Louisiana Believes

Common Core State Standards for Mathematics

TEACHER SELF-LEARNING SERIES

Module 2

Focus and Coherence – The First Two CCSSM Shifts

DEPARTMENT of EDUCATION
Louisiana Believes

COMMON CORE STATE STANDARDS for Mathematics

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Module 2: Focus and Coherence – The First Two CCSSM Shifts

Time Frame: Approximately 75 minutes

Audience: Teachers, principals, and additional school faculty of all grade levels and all content areas

Module Description: This module examines Focus and Coherence, two of the three shifts required for implementation of the Common Core State Standards for Mathematics (CCSSM). Focus and Coherence are the two major design principles of the math standards. The module assumes prior knowledge of the information presented in Module 1.

Course Objectives: By the end of the module, learner will be able to:

a. state the two levels of focus and find examples of each.
b. state the two levels of coherence and find examples of each.
c. use resources to determine major work for a specific grade or course in mathematics.
d. define *CCSSM stream* and give an example which applies to a grade level or course which is of interest to the learner.

Materials Needed to Complete Module: copy of the CCSSM document posted at http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf, Content Emphases by Cluster document based on learner interest (links provided within module), Internet access

Pre-Assessment: Those who can answer the questions below with confidence may want to skip this module.

1. State the two levels of focus and give an example of each.
2. State the two levels of coherence and give an example of each.
3. Where are the critical focus areas for Grades K-8 located and what is their importance?
4. What is the major work for Grade 8 mathematics as determined by PARCC?
5. What is a CCSSM stream? Give an example of a stream that includes at least one domain for a grade or course that is of interest to you.

Introduction

Implementation of the Common Core State Standards for Mathematics (CCSSM) requires three instructional shifts. They are Focus, Coherence, and Rigor. A brief summary of each is provided below:

1. **Focus :** Ensuring that instruction focuses strongly where the Standards focus
2. **Coherence: Think** across grades and **link** to major topics within grades
3. **Rigor**: In major topics, pursue **conceptual understanding**, procedural skill and **fluency**, and **application** with equal intensity

This module examines the shifts of Focus and Coherence, the two major design principles of standards of high achieving countries used to develop the CCSSM.

Introductory Video

The New York State Education Department has posted a video of a presentation made by David Coleman from Student Achievement Partners to New York education leaders at http://neric.welearntube.org/?q=node/149. There are few references to New York's timeline for implementation that should be ignored. This module will expand on the topics that Mr. Coleman mentions in his presentation. Please view the video. It is approximately 20 minutes long.

Understanding Focus

A Writer's Summary of Focus

Watch the short overview video, *The Importance of Focus in Mathematics*, featuring Jason Zimba, one of the lead CSSM writers, at http://tinyurl.com/akua68v. Note: This video is posted on YouTube.

Levels of Focus

There are two levels of focus.

- The first level of focus is knowing what is to be taught at each grade level and what is not. This knowledge will provide teachers time to go deeper with the math that is most important. Compared to the typical state standards of the past, the Common Core State Standards for Mathematics have fewer standards making them more manageable. The CCSSM are clear about what is expected of the teachers and students at each grade level.

- The second level of focus is determining the shape of the content or knowing the major work of each grade. Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.

Determining the Major Work of a Grade using Critical Focus Areas

The CCSSM provide critical focus areas for grades K-8 indicating that the focal areas should account for at least 75% of the instructional time. The CCSSM in high school provide a high-level overview of each conceptual category, but no critical focus areas.

Turn to page 9 of the CCSSM document. Thumb through the document as indicated below:

- The first page of each grade-level section always provides the critical focus areas for that grade. The critical focus areas for Kindergarten on page 9 are shown below. A narrative overview is provided as the first page of each conceptual category in the high school standards. For example, page 58 of the CCSSM contains the narrative overview for the Number and Quantity conceptual category.
- The second page of each grade level section and each high school conceptual category shows a listing of the domains and cluster headings and a listing of the Standards for Mathematical Practice. See pages 10 and 59 for examples.

Many teachers of grades K-8 want to know which standards address the critical focus areas. To do this, each critical focus area must be decomposed and compared to the content of the standards.

Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

The process of aligning the critical focus areas to domains/clusters /standards requires three steps. Steps 1-3 are repeated until all skills and understandings in the critical focus area have been aligned with the CCSSM. To aid in understanding the process, we will target the skills in the Kindergarten critical area 1 that have been outlined with a blue box in the graphic above.

Step 1) Isolate a specific skill or understanding found in one of the focus areas. The skill in the blue box is "comparing sets or numerals."

Step 2) Look for the domain(s) and cluster(s) in which this skill or understanding is most likely to be found. The domains and clusters for Kindergarten are shown to the left below. These can be found on page 10 of the CCSSM.

Since Compare Numbers is a cluster under the Counting and Cardinality domain, it would be reasonable to look in that cluster for standards associated with "comparing sets or numerals."

Before proceeding, take time to read the standards in the Compare Numbers cluster looking for standards that relate to *comparing sets* or *comparing numerals*.

Step 3) Determine which standard(s) in the cluster match the identified skill or understanding. (See self-check answers at the end of this document)

Assignment 1: Read the text of critical focus area 1 that has been outlined with a green box. Identify the domain /cluster/ standards which relate to these skills. (See self-check answers at the end of this document)

Note that the domains provide one way of looking at Focus at the first level as they provide a general picture of what is taught and what is not. The actual standards were used to provide specific information on what is really happening in this grade.

Determining Major Work for a Grade or Course: PARCC Model Content Frameworks for Grades 3-11 and Student Achievement Partners' Cluster Analyses for Grades K-2

PARCC stands for the Partnership for Assessment of Readiness for College and Careers. PARCC is a consortium of states that is developing common assessments for Louisiana and other member states to administer in 2014-15. The PARCC website is www.parcconline.org. Student Achievement Partners (SAP) is a non-profit organization founded by three of the contributing authors of the CCSSM. SAP supports successful implementation of the Common Core Standards by working with teachers to develop tools and makes all resources available at no cost to educators at its website, http://www.achievethecore.org/. (Copy the link into your browser if the link does not work.)

The PARCC Model Content Frameworks for Mathematics Grades 3-11 (http://tinyurl.com/atdtved) was developed with the primary purpose of providing a frame for the PARCC assessments. In this document, PARCC describes content emphases in the standards at the cluster level for each grade 3-8 and for the high school courses Algebra I, Geometry, and Algebra II. The PARCC Frameworks

document is designed to be used in conjunction with the CCSSM document as the frameworks document does not include a listing of the standards referenced. Student Achievement Partners developed similar content emphases for grades K-2.

The Content Emphases by Cluster chart for Grade 3 is shown below. All such charts provide the same type of information with minor variations in how the high school information is displayed.

Content Emphases by Cluster--Grade 3*

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

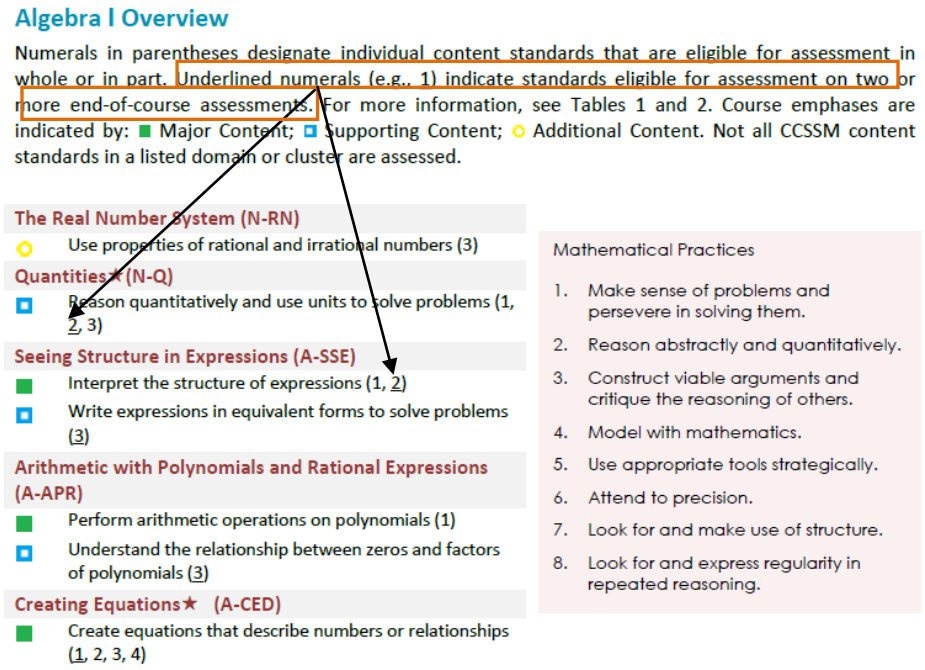
Operations and Algebraic Thinking
■ Represent and solve problems involving multiplication and division.
■ Understand properties of multiplication and the relationship between multiplication and division.
■ Multiply and divide within 100.
■ Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Number and Operations in Base Ten
○ Use place value understanding and properties of operations to perform multi-digit arithmetic.
Number and Operations—Fractions
■ Develop understanding of fractions as numbers.
Measurement and Data
■ Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
◘ Represent and interpret data.
■ Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
○ Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
Geometry
◘ Reason with shapes and their attributes.

