GRADE-LEVEL EXPECTATIONS (GLE) HANDBOOK

MATHEMATICS
GRADES 8–12

LOUISIANA DEPARTMENT OF EDUCATION

CECIL J. PICARD
STATE SUPERINTENDENT OF EDUCATION

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ENGLISH LANGUAGE ARTS COMMITTEE MEMBERS

Walter Brown, Caddo Parish
Beverly Criddle, Orleans Parish
Dory Doud, Caddo Parish
Beth Ferguson, Cameron Parish
Debra Gant, Natchitoches Parish
Brenda Green, Allen Parish
Charyl Guidry, Jefferson Davis Parish
Janis Hill, Ouachita Parish
Alice Himel, Lafourche Parish
Inez Jenkins, East Baton Rouge Parish
Kristin Kaufman Patton, East Baton Rouge Parish
Sandra LaBry, Lafayette Parish
Sandra Landry, Jefferson Parish
Sandra Miller, Caddo Parish
Annette Norman, Calcasieu Parish
Martha Patten, Bossier Parish
Patricia Plauche, Lafayette Parish
Cassie Puckett, Avoyelles Parish
Nancy Romero, Lafayette Parish
Cindy Rushing, Rapides Parish
Karen Ryder, St. Landry Parish
Rachel Sanders, Calcasieu Parish
Rosemary Sanders, St. Charles Parish
Ellen Schexnider, St. Tammany Parish
Kelly Self, Rapides Parish
Patti Trudell, Consortium for Education, Research and Technology
Stacey Viator, Livingston Parish
Martha Waldrop, Jefferson Parish
Daneen Witty, West Baton Rouge Parish

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MATHEMATICS COMMITTEE MEMBERS

Christine Armand, Avoyelles Parish
Tommy Awtry, Louisiana State University – Alexandria
Denise Blume, St. Tammany Parish
George Broussard, Lafourche Parish
Mary Caballero, East Baton Rouge Parish
Judith Cain, Lafayette Parish
Robyn Carlin, Iberville Parish
Michelle Fazio, Natchitoches Parish
Pat Gilbert, Lafayette Parish
Tena Golding, Southeastern Louisiana University
Torrie Guzzetta, Vermilion Parish
Theresa Hamilton, Natchitoches Parish
Carolyn Hannum, Calcasieu Parish
Gail Harvey, Avoyelles Parish
Stacey Jamison, Caddo Parish
Kathy Jordan-Noel, DeSoto Parish
Davis Kieff, Lafourche Parish
Michele Lewis, Ouachita Parish
Ellen Marino, St. Tammany Parish
Leigh Myers, Northwestern State University
Emily Rash, City of Monroe
Kathleen Ross, Jefferson Parish
Louis Schultz, Sr., Southeastern Louisiana University
Kim Shackelford, Ouachita Parish
Sharon Thomas, Jefferson Parish
Mary Thompson, Orleans Parish
Margaretta Thornton, Calcasieu Parish
Jean Ware, Caddo Parish
Mattie White, Orleans Parish
Tracey Winn, Calcasieu Parish

ACKNOWLEDGEMENTS

SCIENCE COMMITTEE MEMBERS

Barbara Armfield, Delta Region Systemic Initiative
Charlotte Bihm, St. Landry Parish
Lawrence Blanchard, Jr., Orleans Parish
Lula Boenig, Jefferson Davis Parish
Michelle Brand-Buchanan, Rapides Parish
Mary Alice Cain, Jefferson Parish
Terri Colclough, Ascension Parish
Tammie Cox, Ouachita Parish
William Daniel, Bossier Parish
Debbie Gauthier, LaSalle Parish
Dana Gonzalez, Orleans Parish
Janet Graff, Caddo Parish
Norma Guillory, Calcasieu Parish
Christa Haymon, Vernon Parish
Terrie Johnson, Bossier Parrish
Mia Kleinpeter, East Baton Rouge Parish
Gary Kratzer, Calcasieu Parish
Errol LaBorde, Avoyelles Parish
LuLu Martin, Caddo Parish
Starr Paul, East Baton Rouge Parish
Laura Ponder, Natchitoches Parish
Heidi Rhea, Tangipahoa Parish
Donna Rigsby, Beauregard Parish
Royal Shelton, Orleans Parish
Joyce Tate, City of Monroe
Ranell Troxler, St. Charles Parish
Pauline Vidrine, St. Landry Parish
Patricia Watts, University of Louisiana-Monroe
Erica Wells, East Baton Rouge Parish
Gene Winegeart, Bossier Parish

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SOCIAL STUDIES COMMITTEE MEMBERS

Martha Antoine, Iberia Parish
Carol Aucoin, Assumption Parish
Lauren Avery, Ascension Parish
Janel Bourgeois, Assumption Parish
Anna Maria Brodie, Iberia Parish
Sarah Burton, Caddo Parish
Jill Crain, Calcasieu Parish
Judd Dupuy, Avoyelles Parish
Paula Foret, Iberia Parish
Amy Graham, Bossier Parish
June Griffin, Orleans Parish
Angela Guillory-Cassimere, St. Landry Parish
Demetria Jones, Orleans Parish
Donna Mancuso, Calcasieu Parish
Elizabeth Manning, Lincoln Parish
Jennifer Mixon, Avoyelles Parish
Ben Necaise, West Feliciana Parish
Paulette Parker, Ascension Parish
Lesia Rawls, Washington Parish
Cathy Seymour, Northwestern State University
Pinelle Spain, Calcasieu Parish
Glen Sutherland, St. Tammany Parish
Jane Texada, Rapides Parish
Ann Trappey, Tangipahoa Parish
Mary Webb Green, Jefferson Parish
Steve Westbrook, Ascension Parish
Bonita Williams, Vernon Parish
Ginger Williams, Caddo Parish

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NATIONAL CONSULTANTS

John A. Dossey, Mathematics
Distinguished University Professor of Mathematics Emeritus
Illinois State University

Linda Farrell, English Language Arts
Reading Street, LLC
Alexandria, VA

Daniel W. Gregg, Social Studies
Connecticut State Department of Education

Susan Hall, English Language Arts
Independent Reading Consultant

Jerry Horn, Science
The Evaluation Center
Western Michigan University

Billie M. Kapp, Social Studies
Connecticut Geographic Alliance

ACKNOWLEDGEMENTS

EXTERNAL REVIEWERS

Arthur Halbrook, Coordinator
Council of Chief State School Officers

English Language Arts
Clarence Bina, North Dakota
Kathleen (Katie) Forbrich, Maryland
Ellen Last, Wisconsin

Mathematics
Douglas A. Grouws, Iowa
Mary Lindquist, Georgia
Carole White, Delaware

Science
Jon Pederson, Oklahoma
Joanne Vazquex, Arizona
Jim Woodland, Nebraska

Social Studies
Fred Czarra, Maryland
Joann Pruett, Delaware
Robert "Bob" Jarvis, Maryland

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LOUISIANA DEPARTMENT OF EDUCATION

Cecil J. Picard
State Superintendent of Education

Rodney Watson
Assistant Superintendent
Office of Student and School Performance

Scott Norton
Division Director
Student Standards and Assessments

Nancy Beben, Section Supervisor
Secondary Standards

Mary Louise Jones, Section Supervisor
Elementary Standards

Claudia Davis, Section Supervisor
Assessment Development and Support

Fen Chou, Education Research Analyst Manager
Assessment Research and Technology

Carolyn Sessions, GLE Project Director
Student Standards and Assessment

Chris Parmentier, Educational Technology Specialist
Assessment Research and Technology

Meg Casper
Public Relations Coordinator

ACKNOWLEDGEMENTS

DIVISION OF STUDENT STANDARDS AND ASSESSMENTS
COMMITTEE COORDINATORS

Nancy Beben, Mathematics
Nikki Bray-Clark, Social Studies
Anne Campbell, Social Studies
Sharon Compton, Mathematics
Susannah Craig, Science
Allison Cruz-Mitchell, Social Studies
Patrick Dobard, Social Studies
David Hopkins, Mathematics
Joni Lacy, English Language Arts
Jean May-Brett, Science
Donna Garrett Moore, English Language Arts
Taylor Powers, English Language Arts
Marybeth Ridgel, Science
Jan Sibley, Mathematics
Ivy Starns, English Language Arts
Beth Strange, English Language Arts
Ann Wilson, Science

LOUISIANA DEPARTMENT OF EDUCATION
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Ivy Alford, Professional Development
Anne Clouatre, Special Populations
Betsy Delahaye, Reading
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Chris Frugé, Legal
Jane Grove, Reading
Nicole J. Honoré, Technology
Kathy Mouton, Professional Development
Kartina Roberts, Reading
Keisha Thomas, Reading
Ugena Whitlock, Professional Development

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LOUISIANA SYSTEMIC INITIATIVE COMMITTEE AD HOC ADVISORS

Faimon A. Roberts II, Science
Lynne D. Tullos, Mathematics

DATA RECOGNITION CORPORATION

Patricia Porter, Project Director
Donniel Johnson, Assistant Project Director
Patricia McDivitt, Lead Content Specialist
Anne Kirpes, English Language Arts Consultant
Alice Golden, English Language Arts Consultant
Christi Dennis, Social Studies Consultant
Todd Werner, Mathematics Consultant
Roy Merritt, Science Consultant
Marilyn Cook, Assessment Consultant

RESOURCES FOR LEARNING

Linda Wurzbach, President
Tracy Laughlin, Project Manager
Amy Guadagnoli, Creative Media Specialist
Lauren Sartain, Research Assistant
Amy Buller, Senior Project Director

DNA CREATIVE SERVICES

David Cardwell, Videographer

GRADE-LEVEL EXPECTATIONS HANDBOOK OVERVIEW

INTRODUCTION

In 1997, rigorous K–12 content standards were approved for mathematics, English language arts, science, social studies, foreign languages, and the arts. In conjunction with the development of these content standards, the Louisiana Department of Education (LDE) developed standards-based tests in English language arts, mathematics, science and social studies for grades 4, 8, 10, and 11. These tests have served as the basis for Louisiana's School and District Accountability System for several years. The development of Grade-Level Expectations (GLEs) in 2003 in English language arts, mathematics, science and social studies was a continuation of Louisiana's effort to expand and extend the content standards. GLEs identify what all students should know or be able to do by the end of a given grade level from prekindergarten through grade 12 in these four content areas.

