Lesson 1: Construct an Equilateral Triangle

Classwork

Opening Exercise

Joe and Marty are in the park playing catch. Tony joins them, and the boys want to stand so that the distance between any two of them is the same. Where do they stand?

How do they figure this out precisely? What tool or tools could they use?

Fill in the blanks below as each term is discussed:

1. ________________________ The _______ between points A and B is the set consisting of A, B, and all points on the line AB between A and B.

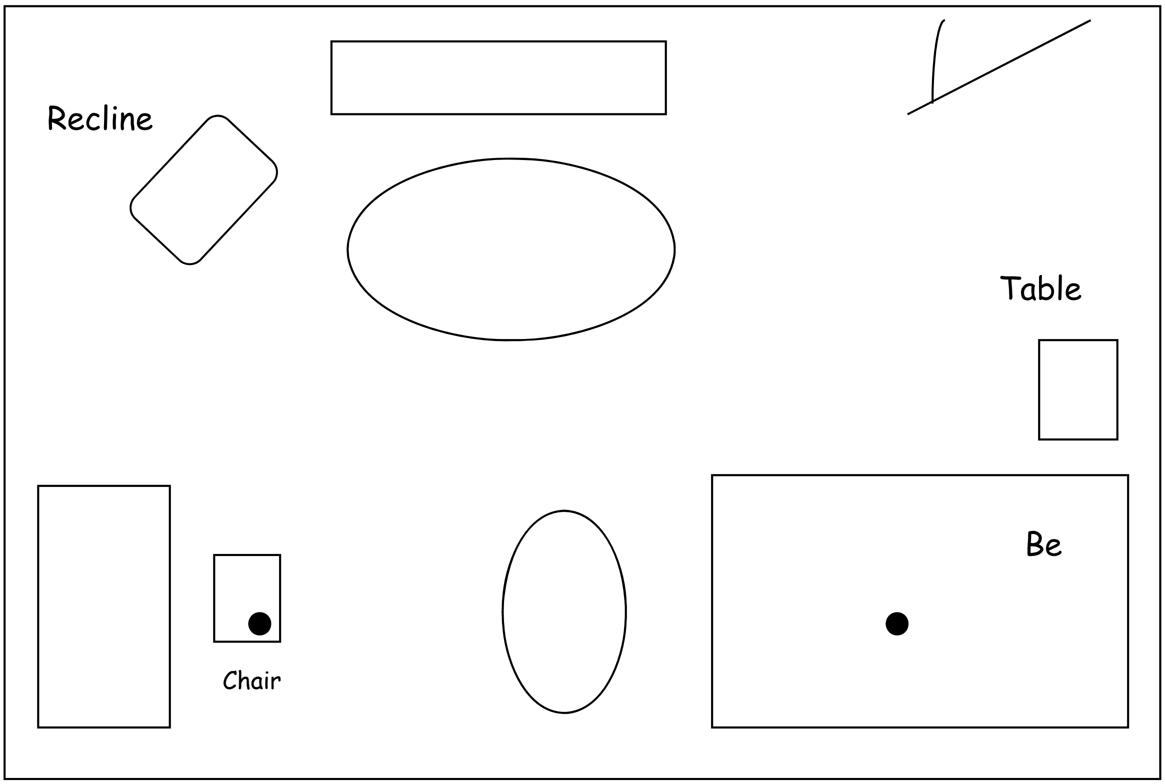
2. ________________________ A segment from the center of a circle to a point on the circle.

3. ________________________ Given a point C in the plane and a number $r > 0$, the _______ with center C and radius r is the set of all points in the plane that are distance r from point C.

Example 1: Sitting Cats

You will need a compass and a straightedge.

Margie has three cats. She has heard that cats in a room position themselves at equal distances from one another and wants to test that theory. Margie notices that Simon, her tabby cat, is in the center of her bed (at S), while JoJo, her Siamese, is lying on her desk chair (at J). If the theory is true, where will she find Mack, her calico cat? Use the scale drawing of Margie's room shown below, together with (only) a compass and straightedge. Place an M where Mack will be if the theory is true.



