

Instructional Vision

To meet these raised expectations, we must clarify our focus on what our students need. Specifically, we must ensure this year that our students...

English language arts

- Comprehend (access) meaningful, on grade level texts
- Speak and write in response to meaningful texts

Math students

- Master priority concepts and practice standards (not just procedures)
- Target remedial content that allows faster on grade level practice

Teacher Leader Summit: Day 1 Ready!

This Summit will prepare teachers to make these shifts beginning the first day of the 14-15 school year. This will include focused training on:

- Student Learning Targets
- Assessment
- Standards, curricula, and instructional strategies

Using

to Teach

Why Conceptual Understanding?

Does anybody teach students who...

- are scared of fractions?
- can't add/subtract fractions?
- can't multiply/divide fractions?
- can't add/subtract signed numbers?
- can't multiply/divide signed numbers?

What other arithmetic do your students have trouble with? Why do students struggle with these topics?

Procedural Skill and Fluency

Kyle Falting & Britton Kilpatrick
contact@universalachievement.com

Goals for Today:

- Establish the need for and power of conceptual understanding
- Examine the explicit coherence and progression from conceptual understanding to procedural skill and fluency in K-8
- Understand how to build lessons using the progressions from K-8 as a model
- Explore the connections between the focus lesson and the Student Achievement Partners' (SAP) Instructional Practice Guide (IPG)



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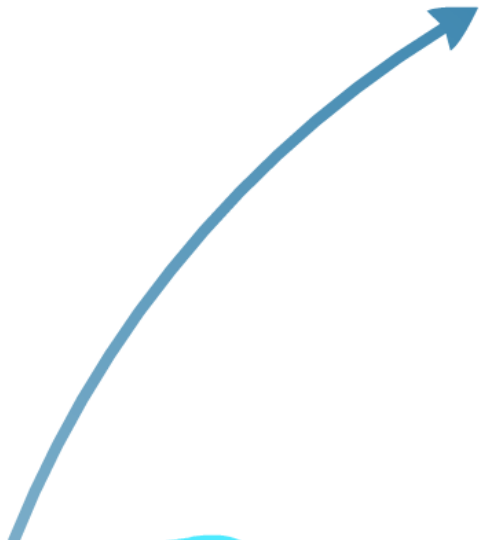
Addition and Subtraction

K.OA.5

Fluently add and subtract within 5.

K.OA.3

Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5=2+3$ and $5=4+1$).



Kindergarten

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1st Grade

1.OA.6

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8+6 = 8+2+4 = 10+4 = 14$); decomposing a number leading to a ten (e.g., $13-4 = 13-3-1 = 10-1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8+4=12$, one knows $12-8=4$); and creating equivalent but easier or known sums (e.g., adding $6+7$ by creating the known equivalent $6+6+1 = 12+1 = 13$).

1.NBT.4

Add within 100, including adding a two-digit number and a one digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and one; and sometimes it is necessary to compose a ten.

1st Grade

2nd Grade

1.OA.6

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8+6 = 8+2+4 = 10+4 = 14$); decomposing a number leading to a ten (e.g., $13-4 = 13-3-1 = 10-1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8+4=12$, one knows $12-8=4$); and creating equivalent but easier or known sums (e.g., adding $6+7$ by creating the known equivalent $6+6+1 = 12+1 = 13$).

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2.NBT.5

Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

2.NBT.7

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2nd Grade

3rd Grade

2.NBT.5

Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

2.NBT.7

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

3.NBT.2

Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NF.

1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by "a" parts of size $1/b$.
3b Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size...
Recognize and generate simple equivalent fractions, e.g., $1/2=2/4$, $4/6=2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

2nd Grade

3rd Grade

4th Grade

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Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

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4.NBT.4

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

4.NF.

1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
3a Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

3rd Grade

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4th Grade

5th Grade

4.NBT.4

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

5.NF.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.)

4.NF.

1 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{n \times a}{n \times b}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

3a Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

5.NBT.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

subtract multi-digit whole numbers using the standard algorithm.

Understand that a fraction a/b is equivalent to $(n \times a)/(n \times b)$ by using visual models to represent the fraction. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

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Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)