PURPOSES OF THE GLE HANDBOOKS

The Louisiana Grade-Level Expectations handbooks include grade-specific information about the GLEs. Each handbook includes introductory information for each content area contained within the handbook, a glossary, and tables that map the relationship between the standards and/or strands, benchmarks, and GLEs. Each handbook also correlates with one of fourteen GLE posters.

GRADE-LEVEL EXPECTATIONS DEVELOPMENT GUIDELINES

Each grade-level expectation is meant to further define a content standard and benchmark(s). There is a progression of specificity; the standards represent broad statements, benchmarks are more specific, and GLEs provide the most detail. Grade-level expectations have been developed from prekindergarten through grade 12.

GLEs do not represent the entire curriculum for a given grade or course. Rather, they represent the core content that should be mastered by the end of a given year by all students. For mastery to be achieved at a given level, it may be necessary for those skills to be introduced at an earlier grade. Similarly, skills will need to be maintained after mastery has occurred.

The GLEs were developed with the following goals in mind:

- to articulate learning from PreK–12
- to be appropriate for the developmental or grade level of students
- to move from the concrete to the abstract
- to attend to prerequisite skills and understandings
- to be specific, but not so specific as to be too small in "grain size" compared with other GLEs for a particular content area

The GLEs were developed with an effort to avoid including:

- statements of curricular activities or instructional strategies
- value-laden concepts and understandings

SUMMARY OF GLE DEVELOPMENT PROCESS

In December 2002, the LDE selected Data Recognition Corporation (DRC) as the contractor for the Grade-Level Expectations Project. The steps in the process of developing the Grade-Level Expectations for English language arts, mathematics, science, and social studies are described below.

1. **Identifying National Consultants.** National consultants representing each content area were selected to provide a national perspective to the GLE project and to assist the LDE and DRC with various steps in the GLE development process.

2. **Conducting Teacher Committee Meetings.** In March and April of 2003, content-area development committees, comprised of approximately 120 Louisiana classroom teachers, administrators, special populations teachers, and resource teachers chosen for their knowledge of standards and curriculum, were convened to assist with the development of the GLEs for English language arts, mathematics, science, and social studies.

3. **Completing Initial drafts.** The draft GLEs for the four content areas were completed and prepared for further review.

4. **Convening Focus Groups.** Two eighty-member groups of educators, nominated by their districts, reviewed the draft GLEs for horizontal and vertical alignments with the standards for English language arts, mathematics, science, and social studies and provided additional input on these initial drafts to the LDE.

5. **Conducting an Online/Electronic Public Review.** An online/electronic public review and feedback system was developed for the GLEs and made available on the LDE Web site. The purpose of this review was to solicit a broad range of feedback on the GLEs from parents, teachers, and other stakeholders.

6. **Completing an External Review.** With assistance from the staff of the Council of Chief State School Officers (CCSSO), twelve content specialists (three per content area) from other states or from universities or educational organizations across the nation were invited to serve on the review committee. In addition, the LDE also invited the national consultants from each content area to participate in the review meeting to answer questions about the development of the GLEs. External review committee members reviewed the GLE documents prior to the meetings, met with national consultants and LDE staff, and provided specific feedback on the GLEs via a written report.

7. **Conducting Final Committee Meeting.** The public comments and suggestions from the online public review were tabulated, and the additional comments and feedback from the external review committee meeting were compiled for sharing with committee members during the third and final GLE development committee meeting. The outcome of the third meeting of content-area committees consisted of suggestions for final edits that would be incorporated into the documents for presentation to the Louisiana State Board of Elementary and Secondary Education.

8. **Obtaining SBESE Board Approval of the GLEs.** In October 2003, the LDE staff presented the GLEs to the Louisiana State Board of Elementary and Secondary Education for review and approval.

9. **Conducting Preworkshops and GLE Awareness Workshops.** Two preworkshops for district supervisors in early December 2003 and fifteen GLE awareness workshops for local school personnel in late January 2004 have been conducted to inform educators about the grade-level expectations and their future role as it relates to curriculum and assessment

CONCLUSION

Louisiana's content standards and benchmarks have guided the Louisiana education reform program for several years. As an extension of the content standards and benchmarks, the GLEs provide a link among instruction, curriculum, and assessment. The primary goal is a common understanding among parents, students, teachers, and the general public about what is expected of Louisiana students as they progress from grade to grade.

MATHEMATICS INTRODUCTION

INTRODUCTION

Grade-Level Expectations (GLEs) are explicit recommendations for what students should know and be able to do as a result of each level of schooling from prekindergarten through grade 12. This degree of specificity is made with the expectation that all students in Louisiana have access to a high-quality instructional program in mathematics. Instructional programs must provide all students with a solid foundation in mathematics, regardless of race or ethnic origin, geographical location, or socioeconomic status. The design, delivery, and assessment of such programs require careful planning, articulation, and coordination.

The GLEs provide a basis for all of these goals. Further, they clearly define what schools and teachers need to focus on in each year of instruction. This focus provides teachers with a quick listing of the main ideas that frame the year's study and, at the same time, helps students see the connections in the mathematics they are studying.

The following list of GLEs provides a comprehensive look at the expectations for all Louisiana mathematics students for each grade level. They define the targets for instruction, assessment, and emphasis during each year of public schooling. Further, they outline the promises that each teacher is striving to make to successive teachers about what their students know and can do. Such attempts at articulation serve to tighten the curriculum; help avoid programs that have great breadth, but no depth; and assist in helping teachers link the mathematics they are teaching to what students have already studied and to what they will be using in other areas of the curriculum.

ORGANIZATIONAL PRINCIPLES

All GLEs in mathematics are organized by grade levels and address one or more benchmarks in the six mathematics strands: Number and Number Relations; Algebra; Geometry; Measurement; Data Analysis, Probability, and Discrete Math; and Patterns, Relations, and Functions.

With the single exception of prekindergarten, there are GLEs representing each of the six strands for each grade level. In addition, the number of GLEs in each strand either increases or decreases depending on the experience of the learner. For example, most of the GLEs in prekindergarten and kindergarten are based on Number and Number Relations, and few are based on the Data Analysis, Probability, and Discrete Math, or Patterns, Relations and Functions strands. By grade 9, the trend reverses as students have completed most of their study of Number.

Not all high school students take the same courses, nor do they take them in the same order. As a result**, the Grade-Level Expectations for grades 9 and 10 represent the content that all students should master before graduation.** When mastery occurs will depend on the order in which courses are taken. The expectations are written with a view toward a greater integration of mathematics learning across these two grades than traditionally has been the case. For schools teaching separate courses in Algebra I and Geometry, the total of the Grade-Level Expectations for grades 9 and 10 should be considered as what students should be able to do by the end of grade 10, rather than thinking of these as separate outcomes for grade 9 and grade 10.

MATHEMATICS INTRODUCTION

The GLEs for grades 11/12 represent the core content for students who enroll in Algebra II and higher courses in preparation for post-secondary education. They reflect the content of collegiate entrance examinations and mathematics found in common applications in such programs of study.

BENCHMARK CODES

Codes at the end of each GLE are used to identify a developmental profile indicator from the *Louisiana Standards for Programs Serving Four-Year-Old Children* (Table 1) and/or benchmarks from the *Louisiana Mathematics Framework* (Table 2). A GLE may apply to more than one benchmark and, as a result, a GLE may have more than one code.

Developmental Profile Indicator Codes: The first part is always PK, which means prekindergarten. The second part indicates the domain and content area (i.e., Cognitive Mathematics). The third part indicates the skill area (i.e., Number, Measurement, Geometry, Data, Patterns) and skill number (e.g., 1, 2).

Table 1. Explanation of Developmental Profile Indicator Codes

Code(s)	Explanation
PK-CM-N1	Prekindergarten, Cognitive Mathematics, Number, Skill 1
PK-CM-M3	Prekindergarten, Cognitive Mathematics, Measurement, Skill 3
PK-CM- G2	Prekindergarten, Cognitive Mathematics, Geometry, Skill 2

Benchmark Codes: Benchmark codes have 3 parts. The first part in the benchmark code refers to the strand (e.g., Number and Number Relations). The second part refers to the benchmark number. The third part refers to the grade cluster (i.e., E, M, H).

Table 2. Explanation of Benchmark Codes

Code	Explanation
N-1-E	Number and Number Relations, Benchmark 1, Elementary
G-5-M	Geometry, Benchmark 5, Middle School
A-3-H	Algebra, Benchmark 3, High School

MATHEMATICS INTRODUCTION

Each of the following GLE grade listings opens with a concise note about the focal emphases of that grade. These emphases serve to shape and mold the program for that individual grade level. Such focal points at each level and the careful articulation of these ideas lead to significant growth in students' abilities to learn and profitably apply mathematics in their professional, vocational, and personal lives.

EIGHTH GRADE INTRODUCTION

Eighth Grade

Grade 8 mathematics sees the final shift from arithmetic to Algebra take place. The work in Number and Number Relations focuses on pulling the work with natural numbers, integers, decimals, and fractions to a mature understanding, and shifting the emphasis to the real number system. More focus is placed on understanding the properties of the real numbers and their operations and using this knowledge as a foundation for understanding elementary Algebra. By the end of grade 8, students have a mastery of all operations with integers, fractions, and decimals. They numerically interpret situations involving small integer exponents and use scientific notation to represent large and small numbers. They employ proportions to handle a wide variety of percentage problems, including those of percent of increase and decrease.

