GRADE-LEVEL EXPECTATIONS (GLE) HANDBOOK

MATHEMATICS
GRADES 5–8

LOUISIANA DEPARTMENT OF EDUCATION

CECIL J. PICARD
STATE SUPERINTENDENT OF EDUCATION

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GRADE-LEVEL EXPECTATIONS HANDBOOK OVERVIEW

INTRODUCTION

In 1997, rigorous K–12 content standards were approved for mathematics, English language arts, science, social studies, foreign languages, and the arts. In conjunction with the development of these content standards, the Louisiana Department of Education (LDE) developed standards-based tests in English language arts, mathematics, science and social studies for grades 4, 8, 10, and 11. These tests have served as the basis for Louisiana's School and District Accountability System for several years. The development of Grade-Level Expectations (GLEs) in 2003 in English language arts, mathematics, science and social studies was a continuation of Louisiana's effort to expand and extend the content standards. GLEs identify what all students should know or be able to do by the end of a given grade level from prekindergarten through grade 12 in these four content areas.

PURPOSES OF THE GLE HANDBOOKS

The Louisiana Grade-Level Expectations handbooks include grade-specific information about the GLEs. Each handbook includes introductory information for each content area contained within the handbook, a glossary, and tables that map the relationship between the standards and/or strands, benchmarks, and GLEs. Each handbook also correlates with one of fourteen GLE posters.

GRADE-LEVEL EXPECTATIONS DEVELOPMENT GUIDELINES

Each grade-level expectation is meant to further define a content standard and benchmark(s). There is a progression of specificity; the standards represent broad statements, benchmarks are more specific, and GLEs provide the most detail. Grade-level expectations have been developed from prekindergarten through grade 12.

GLEs do not represent the entire curriculum for a given grade or course. Rather, they represent the core content that should be mastered by the end of a given year by all students. For mastery to be achieved at a given level, it may be necessary for those skills to be introduced at an earlier grade. Similarly, skills will need to be maintained after mastery has occurred.

The GLEs were developed with the following goals in mind:

- to articulate learning from PreK–12
- to be appropriate for the developmental or grade level of students
- to move from the concrete to the abstract
- to attend to prerequisite skills and understandings
- to be specific, but not so specific as to be too small in "grain size" compared with other GLEs for a particular content area

The GLEs were developed with an effort to avoid including:

- statements of curricular activities or instructional strategies
- value-laden concepts and understandings

SUMMARY OF GLE DEVELOPMENT PROCESS

In December 2002, the LDE selected Data Recognition Corporation (DRC) as the contractor for the Grade-Level Expectations Project. The steps in the process of developing the Grade-Level Expectations for English language arts, mathematics, science, and social studies are described below.

1. **Identifying National Consultants.** National consultants representing each content area were selected to provide a national perspective to the GLE project and to assist the LDE and DRC with various steps in the GLE development process.

2. **Conducting Teacher Committee Meetings.** In March and April of 2003, content-area development committees, comprised of approximately 120 Louisiana classroom teachers, administrators, special populations teachers, and resource teachers chosen for their knowledge of standards and curriculum, were convened to assist with the development of the GLEs for English language arts, mathematics, science, and social studies.

3. **Completing Initial drafts.** The draft GLEs for the four content areas were completed and prepared for further review.

4. **Convening Focus Groups.** Two eighty-member groups of educators, nominated by their districts, reviewed the draft GLEs for horizontal and vertical alignments with the standards for English language arts, mathematics, science, and social studies and provided additional input on these initial drafts to the LDE.

5. **Conducting an Online/Electronic Public Review.** An online/electronic public review and feedback system was developed for the GLEs and made available on the LDE Web site. The purpose of this review was to solicit a broad range of feedback on the GLEs from parents, teachers, and other stakeholders.

6. **Completing an External Review.** With assistance from the staff of the Council of Chief State School Officers (CCSSO), twelve content specialists (three per content area) from other states or from universities or educational organizations across the nation were invited to serve on the review committee. In addition, the LDE also invited the national consultants from each content area to participate in the review meeting to answer questions about the development of the GLEs. External review committee members reviewed the GLE documents prior to the meetings, met with national consultants and LDE staff, and provided specific feedback on the GLEs via a written report.

7. **Conducting Final Committee Meeting.** The public comments and suggestions from the online public review were tabulated, and the additional comments and feedback from the external review committee meeting were compiled for sharing with committee members during the third and final GLE development committee meeting. The outcome of the third meeting of content-area committees consisted of suggestions for final edits that would be incorporated into the documents for presentation to the Louisiana State Board of Elementary and Secondary Education.

8. **Obtaining SBESE Board Approval of the GLEs.** In October 2003, the LDE staff presented the GLEs to the Louisiana State Board of Elementary and Secondary Education for review and approval.

9. **Conducting Preworkshops and GLE Awareness Workshops.** Two preworkshops for district supervisors in early December 2003 and fifteen GLE awareness workshops for local school personnel in late January 2004 have been conducted to inform educators about the grade-level expectations and their future role as it relates to curriculum and assessment

CONCLUSION

Louisiana's content standards and benchmarks have guided the Louisiana education reform program for several years. As an extension of the content standards and benchmarks, the GLEs provide a link among instruction, curriculum, and assessment. The primary goal is a common understanding among parents, students, teachers, and the general public about what is expected of Louisiana students as they progress from grade to grade.

MATHEMATICS INTRODUCTION

INTRODUCTION

Grade-Level Expectations (GLEs) are explicit recommendations for what students should know and be able to do as a result of each level of schooling from prekindergarten through grade 12. This degree of specificity is made with the expectation that all students in Louisiana have access to a high-quality instructional program in mathematics. Instructional programs must provide all students with a solid foundation in mathematics, regardless of race or ethnic origin, geographical location, or socioeconomic status. The design, delivery, and assessment of such programs require careful planning, articulation, and coordination.

The GLEs provide a basis for all of these goals. Further, they clearly define what schools and teachers need to focus on in each year of instruction. This focus provides teachers with a quick listing of the main ideas that frame the year's study and, at the same time, helps students see the connections in the mathematics they are studying.

The following list of GLEs provides a comprehensive look at the expectations for all Louisiana mathematics students for each grade level. They define the targets for instruction, assessment, and emphasis during each year of public schooling. Further, they outline the promises that each teacher is striving to make to successive teachers about what their students know and can do. Such attempts at articulation serve to tighten the curriculum; help avoid programs that have great breadth, but no depth; and assist in helping teachers link the mathematics they are teaching to what students have already studied and to what they will be using in other areas of the curriculum.

ORGANIZATIONAL PRINCIPLES

All GLEs in mathematics are organized by grade levels and address one or more benchmarks in the six mathematics strands: Number and Number Relations; Algebra; Geometry; Measurement; Data Analysis, Probability, and Discrete Math; and Patterns, Relations, and Functions.

With the single exception of prekindergarten, there are GLEs representing each of the six strands for each grade level. In addition, the number of GLEs in each strand either increases or decreases depending on the experience of the learner. For example, most of the GLEs in prekindergarten and kindergarten are based on Number and Number Relations, and few are based on the Data Analysis, Probability, and Discrete Math, or Patterns, Relations and Functions strands. By grade 9, the trend reverses as students have completed most of their study of Number.

Not all high school students take the same courses, nor do they take them in the same order. As a result**, the Grade-Level Expectations for grades 9 and 10 represent the content that all students should master before graduation.** When mastery occurs will depend on the order in which courses are taken. The expectations are written with a view toward a greater integration of mathematics learning across these two grades than traditionally has been the case. For schools teaching separate courses in Algebra I and Geometry, the total of the Grade-Level Expectations for grades 9 and 10 should be considered as what students should be able to do by the end of grade 10, rather than thinking of these as separate outcomes for grade 9 and grade 10.

MATHEMATICS INTRODUCTION

The GLEs for grades 11/12 represent the core content for students who enroll in Algebra II and higher courses in preparation for post-secondary education. They reflect the content of collegiate entrance examinations and mathematics found in common applications in such programs of study.

BENCHMARK CODES

Codes at the end of each GLE are used to identify a developmental profile indicator from the *Louisiana Standards for Programs Serving Four-Year-Old Children* (Table 1) and/or benchmarks from the *Louisiana Mathematics Framework* (Table 2). A GLE may apply to more than one benchmark and, as a result, a GLE may have more than one code.

Developmental Profile Indicator Codes: The first part is always PK, which means prekindergarten. The second part indicates the domain and content area (i.e., Cognitive Mathematics). The third part indicates the skill area (i.e., Number, Measurement, Geometry, Data, Patterns) and skill number (e.g., 1, 2).

Table 1. Explanation of Developmental Profile Indicator Codes

Code(s)	Explanation
PK-CM-N1	Prekindergarten, Cognitive Mathematics, Number, Skill 1
PK-CM-M3	Prekindergarten, Cognitive Mathematics, Measurement, Skill 3
PK-CM- G2	Prekindergarten, Cognitive Mathematics, Geometry, Skill 2

Benchmark Codes: Benchmark codes have 3 parts. The first part in the benchmark code refers to the strand (e.g., Number and Number Relations). The second part refers to the benchmark number. The third part refers to the grade cluster (i.e., E, M, H).