5.NBT.7

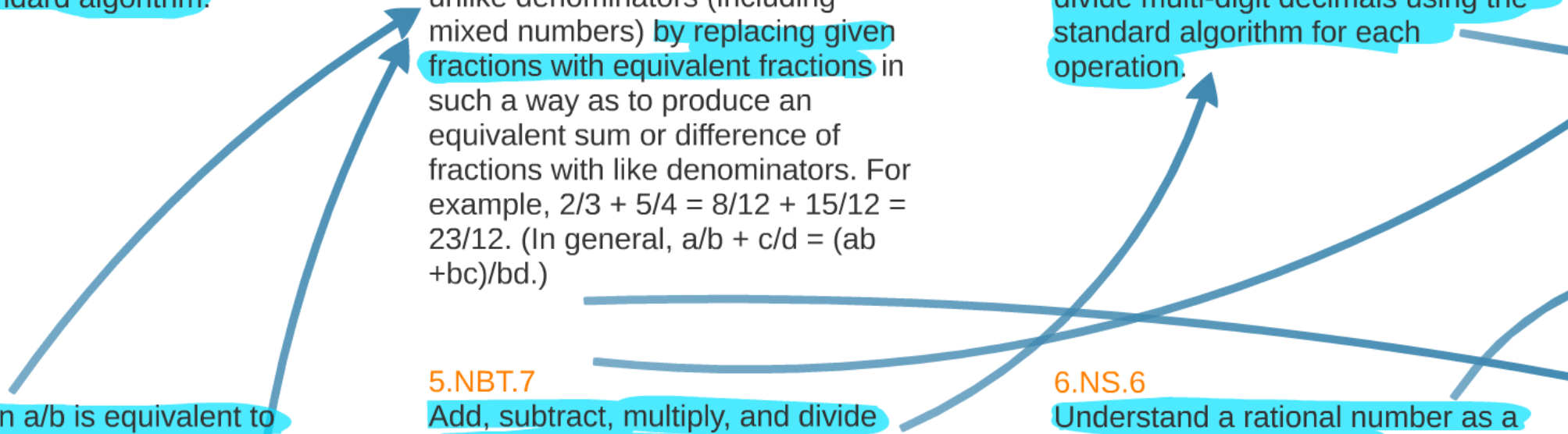
Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

6.NS.3

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

6.NS.6

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates...



Grade

6th Grade

7th Grade

Subtract fractions with different denominators (including mixed numbers) by replacing given fractions with equivalent fractions in order to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{1}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.

Understand and explain to others how to multiply and divide multi-digit whole numbers and multi-digit decimals, using models or drawings and place value, and/or the relationship between addition and multiplication; relate the strategy to a method and explain the process used.

6.NS.3

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

6.NS.6

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates...

7.NS.

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram...
... (d) Apply properties of operations as strategies to add and subtract rational numbers.

How can this same progression (from conceptual understanding to procedural skill and fluency) be applied to high school content?

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A-APR.A.1 - Perform arithmetic operations on polynomials

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

6.EE.3

Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2+x)$ to produce the equivalent expression $6+3x$;

7.EE.1

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

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Year-At-A-Glance

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10	Unit 11	Unit 12
1st nine weeks			2nd nine weeks			3rd nine weeks			4th nine weeks		
A-CED.A.1	A-CED.A.2	A-CED.A.3	F-IF.A.1	A-SSE.B.3c	A-SSE.A.1	A-SSE.A.2	F-IF.B.5	F-IF.B.4	A-REI.D.11	N-Q.A.2	S-ID.C.7
A-CED.A.4	A-REI.D.10	A-REI.C.5	F-IF.A.2	A-CED.A.1	A-APR.A.1	A-REI.A.1	F-IF.B.6	F-IF.B.5	F-IF.B.4	N-Q.A.3	S-ID.C.8
A-REI.B.3	A-REI.D.12	A-REI.C.6	F-IF.A.3	F-LE.A.1		A-REI.B.4	A-SSE.B.3a	F-IF.B.6	F-IF.C.7b	S-ID.A.1	S-ID.C.9
N-Q.A.1	N-Q.A.1	A-REI.D.12	F-IF.B.5	F-LE.A.2		A-SSE.B.3a	A-SSE.B.3b	F-IF.C.7b		S-ID.A.2	N-Q.A.1
N-RN.B.3	F-IF.C.7a		F-BFA.1a	F-LE.A.3		A-SSE.B.3b	A-APR.B.3	F-IF.C.9		S-ID.A.3	S-ID.B.5
			F-BF.B.3	F-LE.B.5			F-IF.C.7a	F-BF.B.3			S-ID.B.6
							F-IF.C.8a				
							F-IF.C.9				
							F-BF.B.3				
Major Clusters			Supporting Clusters				Additional Clusters				
SSE	Seeing Structure in Expressions (1, 2)			N-Q	Quantities (1, 2, 3)		N-RN	The Real Number System (3)			
APR	Arithmetic with Polynomials and Rational Expressions (1)			A-SSE	Seeing Structure in Expressions (3)		F-BF	Building Functions (3)			
CED	Creating Equations (1, 2, 3, 4)			A-APR	Arithmetic with Polynomials and Rational Expressions (3)		S-ID	Interpreting Categorical & Quantitative Data (1, 2, 3)			
REI	Reasoning with Equations and Inequalities (1, 3, 4, 5, 6, 10, 11, 12)			F-IF	Interpreting Functions (7, 8, 9)						
IF	Interpreting Functions (1, 2, 3, 4, 5, 6)			F-BF	Building Functions (1)						
ID	Interpreting Categorical and Quantitative Data (7, 8, 9)			F-LE	Linear, Quadratic, and Exponential Models (1, 2, 3, 5)						
				S-ID	Interpreting Categorical & Quantitative Data (5, 6)						