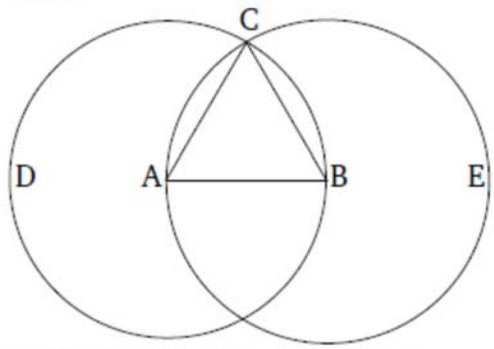
Mathematical Modeling Exercise: Euclid, Proposition 1

Let's see how Euclid approached this problem. Look at his first proposition, and compare his steps with yours.

In this margin, compare your steps with Euclid's.

Proposition 1

To construct an equilateral triangle on a given finite straight-line.



Let AB be the given finite straight-line.

So it is required to construct an equilateral triangle on the straight-line AB.

Let the circle BCD with center A and radius AB have been drawn [Post. 3], and again let the circle ACE with center B and radius BA have been drawn [Post. 3]. And let the straight-lines CA and CB have been joined from the point C, where the circles cut one another,† to the points A and B (respectively) [Post. 1].

And since the point A is the center of the circle CDB, AC is equal to AB [Def. 1.15]. Again, since the point B is the center of the circle CAE, BC is equal to BA [Def. 1.15]. But CA was also shown (to be) equal to AB. Thus, CA and CB are each equal to AB. But things equal to the same thing are also equal to one another [C.N. 1]. Thus, CA is also equal to CB. Thus, the three (straight-lines) CA, AB, and BC are equal to one another.

Thus, the triangle ABC is equilateral, and has been constructed on the given finite straight-line AB. (Which is) the very thing it was required to do.

Problem Set

1. Write a clear set of steps for the construction of an equilateral triangle. Use Euclid's Proposition 1 as a guide.

2. Suppose two circles are constructed using the following instructions:

   Draw circle: Center A, radius AB.

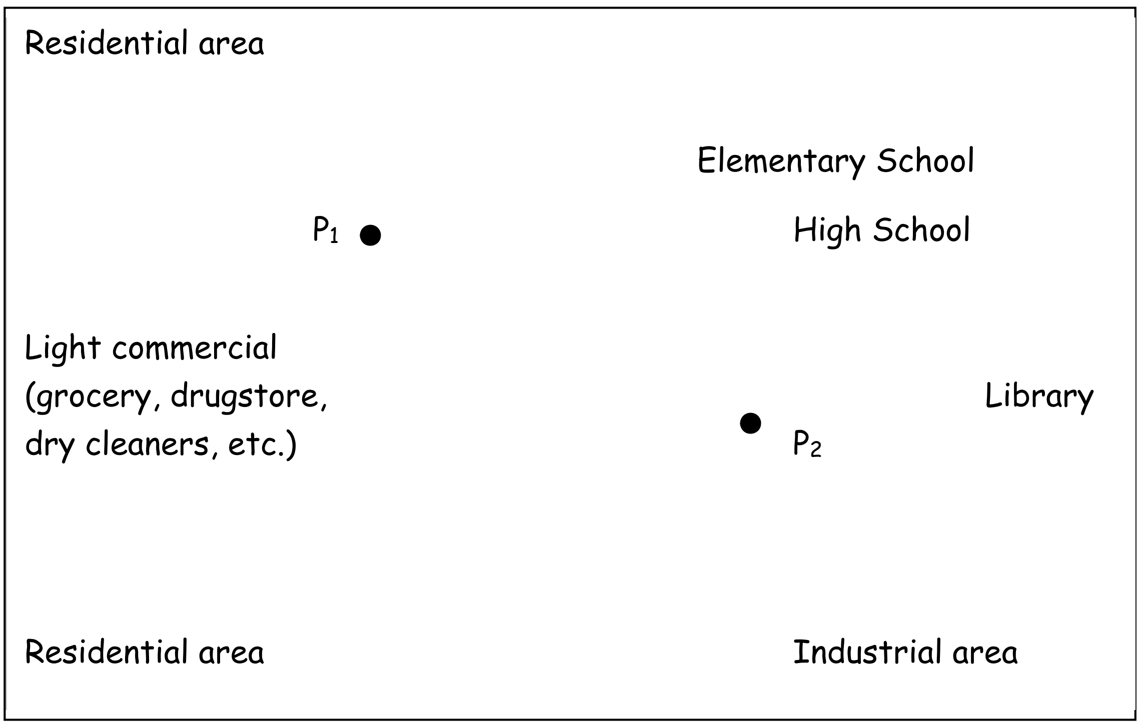
   Draw circle: Center C, radius CD.