Table 2. Explanation of Benchmark Codes

Code	Explanation
N-1-E	Number and Number Relations, Benchmark 1, Elementary
G-5-M	Geometry, Benchmark 5, Middle School
A-3-H	Algebra, Benchmark 3, High School

MATHEMATICS INTRODUCTION

Each of the following GLE grade listings opens with a concise note about the focal emphases of that grade. These emphases serve to shape and mold the program for that individual grade level. Such focal points at each level and the careful articulation of these ideas lead to significant growth in students' abilities to learn and profitably apply mathematics in their professional, vocational, and personal lives.

FIFTH GRADE INTRODUCTION

Fifth Grade

Number and Number Relations work at grade 5 shifts from the focus on whole numbers to an emphasis on fractions and decimals. Students work to compare and represent positive fractions on a number line and to add and subtract fractions with common denominators. Students continue to work with whole numbers to demonstrate computational fluency by year's end and further develop estimation strategies and their ability to test the reasonableness of an answer to a given computation. Work with rates, ratios, and equivalent ratios emerge, but formal development waits for later grades.

By grade 5, students can select appropriate units and tools for measurement situations and perform operations on the resulting measures. In particular, they become proficient in the use of rulers, yard- and meter-sticks, as well as protractors. They can handle elapsed time problems that involve the noon and midnight breaks and solve real-life problems involving time. Students extend their understanding of polygons to the properties of regular polygons and learn about rotational symmetry and basic transformations.

In Data, grade 5 students extend their capabilities to using divided bar and line graphs. Special emphasis is given to the impact of different scales and labels for bar and line graphs. In Probability, grade 5 students start to represent probabilities as common fractions. The grade 5 study of Algebra involves students in an extension of ways of solving sentences involving unknowns. Students are introduced to solving number sentences by mental math, inverse processes (i.e., unwrapping operations), and manipulatives. They also begin to study representations of simple inequalities in one variable on number lines. Students learn to extend patterns consisting of multiples, as well as patterns involving positions of geometric figures.

SAMPLE PAGE AND KEY FOR MATHEMATICS

Strand/ Standard

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-E: constructing number meaning and demonstrating that a number can be expressed in many different forms (e.g., standard notation, number words, number lines, geometrical representation, fractions, and decimals)	1. Read and write place value in word, standard, and expanded form through 1,000,000 (N-1-E) 2. Read, write, compare, and order whole numbers using place value concepts, standard notation, and models through 1,000,000 (N-1-E) (N-3-E) (A-1-E) 3. Illustrate with manipulatives when a number is divisible by 2, 3, 5, or 10 (N-1-E) 4. Know all basic facts for multiplication and division through 12 x 12 and 144 ÷ 12, and recognize factors of composite numbers less than 50 (N-1-E) (N-6-E) (N-7-E) 5. Read, write, and relate decimals through hundredths and connect them with corresponding decimal fractions (N-1-E) 6. Model, read, write, compare, order, and represent fractions with denominators through twelfths using region and set models (N-1-E) (A-1-E) Also see GLE #7.
N-2-E: demonstrating number sense and estimation skills, giving particular attention to common equivalent reference points (i.e., 1/4 = 25% = .25; ½ = 50% = .5; $1 = 100%, etc.)	7. Give decimal equivalents of halves, fourths, and tenths (N-2-E) (N-1-E) 8. Use common equivalent reference points for percents (i.e., ¼, ½, ¾, and 1 whole) (N-2-E) 9. Estimate fractional amounts through twelfths, using pictures, models, and diagrams (N-2-E) Also see GLE #27.

Benchmarks

Grade-Level Expectations (GLEs)

FIFTH GRADE

MATHEMATICS

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-M: demonstrating that a rational number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, and percents)	1. Differentiate between the terms *factor* and *multiple*, and *prime* and *composite* (N-1-M) 2. Recognize, explain, and compute equivalent fractions for common fractions (N-1-M) (N-3-M)
N-2-M: demonstrating number sense and estimation skills to describe, order, and compare rational numbers (e.g., magnitude, integers, fractions, decimals, and percents)	3. Add and subtract fractions with common denominators and use mental math to determine whether the answer is reasonable (N-2-M) 4. Compare positive fractions using number sense, symbols (i.e., <, =, >), and number lines (N-2-M) See GLE #9.
N-3-M: reading, writing, representing, and using rational numbers in a variety of forms (e.g., integers, mixed numbers, and improper fractions)	5. Read, explain, and write a numerical representation for positive improper fractions, mixed numbers, and decimals from a pictorial representation and vice versa (N-3-M) Also see GLE #2.
N-4-M: demonstrating a conceptual understanding of the meaning of the basic arithmetic operations (add, subtract, multiply and divide) and their relationships to each other	6. Select and discuss the correct operation for a given problem involving positive fractions using appropriate language such as *sum*, *difference*, *numerator,* and *denominator* (N-4-M) (N-5-M) See GLE #7.

N-5-M: applying an understanding of rational numbers and arithmetic operations to real-life situations	7. Select, sequence, and use appropriate operations to solve multi-step word problems with whole numbers (N-5-M) (N-4-M) 8. Use the whole number system (e.g., computational fluency, place value, etc.) to solve problems in real-life and other content areas (N-5-M) Also see GLEs #6, #10, and #11.
N-6-M: constructing, using, and explaining procedures to compute and estimate with rational numbers employing mental math strategies	9. Use mental math and estimation strategies to predict the results of computations (i.e., whole numbers, addition and subtraction of fractions) and to test the reasonableness of solutions (N-6-M) (N-2-M) 10. Determine when an estimate is sufficient and when an exact answer is needed in real-life problems using whole numbers (N-6-M) (N-5-M)
N-7-M: selecting and using appropriate computational methods and tools for given situations involving rational numbers (e.g., estimation, or exact computation using mental arithmetic, calculator, computer, or paper and pencil)	
N-8-M: demonstrating a conceptual understanding and applications of proportional reasoning (e.g., determining equivalent ratios, finding a missing term of a given proportion)	11. Explain concepts of ratios and equivalent ratios using models and pictures in real-life problems (e.g., understand that 2/3 means 2 divided by 3) (N-8-M) (N-5-M)
Algebra: In problem-solving investigations, students demonstrate an understanding of concepts and processes that allow them to analyze, represent, and describe relationships among variable quantities and to apply algebraic methods to real-world situations.	
Benchmarks	**Grade-Level Expectations**
A-1-M: demonstrating a conceptual understanding of variables, expressions, equations, and inequalities (e.g., symbolically represent real-world problems as linear terms, equations, or inequalities)	See GLE #13.

A-2-M: modeling and developing methods for solving equations and inequalities (e.g., using charts, graphs, manipulatives, and/or standard algebraic procedures)	12. Find unknown quantities in number sentences by using mental math, backward reasoning, inverse operations (i.e., unwrapping), and manipulatives (e.g., tiles, balance scales) (A-2-M) (A-3-M) 13. Write a number sentence from a given physical model of an equation (e.g., balance scale) (A-2-M) (A-1-M) 14. Find solutions to one-step inequalities and identify positive solutions on a number line (A-2-M) (A-3-M)
A-3-M: representing situations and number patterns with tables, graphs, and verbal and written statements, while exploring the relationships among these representations (e.g., multiple representations for the same situation)	See GLEs #12 and #14.
A-4-M: analyzing tables and graphs to identify relationships exhibited by the data and making generalizations based upon these relationships	See GLE #28.
A-5-M: demonstrating the connection of algebra to the other strands and to real-life situations	

Measurement: In problem-solving investigations, students demonstrate an understanding of the concepts, processes, and real-life applications of measurement.	
Benchmarks	**Grade-Level Expectations**
M-1-M: applying the concepts of length, area, surface area, volume, capacity, weight, mass, money, time, temperature, and rate to real-world experiences	15. Model, measure, and use the names of all common units in the U.S. and metric systems (M-1-M) 16. Apply the concepts of elapsed time in real-life situations and calculate equivalent times across time zones in real-life problems (M-1-M) (M-6-M)

M-2-M demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of measures)	17. Distinguish among the processes of counting, calculating, and measuring and determine which is the most appropriate strategy for a given situation (M-2-M) 18. Estimate time, temperature, weight/mass, and length in familiar situations and explain the reasonableness of answers (M-2-M) 19. Compare the relative sizes of common units for time, temperature, weight, mass, and length in real-life situations (M-2-M) (M-4-M)
M-3-M: selecting appropriate units and tools for tasks by considering the purpose for the measurement and the precision required for the task (e.g., length of a room in feet rather than inches)	20. Identify appropriate tools and units with which to measure time, mass, weight, temperature, and length (M-3-M) 21. Measure angles to the nearest degree (M-3-M)
M-4-M: using intuition and estimation skills to describe, order, and compare formal and informal measures (e.g., ordering cup, pint, quart, gallon; comparing a meter to a yard)	22. Compare and estimate measurements between the U.S. and metric systems in terms of common reference points (e.g., l vs. qt., m vs. yd.) (M-4-M) See GLE #19.
M-5-M: converting from one unit of measurement to another within the same system (Comparisons between systems, customary and metric, should be based on intuitive reference points, not formal computation.)	23. Convert between units of measurement for length, weight, and time, in U.S. and metric, within the same system (M-5-M)
M-6-M: demonstrating the connection of measurement to the other strands and to real-life situations	See GLE #16.