Year-At-A-Glance

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9	Unit 10	Unit 11	Unit 12
Solving Linear Equations and Inequalities in One Variable	Linear Equations and Inequalities in Two Variables	Systems of Linear Equations and Inequalities	Functions	Linear and Exponential Functions	Arithmetic with Polynomials	Solving Quadratic Equations	Quadratic and Other Polynomial Functions	Radical Functions	Piecewise-Defined Functions	Univariate Statistics	Bivariate Statistics: Linear, Quadratic, and Exponential
1st nine weeks			2nd nine weeks			3rd nine weeks			4th nine weeks		
A-CED.A.1	A-CED.A.2	A-CED.A.3	F-IF.A.1	A-SSE.B.3c	A-SSE.A.1	A-SSE.A.2	F-IF.B.5	F-IF.B.4	A-REI.D.11	N-Q.A.2	S-ID.C.7
A-CED.A.4	A-REI.D.10	A-REI.C.5	F-IF.A.2	A-CED.A.1	A-APR.A.1	A-REI.A.1	F-IF.B.6	F-IF.B.5	F-IF.B.4	N-Q.A.3	S-ID.C.8
A-REI.B.3	A-REI.D.12	A-REI.C.6	F-IF.A.3	F-LE.A.1		A-REI.B.4	A-SSE.B.3a	F-IF.B.6	F-IF.C.7b	S-ID.A.1	S-ID.C.9
N-Q.A.1	N-Q.A.1	A-REI.D.12	F-IF.B.5	F-LE.A.2		A-SSE.B.3a	A-SSE.B.3b	F-IF.C.7b		S-ID.A.2	N-Q.A.1
N-RN.B.3	F-IF.C.7a		F-BF.A.1a	F-LE.A.3		A-SSE.B.3b	A-APR.B.3	F-IF.C.9		S-ID.A.3	S-ID.B.5
			F-BF.B.3	F-LE.B.5			F-IF.C.7a	F-BF.B.3			S-ID.B.6
							F-IF.C.8a				
							F-IF.C.9				
							F-BF.B.3				
Major Clusters				Supporting Clusters				Additional Clusters			
A-SSE	Seeing Structure in Expressions (1, 2)			N-Q	Quantities (1, 2, 3)			N-RN	The Real Number System (3)		
A-APR	Arithmetic with Polynomials and Rational Expressions (1)			A-SSE	Seeing Structure in Expressions (3)			F-BF	Building Functions (3)		
A-CED	Creating Equations (1, 2, 3, 4)			A-APR	Arithmetic with Polynomials and Rational Expressions (3)			S-ID	Interpreting Categorical & Quantitative Data (1, 2, 3)		
A-REI	Reasoning with Equations and Inequalities (1, 3, 4, 5, 6, 10, 11, 12)			F-IF	Interpreting Functions (7, 8, 9)						
F-IF	Interpreting Functions (1, 2, 3, 4, 5, 6)			F-BF	Building Functions (1)						
S-ID	Interpreting Categorical and Quantitative Data (7, 8, 9)			F-LE	Linear, Quadratic, and Exponential Models (1, 2, 3, 5)						
				S-ID	Interpreting Categorical & Quantitative Data (5, 6)						

Unit 6	Unit 7	Unit 8	Unit 9
Arithmetic with Polynomials	Solving Quadratic Equations	Quadratic and Other Polynomial Functions	Radical Functions
	3rd nine weeks		
A-SSE.A.1	A-SSE.A.2	F-IF.B.5	F-IF.B.4
A-APR.A.1	A-REI.A.1	F-IF.B.6	F-IF.B.5

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A-CED.A.4	A-REI.D.10	A-REI.C.5	F-IF.A.2	A-CED.A.1	A-APR.A.1	A-REI.A.1	F-IF.B.6	F-IF.B.5	F-IF.B.4	N-Q.A.3	S-ID.C.8
A-REI.B.3	A-REI.D.12	A-REI.C.6	F-IF.A.3	F-LE.A.1		A-REI.B.4	A-SSE.B.3a	F-IF.B.6	F-IF.C.7b	S-ID.A.1	S-ID.C.9
N-Q.A.1	N-Q.A.1	A-REI.D.12	F-IF.B.5	F-LE.A.2		A-SSE.B.3a	A-SSE.B.3b	F-IF.C.7b		S-ID.A.2	N-Q.A.1
N-RN.B.3	F-IF.C.7a		F-BFA.1a	F-LE.A.3		A-SSE.B.3b	A-APR.B.3	F-IF.C.9		S-ID.A.3	S-ID.B.5
			F-BF.B.3	F-LE.B.5			F-IF.C.7a	F-BF.B.3			S-ID.B.6
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				S-ID	Interpreting Categorical & Quantitative Data (5, 6)						

Name: _____

Date: _____

Unit 6

Unit 6 - Lesson 5

Conceptual Understanding (Tier 1)

1. Explain the concept of a monomial and provide evidence to support your explanation.

2. Explain the concept of a polynomial and provide evidence to support your explanation.

3. Explain the process for evaluating the following product using the standard algorithm and evaluate: $12(13)$.

4. How many individual products does it take to multiply $12(13)$ using the standard algorithm? Write each of them below.

