 **Geometry Standard Taught Concurrently:** none	Conceptual Understanding, Procedural Skill and Fluency, Application

Examples and Explanations

Students understand that the $P\ (A\ or\ B)\ =\ P\ (A)\ +\ P\ (B)\ -\ P\ (A\ and\ B)$. Students may recognize that if two events *A* and *B* are mutually exclusive, also called *disjoint*, the rule can be simplified to $P\ (A\ or\ B)\ =\ P\ (A)\ +\ P\ (B)$ since for mutually exclusive events $P\ (A\ and\ B)\ =\ 0$.

Example:

- In a math class of 32 students, 18 are boys and 14 are girls. On a unit test, 5 boys and 7 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

Example:

- At Mom's diner, everyone drinks coffee. Let C= the event that a randomly-selected customer puts cream in their coffee. Let S= the event that a randomly-selected customer puts sugar in their coffee. Suppose that after years of collecting data, Mom has estimated the following probabilities:

  P(C)=0.6

  P(S)=0.5

  P(C or S)=0.7

  Estimate P(C and S) and interpret this value in the context of the problem.[48]

Example:

a. Today there is a 55% chance of rain, a 20% chance of lightning, and a 15% chance of lightning and rain together. Are the two events "rain today" and "lightning today" independent events? Justify your answer.

[48] https://www.illustrativemathematics.org/content-standards/HSS/CP/B/7/tasks/1024

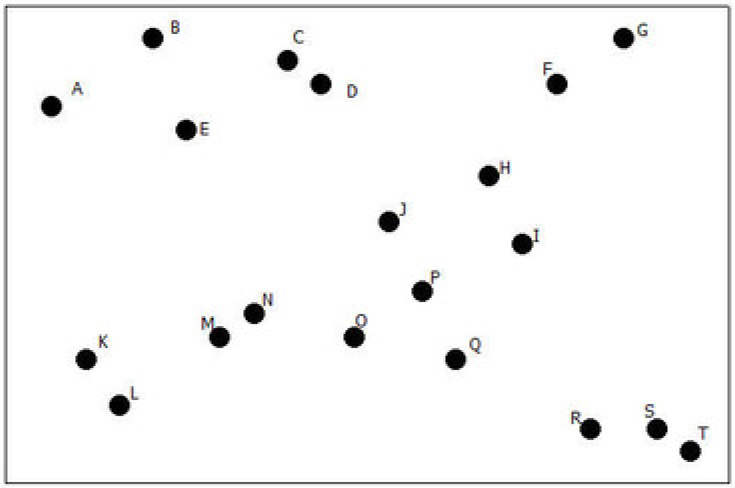
b. Now suppose that today there is a 60% chance of rain, a 15% chance of lightning, and a 20% chance of lightning if it's raining. What is the chance of both rain and lightning today?
c. Now suppose that today there is a 55% chance of rain, a 20% chance of lightning, and a 15% chance of lightning and rain. What is the chance that we will have rain or lightning today?
d. Now suppose that today there is a 50% chance of rain, a 60% chance of rain or lightning, and a 15% chance of rain and lightning. What is the chance that we will have lightning today?[49]

Example:

- Consider the population consisting of the students at a high school who are members of the student council.

  A diagram can be used to visualize this population. To construct such a diagram, a rectangle is drawn to represent the population, and each student on the student council is represented by a labeled point in the rectangle.

  Here is the diagram for this population:



a. How many students are on this student council?
b. The given diagram also represents the sample space for the chance experiment of selecting a student at random from this population. For this chance experiment, what is the probability that student K is selected? Justify your answer.
c. In a diagram, an event can be represented by a shape (often a circle or oval) that encloses the outcomes that make up the event. Consider the following events:

F= the event that the selected student is a freshman

S = the event that the selected student is a sophomore

[49] https://www.illustrativemathematics.org/content-standards/HSS/CP/B/7/tasks/1112

J = the event that the selected student is a junior

Which of the following diagrams could be a representation of these three events? Explain your choice.