Measurement concepts are a major focus of the grade 8 year. At this level students complete their understanding of the volume of prisms, cylinders, spheres, pyramids, and cones. They also encounter and learn to work with the concepts related to the derived measures of density, speed, and monetary equivalences of different currencies. In Geometry, they do related ratio and proportion work in solving problems involving scale drawings and the finding of measurements indirectly with the Pythagorean theorem or its converse. These latter relationships result as extensions of the students' work with similar triangles and angle relationships.

In Data, grade 8 students learn to use methods of representation that allow them to look at the role of variability, or spread, in data sets. They learn to use spreadsheets to analyze and represent data. They learn to represent data sets using box-and-whisker plots and percentiles with and without technological support. They analyze two-variable data in scatterplots and intuitively fit and interpret trend lines fitted to the data. These analyses are used as bases for simple predictions from such data sets. With Probability, grade 8 students are first encountering permutations and combinations in organized settings. These are used both as a basis for probability and as a more general way of counting for discrete settings. Students also extend their work with sample spaces and probability trees to consider the outcomes and chances associated with various events.

In grade 8, students begin to distinguish between linear and nonlinear settings, primarily exponential growth settings in the latter category. They solve and graph the solutions to one- and two-step linear equations and inequalities. Working with linear equations, they graph them in the plane and discuss how the change in parameters affects their appearance when graphed. In patterns, grade 8 students begin to more formally consider sequence-based patterns by determining a numbering structure for terms and then using it to represent the general *nth* term for such sequences.

SAMPLE PAGE AND KEY FOR MATHEMATICS

Strand/ Standard

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-E: constructing number meaning and demonstrating that a number can be expressed in many different forms (e.g., standard notation, number words, number lines, geometrical representation, fractions, and decimals)	1. Read and write place value in word, standard, and expanded form through 1,000,000 (N-1-E) 2. Read, write, compare, and order whole numbers using place value concepts, standard notation, and models through 1,000,000 (N-1-E) (N-3-E) (A-1-E) 3. Illustrate with manipulatives when a number is divisible by 2, 3, 5, or 10 (N-1-E) 4. Know all basic facts for multiplication and division through 12 x 12 and 144 ÷ 12, and recognize factors of composite numbers less than 50 (N-1-E) (N-6-E) (N-7-E) 5. Read, write, and relate decimals through hundredths and connect them with corresponding decimal fractions (N-1-E) 6. Model, read, write, compare, order, and represent fractions with denominators through twelfths using region and set models (N-1-E) (A-1-E) Also see GLE #7.
N-2-E: demonstrating number sense and estimation skills, giving particular attention to common equivalent reference points (i.e., 1/4 = 25% = .25; ½ = 50% = .5; $1 = 100%, etc.)	7. Give decimal equivalents of halves, fourths, and tenths (N-2-E) (N-1-E) 8. Use common equivalent reference points for percents (i.e., ¼, ½, ¾, and 1 whole) (N-2-E) 9. Estimate fractional amounts through twelfths, using pictures, models, and diagrams (N-2-E) Also see GLE #27.

Benchmarks

Grade-Level Expectations (GLEs)

EIGHTH GRADE

MATHEMATICS

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-M: demonstrating that a rational number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, and percents)	1. Compare rational numbers using symbols (i.e., <, ≤ , =, ≥, >) and position on a number line (N-1-M) (N-2-M) 2. Use whole number exponents (0-3) in problem-solving contexts (N-1-M) (N-5-M)
N-2-M: demonstrating number sense and estimation skills to describe, order, and compare rational numbers (e.g., magnitude, integers, fractions, decimals, and percents)	3. Estimate the answer to an operation involving rational numbers based on the original numbers (N-2-M) (N-6-M) Also see GLE #1.
N-3-M: reading, writing, representing, and using rational numbers in a variety of forms (e.g., integers, mixed numbers, and improper fractions)	4. Read and write numbers in scientific notation with positive exponents (N-3-M)
N-4-M: demonstrating a conceptual understanding of the meaning of the basic arithmetic operations (add, subtract, multiply and divide) and their relationships to each other	5. Simplify expressions involving operations on integers, grouping symbols, and whole number exponents using order of operations (N-4-M)
N-5-M: applying an understanding of rational numbers and arithmetic operations to real-life situations	6. Identify missing information or suggest a strategy for solving a real-life, rational-number problem (N-5-M) Also see GLEs #2, #8 and #9.
N-6-M: constructing, using, and explaining procedures to compute and estimate with rational numbers employing mental math strategies	See GLE #3.

N-7-M: selecting and using appropriate computational methods and tools for given situations involving rational numbers (e.g., estimation, or exact computation using mental arithmetic, calculator, computer, or paper and pencil)	
N-8-M: demonstrating a conceptual understanding and applications of proportional reasoning (e.g., determining equivalent ratios, finding a missing term of a given proportion)	7. Use proportional reasoning to model and solve real-life problems (N-8-M) 8. Solve real-life problems involving percentages, including percentages less than 1 or greater than 100 (N-8-M) (N-5-M) 9. Find unit/cost rates and apply them in real-life problems (N-8-M) (N-5-M) (A-5-M) Also see GLEs #18 and #30.
Algebra: In problem-solving investigations, students demonstrate an understanding of concepts and processes that allow them to analyze, represent, and describe relationships among variable quantities and to apply algebraic methods to real-world situations.	
Benchmarks	**Grade-Level Expectations**
A-1-M: demonstrating a conceptual understanding of variables, expressions, equations, and inequalities (e.g., symbolically represent real-world problems as linear terms, equations, or inequalities)	10. Write real-life meanings of expressions and equations involving rational numbers and variables (A-1-M) (A-5-M) 11. Translate real-life situations that can be modeled by linear or exponential relationships to algebraic expressions, equations, and inequalities (A-1-M) (A-4-M) (A-5-M)
A-2-M: modeling and developing methods for solving equations and inequalities (e.g., using charts, graphs, manipulatives, and/or standard algebraic procedures)	12. Solve and graph solutions of multi-step linear equations and inequalities (A-2-M) Also see GLE #14.
A-3-M: representing situations and number patterns with tables, graphs, and verbal and written statements, while exploring the relationships among these representations (e.g., multiple representations for the same situation)	13. Switch between functions represented as tables, equations, graphs, and verbal representations, with and without technology (A-3-M) (P-2-M) (A-4-M) 14. Construct a table of x- and y-values satisfying a linear equation and construct a graph of the line on the coordinate plane (A-3-M) (A-2-M)

A-4-M: analyzing tables and graphs to identify relationships exhibited by the data and making generalizations based upon these relationships	15. Describe and compare situations with constant or varying rates of change (A-4-M) 16. Explain and formulate generalizations about how a change in one variable results in a change in another variable (A-4-M) Also see GLEs #11, #13, and #38.
A-5-M: demonstrating the connection of algebra to the other strands and to real-life situations	See GLEs #9, #10, #11, #29, #38, and #47.

Measurement: In problem-solving investigations, students demonstrate an understanding of the concepts, processes, and real-life applications of measurement.

Benchmarks	Grade-Level Expectations
M-1-M: applying the concepts of length, area, surface area, volume, capacity, weight, mass, money, time, temperature, and rate to real-world experiences	17. Determine the volume and surface area of prisms and cylinders (M-1-M) (G-7-M) 18. Apply rate of change in real-life problems, including density, velocity, and international monetary conversions (M-1-M) (N-8-M) (M-6-M)
M-2-M: demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of measures)	19. Demonstrate an intuitive sense of the relative sizes of common units of volume in relation to real-life applications and use this sense when estimating (M-2-M) (G-1-M)
M-3-M: selecting appropriate units and tools for tasks by considering the purpose for the measurement and the precision required for the task (e.g., length of a room in feet rather than inches)	20. Identify and select appropriate units for measuring volume (M-3-M)
M-4-M: using intuition and estimation skills to describe, order, and compare formal and informal measures (e.g., ordering cup, pint, quart, gallon; comparing a meter to a yard)	21. Compare and estimate measurements of volume and capacity within and between the U.S. and metric systems (M-4-M) (G-1-M)

M-5-M: converting from one unit of measurement to another within the same system (Comparisons between systems, customary and metric, should be based on intuitive reference points, not formal computation.)	22. Convert units of volume/capacity within systems for U.S. and metric units (M-5-M)
M-6-M: demonstrating the connection of measurement to the other strands and to real-life situations	See GLEs #18 and #30.
Geometry: In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings.	
Benchmarks	**Grade-Level Expectations**
G-1-M: using estimation skills to describe, order, and compare geometric measures	See GLEs #19 and #21.
G-2-M: identifying, describing, comparing, constructing, and classifying geometric figures and concepts	23. Define and apply the *terms measure, distance, midpoint, bisect, bisector*, and *perpendicular bisector* (G-2-M) 24. Demonstrate conceptual and practical understanding of symmetry, similarity, and congruence and identify similar and congruent figures (G-2-M)
G-3-M: making predictions regarding transformations of geometric figures (e.g., make predictions regarding translations, reflections, and rotations of common figures)	25. Predict, draw, and discuss the resulting changes in lengths, orientation, angle measures, and coordinates when figures are translated, reflected across horizontal or vertical lines, and rotated on a grid (G-3-M) (G-6-M) 26. Predict, draw, and discuss the resulting changes in lengths, orientation, and angle measures that occur in figures under a similarity transformation (dilation) (G-3-M) (G-6-M)
G-4-M: constructing two- and three-dimensional models	27. Construct polyhedra using 2-dimensional patterns (nets) (G-4-M)