5. Evaluate the following product by applying the distributive property: $12(10 + 3)$. Is the product equivalent to the product you produced in problem #3? Why or why not?

6. Evaluate the following product by applying the distributive property: $(10 + 2)13$. Is the product equivalent to the product you produced in problem #3? Why or why not?

7. Evaluate the following expression by first applying the distributive property: $10(10 + 3) + 2(10 + 3)$. Is the product equivalent to the product you produced in problem #3? Why or why not?

8. When applying the distributive property to $10(10 + 3) + 2(10 + 3)$, did it take the same number of individual products as you produced in problem #3? Why or why not?

9. Could the expression $10(10 + 3) + 2(10 + 3)$ be looked at as the sum of two terms like terms? *Explain.*

10. Is the expression $10(10 + 3) + 2(10 + 3)$ equivalent to the expression $(10 + 2)(10 + 3)$? Why or why not?

11. Explain how to evaluate the expression $(10 + 2)(10 + 3)$ by applying the distributive property and provide evidence to support your explanation.

3. Explain the process for evaluating the following product using the standard algorithm and evaluate: $12(13)$.

4. How many individual products does it take to multiply $12(13)$ using the standard algorithm? Write each of them below.

5. Evaluate the following product by applying the distributive property: $12(10 + 3)$. Is the product equivalent to the product you produced in problem #3? Why or why not?
6. Evaluate the following product by applying the distributive property: $(10 + 2)13$. Is the product equivalent to the product you produced in problem #3? Why or why not?

7. Evaluate the following expression by first applying the distributive property: $10(10 + 3) + 2(10 + 3)$. Is the product equivalent to the product you produced in problem #3? Why or why not?

Date: _____

Unit 6

Lesson 5

Unit 6

8. When applying the distributive property, how many individual products did it take to evaluate $10(10 + 3) + 2(10 + 3)$? Are they the same individual products as you listed in problem #4? Why or why not?

...ing product by applying the
...rty: $12(10 + 3)$. Is the product
...product you produced in problem
...not?

9. Could the expression $10(10 + 3) + 2(10 + 3)$ be looked at as the sum of two terms? If so, are the terms like terms? *Explain.*

...ing product by applying the
...rty: $(10 + 2)13$. Is the product
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11. Explain how to evaluate the expression $(10 + 2)(10 + 3)$ by applying the distributive property and provide evidence to support your explanation.

12. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(7 + 5)(7 + 6)$? Why or why not? *Include the distributive property in your justification.*

13. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(6 + 6)(9 + 4)$? Why or why not? *Include the distributive property in your justification.*

14. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(10 + 2)(15 - 2)$? Why or why not? *Include the distributive property in your justification.*

15. Could you have produced the same product by rewriting the expression $12(13)$ as any expression $(a + b)(c + d)$? Why or why not? *Include the distributive property in your justification.*

16. Following your responses, conjecture can you make about the product in the form $(a + b)(c + d)$? Provide evidence to support your conjecture.

Evaluate each of the following expressions in two different ways:
1. following the standard algorithm
2. rewriting each factor as a sum and applying the distributive property

17. $11(14)$

18. $15(15)$

19. $13(17)$

8. When applying the distributive property, how many individual products did it take to evaluate $10(10 + 3) + 2(10 + 3)$? Are they the same individual products as you listed in problem #4? Why or why not?

9. Could the expression $10(10 + 3) + 2(10 + 3)$ be looked at as the sum of two terms? If so, are the terms like terms? *Explain.*

10. Is the expression $10(10 + 3) + 2(10 + 3)$ equivalent to the expression $(10 + 2)(10 + 3)$? Why or why not?

11. Explain how to evaluate the expression $(10 + 2)(10 + 3)$ by applying the distributive property and provide evidence to support your explanation.

12. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(7 + 5)(7 + 6)$? Why or why not? *Include the distributive property in your justification.*

13. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(6 + 6)(9 + 4)$? Why or why not? *Include the distributive property in your justification.*

14. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(10 + 2)(15 - 2)$? Why or why not? *Include the distributive property in your justification.*
15. Could you have produced the same product by rewriting the expression $12(13)$ as any expression $(a + b)(c + d)$? Why or why not? *Include the distributive property in your justification.*

Lesson 5

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ssion 12(13) as any expression
y or why not? *Include the
y in your justification.*

Unit 6

16. Following your responses to problems #3-11, what
conjecture can you make about evaluating any
product in the form $(a + b)(c + d)$? *Provide
evidence to support your conjecture.*