Diagram 1

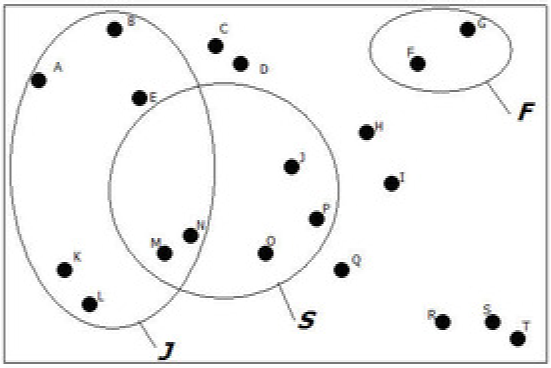
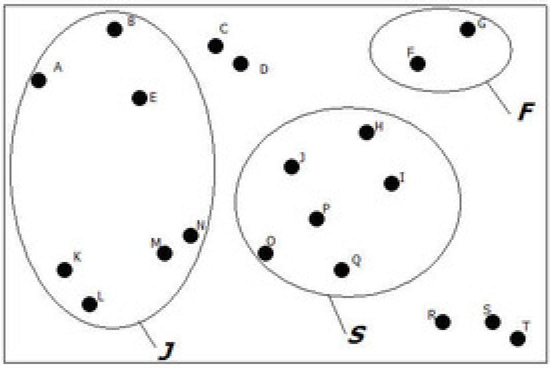


Diagram 2



Use Diagram 2 above to answer questions d – f.

d. If a student is selected at random from the council, what is P(J), the probability that the selected student is a junior? Justify your answer.
e. Is P(F) = P(J)? Explain why or why not.
f. Use Diagram 2 to find the following probabilities:

a.P(F)

b.P(S)

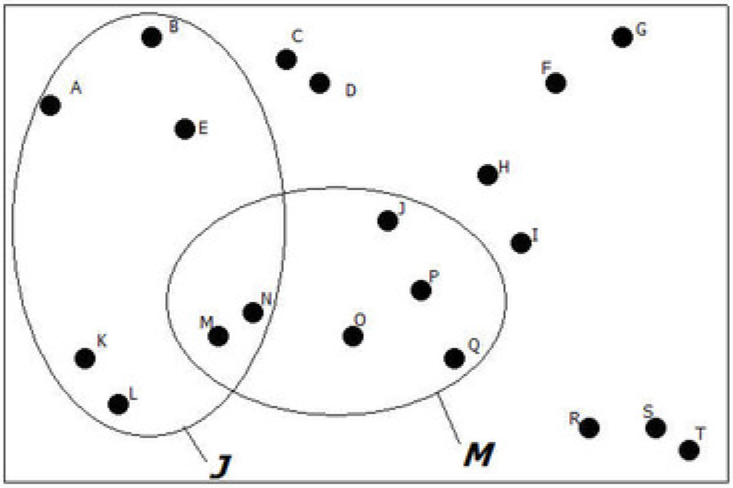
c.P(F or S)

Now consider Diagram 3 shown below. This diagram contains the two events

J = event that the selected student is a junior

M = event that the selected student is male

Diagram 3



g. Shade the region where the two ovals overlap. What event is represented by this shaded region?
h. Use Diagram 3 to find the following probabilities:

a.P(J)

b.P(M)

c.P(J or M)

d.P(J and M)

i. Explain why P(F or S) = P(F) + P(S)(see question 6) but P(J or M) ≠ P(J) + P(M).
j. Can you come up with a way to calculate P(J or M) using the other three probabilities calculated in question h? Write this as a formula in the form

P(J or M) =

k. Explain why your formula in question j makes sense in terms of Diagram 3.
l. In general, suppose A and B are two events. Write a formula for calculating P(A or B) in each of the following two cases:

Case 1:A and B are mutually exclusive (A and B have no outcomes in common)