G-5-M: making and testing conjectures about geometric shapes and their properties	28. Apply concepts, properties, and relationships of adjacent, corresponding, vertical, alternate interior, complementary, and supplementary angles (G-5-M) 29. Solve problems involving lengths of sides of similar triangles (G-5-M) (A-5-M) 30. Construct, interpret, and use scale drawings in real-life situations (G-5-M) (M-6-M) (N-8-M) 31. Use area to justify the Pythagorean theorem and apply the Pythagorean theorem and its converse in real-life problems (G-5-M) (G-7-M) 32. Model and explain the relationship between the dimensions of a rectangular prism and its volume (i.e., how scale change in linear dimension(s) affects volume) (G-5-M)
G-6-M: demonstrating an understanding of the coordinate system (e.g., locate points, identify coordinates, and graph points in a coordinate plane to represent real-world situations)	33. Graph solutions to real-life problems on the coordinate plane (G-6-M) Also see GLEs #25 and #26.
G-7-M: demonstrating the connection of geometry to the other strands and to real-life situations (e.g., applications of the Pythagorean Theorem)	See GLE #17 and #31.
Data Analysis, Probability, and Discrete Math: In problem-solving investigations, students discover trends, formulate conjectures regarding cause-and-effect relationships, and demonstrate critical thinking skills in order to make informed decisions.	
Benchmarks	**Grade-Level Expectations**
D-1-M: systematically collecting, organizing, describing, and displaying data in charts, tables, plots, graphs, and/or spreadsheets	34. Determine what kind of data display is appropriate for a given situation (D-1-M) 35. Match a data set or graph to a described situation, and vice versa (D-1-M) 36. Organize and display data using circle graphs (D-1-M) 37. Collect and organize data using box-and-whisker plots and use the plots to interpret quartiles and range (D-1-M) (D-2-M)

D-2-M: analyzing, interpreting, evaluating, drawing inferences, and making estimations, predictions, decisions, and convincing arguments based on organized data (e.g., analyze data using concepts of mean, median, mode, range, random samples, sample size, bias, and data extremes)	38. Sketch and interpret a trend line (i.e., line of best fit) on a scatterplot (D-2-M) (A-4-M) (A-5-M) 39. Analyze and make predictions from discovered data patterns (D-2-M) 40. Explain factors in a data set that would affect measures of central tendency (e.g., impact of extreme values) and discuss which measure is most appropriate for a given situation (D-2-M) 41. Select random samples that are representative of the population, including sampling with and without replacement, and explain the effect of sampling on bias (D-2-M) (D-4-M) Also see GLE #37.
D-3-M: describing informal thinking procedures (e.g., solving elementary logic problems using Venn diagrams, tables, charts, and/or elementary logic operatives to solve logic problems in real-life situations; reach valid conclusions in elementary logic problems involving "and, or, not, if/then")	
D-4-M: analyzing various counting and enumeration procedures with and without replacement (e.g., find the total number of possible outcomes or possible choices in a given situation)	42. Use lists, tree diagrams, and tables to apply the concept of permutations to represent an ordering with and without replacement (D-4-M) 43. Use lists and tables to apply the concept of combinations to represent the number of possible ways a set of objects can be selected from a group (D-4-M) Also see GLE #41.
D-5-M: comparing experimental probability results with theoretical probability (e.g., representing probabilities of concrete situations as common fractions, investigating single-event and multiple-event probability, using sample spaces, geometric figures, tables, and/or graphs)	44. Use experimental data presented in tables and graphs to make outcome predictions of independent events (D-5-M) 45. Calculate, illustrate, and apply single- and multiple-event probabilities, including mutually exclusive, independent events and non-mutually exclusive, dependent events (D-5-M)
D-6-M: demonstrating the connection of data analysis, probability, and discrete math to other strands and to real-life situations	

Patterns, Relations, and Functions: In problem-solving investigations, students demonstrate an understanding of patterns, relations, and functions that represent and explain real-world situations.	
Benchmarks	**Grade-Level Expectations**
P-1-M: describing, extending, analyzing, and creating a wide variety of numerical, geometrical, and statistical patterns (e.g., skip counting of rational numbers and simple exponential number patterns)	46. Distinguish between and explain when real-life numerical patterns are linear/arithmetic (i.e., grows by addition) or exponential/geometric (i.e., grows by multiplication) (P-1-M) (P-4-M) 47. Represent the n^{th} term in a pattern as a formula and test the representation (P-1-M) (P-2-M) (P-3-M) (A-5-M)
P-2-M: describing and representing relationships using tables, rules, simple equations, and graphs	See GLEs #13 and #47.
P-3-M: analyzing relationships to explain how a change in one quantity results in a change in another (e.g., change in the dimensions of a rectangular solid affects the volume)	48. Illustrate patterns of change in dimension(s) and corresponding changes in volumes of rectangular solids (P-3-M) Also see GLE #47.
P-4-M: demonstrating the pervasive use of patterns, relations, and functions in other strands and in real-life situations	See GLE #46.

NINTH GRADE INTRODUCTION

Ninth Grade

The Grade-Level Expectations for grades 9 and 10 represent the mathematics all students should know before graduation. The expectations are written with a view toward a greater integration of mathematics learning across these two grades than traditionally has been the case. For schools teaching separate courses in Algebra I and Geometry, the total of the Grade-Level Expectations for grades 9 and 10 should be considered as what students should be able to do by the end of grade 10, rather than thinking of these as separate grade 9 and grade 10 outcomes.

In grade 9, students extend their numeric understanding and computational skills to interpret and evaluate a wide range of integer exponents, represent and calculate with large and small numbers in scientific notation form, and to deal with direct variation and related proportional thinking. They show an increased level of numerical fluency, one marked by a real understanding of estimation, choice of computational methods, and an ability to discuss the reasonableness of answers.

In Measurement work, grade 9 students focus on units, their choice for specific situations and their role in describing measurement error, precision, and in determining the accuracy of measurements. In Geometry, grade 9 students extend their understanding of transformations (i.e., translations, reflections, and rotations) by representing their actions in the coordinate plane. The notion of "slope" of a line is considered from both a geometric and an algebraic viewpoint. Students should see it as rate of change and unit difference for growth. In studying Probability, they learn about simulations and their use in estimating probabilities in complex settings.

The focus of grade 9 work in Algebra is the study of linear relations. Here students learn to represent and solve, in a broad and comprehensive way, linear equations, inequalities, and systems of equations. They recognize such relationships in tables, graphs, and verbal forms, using differences and growth rates as the major determining characteristics. They understand the role of intercepts and slopes in interpretation and graphing. They also learn how to represent and find solutions to single equations and systems of 2 and 3 linear equations. The latter are approached through multiple methods, including the use of matrix methods (with technology). Students understand the role of domain and range and what happens to a graph when the parameters in its equation are changed.

NINTH GRADE

MATHEMATICS

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-H: demonstrating an understanding of the real number system	1. Identify and describe differences among natural numbers, whole numbers, integers, rational numbers, and irrational numbers (N-1-H) (N-2-H) (N-3-H) *
N-2-H: demonstrating that a number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, percents, and scientific notation)	2. Evaluate and write numerical expressions involving integer exponents (N-2-H) * 3. Apply scientific notation to perform computations, solve problems, and write representations of numbers (N-2-H) * Also see GLEs #1 and #19.
N-3-H: using number sense to estimate and determine if solutions are reasonable	4. Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H) * Also see GLE #1.
N-4-H: determining whether an exact or approximate answer is necessary	See GLE #4.
N-5-H: selecting and using appropriate computational methods and tools for given situations (e.g., estimation, or exact computation using mental arithmetic, calculator, symbolic manipulator, or paper and pencil)	5. Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H) * 6. Simplify and perform basic operations on numerical expressions involving radicals (e.g., $2\sqrt{3}+5\sqrt{3}=7\sqrt{3}$) (N-5-H) *

*** GLE is assessed on the *i*LEAP test.**

N-6-H: applying ratios and proportional thinking in a variety of situations (e.g., finding a missing term of a proportion)	7. Use proportional reasoning to model and solve real-life problems involving direct and inverse variation (N-6-H) *
N-7-H: justifying reasonableness of solutions and verifying results	See GLE #4.
Algebra: In problem-solving investigations, students demonstrate an understanding of concepts and processes that allow them to analyze, represent, and describe relationships among variable quantities and to apply algebraic methods to real-world situations.	
Benchmarks	**Grade-Level Expectations**
A-1-H: demonstrating the ability to translate real-world situations (e.g., distance versus time relationships, population growth, growth functions for diseases, growth of minimum wage, auto insurance tables) into algebraic expressions, equations, and inequalities and vice versa	8. Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H) * 9. Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)* 10. Identify independent and dependent variables in real-life relationships (A-1-H) * 11. Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)* Also see GLE #25.
A-2-H: recognizing the relationship between operations involving real numbers and operations involving algebraic expressions	12. Evaluate polynomial expressions for given values of the variable (A-2-H) * 13. Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H) * 14. Graph and interpret linear inequalities in one or two variables and systems of linear inequalities (A-2-H) (A-4-H) * Also see GLE # 8.
A-3-H: using tables and graphs as tools to interpret algebraic expressions, equations, and inequalities	15. Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H) *

A-4-H: solving algebraic equations and inequalities using a variety of techniques with the appropriate tools (e.g., hand-held manipulatives, graphing calculator, symbolic manipulator, or pencil and paper)	16. Interpret and solve systems of linear equations using graphing, substitution, elimination, with and without technology, and matrices using technology (A-4-H) Also see GLE #14.

Measurement: In problem-solving investigations, students demonstrate an understanding of the concepts, processes, and real-life applications of measurement.