   Under what conditions (in terms of distances AB, CD, AC) do the circles have

   a. One point in common?
   b. No points in common?
   c. Two points in common?
   d. More than two points in common? Why?

3. *You will need* a compass and straightedge.

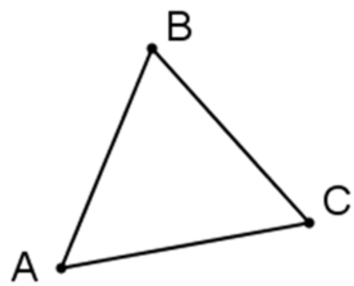
Cedar City boasts two city parks and is in the process of designing a third. The planning committee would like all three parks to be equidistant from one another to better serve the community. A sketch of the city appears below, with the centers of the existing parks labeled as P_1 and P_2. Identify two possible locations for the third park, and label them as P_{3a} and P_{3b} on the map. Clearly and precisely list the mathematical steps used to determine each of the two potential locations.



Lesson 2: Construct an Equilateral Triangle

Exit Ticket

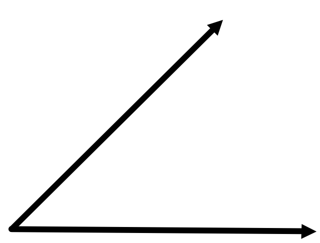
$\triangle ABC$ is shown below. Is it an equilateral triangle? Justify your response.



Mathematical Modeling Exercise 2: Investigate How to Copy an Angle

You will need a compass and a straightedge.

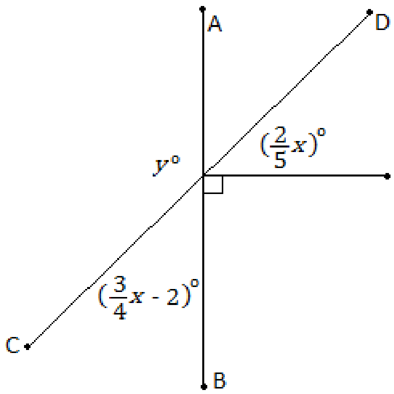
You and your partner will be provided with a list of steps (in random order) needed to copy an angle using a compass and straightedge. Your task is to place the steps in the correct order, then follow the steps to copy the angle below.



Steps needed (in correct order):