Geometry: In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings.	
Benchmarks	**Grade-Level Expectations**
G-1-M: using estimation skills to describe, order, and compare geometric measures	
G-2-M: identifying, describing, comparing, constructing, and classifying geometric figures and concepts	24. Use mathematical terms to classify and describe the properties of 2-dimensional shapes, including circles, triangles, and polygons (G-2-M)
G-3-M: making predictions regarding transformations of geometric figures (e.g., make predictions regarding translations, reflections, and rotations of common figures)	25. Identify and use appropriate terminology for transformations (e.g., *translation* as *slide*, *reflection* as *flip*, and *rotation* as *turn*) (G-3-M) 26. Identify shapes that have rotational symmetry (G-3-M)
G-4-M: constructing two- and three-dimensional models	
G-5-M: making and testing conjectures about geometric shapes and their properties	
G-6-M: demonstrating an understanding of the coordinate system (e.g., locate points, identify coordinates, and graph points in a coordinate plane to represent real-world situations)	27. Identify and plot points on a coordinate grid in the first quadrant (G-6-M)
G-7-M: demonstrating the connection of geometry to the other strands and to real-life situations (e.g., applications of the Pythagorean Theorem)	

Data Analysis, Probability, and Discrete Math: In problem-solving investigations, students discover trends, formulate conjectures regarding cause-and-effect relationships, and demonstrate critical thinking skills in order to make informed decisions.	
Benchmarks	**Grade-Level Expectations**
D-1-M: systematically collecting, organizing, describing, and displaying data in charts, tables, plots, graphs, and/or spreadsheets	28. Use various types of charts and graphs, including double bar graphs, to organize, display, and interpret data and discuss patterns verbally and in writing (D-1-M) (D-2-M) (P-3-M) (A-4-M) 29. Compare and contrast different scales and labels for bar and line graphs (D-1-M) 30. Organize and display data using spreadsheets, with technology (D-1-M)
D-2-M: analyzing, interpreting, evaluating, drawing inferences, and making estimations, predictions, decisions, and convincing arguments based on organized data (e.g., analyze data using concepts of mean, median, mode, range, random samples, sample size, bias, and data extremes)	31. Compare and contrast survey data from two groups relative to the same question (D-2-M) See GLE #28.
D-3-M: describing informal thinking procedures (e.g., solving elementary logic problems using Venn diagrams, tables, charts, and/or elementary logic operatives to solve logic problems in real-life situations; reach valid conclusions in elementary logic problems involving "and, or, not, if/then")	
D-4-M: analyzing various counting and enumeration procedures with and without replacement (e.g., find the total number of possible outcomes or possible choices in a given situation)	

D-5-M: comparing experimental probability results with theoretical probability (e.g., representing probabilities of concrete situations as common fractions, investigating single-event and multiple-event probability, using sample spaces, geometric figures, tables, and/or graphs)	32. Represent probabilities as common fractions and recognize that probabilities fall between 0 and 1, inclusive (D-5-M)
D-6-M: demonstrating the connection of data analysis, probability, and discrete math to other strands and to real-life situations	
Patterns, Relations, and Functions: In problem-solving investigations, students demonstrate an understanding of patterns, relations, and functions that represent and explain real-world situations.	
Benchmarks	**Grade-Level Expectations**
P-1-M: describing, extending, analyzing, and creating a wide variety of numerical, geometrical, and statistical patterns (e.g., skip counting of rational numbers and simple exponential number patterns)	33. Fill in missing elements in sequences of designs, number patterns, positioned figures, and quantities of objects (P-1-M)
P-2-M: describing and representing relationships using tables, rules, simple equations, and graphs	
P-3-M: analyzing relationships to explain how a change in one quantity results in a change in another (e.g., change in the dimensions of a rectangular solid affects the volume)	See GLE #28.
P-4-M: demonstrating the pervasive use of patterns, relations, and functions in other strands and in real-life situations	

SIXTH GRADE INTRODUCTION

Sixth Grade

The grade 6 year is mathematically important for students because it is a year of consolidation and foundation building. During the year, students continue to work with rational numbers in both fraction and decimal forms, and begin to develop a solid basis for work with rates, ratios, and proportions. In Measurement and Geometry, their focus is mastery of the perimeter and areas of triangles and quadrilaterals. They are extending their knowledge of 2-dimensional figures to their 3-dimensional counterparts, as well as developing concepts and skills related to the measurement of angles and using coordinate geometry in the first quadrant to locate points and investigate simple figures.

In the fields of data and chance, grade 6 students extend their capabilities to represent and evaluate data using frequency tables, scatter plots, stem-and-leaf plots, and Venn diagrams. They use mean, median, mode, and range to describe and analyze patterns in data sets. They also employ Venn diagrams to analyze arguments and solve simple problems involving inclusion and exclusion.

Grade 6 students are growing in their capabilities to match algebraic expressions with their verbal counterparts and evaluate such expressions using substitution. They also develop skills to help them relate patterns to tables of values and to their defining rules.

SIXTH GRADE

MATHEMATICS

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-M: demonstrating that a rational number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, and percents)	1. Factor whole numbers into primes (N-1-M) 2. Determine common factors and common multiples for pairs of whole numbers (N-1-M) 3. Find the greatest common factor (GCF) and least common multiple (LCM) for whole numbers in the context of problem-solving (N-1-M) 4. Recognize and compute equivalent representations of fractions and decimals (i.e., halves, thirds, fourths, fifths, eighths, tenths, hundredths) (N-1-M) (N-3-M) 5. Decide which representation (i.e., fraction or decimal) of a positive number is appropriate in a real-life situation (N-1-M) (N-5-M)
N-2-M: demonstrating number sense and estimation skills to describe, order, and compare rational numbers (e.g., magnitude, integers, fractions, decimals, and percents)	6. Compare positive fractions, decimals, and positive and negative integers using symbols (i.e., <, =, >) and number lines (N-2-M)
N-3-M: reading, writing, representing, and using rational numbers in a variety of forms (e.g., integers, mixed numbers, and improper fractions)	7. Read and write numerals and words for decimals through ten-thousandths (N-3-M) 8. Demonstrate the meaning of positive and negative numbers and their opposites in real-life situations (N-3-M) (N-5-M) Also see GLE #4.
N-4-M: demonstrating a conceptual understanding of the meaning of the basic arithmetic operations (add, subtract, multiply and divide) and their relationships to each other	

N-5-M: applying an understanding of rational numbers and arithmetic operations to real-life situations	9. Add and subtract fractions and decimals in real-life situations (N-5-M) Also see GLEs #5 and #8.
N-6-M: constructing, using, and explaining procedures to compute and estimate with rational numbers employing mental math strategies	10. Use and explain estimation strategies to predict computational results with positive fractions and decimals (N-6-M) 11. Mentally multiply and divide by powers of 10 (e.g., 25/10 = 2.5; 12.56 x 100 = 1,256) (N-6-M)
N-7-M: selecting and using appropriate computational methods and tools for given situations involving rational numbers (e.g., estimation, or exact computation using mental arithmetic, calculator, computer, or paper and pencil)	12. Divide 4-digit numbers by 2-digit numbers with the quotient written as a mixed number or a decimal (N-7-M)
N-8-M: demonstrating a conceptual understanding and applications of proportional reasoning (e.g., determining equivalent ratios, finding a missing term of a given proportion)	13. Use models and pictures to explain concepts or solve problems involving ratio, proportion, and percent with whole numbers (N-8-M)
Algebra: In problem-solving investigations, students demonstrate an understanding of concepts and processes that allow them to analyze, represent, and describe relationships among variable quantities and to apply algebraic methods to real-world situations.	
Benchmarks	**Grade-Level Expectations**
A-1-M: demonstrating a conceptual understanding of variables, expressions, equations, and inequalities (e.g., symbolically represent real-world problems as linear terms, equations, or inequalities)	14. Model and identify perfect squares up to 144 (A-1-M) 15. Match algebraic equations and expressions with verbal statements and vice versa (A-1-M) (A-3-M) (A-5-M) (P-2-M)
A-2-M: modeling and developing methods for solving equations and inequalities (e.g., using charts, graphs, manipulatives, and/or standard algebraic procedures)	16. Evaluate simple algebraic expressions using substitution (A-2-M) 17. Find solutions to 2-step equations with positive integer solutions (e.g., $3x - 5 = 13$, $2x + 3x = 20$) (A-2-M)

A-3-M: representing situations and number patterns with tables, graphs, and verbal and written statements, while exploring the relationships among these representations (e.g., multiple representations for the same situation)	See GLEs #15 and #29.
A-4-M: analyzing tables and graphs to identify relationships exhibited by the data and making generalizations based upon these relationships	See GLE #38.
A-5-M: demonstrating the connection of algebra to the other strands and to real-life situations	See GLEs #15 and #20.

Measurement: In problem-solving investigations, students demonstrate an understanding of the concepts, processes, and real-life applications of measurement.