*Evaluate each of the following products two
different ways:
1. following the standard algorithm
2. rewriting each factor as a sum of two numbers
and applying the distributive property*

17. 11(14)

18. 15(15)

19. 13(17)

Lesson 5

20. 18(12)

21. 19(11)

22. 14(16)

23. 21(19)

24. 21(24)

Unit 6

25. 21(29)

26. 17(49)

27. Looking back at prob
support your conjectu
*Explain and provide
explanation.*

28. Could you have produ
rewriting the expressi
 $(10 + 2)(10 + 2 + 1)$?
distributive property

Unit 6

16. Following your responses to problems #3-11, what conjecture can you make about evaluating any product in the form $(a + b)(c + d)$? *Provide evidence to support your conjecture.*

Evaluate each of the following products two different ways:

1. following the standard algorithm

2. rewriting each factor as a sum of two numbers and applying the distributive property

17. $11(14)$

23. 21(19)

24. 21(24)

Lesson 5

Unit 6

25. 21(29)

26. 17(49)

27. Looking back at problem #17-26, did your work support your conjecture from problem #16? Explain and provide evidence to support your explanation.

28. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(10 + 2)(10 + 2 + 1)$? Why or why not? Include the distributive property in your justification.

Lesson 5

29. When applying the distributive property, how many individual products would it take to evaluate $(10 + 2)(10 + 2 + 1)$? Write each of them below.

30. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(10 + 1 + 1)(10 + 2 + 1)$? Why or why not? Include the distributive property in your justification.

31. When applying the distributive property, how many individual products would it take to evaluate $(10 + 1 + 1)(10 + 2 + 1)$? Write each of them below.

32. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(15 - 1 - 1 - 1)(15 - 2)$? Why or why not? Include the distributive property in your justification.

Unit 6

33. When applying the distributive property, how many individual products would it take to evaluate $(15 - 1 - 1 - 1)(15 - 2)$? Write each of them below.

34. Following your responses to problem #30, what conjecture can you make about the number of terms in two sums or differences? Provide evidence to support your conjecture.

35. In the product $(a + b)(c + d)$, what do a , b , c , and d represent any real number? Could they represent any variable? Could they represent any expression? Explain your responses and provide evidence to support your explanations.

36. How is the expression $(x + 2)(x + 3)$ related to the expression $(10 + 2)(10 + 3)$? Explain your response.

27. Looking back at problem #17-26, did your work support your conjecture from problem #16? *Explain and provide evidence to support your explanation.*

28. Could you have produced the same product by rewriting the expression $12(13)$ as the expression $(10 + 2)(10 + 2 + 1)$? Why or why not? *Include the distributive property in your justification.*

Lesson 5

Using the distributive property, how many individual products would it take to evaluate $(15 - 1 - 1 - 1)(15 - 2)$? Write each of them below.

Did you produce the same product by using the distributive property on the expression $12(13)$ as the expression $(10 + 2 + 1)(13)$? Why or why not? *Include evidence to support your conjecture.*

Using the distributive property, how many individual products would it take to evaluate $(15 - 2)(10 + 2 + 1)$? Write each of them below.

Did you produce the same product by using the distributive property on the expression $12(13)$ as the expression $(15 - 2)(10 + 2 + 1)$? Why or why not? *Include evidence to support your justification.*

Unit 6

33. When applying the distributive property, how many individual products would it take to evaluate $(15 - 1 - 1 - 1)(15 - 2)$? Write each of them below.

34. Following your responses to problems #28-33, what conjecture can you make about multiplying two sums or differences? *Provide evidence to support your conjecture.*

35. In the product $(a + b)(c + d)$, could a , b , c , and d represent any real number? Could they represent any variable? Could they represent any monomial? *Explain your responses and provide evidence to support your explanations.*

36. How is the expression $(x + 2)(x + 3)$ different from the expression $(10 + 2)(10 + 3)$?

Lesson 5

37. Can you produce a numerical product for the expression $(x + 2)(x + 3)$? Why or why not?

38. Could you follow the process you described in problem #16 to generate an expression involving only addition of monomials equivalent to the expression $(x + 2)(x + 3)$? If so, generate the equivalent expression.

Evaluate each product by applying the distributive property. Show each step of evaluating the product separately.

39. $(x + 5)(x + 4)$

40. $(x + 2)(x + 7)$

Unit 6

41. $(x + 11)(x - 3)$

42. $(x - 7)(x + 4)$

43. $(x - 5)(x - 4)$

44. $(2x + 5)(x + 4)$

45. $(2x + 5)(3x + 4)$

46. $(-5x + 1)(3x - 7)$

34. Following your responses to problems #28-33, what conjecture can you make about multiplying two sums or differences? *Provide evidence to support your conjecture.*

35. In the product $(a + b)(c + d)$, could a , b , c , and d represent any real number? Could they represent any variable? Could they represent any monomial? *Explain your responses and provide evidence to support your explanations.*

36. How is the expression $(x + 2)(x + 3)$ different from the expression $(10 + 2)(10 + 3)$?

37. Can you produce a numerical product for the expression $(x + 2)(x + 3)$? Why or why not?
38. Could you follow the process you described in problem #16 to generate an expression involving only addition of monomials equivalent to the expression $(x + 2)(x + 3)$? If so, generate the equivalent expression.

