Rigor

To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor: (1) conceptual understanding, (2) procedural skill and fluency, and (3) applications. The word "rigor" isn't a code word for just one of these three; rather, it means equal intensity in all three. The word "understand" is used in the Standards to set explicit expectations for conceptual understanding, and the phrase "real-world problems" and the star symbol (⋆) are used to set expectations and flag opportunities for applications and modeling. (Modeling is a Standard for Mathematical Practice as well as a content category in High School.) The High School content standards do not set explicit expectations for fluency, but fluency is important in high school mathematics.

The Standards for Mathematical Practice set expectations for using mathematical language and representations to reason, solve problems, and model. These expectations are related to fluency: precision in the use of language, seeing structure in expressions, and reasoning from the concrete to the abstract correspond to high orders of fluency in the acquisition of mathematical language, especially in the form of symbolic expressions and graphs. High School mathematics builds new and more sophisticated fluencies on top of the earlier fluencies from K-8 that centered on numerical calculation.

To date, curricula have not always been balanced in their approach to these three aspects of rigor. Some curricula stress fluency in computation without acknowledging the role of conceptual understanding in attaining fluency and making algorithms more learnable. Some stress conceptual understanding without acknowledging that fluency requires separate classroom work of a different nature. Some stress pure mathematics without acknowledging that applications can be highly motivating for students and that a mathematical education should make students fit for more than just their next mathematics course. At another extreme, some curricula focus on applications, without acknowledging that math doesn't teach itself.

The Standards do not take sides in these ways, but rather they set high expectations for all three components of rigor in the major work of each grade. Of course, that makes it necessary that we focus—otherwise we are asking teachers and students to do more with less.

What's All This Talk about Rigor?

By NCTM President Linda M. Gojak
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Recently, I had a conversation with a group of math coaches who are working with elementary teachers on implementation of the Common Core Standards for Mathematics. The discussion turned to a description of rigor in the classroom. The coaches commented that many of their teachers were confused by exactly what was meant by teaching and learning with rigor. The coaches weren't sure how to respond.

Rigor in the Common Core State Standards
The word "rigor" is widely used in policy discussions, but it's rarely understood or defined, and often it merely passes as code for "better." It is interesting that the term "rigor" does not appear in the Common Core State Standards for Mathematics, although it is certainly implied. "Rigor" appears multiple times in the U.S. Department of Education's paper, "A Blueprint for Reform: The Reauthorization of the Elementary and Secondary Education Act," as well as its recent document, "ESEA Flexibility"—both of which include a call for rigorous academic content standards.

Rigor in Instruction
The coaches and I began our work of exploring the notion of rigor with an online search of the word "rigor." The thesaurus led us to a list of synonyms, including "affliction," "inflexibility," "difficulty," "severity," "rigidity," "suffering," and "traditionalism"—none of which describe characteristics of rigorous mathematics instruction. No wonder the teachers were confused! However, two additional words included in the list—"thoroughness"and "tenacity"—provided avenues for some serious thought about what "rigor"implies. We generated the following chart, which led to an interesting discussion with the classroom teachers. There are certainly other characteristics that can be added to the list.

Learning experiences that involve rigor …	**Experiences that do not involve rigor …**
challenge students	are more "difficult," with no purpose (for example, adding 7ths and 15ths without a real context)
require effort and tenacity by students	require minimal effort
focus on quality (rich tasks)	focus on quantity (more pages to do)
include entry points and extensions for all students	are offered only to gifted students
are not always tidy, and can have multiple paths to possible solutions	are scripted, with a neat path to a solution
provide connections among mathematical ideas	do not connect to other mathematical ideas

contain rich mathematics that is relevant to students	contain routine procedures with little relevance
develop strategic and flexible thinking	follow a rote procedure
encourage reasoning and sense making	require memorization of rules and procedures without understanding
expect students to be actively involved in their own learning	often involve teachers doing the work while students watch

Rigor Involves Everyone

Rigor involves all partners in teaching and learning. Teachers must consider rigor in planning lessons, tasks, and assignments. Rigorous lessons build on and extend prior knowledge. They encourage productive struggling. Although the objective of a lesson should be clear in the teacher's mind, the lesson should not focus on one correct path to a solution or even one correct answer. A rigorous lesson embraces the messiness of a good mathematics task and the deep learning that it has the potential to achieve.

Students who are successful in a rigorous learning environment take responsibility for their learning. They learn to reflect on their thinking. They persist in solving a problem when the path to solution is not immediately obvious. They recognize when they are not on the correct path and need to switch directions during the solution process. Students must learn to ask productive questions rather than expecting to be shown how to proceed. (And, teachers must answer those questions with just enough information to move students forward while preserving the challenge of the task!

Rigorous teaching and learning require rigorous formative assessment throughout a unit so the teacher knows what the student has learned and can plan additional activities, or adjust them, to address student needs. Students also have a role in formative assessment—they must approach tasks with tenacity and ask clarifying questions when they are unsure how to proceed. All assessments must include opportunities for students to demonstrate the processes and practices in their approach to doing mathematics. Good formative assessment can be incorporated into daily instruction and prepare students for the summative assessments that take place at certain points throughout the unit of study.

Moving toward Rigor

How can we support classroom teachers and pre-service teachers (pre-K–16) in working toward greater rigor in mathematics instruction? Professional development experiences that model rigor through the use of rich tasks, rich discourse, and good questions allow teachers to experience rigorous instruction. When selecting tasks, teachers must be sure that mathematical ideas are explicit and the connections are clear. The days of a few word problems at the end of multiple skill exercises in the textbook are over! Concepts must be introduced and explored in contexts that are interesting and motivating for students. Tasks must provide entry points for all students, offer them well-defined opportunities to make connections to other mathematics, and include both opportunities and expectations for them to develop deeper understanding. The focus and coherence of the Common Core State Standards lead the way to rigorous instruction. It is time for us to begin the journey.