Benchmarks	Grade-Level Expectations
M-1-M: applying the concepts of length, area, surface area, volume, capacity, weight, mass, money, time, temperature, and rate to real-world experiences	18. Measure length and read linear measurements to the nearest sixteenth-inch and mm (M-1-M) 19. Calculate perimeter and area of triangles, parallelograms, and trapezoids (M-1-M) 20. Calculate, interpret, and compare rates such as $/lb., mpg, and mph (M-1-M) (A-5-M)
M-2-M: demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of measures)	21. Demonstrate an intuitive sense of relative sizes of common units for length and area of familiar objects in real-life problems (e.g., estimate the area of a desktop in square feet, the average adult is between 1.5 and 2 meters tall) (M-2-M) (G-1-M) 22. Estimate perimeter and area of any 2-dimensional figure (regular and irregular) using standard units (M-2-M) Also See GLE #31.

M-3-M: selecting appropriate units and tools for tasks by considering the purpose for the measurement and the precision required for the task (e.g., length of a room in feet rather than inches)	23. Identify and select appropriate units to measure area (M-3-M)
M-4-M: using intuition and estimation skills to describe, order, and compare formal and informal measures (e.g., ordering cup, pint, quart, gallon; comparing a meter to a yard)	
M-5-M: converting from one unit of measurement to another within the same system (Comparisons between systems, customary and metric, should be based on intuitive reference points, not formal computation.)	
M-6-M: demonstrating the connection of measurement to the other strands and to real-life situations	

Geometry: In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings.

Benchmarks	**Grade-Level Expectations**
G-1-M: using estimation skills to describe, order, and compare geometric measures	See GLEs #21 and #26.

G-2-M: identifying, describing, comparing, constructing, and classifying geometric figures and concepts	24. Use mathematical terms to describe the basic properties of 3-dimensional objects (edges, vertices, faces, base, etc.) (G-2-M) 25. Relate polyhedra to their 2-dimensional shapes by drawing or sketching their faces (G-2-M) (G-4-M) 26. Apply concepts, properties, and relationships of points, lines, line segments, rays, diagonals, circles, and right, acute, and obtuse angles and triangles in real-life situations, including estimating sizes of angles (G-2-M) (G-5-M) (G-1-M)
G-3-M: making predictions regarding transformations of geometric figures (e.g., make predictions regarding translations, reflections, and rotations of common figures)	27. Make and test predictions regarding tessellations with geometric shapes (G-3-M)
G-4-M: constructing two- and three-dimensional models	See GLE #25.
G-5-M: making and testing conjectures about geometric shapes and their properties	See GLE #26.
G-6-M: demonstrating an understanding of the coordinate system (e.g., locate points, identify coordinates, and graph points in a coordinate plane to represent real-world situations)	28. Use a rectangular grid and ordered pairs to plot simple shapes and find horizontal and vertical lengths and area (G-6-M)
G-7-M: demonstrating the connection of geometry to the other strands and to real-life situations (e.g., applications of the Pythagorean Theorem)	

Data Analysis, Probability, and Discrete Math: In problem-solving investigations, students discover trends, formulate conjectures regarding cause-and-effect relationships, and demonstrate critical thinking skills in order to make informed decisions.	
Benchmarks	**Grade-Level Expectations**
D-1-M: systematically collecting, organizing, describing, and displaying data in charts, tables, plots, graphs, and/or spreadsheets	29. Collect, organize, label, display, and interpret data in frequency tables, stem-and-leaf plots, and scatter plots and discuss patterns in the data verbally and in writing (D-1-M) (D-2-M) (A-3-M)
D-2-M: analyzing, interpreting, evaluating, drawing inferences, and making estimations, predictions, decisions, and convincing arguments based on organized data (e.g., analyze data using concepts of mean, median, mode, range, random samples, sample size, bias, and data extremes)	30. Describe and analyze trends and patterns observed in graphic displays (D-2-M) 31. Demonstrate an understanding of precision, accuracy, and error in measurement (D-2-M) (M-2-M) 32. Calculate and discuss mean, median, mode, and range of a set of discrete data to solve real-life problems (D-2-M) Also see GLE #29.
D-3-M: describing informal thinking procedures (e.g., solving elementary logic problems using Venn diagrams, tables, charts, and/or elementary logic operatives to solve logic problems in real-life situations; reach valid conclusions in elementary logic problems involving "and, or, not, if/then")	33. Create and use Venn diagrams with two overlapping categories to solve counting logic problems (D-3-M)
D-4-M: analyzing various counting and enumeration procedures with and without replacement (e.g., find the total number of possible outcomes or possible choices in a given situation)	34. Use lists, tree diagrams, and tables to determine the possible combinations from two disjoint sets when choosing one item from each set (D-4-M)

D-5-M comparing experimental probability results with theoretical probability (e.g., representing probabilities of concrete situations as common fractions, investigating single-event and multiple-event probability, using sample spaces, geometric figures, tables, and/or graphs)	35. Illustrate and apply the concept of complementary events (D-5-M) 36. Apply the meaning of *equally likely* and *equally probable* to real-life situations (D-5-M) (D-6-M)
D-6-M demonstrating the connection of data analysis, probability, and discrete math to other strands and to real-life situations	See GLE #36.
Patterns, Relations, and Functions: In problem-solving investigations, students demonstrate an understanding of patterns, relations, and functions that represent and explain real-world situations.	
Benchmarks	**Grade-Level Expectations**
P-1-M describing, extending, analyzing, and creating a wide variety of numerical, geometrical, and statistical patterns (e.g., skip counting of rational numbers and simple exponential number patterns)	37. Describe, complete, and apply a pattern of differences found in an input-output table (P-1-M) (P-2-M) (P-3-M)
P-2-M describing and representing relationships using tables, rules, simple equations, and graphs	See GLEs #15 and #37.
P-3-M analyzing relationships to explain how a change in one quantity results in a change in another (e.g., change in the dimensions of a rectangular solid affects the volume)	38. Describe patterns in sequences of arithmetic and geometric growth and now-next relationships (i.e., growth patterns where the next term is dependent on the present term) with numbers and figures (P-3-M) (A-4-M) Also see GLE #37.
P-4-M demonstrating the pervasive use of patterns, relations, and functions in other strands and in real-life situations	

SEVENTH GRADE INTRODUCTION

Seventh Grade

Grade 7 is the year in which students consolidate their knowledge of arithmetic and begin the transition to approaching problems through algebraic means. In Number and Number Relations, students finish their study of fractions with mastery of multiplication and division of fractions and decimals and consider the ordering and representation of such numbers in practical settings. They extend their understanding of rates, ratios, and proportions and their applications to setting up and solving percent problems. They are able to distinguish between a unit rate and general ratio, and can apply both. They can set up and solve simple proportion problems.

In Measurement, grade 7 students extend their knowledge of perimeter and area concepts to confront irregular figures comprised of two or more simple shapes that they have worked with before. They also apply their work with ratio and proportion concepts to the conversion of measures within systems for both U.S. and metric measures. In Geometry, grade 7 students focus on circles and their subparts, including the development of π and its use in finding the circumference and area of circles. Students also solidify their knowledge of angles, angle measure in a triangle, and the classification of triangles according to their angle measures.

In Data and Probability, grade 7 students study the concept of the nature of the data (i.e., outliers, clusters, gaps) and the beginning of the study of distributions. Grade 7 students also progress to the study of 3-circle Venn diagram situations and the accompanying logical relationships. In dealing with probability, they move to the comparison of data resulting from experimental and theoretical analyses of similar situations.

Grade 7 students extend their knowledge of algebraic equations and inequalities by translating and matching equivalent statements in verbal or written and symbolic forms. Their solution strategies are extended to the use of mental math and more structured algebraic approaches from the more arithmetic methods they have used before. By the end of the year, they should be solving both one- and two-step linear equations with ease. In dealing with Patterns and Functions, they understand function machines, recognize the input-output natures of such representations, and know how to graph the resulting ordered pairs on a coordinate graph. As part of this, they should be dealing with the differences between arithmetic (i.e., additive/linear) and geometric (i.e., multiplicative/ exponential) growth situations.

SEVENTH GRADE

MATHEMATICS

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-M: demonstrating that a rational number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, and percents)	1. Recognize and compute equivalent representations of fractions, decimals, and percents (i.e., halves, thirds, fourths, fifths, eighths, tenths, hundredths) (N-1-M)
N-2-M: demonstrating number sense and estimation skills to describe, order, and compare rational numbers (e.g., magnitude, integers, fractions, decimals, and percents)	2. Compare positive fractions, decimals, percents, and integers using symbols (i.e., $<$, $\leq$, $=$, $\geq$, $>$) and position on a number line (N-2-M)
N-3-M: reading, writing, representing, and using rational numbers in a variety of forms (e.g., integers, mixed numbers, and improper fractions)	See GLE #7.
N-4-M: demonstrating a conceptual understanding of the meaning of the basic arithmetic operations (add, subtract, multiply and divide) and their relationships to each other	3. Solve order of operations problems involving grouping symbols and multiple operations (N-4-M) 4. Solve order of operations problems involving grouping symbols and multiple operations (N-4-M) Also see GLE #7.