Evaluate each product by applying the distributive property. Show each step of evaluating the product separately.

39. $(x + 5)(x + 4)$

40. $(x + 2)(x + 7)$

Lesson 5

numerical product for the
(3)? Why or why not?

process you described in
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mials equivalent to the
(3)? If so, generate the

by applying the distributive
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Unit 6

41. $(x + 11)(x - 3)$

42. $(x - 7)(x + 4)$

43. $(x - 5)(x - 4)$

44. $(2x + 5)(x + 4)$

45. $(2x + 5)(3x + 4)$

46. $(-5x + 1)(3x - 7)$

Lesson 5

47. $\left(4x + \frac{1}{4}\right)\left(8x + \frac{3}{4}\right)$

48. $(x + 17)(x - 17)$

49. $(x - 9)(x - 9)$

50. $(x^2 + 7)(x - 13)$

51. $(x^2 - x)(-5x + 11)$

Unit 6

52. $(x + 1)(x^2 + x + 1)$

53. $(x + 2)(x^2 + 3x + 6)$

54. $(x - 4)(2x^2 + 8x - 7)$

55. $(x^2 + x + 1)(3x - 8)$

Procedural Skill and Fluency

Simplify each of the following.

59. $(a + 7)(a + 9)$

60. $(a - 7)(a + 9)$

$$44. \quad (2x + 5)(x + 4)$$

$$45. \quad (2x + 5)(3x + 4)$$

$$50. \quad (x^2 + 7)(x - 13)$$

$$51. \quad (x^2 - x)(-5x + 11)$$

52. $(x+1)(x^2+x+1)$

53. $(x+2)(x^2+3x+6)$

54. $(x-4)(2x^2+8x-7)$

55. $(x^2+x+1)(3x-8)$

56. $(x^2-5)(3x^2-6x+9)$

57. $(x^2-x+3)(x^2+x+2)$

58. $(4x^2+3x+2)(5x^2-2x+3)$

63. $(a-8)(a+6)$

64. $(a-8)(a-6)$

65. $(a+8)(a-6)$

66. $(a+8)(a+6)$

67. $(2a-5)(4a+3)$

68. $(2a+5)(4a-3)$

Procedural Skill and Fluency (Tier 2)*Simplify each of the following expressions.*

59. $(a+7)(a+9)$

60. $(a-7)(a+9)$

61. $(a+7)(a-9)$

62. $(a-7)(a-9)$

69. $(2a+5)(4a+3)$

70. $(2a-5)(4a-3)$

$$54. (x - 4)(2x^2 + 8x - 7)$$

$$55. (x^2 + x + 1)(3x - 8)$$

$$57. \quad (x^2 - x + 3)(x^2 + x + 2)$$

$$58. \quad (4x^2 + 3x + 2)(5x^2 - 2x + 3)$$

Procedural Skill and Fluency (Tier 2)

Simplify each of the following expressions.

59. $(a + 7)(a + 9)$

60. $(a - 7)(a + 9)$

Lesson 5**Unit 6**

$(-6x+9)$

63. $(a-8)(a+6)$

64. $(a-8)(a-6)$

(x^2+x+2)

65. $(a+8)(a-6)$

66. $(a+8)(a+6)$

$(5x^2-2x+3)$

67. $(2a-5)(4a+3)$

68. $(2a+5)(4a-3)$

69. $(2a+5)(4a+3)$

70. $(2a-5)(4a-3)$

Lesson 5**Unit 6**

71. $(a+4)(a^2+a+12)$

72. $(a^2+4)(a^2+a+12)$

73. $(a^2+4a)(a^2+a+12)$

74. $(a^2+4a+1)(a^2+a+12)$

75. $(a+b)(a+b)$

76. $(2a+b)(a+2b)$

77. $(2a+3b)(3a+2b)$

78. $(a+b)(a+b+c)$

79. $(a+b+c)(a+b)$

80. $(a+b+c)(a+b+c)$

$$67. (2a - 5)(4a + 3)$$

$$68. (2a + 5)(4a - 3)$$

$$69. (2a + 5)(4a + 3)$$

$$74. \quad (a^2 + 4a + 1)(a^2 + a + 12)$$

$$75. \quad (a + b)(a + b)$$

Lesson 5

Unit 6

Lesson 5

77. $(2a + 3b)(3a + 2b)$

78. $(a + b)(a + b + c)$

79. $(a + b + c)(a + b)$

80. $(a + b + c)(a + b + c)$

$$79. (a + b + c)(a + b)$$

$$80. (a + b + c)(a + b + c)$$

CCSS INSTRUCTIONAL PRACTICE GUIDE

MATH

SUBJECT

K–8

GRADES


LESSON


GUIDE TYPE

This guide provides specific Core Actions teachers take when they are implementing the Common Core State Standards (CCSS) in daily planning and practice. It also includes indicators of what teachers are doing – and students are demonstrating – when those Core Actions are displayed. Designed as a developmental tool for teachers and those who support teachers, it can be used for planning, reflection, collaboration, and coaching. Refer to the CCSS for Mathematics (corestandards.org/math) as necessary.