Benchmarks	Grade-Level Expectations
M-1-H: selecting and using appropriate units, techniques, and tools to measure quantities in order to achieve specified degrees of precision, accuracy, and error (or tolerance) of measurements	17. Distinguish between precision and accuracy (M-1-H) * 18. Demonstrate and explain how the scale of a measuring instrument determines the precision of that instrument (M-1-H) * 19. Use significant digits in computational problems (M-1-H) (N-2-H) * 20. Demonstrate and explain how relative measurement error is compounded when determining absolute error (M-1-H) (M-2-H) (M-3-H) * Also see GLE #21.
M-2-H: demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of results as related to area, volume, mass, rate, and distance)	21. Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H) * Also see GLE #20.
M-3-H: estimating, computing, and applying physical measurement using suitable units (e.g., calculate perimeter and area of plane figures, surface area and volume of solids presented in real-world situations)	See GLEs #20 and #21.
M-4-H: demonstrating the concept of measurement as it applies to real-world experiences	22. Solve problems using indirect measurement (M-4-H) *

Geometry: In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings.	
Benchmarks	**Grade-Level Expectations**
G-1-H: identifying, describing, comparing, constructing, and classifying geometric figures in two and three dimensions using technology where appropriate to explore and make conjectures about geometric concepts and figures	
G-2-H: representing and solving problems using geometric models and the properties of those models (e.g., Pythagorean Theorem or formulas involving radius, diameter, and circumference)	23. Use coordinate methods to solve and interpret problems (e.g., slope as rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H) *
G-3-H: solving problems using coordinate methods, as well as synthetic and transformational methods (e.g., transform on a coordinate plane a design found in real-life situations)	24. Graph a line when the slope and a point or when two points are known (G-3-H) * 25. Explain slope as a representation of "rate of change" (G-3-H) (A-1-H) * 26. Perform translations and line reflections on the coordinate plane (G-3-H) * Also see GLEs #13 and #23.
G-4-H: using inductive reasoning to predict, discover, and apply geometric properties and relationships (e.g., patty paper constructions, sum of the angles in a polygon)	
G-5-H: classifying figures in terms of congruence and similarity and applying these relationships	
G-6-H: demonstrating deductive reasoning and mathematical justification (e.g., oral explanation, informal proof, and paragraph proof)	

Data Analysis, Probability, and Discrete Math: In problem-solving investigations, students discover trends, formulate conjectures regarding cause-and-effect relationships, and demonstrate critical thinking skills in order to make informed decisions.	
Benchmarks	**Grade-Level Expectations**
D-1-H: designing and conducting statistical experiments that involve the collection, representation, and analysis of data in various forms (Analysis should reflect an understanding of factors such as: sampling, bias, accuracy, and reasonableness of data.)	27. Determine the most appropriate measure of central tendency for a set of data based on its distribution (D-1-H) 28. Identify trends in data and support conclusions by using distribution characteristics such as patterns, clusters, and outliers (D-1-H) (D-6-H) (D-7-H) 29. Create a scatter plot from a set of data and determine if the relationship is linear or nonlinear (D-1-H) (D-6-H) (D-7-H)
D-2-H: recognizing data that relate two variables as linear, exponential, or otherwise in nature (e.g., match a data set, linear or non-linear, to a graph and vice versa)	See GLE #9.
D-3-H: using simulations to estimate probabilities (e.g., lists and tree diagrams)	30. Use simulations to estimate probabilities (D-3-H) (D-5-H)
D-4-H: demonstrating an understanding of the calculation of finite probabilities using permutations, combinations, sample spaces, and geometric figures	31. Define probability in terms of sample spaces, outcomes, and events (D-4-H) 32. Compute probabilities using geometric models and basic counting techniques such as combinations and permutations (D-4-H) 33. Explain the relationship between the probability of an event occurring, and the odds of an event occurring and compute one given the other (D-4-H)
D-5-H: recognizing events as dependent or independent in nature and demonstrating techniques for computing multiple-event probabilities	See GLE #30.
D-6-H: recognizing and answering questions about data that are normally or non-normally distributed	See GLEs #28 and #29.

D-7-H: making inferences from data that are organized in charts, tables, and graphs (e.g., pictograph; bar, line, or circle graph; stem-and-leaf plot or scatter plot)	See GLEs #28 and #29.
D-8-H: using logical thinking procedures, such as flow charts, Venn diagrams, and truth tables	34. Follow and interpret processes expressed in flow charts (D-8-H) *
D-9-H: using discrete math to model real-life situations (e.g., fair games or elections, map coloring)	
Patterns, Relations, and Functions: In problem-solving investigations, students demonstrate an understanding of patterns, relations, and functions that represent and explain real-world situations.	
Benchmarks	**Grade-Level Expectations**
P-1-H: modeling the concepts of variables, functions, and relations as they occur in the real world and using the appropriate notation and terminology	35. Determine if a relation is a function and use appropriate function notation (P-1-H) 36. Identify the domain and range of functions (P-1-H) 37. Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H) * Also see GLE #15.
P-2-H: translating between tabular, symbolic, or graphic representations of functions	See GLE #15.
P-3-H: recognizing behavior of families of elementary functions, such as polynomial, trigonometric, and exponential functions, and, where appropriate, using graphing technologies to represent them	38. Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H) *

P-4-H: analyzing the effects of changes in parameters (e.g., coefficients and constants) on the graphs of functions, using technology whenever possible	39. Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H) * 40. Explain how the graph of a linear function changes as the coefficients or constants are changed in the function's symbolic representation (P-4-H) *
P-5-H: analyzing real-world relationships that can be modeled by elementary functions	See GLEs # 9 and #37.

TENTH GRADE INTRODUCTION

Tenth Grade

The Grade-Level Expectations for grades 9 and 10 represent the mathematics all students should know before graduation. The expectations are written with a view toward a greater integration of mathematics learning across these two grades than traditionally has been the case. For schools teaching separate courses in Algebra I and Geometry, the total of the Grade-Level Expectations for grades 9 and 10 should be considered as what students should be able to do by the end of grade 10, rather than thinking of these as separate grade 9 and grade 10 outcomes.

Students in grade 10 continue the mastery of the concepts that constitute the grade 9/10 segment of the secondary mathematics curriculum. Here the focus is more geometric in nature, but the connections between Algebra and Geometry are highlighted, along with the corresponding topics in Measurement and Data. In Number and Number Relations, grade 10 students represent and evaluate expressions involving radicals with integral powers and roots. They develop an ability to estimate roots when they fall between integer values. They also simplify and solve ratios and proportions related to similar triangles and trigonometric relationships. Finally, they deepen their understanding of the nature of irrational numbers, such as π and $\sqrt{2}$, as infinite, non-terminating, non-repeating decimals. Much of this work is related to the growth of measurement ideas associated with similar figures, including scale drawings and trigonometry.

In grade 10, students develop an understanding of the Euclidean geometry relationships for points, segments, lines, angles, and planes. Students learn to form and test conjectures, as well as to prove and disprove statements based on logical principles. They can analyze figures under transformations and determine which attributes change and which do not and explain why. Grade 10 students also develop a solid foundation of postulates and theorems that guide their investigation and understanding of spatial situations. Central to these are the theorems that relate angle measures, areas and volumes, and those dealing with congruent and similar figures.

In data settings, grade 10 students extend their data representation modes to include matrix models. In Probability, they extend their understanding of independent events and experimental data to include tables and graphs, as well as geometric probability representations for conditional events. Grade 10 students make the link between the geometric and algebraic representations for parallel and perpendicular lines. They also use their knowledge of linear equations to investigate geometric relationships in the coordinate plane. They understand and use the line of best fit to interpret data in scatterplots. They also examine data from geometric settings for growth patterns reflective of linear or exponential change. Students learn how changes in an object's linear dimensions affect its perimeter, area, and volume.

TENTH GRADE

MATHEMATICS

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-H: demonstrating an understanding of the real number system	
N-2-H: demonstrating that a number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, percents, and scientific notation)	1. Simplify and determine the value of radical expressions (N-2-H) (N-7-H)
N-3-H: using number sense to estimate and determine if solutions are reasonable	2. Predict the effect of operations on real numbers (e.g., the quotient of a positive number divided by a positive number less than 1 is greater than the original dividend) (N-3-H) (N-7-H)
N-4-H: determining whether an exact or approximate answer is necessary	
N-5-H: selecting and using appropriate computational methods and tools for given situations (e.g., estimation, or exact computation using mental arithmetic, calculator, symbolic manipulator, or paper and pencil)	

N-6-H: applying ratios and proportional thinking in a variety of situations (e.g., finding a missing term of a proportion)	3. Define *sine*, *cosine*, and *tangent* in ratio form and calculate them using technology (N-6-H) 4. Use ratios and proportional reasoning to solve a variety of real-life problems including similar figures and scale drawings (N-6-H) (M-4-H) Also see GLE #8.
N-7-H: justifying reasonableness of solutions and verifying results	See GLEs #1, #2 and #23.
Algebra: In problem-solving investigations, students demonstrate an understanding of concepts and processes that allow them to analyze, represent, and describe relationships among variable quantities and to apply algebraic methods to real-world situations.	
Benchmarks	**Grade-Level Expectations**
A-1-H: demonstrating the ability to translate real-world situations (e.g., distance versus time relationships, population growth, growth functions for diseases, growth of minimum wage, auto insurance tables) into algebraic expressions, equations, and inequalities and vice versa	
A-2-H: recognizing the relationship between operations involving real numbers and operations involving algebraic expressions	5. Write the equation of a line of best fit for a set of 2-variable real-life data presented in table or scatterplot form, with or without technology (A-2-H) (D-2-H)
A-3-H: using tables and graphs as tools to interpret algebraic expressions, equations, and inequalities	6. Write the equation of a line parallel or perpendicular to a given line through a specific point (A-3-H) (G-3-H) Also see GLE #27.

A-4-H: solving algebraic equations and inequalities using a variety of techniques with the appropriate tools (e.g., hand-held manipulatives, graphing calculator, symbolic manipulator, or pencil and paper)	
Measurement: In problem-solving investigations, students demonstrate an understanding of the concepts, processes, and real-life applications of measurement.	
Benchmarks	**Grade-Level Expectations**
M-1-H: selecting and using appropriate units, techniques, and tools to measure quantities in order to achieve specified degrees of precision, accuracy, and error (or tolerance) of measurements	
M-2-H: demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of results as related to area, volume, mass, rate, and distance)	
M-3-H: estimating, computing, and applying physical measurement using suitable units (e.g., calculate perimeter and area of plane figures, surface area and volume of solids presented in real-world situations)	7. Find volume and surface area of pyramids, spheres, and cones (M-3-H) (M-4-H)
M-4-H: demonstrating the concept of measurement as it applies to real-world experiences	8. Model and use trigonometric ratios to solve problems involving right triangles (M-4-H) (N-6-H) Also see GLEs #4, #7 and #18.

Geometry: In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings.