N-5-M: applying an understanding of rational numbers and arithmetic operations to real-life situations	5. Multiply and divide positive fractions and decimals (N-5-M) 6. Set up and solve simple percent problems using various strategies, including mental math (N-5-M) (N-6-M) (N-8-M) 7. Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M) Also see GLE #9.
N-6-M: constructing, using, and explaining procedures to compute and estimate with rational numbers employing mental math strategies	8. Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates (N-6-M) (N-7-M) Also see GLE #6.
N-7-M: selecting and using appropriate computational methods and tools for given situations involving rational numbers (e.g., estimation, or exact computation using mental arithmetic, calculator, computer, or paper and pencil)	9. Determine when an estimate is sufficient and when an exact answer is needed in real-life problems using decimals and percents (N-7-M) (N-5-M) Also see GLE #8.
N-8-M: demonstrating a conceptual understanding and applications of proportional reasoning (e.g., determining equivalent ratios, finding a missing term of a given proportion)	10. Determine and apply rates and ratios (N-8-M) 11. Use proportions involving whole numbers to solve real-life problems (N-8-M) Also see GLE #6.

Algebra: In problem-solving investigations, students demonstrate an understanding of concepts and processes that allow them to analyze, represent, and describe relationships among variable quantities and to apply algebraic methods to real-world situations.

Benchmarks	Grade-Level Expectations
A-1-M: demonstrating a conceptual understanding of variables, expressions, equations, and inequalities (e.g., symbolically represent real-world problems as linear terms, equations, or inequalities)	12. Evaluate algebraic expressions containing exponents (especially 2 and 3) and square roots, using substitution (A-1-M) 13. Determine the square root of perfect squares and mentally approximate other square roots by identifying the two whole numbers between which they fall (A-1-M) 14. Write a real-life meaning of a simple algebraic equation or inequality, and vice versa (A-1-M) (A-5-M) 15. Match algebraic inequalities with equivalent verbal statements and vice versa (A-1-M)
A-2-M: modeling and developing methods for solving equations and inequalities (e.g., using charts, graphs, manipulatives, and/or standard algebraic procedures)	16. Solve one- and two-step equations and inequalities (with one variable) in multiple ways (A-2-M) 17. Graph solutions sets of one-step equations and inequalities as points, or open and closed rays on a number line (e.g., $x = 5$, $x < 5$, $x \leq 5$, $x > 5$, $x \geq 5$) (A-2-M)
A-3-M: representing situations and number patterns with tables, graphs, and verbal and written statements, while exploring the relationships among these representations (e.g., multiple representations for the same situation)	18. Describe linear, multiplicative, or changing growth relationships (e.g., 1, 3, 6, 10, 15, 21, …) verbally and algebraically (A-3-M) (A-4-M) (P-1-M)
A-4-M: analyzing tables and graphs to identify relationships exhibited by the data and making generalizations based upon these relationships	19. Use *function machines* to determine and describe the rule that generates outputs from given inputs (A-4-M) (P-3-M) Also see GLE #18.
A-5-M demonstrating the connection of algebra to the other strands and to real-life situations.	See GLEs #14 and #29.

Measurement: In problem-solving investigations, students demonstrate an understanding of the concepts, processes, and real-life applications of measurement.

Benchmarks	Grade-Level Expectations
M-1-M: applying the concepts of length, area, surface area, volume, capacity, weight, mass, money, time, temperature, and rate to real-world experiences	20. Determine the perimeter and area of composite plane figures by subdivision and area addition (M-1-M) (G-7-M)
M-2-M: demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of measures)	
M-3-M: selecting appropriate units and tools for tasks by considering the purpose for the measurement and the precision required for the task (e.g., length of a room in feet rather than inches)	
M-4-M: using intuition and estimation skills to describe, order, and compare formal and informal measures (e.g., ordering cup, pint, quart, gallon; comparing a meter to a yard)	21. Compare and order measurements within and between the U.S. and metric systems in terms of common reference points (e.g., weight/mass and area) (M-4-M) (G-1-M)
M-5-M: converting from one unit of measurement to another within the same system (Comparisons between systems, customary and metric, should be based on intuitive reference points, not formal computation.)	22. Convert between units of area in U.S. and metric units within the **same** system (M-5-M) 23. Demonstrate an intuitive sense of comparisons between degrees Fahrenheit and Celsius in real-life situations using common reference points (M-5-M)
M-6-M: demonstrating the connection of measurement to the other strands and to real-life situations	See GLE #28.

Geometry: In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings.

Benchmarks	Grade-Level Expectations
G-1-M: using estimation skills to describe, order, and compare geometric measures	See GLE #21.
G-2-M: identifying, describing, comparing, constructing, and classifying geometric figures and concepts	24. Identify and draw angles (using protractors), circles, diameters, radii, altitudes, and 2-dimensional figures with given specifications (G-2-M)
G-3-M: making predictions regarding transformations of geometric figures (e.g., make predictions regarding translations, reflections, and rotations of common figures)	25. Draw the results of reflections and translations of geometric shapes on a coordinate grid (G-3-M)
G-4-M: constructing two- and three-dimensional models	
G-5-M: making and testing conjectures about geometric shapes and their properties	26. Recognize π as the ratio between the circumference and diameter of any circle (i.e., $\pi = C/d$ or $\pi = C/2r$) (G-5-M) 27. Model and explain the relationship between perimeter and area (how scale change in a linear dimension affects perimeter and area) and between circumference and area of a circle (G-5-M) 28. Determine the radius, diameter, circumference, and area of a circle and apply these measures in real-life problems (G-5-M) (G-7-M) (M-6-M)
G-6-M: demonstrating an understanding of the coordinate system (e.g., locate points, identify coordinates, and graph points in a coordinate plane to represent real-world situations)	29. Plot points on a coordinate grid in all 4 quadrants and locate the coordinates of a missing vertex in a parallelogram (G-6-M) (A-5-M)
G-7-M: demonstrating the connection of geometry to the other strands and to real-life situations (e.g., applications of the Pythagorean Theorem)	30. Apply the knowledge that the measures of the interior angles in a triangle add up to 180 degrees (G-7-M) Also see GLEs #20 and #28.

Data Analysis, Probability, and Discrete Math: In problem-solving investigations, students discover trends, formulate conjectures regarding cause-and-effect relationships, and demonstrate critical thinking skills in order to make informed decisions.	
Benchmarks	**Grade-Level Expectations**
D-1-M: systematically collecting, organizing, describing, and displaying data in charts, tables, plots, graphs, and/or spreadsheets	
D-2-M: analyzing, interpreting, evaluating, drawing inferences, and making estimations, predictions, decisions, and convincing arguments based on organized data (e.g., analyze data using concepts of mean, median, mode, range, random samples, sample size, bias, and data extremes)	31. Analyze and interpret circle graphs, and determine when a circle graph is the most appropriate type of graph to use (D-2-M) 32. Describe data in terms of patterns, clustered data, gaps, and outliers (D-2-M) 33. Analyze discrete and continuous data in real-life applications (D-2-M) (D-6-M)
D-3-M: describing informal thinking procedures (e.g., solving elementary logic problems using Venn diagrams, tables, charts, and/or elementary logic operatives to solve logic problems in real-life situations; reach valid conclusions in elementary logic problems involving "and, or, not, if/then")	34. Create and use Venn diagrams with three overlapping categories to solve counting logic problems (D-3-M) 35. Use informal thinking procedures of elementary logic involving *if/then* statements (D-3-M)
D-4-M: analyzing various counting and enumeration procedures with and without replacement (e.g., find the total number of possible outcomes or possible choices in a given situation)	36. Apply the fundamental counting principle in real-life situations (D-4-M)
D-5-M: comparing experimental probability results with theoretical probability (e.g., representing probabilities of concrete situations as common fractions, investigating single-event and multiple-event probability, using sample spaces, geometric figures, tables, and/or graphs)	37. Determine probability from experiments and from data displayed in tables and graphs (D-5-M) 38. Compare theoretical and experimental probability in real-life situations (D-5-M)

D-6-M: demonstrating the connection of data analysis, probability, and discrete math to other strands and to real-life situations	See GLE #33.
Patterns, Relations, and Functions: In problem-solving investigations, students demonstrate an understanding of patterns, relations, and functions that represent and explain real-world situations.	
Benchmarks	**Grade-Level Expectations**
P-1-M: describing, extending, analyzing, and creating a wide variety of numerical, geometrical, and statistical patterns (e.g., skip counting of rational numbers and simple exponential number patterns)	39. Analyze and describe simple exponential number patterns (e.g., 3, 9, 27 or 3^1, 3^2, 3^3) (P-1-M) 40. Analyze and verbally describe real-life additive and multiplicative patterns involving fractions and integers (P-1-M) (P-4-M) Also see GLE #18.
P-2-M: describing and representing relationships using tables, rules, simple equations, and graphs	
P-3-M: analyzing relationships to explain how a change in one quantity results in a change in another (e.g., change in the dimensions of a rectangular solid affects the volume)	41. Illustrate patterns of change in length(s) of sides and corresponding changes in areas of polygons (P-3-M) Also see GLE #19.
P-4-M: demonstrating the pervasive use of patterns, relations, and functions in other strands and in real-life situations	See GLE #40.

EIGHTH GRADE INTRODUCTION

Eighth Grade

Grade 8 mathematics sees the final shift from arithmetic to Algebra take place. The work in Number and Number Relations focuses on pulling the work with natural numbers, integers, decimals, and fractions to a mature understanding, and shifting the emphasis to the real number system. More focus is placed on understanding the properties of the real numbers and their operations and using this knowledge as a foundation for understanding elementary Algebra. By the end of grade 8, students have a mastery of all operations with integers, fractions, and decimals. They numerically interpret situations involving small integer exponents and use scientific notation to represent large and small numbers. They employ proportions to handle a wide variety of percentage problems, including those of percent of increase and decrease.