Lesson 6: Solve for Unknown Angles—Angles and Lines at a Point

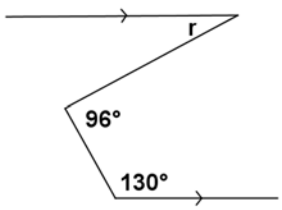
Exercise 10



x = ________ y = ________

Lesson 7: Solve for Unknown Angles—Transversals

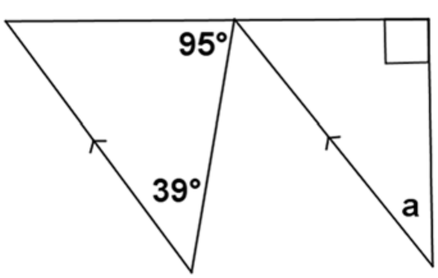
Exercise 10



m∠r = ____________________

Lesson 8: Solve for Unknown Angles—Angles in a Triangle

Problem Set #1

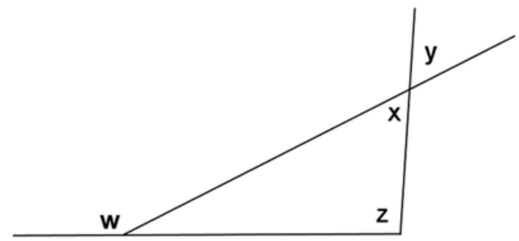


$m\angle a$ = ________

Lesson 9: Unknown Angle Proofs—Writing Proofs

Exercise 2b

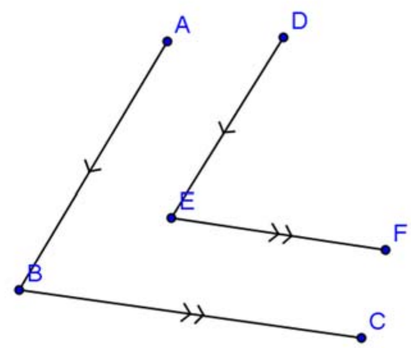
Given the diagram to the right, prove that $\mathrm{m}\angle w = \mathrm{m}\angle y + \mathrm{m}\angle z$.



Lesson 10: Unknown Angle Proofs—Proofs with Constructions

Problem Set

1. In the figure to the right, $\overline{AB} \parallel \overline{DE}$ and $\overline{BC} \parallel \overline{EF}$.
   Prove that $\mathrm{m}\angle ABC = \mathrm{m}\angle DEF$.



Lesson 11: Unknown Angle Proofs – Proofs of Known Facts

Problem Set

2. A theorem states that *in a plane, if a line is perpendicular to one of two parallel lines and intersects the other, then it is perpendicular to the other of the two parallel lines.*

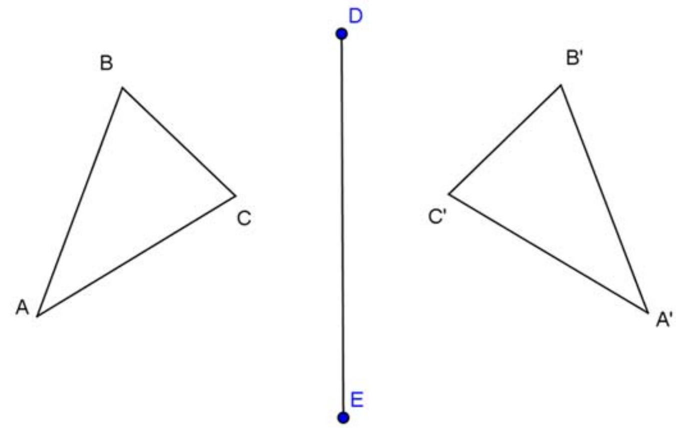
   Prove this theorem. (a) Construct and label an appropriate figure, (b) state the given information and the theorem to be proven, and (c) list the necessary steps to demonstrate the proof.

Lesson 14: Reflections

Exploratory Challenge

$\triangle ABC$ is reflected across $\overline{DE}$ and maps onto $\triangle A'B'C'$.

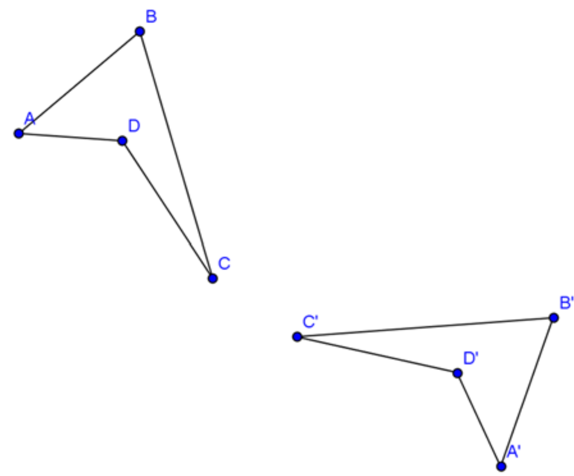
Use your compass and straightedge to construct the perpendicular bisector of each of the segments connecting A to A', B to B', and C to C'. What do you notice about these perpendicular bisectors?