The above chart is similar to the Grade 3 Domain and Clusters information found on page 22 in the CCSSM; however, the clusters have been color coded and designated as **Major (green squares)**, **Supporting (blue squares) and Additional (yellow circles).** Although these designations are used to make relative emphases more transparent and useful, every standard is important and all should be taught. This chart is designed to show the relationship among the standards, specifically the connections and how standards support one another. Below is a brief summary of information found on page 14 of the PARCC Model Content Frameworks regarding the rationale for the designations:

- Not all of the content in a given grade is emphasized equally in the standards.
- Some clusters require greater emphasis based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
- An intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.
- All standards will be eligible for inclusion on PARCC assessments.
    - Major Clusters will be a majority of the assessment.
    - Supporting Clusters will be assessed through their success at supporting the Major Clusters.
    - Additional Clusters will be assessed as well.
    - The assessments will strongly focus where the standards strongly focus.

Content Emphases by Cluster for Algebra I and Algebra II

There is a slight variation in the presentation of content emphases for Algebra I and Algebra II. This variation is due to the fact that some standards are taught in both courses. The sentences that have been outlined in orange in the first paragraph in the screenshot below indicate that standards which are to be taught in both courses are underlined. Two arrows have been drawn to show examples of where the underlining occurs. Table 2 in the PARCC Model Content Frameworks, Assessment Limits for Standards Assessed on More Than One End-of-Course Test, explains how such standards will be assessed on end-of-course tests for these two courses. Table 2 should be used in conjunction with the content emphases to get a complete picture of the Major, Supporting, and Additional content for Algebra I and Algebra II.



Algebra I Overview

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. For more information, see Tables 1 and 2. Course emphases are indicated by: ■ Major Content; ◘ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)
- ○ Use properties of rational and irrational numbers (3)

Quantities★ (N-Q)
- ◘ Reason quantitatively and use units to solve problems (1, 2, 3)

Seeing Structure in Expressions (A-SSE)
- ■ Interpret the structure of expressions (1, 2)
- ◘ Write expressions in equivalent forms to solve problems (3)

Arithmetic with Polynomials and Rational Expressions (A-APR)
- ■ Perform arithmetic operations on polynomials (1)
- ◘ Understand the relationship between zeros and factors of polynomials (3)

Creating Equations★ (A-CED)
- ■ Create equations that describe numbers or relationships (1, 2, 3, 4)

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Engaging with the Content

This activity is designed to examine the Content Emphases by Cluster documents in order to determine the major work for one of the grade spans K-2, 3-5, 6-8, or one of the high school courses (Algebra I, Geometry, Algebra II).

1. Download or open the documents from links provided above. Note: Linked files for high school courses also contain the Assessment Limits found in Table 2.
2. Study the emphases for the chosen grade span or course.
3. Answer the following questions:
   a. How would you summarize the major work of your grade span or course?
   b. What would you have expected to be a part of the major work for your grade span or course that is not?

c. Give an example of how you would change your approach to teaching something designated as supporting the major work, instead of teaching it as a discrete topic?

Answers to the Engaging in the Content activity are provided in the Self-Check section at the end of this module. Bill McCallum calls attention to specific changes at each level in the video, *The Mathematics Standards: Key Changes and their Evidence*, posted at http://tinyurl.com/adf7alv. Because the engagement activity was designed to focus on one grade cluster or course, this video provides a summary for all grades.