Measurement concepts are a major focus of the grade 8 year. At this level students complete their understanding of the volume of prisms, cylinders, spheres, pyramids, and cones. They also encounter and learn to work with the concepts related to the derived measures of density, speed, and monetary equivalences of different currencies. In Geometry, they do related ratio and proportion work in solving problems involving scale drawings and the finding of measurements indirectly with the Pythagorean theorem or its converse. These latter relationships result as extensions of the students' work with similar triangles and angle relationships.

In Data, grade 8 students learn to use methods of representation that allow them to look at the role of variability, or spread, in data sets. They learn to use spreadsheets to analyze and represent data. They learn to represent data sets using box-and-whisker plots and percentiles with and without technological support. They analyze two-variable data in scatterplots and intuitively fit and interpret trend lines fitted to the data. These analyses are used as bases for simple predictions from such data sets. With Probability, grade 8 students are first encountering permutations and combinations in organized settings. These are used both as a basis for probability and as a more general way of counting for discrete settings. Students also extend their work with sample spaces and probability trees to consider the outcomes and chances associated with various events.

In grade 8, students begin to distinguish between linear and nonlinear settings, primarily exponential growth settings in the latter category. They solve and graph the solutions to one- and two-step linear equations and inequalities. Working with linear equations, they graph them in the plane and discuss how the change in parameters affects their appearance when graphed. In patterns, grade 8 students begin to more formally consider sequence-based patterns by determining a numbering structure for terms and then using it to represent the general *nth* term for such sequences.

EIGHTH GRADE

MATHEMATICS

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-M: demonstrating that a rational number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, and percents)	1. Compare rational numbers using symbols (i.e., <, ≤ , =, ≥, >) and position on a number line (N-1-M) (N-2-M) 2. Use whole number exponents (0-3) in problem-solving contexts (N-1-M) (N-5-M)
N-2-M: demonstrating number sense and estimation skills to describe, order, and compare rational numbers (e.g., magnitude, integers, fractions, decimals, and percents)	3. Estimate the answer to an operation involving rational numbers based on the original numbers (N-2-M) (N-6-M) Also see GLE #1.
N-3-M: reading, writing, representing, and using rational numbers in a variety of forms (e.g., integers, mixed numbers, and improper fractions)	4. Read and write numbers in scientific notation with positive exponents (N-3-M)
N-4-M: demonstrating a conceptual understanding of the meaning of the basic arithmetic operations (add, subtract, multiply and divide) and their relationships to each other	5. Simplify expressions involving operations on integers, grouping symbols, and whole number exponents using order of operations (N-4-M)
N-5-M: applying an understanding of rational numbers and arithmetic operations to real-life situations	6. Identify missing information or suggest a strategy for solving a real-life, rational-number problem (N-5-M) Also see GLEs #2, #8 and #9.
N-6-M: constructing, using, and explaining procedures to compute and estimate with rational numbers employing mental math strategies	See GLE #3.

N-7-M: selecting and using appropriate computational methods and tools for given situations involving rational numbers (e.g., estimation, or exact computation using mental arithmetic, calculator, computer, or paper and pencil)	
N-8-M: demonstrating a conceptual understanding and applications of proportional reasoning (e.g., determining equivalent ratios, finding a missing term of a given proportion)	7. Use proportional reasoning to model and solve real-life problems (N-8-M) 8. Solve real-life problems involving percentages, including percentages less than 1 or greater than 100 (N-8-M) (N-5-M) 9. Find unit/cost rates and apply them in real-life problems (N-8-M) (N-5-M) (A-5-M) Also see GLEs #18 and #30.
Algebra: In problem-solving investigations, students demonstrate an understanding of concepts and processes that allow them to analyze, represent, and describe relationships among variable quantities and to apply algebraic methods to real-world situations.	
Benchmarks	**Grade-Level Expectations**
A-1-M: demonstrating a conceptual understanding of variables, expressions, equations, and inequalities (e.g., symbolically represent real-world problems as linear terms, equations, or inequalities)	10. Write real-life meanings of expressions and equations involving rational numbers and variables (A-1-M) (A-5-M) 11. Translate real-life situations that can be modeled by linear or exponential relationships to algebraic expressions, equations, and inequalities (A-1-M) (A-4-M) (A-5-M)
A-2-M: modeling and developing methods for solving equations and inequalities (e.g., using charts, graphs, manipulatives, and/or standard algebraic procedures)	12. Solve and graph solutions of multi-step linear equations and inequalities (A-2-M) See GLE #14.
A-3-M: representing situations and number patterns with tables, graphs, and verbal and written statements, while exploring the relationships among these representations (e.g., multiple representations for the same situation)	13. Switch between functions represented as tables, equations, graphs, and verbal representations, with and without technology (A-3-M) (P-2-M) (A-4-M) 14. Construct a table of *x*- and *y*-values satisfying a linear equation and construct a graph of the line on the coordinate plane (A-3-M) (A-2-M)

A-4-M: analyzing tables and graphs to identify relationships exhibited by the data and making generalizations based upon these relationships	15. Describe and compare situations with constant or varying rates of change (A-4-M) 16. Explain and formulate generalizations about how a change in one variable results in a change in another variable (A-4-M) Also see GLEs #11, #13, and #38.
A-5-M: demonstrating the connection of algebra to the other strands and to real-life situations	See GLEs #9, #10, #11, #29, #38, and #47.

Measurement: In problem-solving investigations, students demonstrate an understanding of the concepts, processes, and real-life applications of measurement.

Benchmarks	Grade-Level Expectations
M-1-M: applying the concepts of length, area, surface area, volume, capacity, weight, mass, money, time, temperature, and rate to real-world experiences	17. Determine the volume and surface area of prisms and cylinders (M-1-M) (G-7-M) 18. Apply rate of change in real-life problems, including density, velocity, and international monetary conversions (M-1-M) (N-8-M) (M-6-M)
M-2-M: demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of measures)	19. Demonstrate an intuitive sense of the relative sizes of common units of volume in relation to real-life applications and use this sense when estimating (M-2-M) (G-1-M)
M-3-M: selecting appropriate units and tools for tasks by considering the purpose for the measurement and the precision required for the task (e.g., length of a room in feet rather than inches)	20. Identify and select appropriate units for measuring volume (M-3-M)
M-4-M: using intuition and estimation skills to describe, order, and compare formal and informal measures (e.g., ordering cup, pint, quart, gallon; comparing a meter to a yard)	21. Compare and estimate measurements of volume and capacity within and between the U.S. and metric systems (M-4-M) (G-1-M)

M-5-M: converting from one unit of measurement to another within the same system (Comparisons between systems, customary and metric, should be based on intuitive reference points, not formal computation.)	22. Convert units of volume/capacity within systems for U.S. and metric units (M-5-M)
M-6-M: demonstrating the connection of measurement to the other strands and to real-life situations	See GLEs #18 and #30.
Geometry: In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings.	
Benchmarks	**Grade-Level Expectations**
G-1-M: using estimation skills to describe, order, and compare geometric measures	Also see GLEs #19 and #21.
G-2-M: identifying, describing, comparing, constructing, and classifying geometric figures and concepts	23. Define and apply the *terms measure, distance, midpoint, bisect, bisector*, and *perpendicular bisector* (G-2-M) 24. Demonstrate conceptual and practical understanding of symmetry, similarity, and congruence and identify similar and congruent figures (G-2-M)
G-3-M: making predictions regarding transformations of geometric figures (e.g., make predictions regarding translations, reflections, and rotations of common figures)	25. Predict, draw, and discuss the resulting changes in lengths, orientation, angle measures, and coordinates when figures are translated, reflected across horizontal or vertical lines, and rotated on a grid (G-3-M) (G-6-M) 26. Predict, draw, and discuss the resulting changes in lengths, orientation, and angle measures that occur in figures under a similarity transformation (dilation) (G-3-M) (G-6-M)
G-4-M: constructing two- and three-dimensional models	27. Construct polyhedra using 2-dimensional patterns (nets) (G-4-M)

G-5-M: making and testing conjectures about geometric shapes and their properties	28. Apply concepts, properties, and relationships of adjacent, corresponding, vertical, alternate interior, complementary, and supplementary angles (G-5-M) 29. Solve problems involving lengths of sides of similar triangles (G-5-M) (A-5-M) 30. Construct, interpret, and use scale drawings in real-life situations (G-5-M) (M-6-M) (N-8-M) 31. Use area to justify the Pythagorean theorem and apply the Pythagorean theorem and its converse in real-life problems (G-5-M) (G-7-M) 32. Model and explain the relationship between the dimensions of a rectangular prism and its volume (i.e., how scale change in linear dimension(s) affects volume) (G-5-M)
G-6-M: demonstrating an understanding of the coordinate system (e.g., locate points, identify coordinates, and graph points in a coordinate plane to represent real-world situations)	33. Graph solutions to real-life problems on the coordinate plane (G-6-M) Also see GLEs #25 and #26.
G-7-M: demonstrating the connection of geometry to the other strands and to real-life situations (e.g., applications of the Pythagorean Theorem)	See GLE #17 and #31.
Data Analysis, Probability, and Discrete Math: In problem-solving investigations, students discover trends, formulate conjectures regarding cause-and-effect relationships, and demonstrate critical thinking skills in order to make informed decisions.	
Benchmarks	**Grade-Level Expectations**
D-1-M: systematically collecting, organizing, describing, and displaying data in charts, tables, plots, graphs, and/or spreadsheets	34. Determine what kind of data display is appropriate for a given situation (D-1-M) 35. Match a data set or graph to a described situation, and vice versa (D-1-M) 36. Organize and display data using circle graphs (D-1-M) 37. Collect and organize data using box-and-whisker plots and use the plots to interpret quartiles and range (D-1-M) (D-2-M)