Example 1

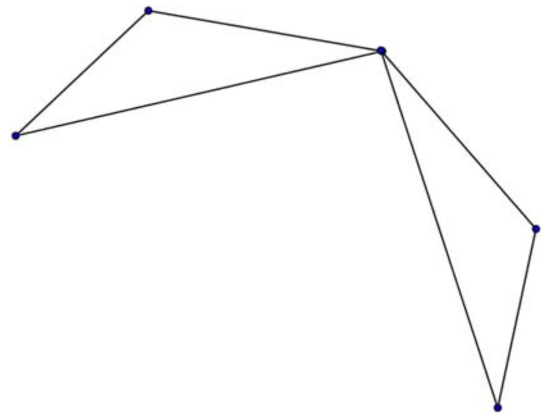
Construct the segment that represents the line of reflection for quadrilateral $ABCD$ and its image $A'B'C'D'$.

What is true about each point on $ABCD$ and its corresponding point on $A'B'C'D'$?

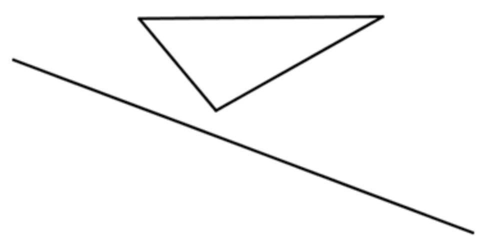


Exit Ticket

1. Construct the line of reflection for the figures.



2. Reflect the given figure across the line of reflection provided.



Lesson 15: Rotations, Reflections, and Symmetry

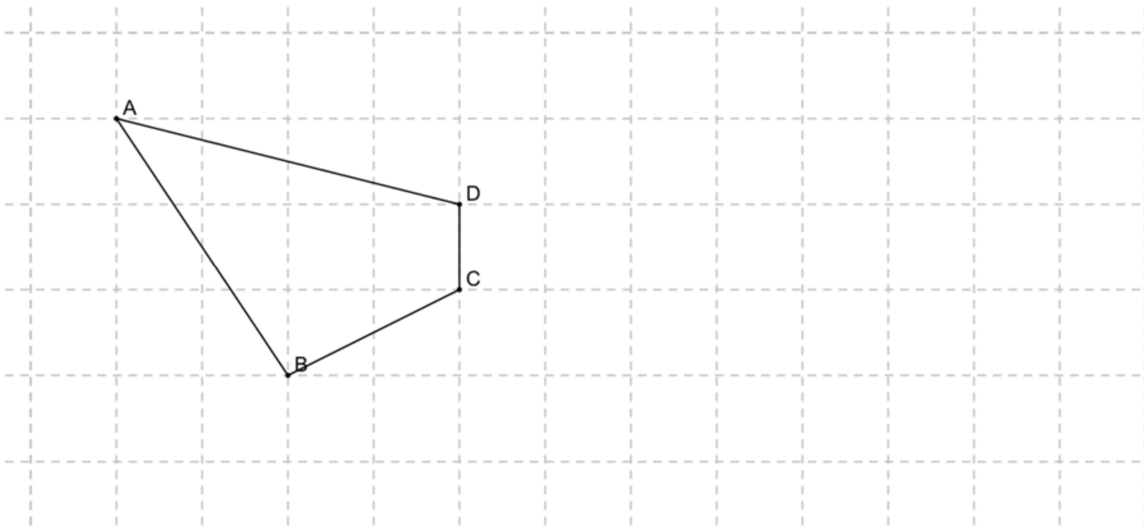
Exit Ticket

What is the relationship between a rotation and reflection? Sketch a diagram that supports your explanation.

Lesson 16: Translations

Exit Ticket

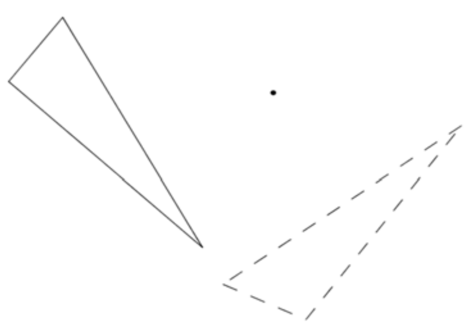
Translate the image one unit down and three units right. Draw the vector that defines the translation.



Lesson 17: Characterize Points on a Perpendicular Bisector

Example 3

Find the center of rotation for the transformation below. How are perpendicular bisectors a major part of finding the center of rotation? Why are they essential?