Benchmarks	Grade-Level Expectations
G-1-H: identifying, describing, comparing, constructing, and classifying geometric figures in two and three dimensions using technology where appropriate to explore and make conjectures about geometric concepts and figures	9. Construct 2- and 3-dimensional figures when given the name, description, or attributes, with and without technology (G-1-H) 10. Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and *n*-gons), with and without technology (G-1-H) (G-4-H) (G-6-H)
G-2-H: representing and solving problems using geometric models and the properties of those models (e.g., Pythagorean Theorem or formulas involving radius, diameter, and circumference)	11. Determine angle measurements using the properties of parallel, perpendicular, and intersecting lines in a plane (G-2-H) 12. Apply the Pythagorean theorem in both abstract and real-life settings (G-2-H) 13. Solve problems and determine measurements involving chords, radii, arcs, angles, secants, and tangents of a circle (G-2-H)
G-3-H: solving problems using coordinate methods, as well as synthetic and transformational methods (e.g., transform on a coordinate plane a design found in real-life situations)	14. Develop and apply coordinate rules for translations and reflections of geometric figures (G-3-H) 15. Draw or use other methods, including technology, to illustrate dilations of geometric figures (G-3-H) 16. Represent and solve problems involving distance on a number line or in the plane (G-3-H) Also see GLE #6.
G-4-H: using inductive reasoning to predict, discover, and apply geometric properties and relationships (e.g., patty paper constructions, sum of the angles in a polygon)	17. Compare and contrast inductive and deductive reasoning approaches to justify conjectures and solve problems (G-4-H) (G-6-H) Also see GLE #10.

G-5-H: classifying figures in terms of congruence and similarity and applying these relationships	18. Determine angle measures and side lengths of right and similar triangles using trigonometric ratios and properties of similarity, including congruence (G-5-H) (M-4-H)
G-6-H: demonstrating deductive reasoning and mathematical justification (e.g., oral explanation, informal proof, and paragraph proof)	19. Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H) Also see GLEs #10, #17 and #23.
Data Analysis, Probability, and Discrete Math: In problem-solving investigations, students discover trends, formulate conjectures regarding cause-and-effect relationships, and demonstrate critical thinking skills in order to make informed decisions.	
Benchmarks	**Grade-Level Expectations**
D-1-H: designing and conducting statistical experiments that involve the collection, representation, and analysis of data in various forms (Analysis should reflect an understanding of factors such as: sampling, bias, accuracy, and reasonableness of data.)	
D-2-H: recognizing data that relate two variables as linear, exponential, or otherwise in nature (e.g., match a data set, linear or non-linear, to a graph and vice versa)	20. Show or justify the correlation (match) between a linear or non-linear data set and a graph (D-2-H) (P-5-H) Also see GLE #5.
D-3-H: using simulations to estimate probabilities (e.g., lists and tree diagrams)	
D-4-H: demonstrating an understanding of the calculation of finite probabilities using permutations, combinations, sample spaces, and geometric figures	21. Determine the probability of conditional and multiple events, including mutually and non-mutually exclusive events (D-4-H) (D-5-H)

D-5-H: recognizing events as dependent or independent in nature and demonstrating techniques for computing multiple-event probabilities	See GLE #21.
D-6-H: recognizing and answering questions about data that are normally or non-normally distributed	
D-7-H: making inferences from data that are organized in charts, tables, and graphs (e.g., pictograph; bar, line, or circle graph; stem-and-leaf plot or scatter plot)	22. Interpret and summarize a set of experimental data presented in a table, bar graph, line graph, scatterplot, matrix, or circle graph (D-7-H)
D-8-H: using logical thinking procedures, such as flow charts, Venn diagrams, and truth tables	23. Draw and justify conclusions based on the use of logic (e.g., conditional statements, converse, inverse, contrapositive) (D-8-H) (G-6-H) (N-7-H)
D-9-H: using discrete math to model real-life situations (e.g., fair games or elections, map coloring)	24. Use counting procedures and techniques to solve real-life problems (D-9-H) 25. Use discrete math to model real life situations (e.g., fair games, elections) (D-9-H)
Patterns, Relations, and Functions: In problem-solving investigations, students demonstrate an understanding of patterns, relations, and functions that represent and explain real-world situations.	
Benchmarks	**Grade-Level Expectations**
P-1-H: modeling the concepts of variables, functions, and relations as they occur in the real world and using the appropriate notation and terminology	26. Generalize and represent patterns symbolically, with and without technology (P-1-H)
P-2-H: translating between tabular, symbolic, or graphic representations of functions	27. Translate among tabular, graphical, and symbolic representations of patterns in real-life situations, with and without technology (P-2-H) (P-3-H) (A-3-H)

P-3-H: recognizing behavior of families of elementary functions, such as polynomial, trigonometric, and exponential functions, and, where appropriate, using graphing technologies to represent them	See GLE #27.
P-4-H: analyzing the effects of changes in parameters (e.g., coefficients and constants) on the graphs of functions, using technology whenever possible	
P-5-H: analyzing real-world relationships that can be modeled by elementary functions	See GLE #20.

ELVENTH AND TWELFTH GRADES INTRODUCTION

Eleventh and Twelfth Grade

These expectations are held for students who continue their study of mathematics beyond grade 10 in preparation for higher education or continued study after high school. They reflect the content of collegiate entrance examinations and mathematics found in common applications in such programs of study.

In Number and Number Relations, students in grades 11/12 extend their knowledge of number systems to the complex numbers, learning how to perform basic operations and represent such numbers in the complex plane. These students also develop a rich understanding of the relationship between exponential and logarithmic notation for numbers. Finally, their conceptual and procedural understanding of radical expressions is extended to include those with rational exponents.

In Measurement, grades 11/12 students extend their study of angle measurement to include measurement in radians. They learn the nature of such measurement and the reasons for it, as well as how to convert between radian and degree measures. They also extend their ability to solve problems in trigonometry through the application and interpretation of results related to the Law of Cosines and Law of Sines. Students become more familiar with the coordinate representation of geometric objects, including lines, circles, and conic sections (i.e., parabolas, ellipses, and hyperbolas). In particular, they can relate the nature of these conic sections and their relationships to plane sections of a cone. In Data work, grade 11/12 students come to understand the need for randomization in using survey data to avoid bias in describing possible population characteristics. In the study of Data at these grades, they see both normal and non-normal distributions of data. These students' knowledge of trend lines is extended to linear regression. They learn to interpret such lines with and without the use of technology. In discrete mathematics settings, they encounter work with Euler and Hamiltonian circuits on networks and their applications to problem solving.

Grades 11/12 students master quadratic relationships and learn to represent them in a variety of ways. They can solve quadratic equations as well as quadratic inequalities in a variety of ways, both with and without technology. Their knowledge of the solution of systems is extended to those involving both linear and quadratic equations. These students also develop a strong understanding of exponential functions and their applications. They use logarithms and other methods to find solutions to such equations. Finally, in patterns and relations, these students develop a strong understanding of polynomial relationships and the global behavior of such relationships, using their coefficients and graphical representations to talk about their extreme values, zeros, and end behavior.

ELEVENTH/TWELFTH GRADE

MATHEMATICS

Number and Number Relations: In problem-solving investigations, students demonstrate an understanding of the real number system and communicate the relationships within that system using a variety of techniques and tools.

Benchmarks	Grade-Level Expectations
N-1-H: demonstrating an understanding of the real number system	1. Read, write, and perform basic operations on complex numbers (N-1-H) (N-5-H)
N-2-H: demonstrating that a number can be expressed in many forms, and selecting an appropriate form for a given situation (e.g., fractions, decimals, percents, and scientific notation)	2. Evaluate and perform basic operations on expressions containing rational exponents (N-2-H) 3. Describe the relationship between exponential and logarithmic equations (N-2-H)
N-3-H: using number sense to estimate and determine if solutions are reasonable	
N-4-H: determining whether an exact or approximate answer is necessary	
N-5-H: selecting and using appropriate computational methods and tools for given situations (e.g., estimation, or exact computation using mental arithmetic, calculator, symbolic manipulator, or paper and pencil)	See GLE #1.
N-6-H: applying ratios and proportional thinking in a variety of situations (e.g., finding a missing term of a proportion)	
N-7-H: justifying reasonableness of solutions and verifying results	

Algebra: In problem-solving investigations, students demonstrate an understanding of concepts and processes that allow them to analyze, represent, and describe relationships among variable quantities and to apply algebraic methods to real-world situations.

Benchmarks	Grade-Level Expectations
A-1-H: demonstrating the ability to translate real-world situations (e.g., distance versus time relationships, population growth, growth functions for diseases, growth of minimum wage, auto insurance tables) into algebraic expressions, equations, and inequalities and vice versa	4. Translate and show the relationships among non-linear graphs, related tables of values, and algebraic symbolic representations (A-1-H)
A-2-H: recognizing the relationship between operations involving real numbers and operations involving algebraic expressions	5. Factor simple quadratic expressions including general trinomials, perfect squares, difference of two squares, and polynomials with common factors (A-2-H)
A-3-H: using tables and graphs as tools to interpret algebraic expressions, equations, and inequalities	6. Analyze functions based on zeros, asymptotes, and local and global characteristics of the function (A-3-H) 7. Explain, using technology, how the graph of a function is affected by change of degree, coefficient, and constants in polynomial, rational, radical, exponential, and logarithmic functions (A-3-H) 8. Categorize non-linear graphs and their equations as quadratic, cubic, exponential, logarithmic, step function, rational, trigonometric, or absolute value (A-3-H) (P-5-H)
A-4-H: solving algebraic equations and inequalities using a variety of techniques with the appropriate tools (e.g., hand-held manipulatives, graphing calculator, symbolic manipulator, or pencil and paper)	9. Solve quadratic equations by factoring, completing the square, using the quadratic formula, and graphing (A-4-H) 10. Model and solve problems involving quadratic, polynomial, exponential, logarithmic, step function, rational, and absolute value equations using technology (A-4-H)

Measurement: In problem-solving investigations, students demonstrate an understanding of the concepts, processes, and real-life applications of measurement.