D-2-M: analyzing, interpreting, evaluating, drawing inferences, and making estimations, predictions, decisions, and convincing arguments based on organized data (e.g., analyze data using concepts of mean, median, mode, range, random samples, sample size, bias, and data extremes)	38. Sketch and interpret a trend line (i.e., line of best fit) on a scatterplot (D-2-M) (A-4-M) (A-5-M) 39. Analyze and make predictions from discovered data patterns (D-2-M) 40. Explain factors in a data set that would affect measures of central tendency (e.g., impact of extreme values) and discuss which measure is most appropriate for a given situation (D-2-M) 41. Select random samples that are representative of the population, including sampling with and without replacement, and explain the effect of sampling on bias (D-2-M) (D-4-M) Also see GLE #37.
D-3-M: describing informal thinking procedures (e.g., solving elementary logic problems using Venn diagrams, tables, charts, and/or elementary logic operatives to solve logic problems in real-life situations; reach valid conclusions in elementary logic problems involving "and, or, not, if/then")	
D-4-M: analyzing various counting and enumeration procedures with and without replacement (e.g., find the total number of possible outcomes or possible choices in a given situation)	42. Use lists, tree diagrams, and tables to apply the concept of permutations to represent an ordering with and without replacement (D-4-M) 43. Use lists and tables to apply the concept of combinations to represent the number of possible ways a set of objects can be selected from a group (D-4-M) Also see GLE #41.
D-5-M: comparing experimental probability results with theoretical probability (e.g., representing probabilities of concrete situations as common fractions, investigating single-event and multiple-event probability, using sample spaces, geometric figures, tables, and/or graphs)	44. Use experimental data presented in tables and graphs to make outcome predictions of independent events (D-5-M) 45. Calculate, illustrate, and apply single- and multiple-event probabilities, including mutually exclusive, independent events and non-mutually exclusive, dependent events (D-5-M)
D-6-M: demonstrating the connection of data analysis, probability, and discrete math to other strands and to real-life situations	

Patterns, Relations, and Functions: In problem-solving investigations, students demonstrate an understanding of patterns, relations, and functions that represent and explain real-world situations.

Benchmarks	Grade-Level Expectations
P-1-M: describing, extending, analyzing, and creating a wide variety of numerical, geometrical, and statistical patterns (e.g., skip counting of rational numbers and simple exponential number patterns)	46. Distinguish between and explain when real-life numerical patterns are linear/arithmetic (i.e., grows by addition) or exponential/geometric (i.e., grows by multiplication) (P-1-M) (P-4-M) 47. Represent the n^{th} term in a pattern as a formula and test the representation (P-1-M) (P-2-M) (P-3-M) (A-5-M)
P-2-M: describing and representing relationships using tables, rules, simple equations, and graphs	See GLEs #13 and #47.
P-3-M: analyzing relationships to explain how a change in one quantity results in a change in another (e.g., change in the dimensions of a rectangular solid affects the volume)	48. Illustrate patterns of change in dimension(s) and corresponding changes in volumes of rectangular solids (P-3-M) Also see GLE #47.
P-4-M: demonstrating the pervasive use of patterns, relations, and functions in other strands and in real-life situations	See GLE #46.

GLOSSARY

MATHEMATICS GLOSSARY

Absolute Error — The difference between a measured value and the actual value. For example, when the thermometer reads 75º, the actual temperature might be 73º. The absolute error is 2 degrees.

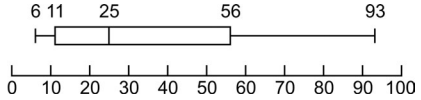
Accuracy — The extent to which a measurement/value/quantity conforms to an actual fact. For example, "The accuracy of this estimate will ensure there is enough water in the container without it overflowing."

Associative Property — A property, applicable to addition and multiplication, which states that it does not matter how numbers are grouped if they are all being added or all being multiplied (e.g., 2 + (4 + 9) = (2 + 4) + 9).

Asymptote — A line or curve that is approached (but never actually reached) by a function (e.g., the *x*-axis is an asymptote of the function $f(x) = \frac{1}{x}$).

Backward Reasoning — The reasoning involved when one assumes a conclusion is true and then works (or reasons) backwards to the evidence that the conclusion is true.

Box and Whiskers Plot — A graph in which five elements in a set of data are specifically marked: minimum value, first quartile, median value, third quartile and maximum value. This plot indicates a minimum of 6, first quartile of 11, median of 25, third quartile of 56 and maximum value of 93.



Combinations — Sets containing a certain number of objects selected from another set. The combinations of three items from the set $\{a,b,c,d\}$ are: $\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}$. The sets *{a, b, c}* and *{b, a, c}* are considered to be the same sets.

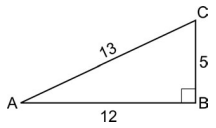
GLOSSARY

Common Equivalent Reference Points	Common fractions with decimal equivalents that can be used to approximate other nearby values. For example, ¼ = 0.25, ½ = 0.5, and ¾ =0 .75. Since 4/10 is slightly less than ½, its decimal equivalent will be slightly less than 0.5.
Commutative Property	A property, applicable to addition and multiplication; this property states that it does not matter in which order numbers are added or multiplied. (e.g., 5 + 6 = 6 + 5 and 8 x 9 = 9 x 8).
Complementary Events	Events that, when combined, constitute all possibilities. In the study of mathematics, the set of all complementary events constitutes the universal set. For example, writing an even integer or writing an odd integer are complementary events, since every possible integer is either even or odd.
Computational Fluency	A level of skill reached when a person is able to execute an algorithm or procedure efficiently and correctly without assistance.
Congruent	Figures that have the same size and shape.
Conic Sections	Figures that result from the intersection of a plane and a double-napped cone. The conic sections are a circle, an ellipse, a parabola, and a hyperbola.
Contrapositive	Given a statement of the form, "If P, then Q," the contrapositive is the statement, "If not Q, then not P." The contrapositive is logically equivalent to the original statement. Given the statement, "If it rained, then Bob went inside," the contrapositive is, "If Bob did not go inside, then it did not rain."
Converse	Given a statement of the form, "If P, then Q," the converse is the statement, "If Q, then P." The converse is not necessarily logically equivalent to the original statement. For example, given the statement, "If it rained, then Bob went inside," the converse is, "If Bob went inside, then it rained."
Coordinate Systems	Systems used to locate points using lines or points.

GLOSSARY

Cosine Ratio

The cosine of an angle in a right triangle is the ratio of the length of the adjacent side to the length of the hypotenuse. For example, in triangle ABC shown below, the cosine of angle A $\left[\cos(A)\right]$ is the ratio $\frac{12}{13}$.

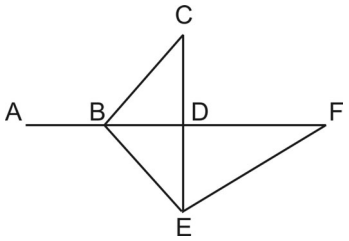


Degenerate Conics

Figures that result from the intersection of a plane and a double-napped cone, but are not "true" conic sections. The degenerate conics are a point, a line, and two intersecting lines.

Euler Path

A path that connects two vertices of a graph and travels each path in the graph exactly once. For example, the path A-B-C-D-B-E-D-F-E is an Euler path.



Dependent Events

Events that influence each other. If a bag contains three red marbles and two green marbles, randomly picking a red marble from the bag and then randomly picking a green marble from the bag (without replacing the red marble) are dependent events.

Dilation

A transformation that maps lines onto parallel lines. A dilation may be a translation (no size change) or an enlargement. The image on a movie screen represents a dilation of the image on the film.

GLOSSARY

Discrete Mathematics

The branch of mathematics dealing with situations in which there are a finite or countable number of values or objects (i.e., not continuous). For example, since no fractional values are applicable, determining the combination of colors that can be used in a drawing requires application of discrete mathematics. Three colors, a discrete number, is reasonable. $3\frac{1}{2}$ and other fractional numbers of colors do not make sense and are not reasonable solutions.

Distributive Property of Multiplication Over Addition

Property that states that for numbers *a, b,* and *c*, $a(b+c) = ab + ac$.

Expanded Form

The form of a number written as a sum to show place value. For example, the expanded form of 367 is 300 + 60 + 7.

Frequency Table

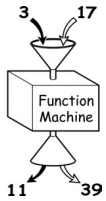
A table that lists how often different outcomes occur. Below is a frequency table representing the results of a poll about students' favorite fruits.