Lesson 20: Applications of Congruence in Terms of Rigid Motions

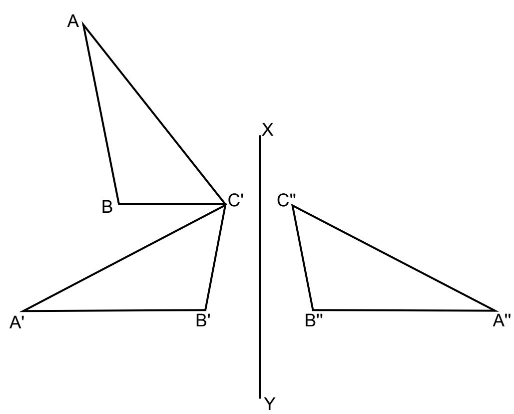
Problem Set #3

Give an example of two triangles and a correspondence between their vertices such that (a) one angle in the first is congruent to the corresponding angle in the second and (b) two sides of the first are congruent to the corresponding sides of the second, but (c) the triangles themselves are not congruent.

Lesson 21: Correspondence and Transformations

Exit Ticket

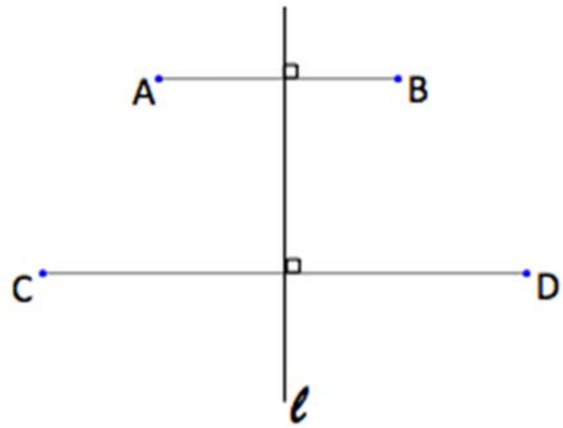
Complete the table based on the series of rigid motions performed on $\triangle ABC$ below.



Sequence of rigid motions (2)	
Composition in function notation	
Sequence of corresponding sides	
Sequence of corresponding angles	
Triangle congruence statement	

Example from Mid-Module Assessment

6. Given in the figure below, line l is the perpendicular bisector of $\overline{AB}$ and of $\overline{CD}$.



a. Show $\overline{AC} \cong \overline{BD}$ using rigid motions.

b. Show $\angle ACD \cong \angle BDC$.

c. Show $\overline{AB} \parallel \overline{CD}$.

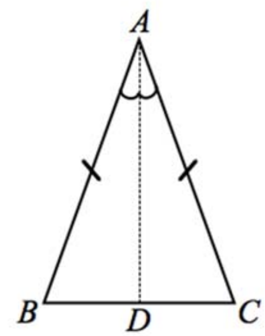
Lesson 23: Base Angles of Isosceles Triangles

Discussion

Prove Base Angles of an Isosceles are Congruent: Transformations

Given: Isosceles $\triangle ABC$, with $AB = AC$.

Prove: $\angle B \cong \angle C$.



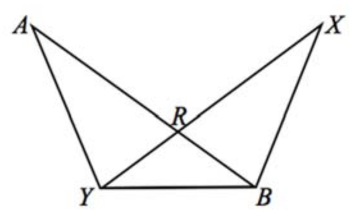
Construction: Draw the angle bisector $\overrightarrow{AD}$ of $\angle A$, where D is the intersection of the bisector and $\overline{BC}$. We need to show that rigid motions will map point B to point C and point C to point B.

Lesson 24: Congruence Criteria for Triangles—ASA and SSS

Exercises

Based on the information provided, determine whether a congruence exists between triangles. If a congruence exists between triangles or if multiple congruencies exist, state the congruencies and the criteria used to determine them.