Benchmarks	Grade-Level Expectations
M-1-H: selecting and using appropriate units, techniques, and tools to measure quantities in order to achieve specified degrees of precision, accuracy, and error (or tolerance) of measurements	11. Calculate angle measures in degrees, minutes, and seconds (M-1-H) 12. Explain the unit circle basis for radian measure and show its relationship to degree measure of angles (M-1-H)
M-2-H: demonstrating an intuitive sense of measurement (e.g., estimating and determining reasonableness of results as related to area, volume, mass, rate, and distance)	
M-3-H: estimating, computing, and applying physical measurement using suitable units (e.g., calculate perimeter and area of plane figures, surface area and volume of solids presented in real-world situations)	
M-4-H: demonstrating the concept of measurement as it applies to real-world experiences	13. Identify and apply the unit circle definition to trigonometric functions and use this definition to solve real-life problems (M-4-H) 14. Use the Law of Sines and the Law of Cosines to solve problems involving triangle measurements (M-4-H)

Geometry: In problem-solving investigations, students demonstrate an understanding of geometric concepts and applications involving one-, two-, and three-dimensional geometry, and justify their findings.	
Benchmarks	**Grade-Level Expectations**
G-1-H: identifying, describing, comparing, constructing, and classifying geometric figures in two and three dimensions using technology where appropriate to explore and make conjectures about geometric concepts and figures	15. Identify conic sections, including the degenerate conics, and describe the relationship of the plane and double-napped cone that forms each conic (G-1-H)
G-2-H: representing and solving problems using geometric models and the properties of those models (e.g., Pythagorean Theorem or formulas involving radius, diameter, and circumference)	
G-3-H: solving problems using coordinate methods, as well as synthetic and transformational methods (e.g., transform on a coordinate plane a design found in real-life situations)	16. Represent translations, reflections, rotations, and dilations of plane figures using sketches, coordinates, vectors, and matrices (G-3-H)
G-4-H: using inductive reasoning to predict, discover, and apply geometric properties and relationships (e.g., patty paper constructions, sum of the angles in a polygon)	
G-5-H: classifying figures in terms of congruence and similarity and applying these relationships	
G-6-H: demonstrating deductive reasoning and mathematical justification (e.g., oral explanation, informal proof, and paragraph proof)	

Data Analysis, Probability, and Discrete Math: In problem-solving investigations, students discover trends, formulate conjectures regarding cause-and-effect relationships, and demonstrate critical thinking skills in order to make informed decisions.	
Benchmarks	**Grade-Level Expectations**
D-1-H: designing and conducting statistical experiments that involve the collection, representation, and analysis of data in various forms (Analysis should reflect an understanding of factors such as: sampling, bias, accuracy, and reasonableness of data.)	17. Discuss the differences between samples and populations (D-1-H) 18. Devise and conduct well-designed experiments/surveys involving randomization and considering the effects of sample size and bias (D-1-H)
D-2-H: recognizing data that relate two variables as linear, exponential, or otherwise in nature (e.g., match a data set, linear or non-linear, to a graph and vice versa)	19. Correlate/match data sets or graphs and their representations and classify them as exponential, logarithmic, or polynomial functions (D-2-H) 20. Interpret and explain, with the use of technology, the regression coefficient and the correlation coefficient for a set of data (D-2-H)
D-3-H: using simulations to estimate probabilities (e.g., lists and tree diagrams)	
D-4-H: demonstrating an understanding of the calculation of finite probabilities using permutations, combinations, sample spaces, and geometric figures	
D-5-H: recognizing events as dependent or independent in nature and demonstrating techniques for computing multiple-event probabilities	
D-6-H: recognizing and answering questions about data that are normally or non-normally distributed	21. Describe and interpret displays of normal and non-normal distributions (D-6-H)
D-7-H: making inferences from data that are organized in charts, tables, and graphs (e.g., pictograph; bar, line, or circle graph; stem-and-leaf plot or scatter plot)	22. Explain the limitations of predictions based on organized sample sets of data (D-7-H)

D-8-H: using logical thinking procedures, such as flow charts, Venn diagrams, and truth tables	
D-9-H: using discrete math to model real-life situations (e.g., fair games or elections, map coloring)	23. Represent data and solve problems involving Euler and Hamiltonian paths (D-9-H)
Patterns, Relations, and Functions: In problem-solving investigations, students demonstrate an understanding of patterns, relations, and functions that represent and explain real-world situations.	
Benchmarks	**Grade-Level Expectations**
P-1-H: modeling the concepts of variables, functions, and relations as they occur in the real world and using the appropriate notation and terminology	24. Model a given set of real-life data with a non-linear function (P-1-H) (P-5-H) 25. Apply the concept of a function and function notation to represent and evaluate functions (P-1-H) (P-5-H)
P-2-H: translating between tabular, symbolic, or graphic representations of functions	26. Represent and solve problems involving n^{th} terms and sums for arithmetic and geometric series (P-2-H)
P-3-H: recognizing behavior of families of elementary functions, such as polynomial, trigonometric, and exponential functions, and, where appropriate, using graphing technologies to represent them	27. Compare and contrast the properties of families of polynomial, rational, exponential, and logarithmic functions, with and without technology (P-3-H)
P-4-H: analyzing the effects of changes in parameters (e.g., coefficients and constants) on the graphs of functions, using technology whenever possible	28. Represent and solve problems involving the translation of functions in the coordinate plane (P-4-H)
P-5-H: analyzing real-world relationships that can be modeled by elementary functions	29. Determine the family or families of functions that can be used to represent a given set of real-life data, with and without technology (P-5-H) Also see GLEs #8, #24, and #25.

GLOSSARY

MATHEMATICS GLOSSARY

Absolute Error — The difference between a measured value and the actual value. For example, when the thermometer reads 75º, the actual temperature might be 73º. The absolute error is 2 degrees.

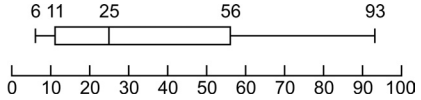
Accuracy — The extent to which a measurement/value/quantity conforms to an actual fact. For example, "The accuracy of this estimate will ensure there is enough water in the container without it overflowing."

Associative Property — A property, applicable to addition and multiplication, which states that it does not matter how numbers are grouped if they are all being added or all being multiplied (e.g., 2 + (4 + 9) = (2 + 4) + 9).

Asymptote — A line or curve that is approached (but never actually reached) by a function (e.g., the *x*-axis is an asymptote of the function $f(x) = \frac{1}{x}$).

Backward Reasoning — The reasoning involved when one assumes a conclusion is true and then works (or reasons) backwards to the evidence that the conclusion is true.

Box and Whiskers Plot — A graph in which five elements in a set of data are specifically marked: minimum value, first quartile, median value, third quartile and maximum value. This plot indicates a minimum of 6, first quartile of 11, median of 25, third quartile of 56 and maximum value of 93.



Combinations — Sets containing a certain number of objects selected from another set. The combinations of three items from the set $\{a,b,c,d\}$ are: $\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}$. The sets *{a, b, c}* and *{b, a, c}* are considered to be the same sets.

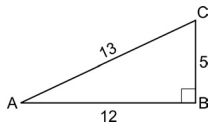
GLOSSARY

Common Equivalent Reference Points	Common fractions with decimal equivalents that can be used to approximate other nearby values. For example, ¼ = 0.25, ½ = 0.5, and ¾ =0 .75. Since 4/10 is slightly less than ½, its decimal equivalent will be slightly less than 0.5.
Commutative Property	A property, applicable to addition and multiplication; this property states that it does not matter in which order numbers are added or multiplied. (e.g., 5 + 6 = 6 + 5 and 8 x 9 = 9 x 8).
Complementary Events	Events that, when combined, constitute all possibilities. In the study of mathematics, the set of all complementary events constitutes the universal set. For example, writing an even integer or writing an odd integer are complementary events, since every possible integer is either even or odd.
Computational Fluency	A level of skill reached when a person is able to execute an algorithm or procedure efficiently and correctly without assistance.
Congruent	Figures that have the same size and shape.
Conic Sections	Figures that result from the intersection of a plane and a double-napped cone. The conic sections are a circle, an ellipse, a parabola, and a hyperbola.
Contrapositive	Given a statement of the form, "If P, then Q," the contrapositive is the statement, "If not Q, then not P." The contrapositive is logically equivalent to the original statement. Given the statement, "If it rained, then Bob went inside," the contrapositive is, "If Bob did not go inside, then it did not rain."
Converse	Given a statement of the form, "If P, then Q," the converse is the statement, "If Q, then P." The converse is not necessarily logically equivalent to the original statement. For example, given the statement, "If it rained, then Bob went inside," the converse is, "If Bob went inside, then it rained."
Coordinate Systems	Systems used to locate points using lines or points.

GLOSSARY

Cosine Ratio

The cosine of an angle in a right triangle is the ratio of the length of the adjacent side to the length of the hypotenuse. For example, in triangle ABC shown below, the cosine of angle A $\left[\cos(A)\right]$ is the ratio $\frac{12}{13}$.

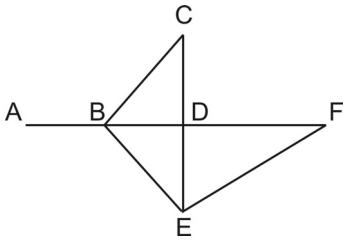


Degenerate Conics

Figures that result from the intersection of a plane and a double-napped cone, but are not "true" conic sections. The degenerate conics are a point, a line, and two intersecting lines.

Euler Path

A path that connects two vertices of a graph and travels each path in the graph exactly once. For example, the path A-B-C-D-B-E-D-F-E is an Euler path.



Dependent Events

Events that influence each other. If a bag contains three red marbles and two green marbles, randomly picking a red marble from the bag and then randomly picking a green marble from the bag (without replacing the red marble) are dependent events.

Dilation

A transformation that maps lines onto parallel lines. A dilation may be a translation (no size change) or an enlargement. The image on a movie screen represents a dilation of the image on the film.