Case 2:A and B are not mutually exclusive

m. The formula for Case 2 in question l is called the General Addition Rule. This rule also works for calculating P(A or B) when A and B are mutually exclusive. Explain why this is the case.
n. Write a sentence or two explaining how to calculate the probability of the union of two events.[50]

[50] .https://www.illustrativemathematics.org/content-standards/HSS/CP/B/7/tasks/1885

Assessment Connections

Assessment Item Type(s): Type I

Evidence Statement(s): LEAP.I.GM.5

Achievement Level Descriptor				
Content	**Advanced**	**Mastery**	**Basic**	**Approaching Basic**
Probability LEAP.I.GM.5	Recognizes, determines and uses conditional probability and independence in multi-step contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes, determines and uses conditional probability and independence in contextual problems, using appropriate set language and appropriate representations, including two-way frequency tables.	Recognizes and determines conditional probability and independence in contextual problems.	Recognizes and determines independence in contextual problems.
	Applies the Addition Rule of probability.			

Cross Content Connections

Mathematics supports understanding and skills across all disciplines. The ability to apply mathematics learning across disciplines allows students to transform abstract concepts into practical tools, deepen engagement, think critically, and apply mathematics to the real world.

English Language Arts

The connection between math and English language arts (ELA) begins in disciplinary literacy, which requires literacy skills in technical subjects, including the ability to decode complex word problems as narrative structures. Students must master the language of math by using ELA skills like reading comprehension and argumentative writing to explain their reasoning, prove theorems, and articulate the logic behind their numerical solutions.

Science

K-12 math and science are deeply integrated, with mathematics serving as the essential language that allows students to quantify, model, and analyze real-world phenomena they observe in science. As students progress, they use mathematics and computational thinking, ranging from simple data graphing to complex algebraic modeling, to turn raw scientific observations into evidence-based conclusions and predictable patterns. Explicit connections between the LSSM and Louisiana Student Standards for Science (LSSS) can be found on the Connections to K-12 ELA and Math document within the LSSS files.

Computer Science

The integration of mathematics and computer science is codified through state standards that treat computational thinking as a practical application of algebraic logic and problem-solving. Students use mathematical foundations, such as functions and data analysis, to build algorithms and write code that solves real-world challenges in Louisiana's growing technology and energy sectors. The Louisiana CS Frameworks make explicit connections and embed technology tools such as artificial intelligence to support students as they engage in math and computer science learning. Students can strategically use artificial intelligence tools to explore math ideas, test conjectures, analyze patterns and trends, and refine their thinking. The appropriate and skillful use of artificial intelligence in math requires students to use their mathematical reasoning to validate and critique artificial intelligence outputs in addition to naming how the use of artificial intelligence supports and extends their thinking, understanding, and reasoning about concepts, ensuring the student maintains ownership of the mathematics, and leverages artificial intelligence as a resource.

Social Studies

The intersection of math and social studies is most evident in the analysis of historical data and economic trends, allowing students to quantify the impact of events such as the Great Flood of 1927 or the evolution of Louisiana's energy industry. By applying statistical tools to demographic shifts and geographic mapping, students move beyond historical content standards to demonstrate understanding of the skills and practices standards for social studies. Identifying and interpreting mathematical patterns strengthens students' ability to recognize trends that have shaped the geographical, economic, political, and cultural landscape of our state, nation, and world.