Understanding Coherence

A Writer's Perspective

Watch the short overview video, *The Importance of Coherence in Mathematics*, featuring William McCallum, one of the lead CSSM writers, at http://tinyurl.com/aaejwbj. Note: This video is posted on YouTube.

Levels of Coherence

Coherence is about having math make sense to students. As with focus, there are two levels of coherence.

- One level is *coherence of topics within a grade* as the Standards direct us to have students reinforce a major topic in a grade by utilizing a supporting topic. Coherence within a grade requires meaningful introduction to topics in the same grade that complement each other.

- A second level is *coherence of topics across grades* as evidenced when the CCSSM direct us to have students apply learning from a previous grade to learn a new topic. Coherence across grades is also reflected in thoughtfully laid out progressions of mathematics that are meaningful and make sense.

Examples of Coherence of Topics Within Grades

- Grades 1-5: Represent and Interpret Data

  In every case in Grades 1-5, the cluster, Represent and Interpret Data, is a supporting cluster. The clear intent is that that representing and interpreting data should support and deepen the understanding of the major work of the grade.

  In Grade 2, standard 2.NBT.B.5 requires students to fluently add and subtract within 100. Standard 2.MD.B.5 strengthens this by asking students to solve word problems involving length as a context for adding and subtracting within 100. These standards support each other so math makes sense to students as this connection helps to eliminate endless lists of discrete topics to learn.

In Grade 4, this is further evidenced as shown in 4.MD.B.4 which is copied below.

> Make a line plot to display a data set of measurements in fractions of a unit (½, ¼, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

In addition to requiring that students make line plots of measurements, the Grade 4 example indicates that the use of line plots is one way to reinforce the major work of adding and subtracting fractions with like denominators. As such, the study of line plots is not a discrete unrelated topic, but a vehicle for students to practice their newly learned skill of adding and subtracting fractions, as well as a vehicle for making sense of the sums and differences.

- Grade 6: Solve problems by graphing in all 4 quadrants.
  Having students graph in all 4 quadrants for the first time (6.NS.C.8) supports the understanding of system of rational numbers (6.NS.C.5, 6, 7).

- Grade 8: "Understand the connections between proportional relationships, lines and linear equations."
  Cluster 8.EE.B demands that students actually understand *the connections* between proportional relationships, lines**,** and linear equations. This is not left up to chance, but is written in the standards.

- Algebra I: Defining and analyzing relationships between two quantities
  Many standards are well connected particularly when modeling real-life applications. Being able to define appropriate quantities for the purpose of descriptive modeling (N-Q.A.2) is critical to interpreting key features of graphs and tables that model a relationship between two quantities (F-IF.B.4) and to creating equations in two variables to represent those relationships (A.CED.2).

- Geometry: Understanding congruence in rigid motions
  The CCSSM approach to proving congruence is based on rigid transformations, making clusters G-CO.A and G-CO.B dependent upon one another.