GLOSSARY

Discrete Mathematics

The branch of mathematics dealing with situations in which there are a finite or countable number of values or objects (i.e., not continuous). For example, since no fractional values are applicable, determining the combination of colors that can be used in a drawing requires application of discrete mathematics. Three colors, a discrete number, is reasonable. $3\frac{1}{2}$ and other fractional numbers of colors do not make sense and are not reasonable solutions.

Distributive Property of Multiplication Over Addition

Property that states that for numbers *a, b,* and *c*, $a(b+c) = ab + ac$.

Expanded Form

The form of a number written as a sum to show place value. For example, the expanded form of 367 is 300 + 60 + 7.

Frequency Table

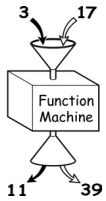
A table that lists how often different outcomes occur. Below is a frequency table representing the results of a poll about students' favorite fruits.

FAVORITE FRUITS	
Fruit	**Frequency**
apple	5
banana	9
orange	5
strawberry	7

GLOSSARY

Function Machine

A simple way to think of a function. You provide one input value and the machine (function) produces one output value. The diagram below shows a function machine which doubles and adds five to each number placed into the machine.

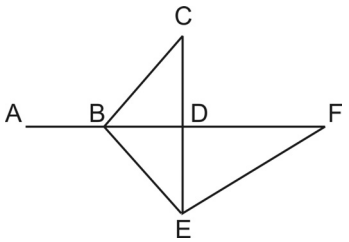


Fundamental Counting Principle

The principle that states that, "If there are **r** ways to do one thing and **s** ways to do another, and **t** ways to do a third thing, and so on, then the number of ways of doing all those things at once is **r** x **s** x **t** x …". Suppose a license plate lists a sequence of three letters followed by three digits. There are a total of $26 \bullet 26 \bullet 26 \bullet 10 \bullet 10 \bullet 10 = 17,576,000$ possible, different license plates.

Hamiltonian Path

A path that connects two vertices of a graph and visits each vertex in the graph exactly once. The path A-B-C-D-E-F is a Hamiltonian path.



Independent Events

Events that have no influence on each other. For example, flipping "tails" with a coin and rolling a four with a die are independent events.

GLOSSARY

Inverse — Given a statement of the form, "If P, then Q," the inverse of the statement is, "If not P, then not Q." The inverse is not necessarily logically equivalent to the original statement. Given the statement, "If it rained, then Bob went inside," the inverse is, "If it didn't rain, then Bob didn't go inside."

Inverse Operations — Operations that "undo" or are opposites of one another are inverse operations (e.g., addition and subtraction, multiplication and division).

Line of Symmetry — A line that divides a geometric object into two congruent halves.

Manipulatives — Concrete, physical objects used to help illustrate mathematical concepts.

Matrix (*pl.* Matrices) — A set of values arranged in a rectangular array. For example, the coefficients of the expressions $3x^2 + 7x + 9$, $11x^2 + (-8x) + 52$, and $-4x^2 + 21x + (-7)$ are represented in the matrix $\begin{bmatrix} 3 & 7 & 9 \\ 11 & -8 & 52 \\ -4 & 21 & -7 \end{bmatrix}$.

Mental Math — Computations and estimations performed without the aid of paper and pencil.

Mutually Exclusive Events — Two or more events, each of which precludes all the others. For example, the people voting in a presidential election and the people who are celebrating their 15^{th} birthday on that election day are mutually exclusive events.

Ordinal Number — A number that denotes position in a sequence. In the sentence, "Susan was the 6^{th} person in line," 6^{th} is an ordinal.

Outcomes — Results that are possible from an experiment or simulation. For example, the possible outcomes of rolling a six-sided number cube are rolls of 1, 2, 3, 4, 5, and 6.

Perfect Square — A number that can be written as the square of one of its factors. For example, the number 36 is a perfect square since it is the product of 6 and 6 (i.e., $6 \bullet 6 = 36$). The number 36 is said to be the square of 6.

GLOSSARY

Permutations — The ordered arrangements of the elements of a set. For example, the permutations of the list $\{A, B, C\}$ are (1) $\{A, B, C\}$ (2) $\{A, C, B\}$ (3) $\{B, A, C\}$ (4) $\{B, C, A\}$ (5) $\{C, A, B\}$ and (6) $\{C, B, A\}$

Pictographs — A visual representation of statistical data that uses pictures to indicate value or quantity.

Picture Graphs — See **pictograph**.

Polyhedron (*pl.* Polyhedra) — A three dimensional object with faces that are plane polygons. Cubes are polyhedra in which each face is a square.

Polynomial Expression — A mathematical expression that is the sum of terms, each of which is the product of a constant and a non-negative power of a variable or variables. For example, the expression $5x^7 + 11x^2 + 7x + (-3)$ is a polynomial expression.

Precision — The degree of specificity to which a measurement/value/quantity is determined. For example, "The measurement is precise to the nearest millimeter."

Probability — A number between 0 and 1, inclusive, which indicates the likelihood of an event occurring. For example, the probability of rolling a 1 on a fair, six-sided number cube is 1/6.

Pythagorean Theorem — The theorem that states a triangle is a right triangle if, and only if, the sum of the squares of the two sides is equal to the square of the hypotenuse. For example, the measures of the sides and hypotenuse of a triangle are 6 in., 8 in., and 10 in. Therefore, the triangle is a right triangle since $6^2 + 8^2 = 10^2$.

Quartiles — The three values that divide a set of data into four intervals with an equal number of elements in each interval. In the set of values $\{1,2,3,4,5,6,7\}$, the quartiles are 2 (1^{st}), 4 (median) and 6 (3^{rd}).

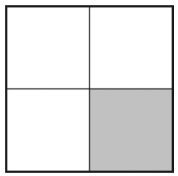
Range — The absolute difference between the greatest and least value in a set of data. For example, the range of the data set $\{7,8,12,17,23\}$ is $16\,(23 - 7)$.

GLOSSARY

Rational Number — Any number that can be written as a fraction in which both the numerator is an integer and the denominator is a natural number. For example, 3/7 is a rational number since it can be written as a fraction.

Rectangular Array — An arrangement of objects to aid in understanding multiplication. For example, four rows with three pieces of candy in each row can be used to illustrate 3 x 4 = 12 and 4 x 3 = 12.

Region Model — A geometric figure divided into equally-sized parts to illustrate fractional concepts. For example, the model below could be used to illustrate ¼ (the shaded region) or ¾ (the unshaded region).



Related Turn-around Pairs — Addition and multiplication facts that are the same except for the order in which the numbers are added or multiplied. For example, 2 + 3 = 5 and 3 + 2 = 5 is a related turn-around pair.

Rotational Symmetry — The property of an object such that after the object is rotated a number of degrees (less than 360°), the object is identical to the original object. For example, a square has rotational symmetry because it can be rotated 90 degrees and then appears identical to the original square.

Scatter Plot — A two-dimensional graph of a collection of points.

Sequences of Arithmetic Growth — A sequence of numbers in which the difference between successive terms is constant and the value of successive terms is increasing. For example, the pattern 4, 7, 10, 13, 16, 19, … is an arithmetic sequence with a constant difference of 3.

Sequence of Geometric Growth — A sequence of numbers in which the ratio of successive elements is constant and the value of successive elements is increasing. For example, the pattern 3, 6, 12, 24, 48, 96, 192, … is a geometric sequence with a constant ratio of 2.

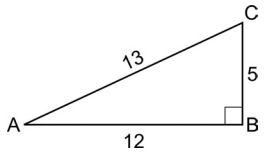
GLOSSARY

Set Model — A model for fractions in which the fraction is shown by a subset of highlighted objects out of the total set of objects. For example, one-third might be represented as • o o.

Similarity — The quality of having the same shape and proportional corresponding linear measurements. For example, the triangles are the same shape, but each side of the one triangle is six times as long as the corresponding side of the other triangle. Also, a marble and a bowling ball are not the same size, but they are similar.

Simulation — A representation (physical, written, verbal, graphic, procedural, experimental, etc.) of a situation or event(s). For example, the results of turning a spinner that is divided into ten equivalent parts marked zero (0) through nine (9) can be used to simulate the sequence of numbers on a license plate.

Sine Ratio — The sine of an angle in a right triangle is the ratio of the length of the opposite side to the length of the hypotenuse. For example, in triangle ABC shown below, the sine of angle A $\left[\sin(A)\right]$ in triangle ABC is the ratio $\frac{5}{13}$.



Skip-counting — Counting forward or backwards by a number other than 1. For example, starting at 2 and skip-counting forward by 2s would result in the sequence: 2, 4, 6, 8, …

Standard Form — The form in which numbers are traditionally written. For example, 367 is in standard form.

GLOSSARY

Stem and Leaf Plot

A data display consisting of "stems," which are the data with the last digit removed, and "leaves," which are the last digits of the data.

Stem	Leaves
3	4 4 6 8
4	0 3 6 6 7
5	1 1 3 4

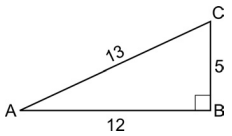
Key: 4 | 3 = 43

Symbolic Representation

A representation that uses symbols to model a situation or event. For example, the circumference of a circle is found by multiplying the diameter by pi. The symbolic representation that models this calculation is $C = \pi d$.

Tangent Ratio

The tangent of an angle in a right triangle is the ratio of the length of the opposite side to the length of the adjacent side. For example, in triangle ABC shown below, the tangent of angle A $\left[\tan(A)\right]$ in triangle ABC is the ratio $\frac{5}{12}$.



Tessellations

A covering of the plane using the same, non-overlapping shapes.

Trend Line (line of best fit)

The line, or equation of a line, that best represents the trend formed by the points in a scatter plot.

Unwrapping

The act of solving an equation by using inverse operations and knowing which operation to use first. This concept is sometimes associated with the concept of wrapping and unwrapping a present. When you unwrap, the first step is to "undo" the last step used when the gift was wrapped. For example, to solve the equation $4x + 2 = 6$, you would subtract 2 from both sides of the equation to "unwrap" the x.