The Shifts required by the Common Core State Standards for Mathematics are¹:

 **Focus:** Focus strongly where the Standards focus.

 **Coherence:** Think across grades, and link to major topics within grades.

 **Rigor:** In major topics pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

The Core Actions should be evident in planning and observable in instruction. For each lesson, evidence might include: lesson plan, problems and exercises, tasks and assessments, teacher instruction, student discussion and behavior, and student work. Although most or all indicators will be observable in a portion of a lesson, when necessary some indicators may be left blank.

STUDENT
ACHIEVEMENT
PARTNERS

Visit achievethecore.org/coaching-tool to use the digital version of the Instructional Practice Guide.

Date

Teacher / Instructor Name

School

Observer Name

Grade / Class Period / Section

Topic / Lesson / Unit

Standard(s) Addressed in this Lesson

Circle the aspect(s) of rigor targeted in the standard(s) addressed in this lesson¹:

Conceptual understanding

Procedural skill and fluency

Application

1. Refer to Common Core Shifts at a Glance (achievethecore.org/mathshifts) and the K-8 Publishers' Criteria for the Common Core State Standards for Mathematics (achievethecore.org/publisherscriteria) for additional information about the Shifts required by the CCSS.

SUMMARY OF CORE ACTIONS

Core Action 1

Ensure the work of the lesson reflects the Shifts required by the CCSS.

Indicators

- A. The lesson focuses on the depth of grade-level content.
- B. The lesson intentionally relates new concepts to prior learning.
- C. The lesson intentionally targets the aspect(s) of rigor being addressed.

Core Action 2

Employ instructional practices that allow all students to learn.

Indicators

- A. The teacher makes the mathematics of the lesson accessible to all students.
- B. The teacher provides opportunities for students to explain their thinking.
- C. The teacher uses variation in students' solution methods.
- D. The teacher checks for understanding throughout the lesson.
- E. The teacher summarizes the mathematics with student input.

Core Action 3

Provide all students with opportunities to exhibit mathematical proficiency.

Indicators

- A. The teacher poses high quality questions and provides time for students to think and discuss. Students share their developing thinking about the problem.
- B. The teacher uses strategies to keep all students engaged. Even after reaching a point of frustration, students continue to work on the problem.
- C. The teacher establishes a classroom culture in which students elaborate with a second sentence (spoken or written) their first sentence.
- D. The teacher creates the conditions for student collaboration. Students talk about and ask questions about each other's work.
- E. The teacher connects students' informal language to mathematical language. Students use precise mathematical language in their work.
- F. The teacher establishes a classroom culture in which students use appropriate tools strategically when solving a problem. Students use appropriate tools strategically when solving a problem.
- G. The teacher asks students to explain and justify their work. Student work includes revisions, especially revisions to their work.

This guide provides specific Core Actions teachers take when they are implementing the Common Core State Standards (CCSS) in daily planning and practice. It also includes indicators of what teachers are doing – and students are demonstrating – when those Core Actions are displayed. Designed as a developmental tool for teachers and those who support teachers, it can be used for planning, reflection, collaboration, and coaching. Refer to the CCSS for Mathematics (corestandards.org/math) as necessary.

The Core Actions should be evident in planning and observable in instruction. For each lesson, evidence might include: lesson plan, problems and exercises, tasks and assessments, teacher instruction, student discussion and behavior, and student work. Although most or all indicators will be observable in a portion of a lesson, when necessary some indicators may be left blank.

Date _____

Teacher / Instructor Name _____

School _____

Observer Name _____

Grade / Class Period / Section _____

Topic / Lesson / Unit _____

Standard(s) Addressed in this Lesson _____

SUMMARY OF CORE ACTIONS

Core Action 1

Ensure the work of the lesson reflects the Shifts required by the CCSS for Mathematics.

Indicators

- A. The lesson focuses on the depth of grade-level cluster(s), grade-level content standard(s) or part(s) thereof.
- B. The lesson intentionally relates new concepts to students' prior skills and knowledge.
- C. The lesson intentionally targets the aspect(s) of rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.

Core Action 2

Employ instructional practices that allow all students to master the content of the lesson.

Indicators

- A. The teacher makes the mathematics of the lesson explicit by using explanations, representations, and/or examples.
- B. The teacher provides opportunities for students to work with and practice grade-level problems and exercises.
- C. The teacher uses variation in students' solution methods to strengthen all students' understanding of the content.
- D. The teacher checks for understanding throughout the lesson using informal, but deliberate methods (such as questioning or assigning short problems).
- E. The teacher summarizes the mathematics with references to student work and discussion in order to reinforce the focus of the lesson.