Coherence of Topics Across Grades

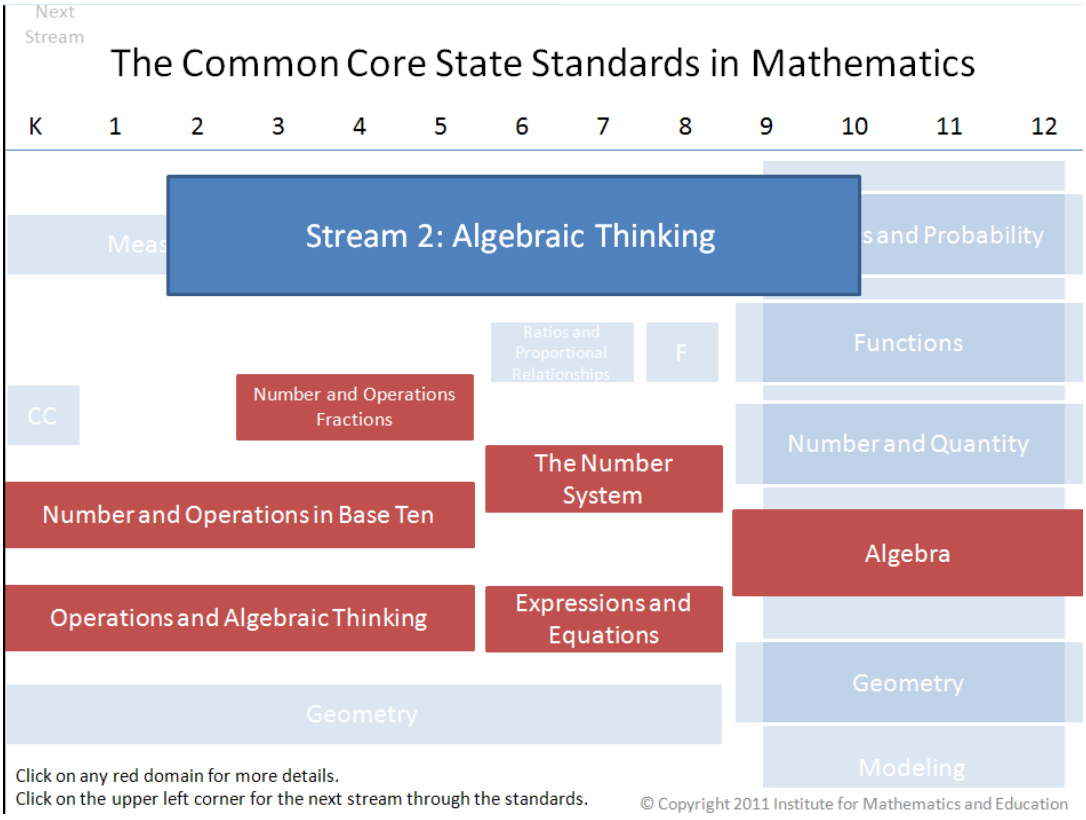
As noted earlier, coherence is one of the major design principles of the standards**.** The *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics* (http://tinyurl.com/azb7gb2) states:

> *Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations. The standards define progressions of learning that leverage these principles as they build knowledge over the grades.*

When people talk about coherence, they often talk about making connections between topics. The most important connections are vertical: the links from one grade to the next that allow students to progress in their mathematical education. That is why it is critical to think across grades and examine the progressions in the standards to see how major content develops over time.

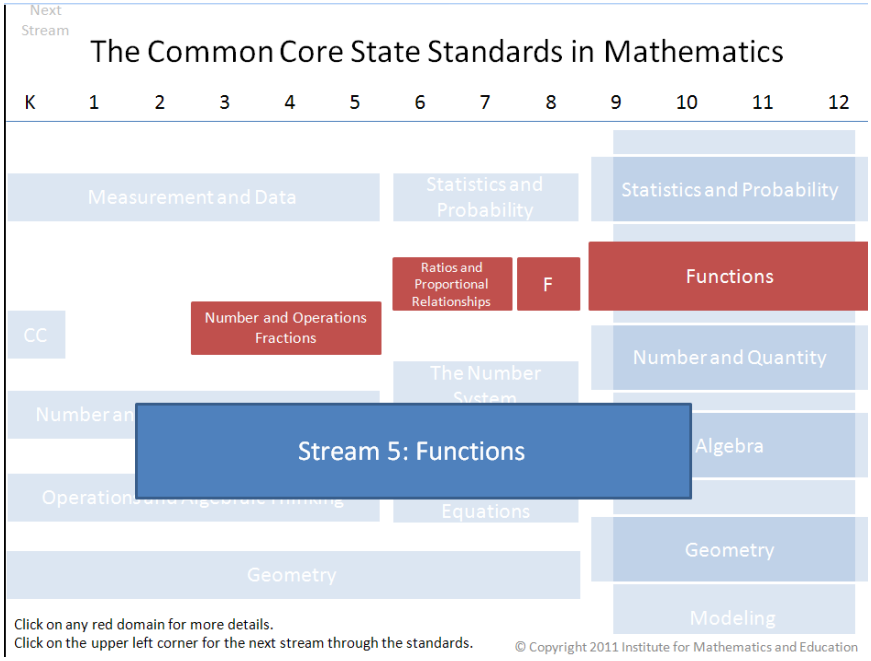
CCSSM writers are in the process of writing documents to describe these cross-grade progressions[1] of learning for each domain. They have also created a CCSSM Visable Map as a resource to explain the streams within the content standards that flow across grade levels. The map is posted at http://tinyurl.com/a3g7aef. Take a moment to review this information. Screenshots of the CCSSM Visable Map are provided below

The slide below shows that domains related to Algebraic Thinking begin in Kindergarten and are found in all grades. The domains of Numbers and Operations in Base Ten and Numbers and Operations Fractions are connected to understanding algebraic concepts. Clicking on any domain will provide more details about the domain.