FAVORITE FRUITS	
Fruit	**Frequency**
apple	5
banana	9
orange	5
strawberry	7

GLOSSARY

Function Machine

A simple way to think of a function. You provide one input value and the machine (function) produces one output value. The diagram below shows a function machine which doubles and adds five to each number placed into the machine.

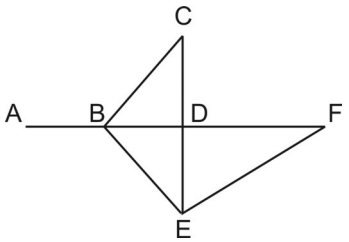


Fundamental Counting Principle

The principle that states that, "If there are **r** ways to do one thing and **s** ways to do another, and **t** ways to do a third thing, and so on, then the number of ways of doing all those things at once is **r** x **s** x **t** x …". Suppose a license plate lists a sequence of three letters followed by three digits. There are a total of $26 \bullet 26 \bullet 26 \bullet 10 \bullet 10 \bullet 10 = 17,576,000$ possible, different license plates.

Hamiltonian Path

A path that connects two vertices of a graph and visits each vertex in the graph exactly once. The path A-B-C-D-E-F is a Hamiltonian path.



Independent Events

Events that have no influence on each other. For example, flipping "tails" with a coin and rolling a four with a die are independent events.

GLOSSARY

Inverse — Given a statement of the form, "If P, then Q," the inverse of the statement is, "If not P, then not Q." The inverse is not necessarily logically equivalent to the original statement. Given the statement, "If it rained, then Bob went inside," the inverse is, "If it didn't rain, then Bob didn't go inside."

Inverse Operations — Operations that "undo" or are opposites of one another are inverse operations (e.g., addition and subtraction, multiplication and division).

Line of Symmetry — A line that divides a geometric object into two congruent halves.

Manipulatives — Concrete, physical objects used to help illustrate mathematical concepts.

Matrix (*pl.* Matrices) — A set of values arranged in a rectangular array. For example, the coefficients of the expressions $3x^2 + 7x + 9$, $11x^2 + (-8x) + 52$, and $-4x^2 + 21x + (-7)$ are represented in the matrix $\begin{bmatrix} 3 & 7 & 9 \\ 11 & -8 & 52 \\ -4 & 21 & -7 \end{bmatrix}$.

Mental Math — Computations and estimations performed without the aid of paper and pencil.

Mutually Exclusive Events — Two or more events, each of which precludes all the others. For example, the people voting in a presidential election and the people who are celebrating their 15^{th} birthday on that election day are mutually exclusive events.

Ordinal Number — A number that denotes position in a sequence. In the sentence, "Susan was the 6^{th} person in line," 6^{th} is an ordinal.

Outcomes — Results that are possible from an experiment or simulation. For example, the possible outcomes of rolling a six-sided number cube are rolls of 1, 2, 3, 4, 5, and 6.

Perfect Square — A number that can be written as the square of one of its factors. For example, the number 36 is a perfect square since it is the product of 6 and 6 (i.e., $6 \bullet 6 = 36$). The number 36 is said to be the square of 6.

GLOSSARY

Permutations — The ordered arrangements of the elements of a set. For example, the permutations of the list $\{A, B, C\}$ are (1) $\{A, B, C\}$ (2) $\{A, C, B\}$ (3) $\{B, A, C\}$ (4) $\{B, C, A\}$ (5) $\{C, A, B\}$ and (6) $\{C, B, A\}$

Pictographs — A visual representation of statistical data that uses pictures to indicate value or quantity.

Picture Graphs — See **pictograph**.

Polyhedron (*pl.* Polyhedra) — A three dimensional object with faces that are plane polygons. Cubes are polyhedra in which each face is a square.

Polynomial Expression — A mathematical expression that is the sum of terms, each of which is the product of a constant and a non-negative power of a variable or variables. For example, the expression $5x^7 + 11x^2 + 7x + (-3)$ is a polynomial expression.

Precision — The degree of specificity to which a measurement/value/quantity is determined. For example, "The measurement is precise to the nearest millimeter."

Probability — A number between 0 and 1, inclusive, which indicates the likelihood of an event occurring. For example, the probability of rolling a 1 on a fair, six-sided number cube is 1/6.

Pythagorean Theorem — The theorem that states a triangle is a right triangle if, and only if, the sum of the squares of the two sides is equal to the square of the hypotenuse. For example, the measures of the sides and hypotenuse of a triangle are 6 in., 8 in., and 10 in. Therefore, the triangle is a right triangle since $6^2 + 8^2 = 10^2$.

Quartiles — The three values that divide a set of data into four intervals with an equal number of elements in each interval. In the set of values $\{1,2,3,4,5,6,7\}$, the quartiles are 2 (1^{st}), 4 (median) and 6 (3^{rd}).

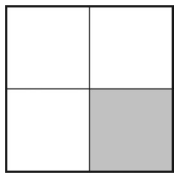
Range — The absolute difference between the greatest and least value in a set of data. For example, the range of the data set $\{7,8,12,17,23\}$ is $16\,(23 - 7)$.

GLOSSARY

Rational Number — Any number that can be written as a fraction in which both the numerator is an integer and the denominator is a natural number. For example, 3/7 is a rational number since it can be written as a fraction.

Rectangular Array — An arrangement of objects to aid in understanding multiplication. For example, four rows with three pieces of candy in each row can be used to illustrate 3 x 4 = 12 and 4 x 3 = 12.

Region Model — A geometric figure divided into equally-sized parts to illustrate fractional concepts. For example, the model below could be used to illustrate ¼ (the shaded region) or ¾ (the unshaded region).



Related Turn-around Pairs — Addition and multiplication facts that are the same except for the order in which the numbers are added or multiplied. For example, 2 + 3 = 5 and 3 + 2 = 5 is a related turn-around pair.

Rotational Symmetry — The property of an object such that after the object is rotated a number of degrees (less than 360°), the object is identical to the original object. For example, a square has rotational symmetry because it can be rotated 90 degrees and then appears identical to the original square.

Scatter Plot — A two-dimensional graph of a collection of points.

Sequences of Arithmetic Growth — A sequence of numbers in which the difference between successive terms is constant and the value of successive terms is increasing. For example, the pattern 4, 7, 10, 13, 16, 19, … is an arithmetic sequence with a constant difference of 3.

Sequence of Geometric Growth — A sequence of numbers in which the ratio of successive elements is constant and the value of successive elements is increasing. For example, the pattern 3, 6, 12, 24, 48, 96, 192, … is a geometric sequence with a constant ratio of 2.

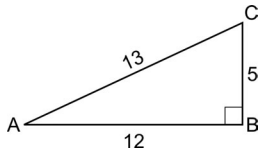
GLOSSARY

Set Model — A model for fractions in which the fraction is shown by a subset of highlighted objects out of the total set of objects. For example, one-third might be represented as • o o.

Similarity — The quality of having the same shape and proportional corresponding linear measurements. For example, the triangles are the same shape, but each side of the one triangle is six times as long as the corresponding side of the other triangle. Also, a marble and a bowling ball are not the same size, but they are similar.

Simulation — A representation (physical, written, verbal, graphic, procedural, experimental, etc.) of a situation or event(s). For example, the results of turning a spinner that is divided into ten equivalent parts marked zero (0) through nine (9) can be used to simulate the sequence of numbers on a license plate.

Sine Ratio — The sine of an angle in a right triangle is the ratio of the length of the opposite side to the length of the hypotenuse. For example, in triangle ABC shown below, the sine of angle A $\left[\sin(A)\right]$ in triangle ABC is the ratio $\frac{5}{13}$.



Skip-counting — Counting forward or backwards by a number other than 1. For example, starting at 2 and skip-counting forward by 2s would result in the sequence: 2, 4, 6, 8, …

Standard Form — The form in which numbers are traditionally written. For example, 367 is in standard form.

GLOSSARY

Stem and Leaf Plot

A data display consisting of "stems," which are the data with the last digit removed, and "leaves," which are the last digits of the data.

Stem	Leaves
3	4 4 6 8
4	0 3 6 6 7
5	1 1 3 4

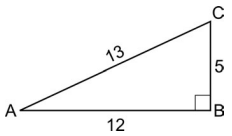
Key: 4 | 3 = 43

Symbolic Representation

A representation that uses symbols to model a situation or event. For example, the circumference of a circle is found by multiplying the diameter by pi. The symbolic representation that models this calculation is $C = \pi d$.

Tangent Ratio

The tangent of an angle in a right triangle is the ratio of the length of the opposite side to the length of the adjacent side. For example, in triangle ABC shown below, the tangent of angle A $\left[\tan(A)\right]$ in triangle ABC is the ratio $\frac{5}{12}$.



Tessellations

A covering of the plane using the same, non-overlapping shapes.

Trend Line (line of best fit)

The line, or equation of a line, that best represents the trend formed by the points in a scatter plot.

Unwrapping

The act of solving an equation by using inverse operations and knowing which operation to use first. This concept is sometimes associated with the concept of wrapping and unwrapping a present. When you unwrap, the first step is to "undo" the last step used when the gift was wrapped. For example, to solve the equation $4x + 2 = 6$, you would subtract 2 from both sides of the equation to "unwrap" the x.