3. Given: $RY = RB,\ AR = XR$.

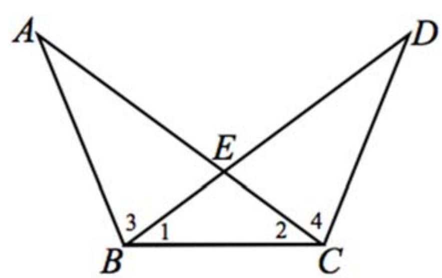


Lesson 26: Triangle Congruency Proofs

Exercises

5. Given: $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$.

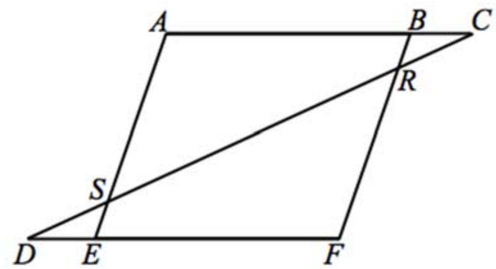
   Prove: $\overline{AC} \cong \overline{BD}$.



Lesson 28: Properties of Parallelograms

Problem Set

5. Given: Parallelogram $ABFE$, $CR = DS$.
   Prove: $BR = SE$.



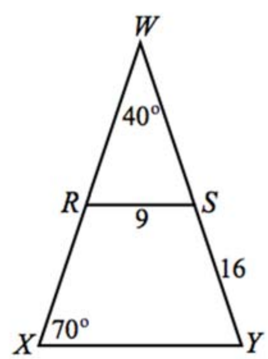
Lesson 29: Special Lines in Triangles

Exit Ticket

Use the properties of midsegments to solve for the unknown value in each question.

R and S are the midpoints of $\overline{XW}$ and $\overline{WY}$, respectively.

What is the perimeter of $\triangle WXY$? ___________________



Lesson 30: Special Lines in Triangles

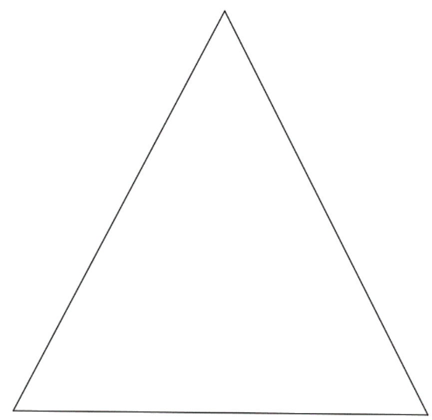
Problem Set

Ty is building a model of a hang glider using the template below. To place his supports accurately, Ty needs to locate the center of gravity on his model.

6. Use your compass and straightedge to locate the center of gravity on Ty's model.

7. Explain what the center of gravity represents on Ty's model.

8. Describe the relationship between the longer and shorter sections of the line segments you drew as you located the center of gravity.

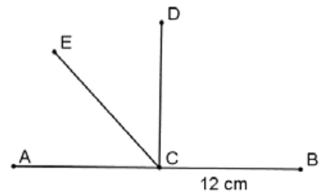


Lesson 31: Construct a Square and a Nine-Point Circle

Exit Ticket

Construct a square $ABCD$ and a square $AXYZ$ so that $\overline{AB}$ contains X and $\overline{AD}$ contains Z.

Lesson 33: Review of the Assumptions

The perpendicular bisector of a segment is the line that passes through the midpoint of a line segment and is perpendicular to the line segment.	In the diagram below, $\overline{DC}$ is the $\perp$ bisector of $\overline{AB}$, and $\overline{CE}$ is the angle bisector of $\angle ACD$. Find the measures of $\overline{AC}$ and $\angle ECD$. 	

Lesson 34: Review of the Assumptions

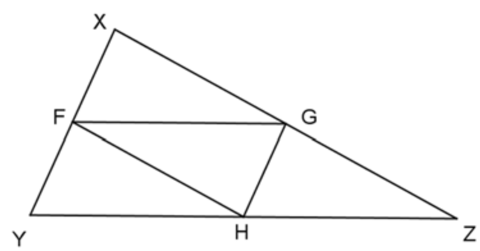
Problem Set

3. $XY = 12$

   $XZ = 20$

   $ZY = 24$

   F, G, and H are midpoints of the sides on which they are located. Find the perimeter of $\triangle FGH$. Justify your solution.