[1] As progression documents are written, they are posted at http://ime.math.arizona.edu/progressions/. Future modules are planned to examine these more closely.

In contrast, the slide below indicates that the Functions stream begins in Grade 3 with the domain Number and Operations Fractions, but the CCSSM in Grades K-2 do not align with this stream.



Examples of Coherence of Topics Across Grades

Cluster headings across grades within the same domain or domains that are part of the cross-grade streams provide insight as to specific examples of coherence across grades. Below are cluster headings related to operations with numbers and algebraic expressions. It is evident that the standards build on students' prior knowledge and understanding.

Grades 1 and 2:	Use place value understanding and properties of operations to add and subtract.
Grades 3 and 4:	Use place value understanding and properties of operations to perform multi-digit arithmetic.
Grade 5:	Perform operations with multi-digit whole numbers and with decimals to hundredths. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
Grade 6:	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
Grade7:	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
Grade 8:	Work with radicals and integer exponents.

Algebra I: Perform arithmetic operations on polynomials

Algebra II: Rewrite rational expressions

In the introductory video for this module, David Coleman urges participants to look at the progression of fractions based on their importance in helping students to be successful in algebra in high school. This activity is designed to provide a structure for doing so.

Coherence Activity

Instructions: In the space below, copy all of the standards related to multiplication and division of fractions and note how coherence is evident in these standards. Note also standards that are outside of the Number and Operations—Fractions domain but are related to, or in support of, fractions. Listing these is optional. Answers are provided on the last page of the module.

Grade	Standard #	Summary of the Standard (If the standard has sub-parts, summarize each sub-part.)

Conclusion

This module presented two of three instructional shifts required for implementation of the Common Core State Standards in Mathematics: Focus and Coherence. Watch the video posted at http://tinyurl.com/b3uc5we to see the writers' views on how these two major design principles will help teachers in their daily work.

You should now be able to answer the Pre-Assessment questions with ease. Answers to the self-check questions are on the next page.

Self-Check Answers to Assignments and Activities found in Module 2:

- Step 3, page 4: K.CC. C.6 and K.CC. C.7
- Assignment 1, page 4: K.CC.B.4a, K.CC.B.4b, K.CC.B.5.
- Engaging with the Content Activity, page 6

  3a.

  K–2 .....Addition and subtraction - concepts, skills, and problem solving
  3–5 .....Multiplication and division of whole numbers and fractions - concepts, skills, and problem solving
  6 .....Ratios and proportional relationships; early expressions and equations
  7 .....Ratios and proportional relationships; arithmetic of rational numbers
  8 .....Linear algebra; linear functions
  Algebra I .....Linear and Quadratic Relationships and Models, Introduction to Functions
  Geometry .....Congruence and Similarity – Understanding, Proof of Theorems, and Models
  Algebra II .....Polynomial Equations, Inequalities, and Functions in modeling contexts

  3b. answers will vary

  3c. answers will vary, but the intent of the CCSSM is to connect the supporting content to the major content in a significant way
- Coherence Activity, page 11

Grade	Standard #	Summary of the Standard (If the standard has sub-parts, summarize each sub-part.)
3	various	Standards that relate to the foundations of being able to multiply and divide fractions found in 3.OA and 3.NF
4	4.NF.1	Recognize & generate equivalent fractions
4	4.NF.4 a, b & c	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number
4	4.MD.2	Use 4 operations to solve word problems…. involving simple fractions
5	5.NF.3	Interpreting a fraction as division of the numerator by the denominator.
5	5.NF.4	Apply & extend previous understandings of multiplication to multiply a fraction or whole number by a fraction
5	5.NF.5	Interpret multiplication as scaling (resizing)
5	5.NF.6	Solve real word problems involving multiplication of fractions and mixed numbers
5	5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions
5	5.MD.2	Use operations on fractions to solve problems involving information presented in line plots
6	6.NS.1	Apply & extend previous understandings of multiplication and division to divide fractions by fractions
6	6.G.2	Find volume of a right rectangular prism with fractional edge lengths….
7	various	Standards that relate to ratio and proportion (7.RP) and standards extending fractions to rational numbers 7.NS.1-3