Leveraging Lower Grade Standards to Support Student Learning

Grade 4 Standards

○ 4.DM.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle.
b. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
c. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

Return to ◻ GM: G-CO.A.1

○ 4.DM.C.6 Measure angles in whole-number degrees using a standard 180° protractor. *Return to* ◻ GM: G-CO.D.12

○ 4.DM.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.

a. Solve addition and subtraction real-world mathematical tasks to find unknown angles on a diagram, e.g., by using an equation with a letter for the unknown angle measure. *Return to* ■ GM: G-CO.C.9

○ 4.GL.A.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. *Return to* ◻ GM: G-CO.A.1

○ 4.GL.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.

a. Recognize right triangles as a category, and identify right triangles. *Return to* ◻ GM: G-CO.A.1

Grade 5 Standards

○ 5.GL.B.3 Analyze and relate attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *Return to* ■ GM: G-CO.C.11

Grade 6 Standards

6.GL.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. *Return to* GM: G-MG.A.1

Grade 7 Standards

7.GL.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. *Return to* GM: G-MG.A.3

7.GL.A.2 Construct triangles with given conditions. Understand the possible side lengths and angle measures that determine one and only one triangle, more than one triangle, or no triangle.
Return to GM: G-CO.C.10, GM: G-CO.D.12

7.GL.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. *Return to* GM: G-GMD.B.4

7.GL.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. *Return to* GM: G-GMD.A.1

7.GL.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. *Return to* GM: G-CO.C.9, GM: G-SRT.C.7

7.GL.B.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (Pyramids limited to surface area only.) *Return to* GM: G-MG.A.1, GM: G-MG.A.2, GM: G-MG.A.3

7.DA.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
c. Design and use a simulation to generate frequencies for compound events.
    - *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Return to GM: S-CP.A.1

Grade 8 Standards

■ 8.AR.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*. *Return to* ■ GM: G-GPE.B.5

■ 8.PF.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; categorize functions as linear or nonlinear when given equations, graphs, or tables.

- *For example, the function $A = s^2$ giving the area of a square as a function of its side length, is not linear because its graph contains the points (1, 1), (2, 4), and (3, 9), which are not on a straight line. Return to* ■ GM: G-GPE.B.5

■ 8.GL.A.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

Return to □ GM: G-CO.A.2, □ GM: G-CO.A.4

■ 8.GL.A.2 Explain that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Rotations are only about the origin, and reflections are only over the *y*-axis and *x*-axis in Grade 8.) *Return to* □ GM: G-CO.A.2, □ GM: G-CO.A.3, □ GM: G-CO.A.5, ■ GM: G-CO.B.6, ■ GM: G-CO.B.7, ■ GM: G-CO.B.8

■ 8.GL.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (Rotations are only about the origin, dilations only use the origin as the center of dilation, and reflections are only over the *y*-axis and *x*-axis in Grade 8.)
Return to □ GM: G-CO.A.2, □ GM: G-CO.A.3, □ GM: G-CO.A.4, □ GM: G-CO.A.5

■ 8.GL.A.4 Explain that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Rotations are only about the origin, dilations only use the origin as the center of dilation, and reflections are only over the *y*-axis and *x*-axis in Grade 8.)
Return to □ GM: G-CO.A.2, ■ GM: G-SRT.A.1, ■ GM: G-SRT.A.2, ■ GM: G-SRT.A.3

■ 8.GL.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

- *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. Return to* ■ GM: G-CO.C.9, ■ GM: G-CO.C.10, ■ GM: G-SRT.A.3

■ 8.GL.B.6 Explain a proof of the Pythagorean Theorem and its converse using the area of squares. *Return to* ■ GM: G-SRT.B.4

■ 8.GL.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. *Return to* ■ GM: G-SRT.C.8

■ 8.GL.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. *Return to* ○ GM: G-GPE.A.1, ■ GM: G-GPE.B.4. ■ GM: GPE.B.7

○ 8.GL.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. *Return to* ○ GM: G-GMD.A.3, ■ GM: G-MG.A.2, ■ GM: G-MG.A.3

Algebra I Course Standards

■ A1: A-REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

Return to ○ GM: G-GPE.A.1

○ A1: F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative). Without technology, find the value of k given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where $f(x)$ is a linear, quadratic, piecewise linear (to include absolute value) or exponential function.
Return to □ GM: G-CO.A.2

□ A1: S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. *Return to* ○ GM: S-CP.A.4