Core Action 3

Provide all students with opportunities to exhibit mathematical practices in connection with the content of the lesson.

Indicators

- A. The teacher poses high quality questions and problems that prompt students to share their developing thinking about the content of the lesson.
Students share their developing thinking about the content of the lesson.
- B. The teacher uses strategies to keep all students persevering with challenging problems.
Even after reaching a point of frustration, students persist in efforts to solve challenging problems.
- C. The teacher establishes a classroom culture in which students explain their thinking.
Students elaborate with a second sentence (spontaneously or prompted by the teacher or another student) to explain their thinking and connect it to their first sentence.
- D. The teacher creates the conditions for student conversations where students are encouraged to talk about each other's thinking.
Students talk about and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.
- E. The teacher connects students' informal language to precise mathematical language appropriate to their grade.
Students use precise mathematical language in their explanations and discussions.
- F. The teacher establishes a classroom culture in which students choose and use appropriate tools when solving a problem.
Students use appropriate tools strategically when solving a problem.
- G. The teacher asks students to explain and justify work and provides feedback that helps students revise initial work.
Student work includes revisions, especially revised explanations and justifications.

Circle the aspect(s) of rigor targeted in the standard(s) addressed in this lesson:

- Conceptual understanding
- Procedural skill and fluency
- Application

Refer to Common Core Shifts at a Glance (achievethecore.org/mathshifts) and the K-8 Publishers' Criteria for the Common Core State Standards for Mathematics (achievethecore.org/publisherscriteria) for additional information about the Shifts required by the CCSS.

For each indicator, circle the appropriate rating to support the rating.

Core Action 1

Ensure the work of the lesson reflects the Shifts required by the CCSS for Mathematics.

Indicators

- A. The lesson focuses on the depth of grade-level content standard(s) or part(s) thereof.

- B. The lesson intentionally relates new concepts to students' prior skills and knowledge.

- C. The lesson intentionally targets the aspect(s) of rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.

For each indicator, circle the appropriate rating based on what was observed during the lesson. Provide specific evidence to support the rating.

Core Action 1

Ensure the work of the lesson reflects the Shifts required by the CCSS for Mathematics.

Indicators

A. The lesson focuses on the depth of grade-level cluster(s), grade-level content standard(s) or part(s) thereof.

YES The lesson focuses only on mathematics within the grade-level standards and fully reflects the depth of the grade-level cluster(s), grade-level content standard(s) or part(s) thereof.

NO The lesson focuses on mathematics outside the grade-level standards or superficially reflects the grade-level cluster(s), grade-level content standard(s) or part(s) thereof.

B. The lesson intentionally relates new concepts to students' prior skills and knowledge.

YES The lesson explicitly builds on students' prior skills and knowledge and students articulate these connections.

NO The lesson contains no meaningful connections to students' prior skills and knowledge.

C. The lesson intentionally targets the aspect(s) of rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.

Circle the aspect(s) of rigor targeted in this lesson:
 Conceptual understanding Procedural skill and fluency Application

YES The lesson explicitly targets the aspect(s) of rigor called for by the standard(s) being addressed.

NO The lesson targets aspects of rigor that are not appropriate for the standard(s) being addressed.

ENSURE THE WORK OF THE LESSON REFLECTS THE SKILLS REQUIRED BY U

Indicators

- A. The lesson focuses on the depth of grade-level cluster(s), grade-level content standard(s) or part(s) thereof.
-
-
-

- YES** The lesson focuses only on mathematics within the grade-level standards and fully reflects the depth of the grade-level cluster(s), grade-level content standard(s) or part(s) thereof.
- NO** The lesson focuses on mathematics outside the grade-level standards or superficially reflects the grade-level cluster(s), grade-level content standard(s) or part(s) thereof.
-
-
-

B. The lesson intentionally relates new concepts to students' prior skills and knowledge.

YES The lesson explicitly builds on students' prior skills and knowledge and students articulate these connections.

NO The lesson contains no meaningful connections to students' prior skills and knowledge.

C. The lesson intentionally targets the aspect(s) of rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.

Circle the aspect(s) of rigor targeted in this lesson:

Conceptual understanding Procedural skill and fluency Application

YES The lesson explicitly targets the aspect(s) of rigor called for by the standard(s) being addressed.

NO The lesson targets aspects of rigor that are not appropriate for the standard(s) being addressed.

Using

to Teach

Why Conceptual Understanding?

Does anybody teach students who...

- are scared of fractions?
- can't add/subtract fractions?
- can't multiply/divide fractions?
- can't add/subtract signed numbers?
- can't multiply/divide signed numbers?

What other arithmetic do your students have trouble with? Why do students struggle with these topics?

Procedural Skill and Fluency

Kyle Falting & Britton Kilpatrick
contact@universalachievement.com