Geometry Module 1: End-of-Module Assessment

1. Each of the illustrations on the next page shows in black a plane figure consisting of the letters F, R, E, and D evenly spaced and arranged in a row. In each illustration, an alteration of the black figure is shown in gray. In some of the illustrations, the gray figure is obtained from the black figure by a geometric transformation consisting of a single rotation. In others, this is not the case.

    a. Which illustrations show a single rotation?

    b. Some of the illustrations are not rotations or even a sequence of rigid transformations. Select one such illustration and use it to explain why it is not a sequence of rigid transformations.

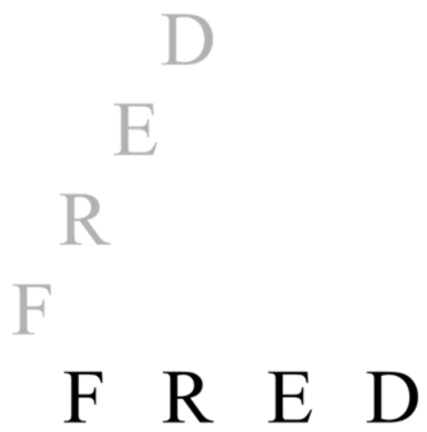


Illustration 1

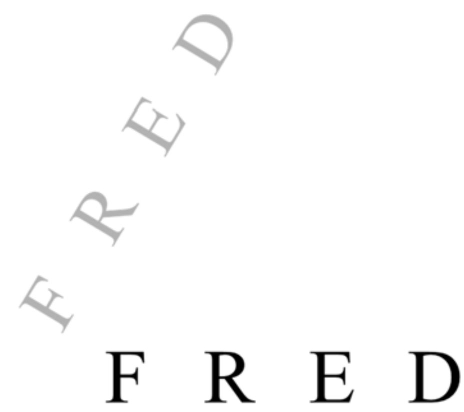


Illustration 2

Illustration 3

Illustration 4

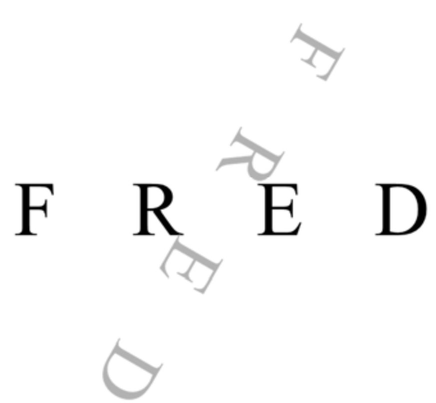


Illustration 5

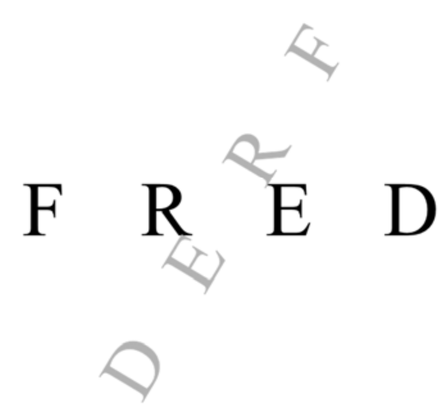
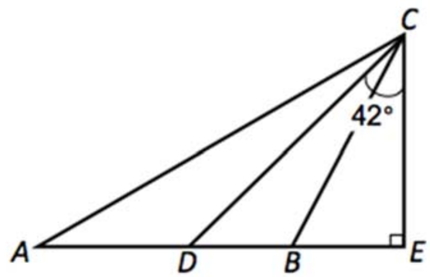


Illustration 6

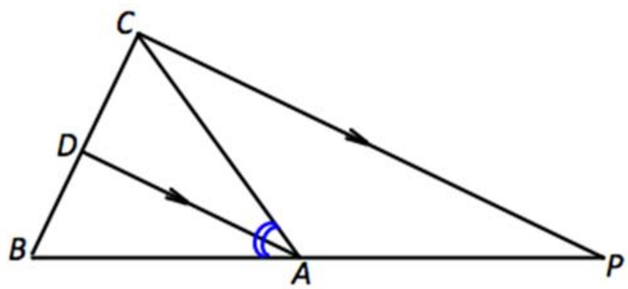
2. In the figure below, $\overline{CD}$ bisects $\angle ACB$, $AB = BC$, $\angle BEC = 90°$, and $\angle DCE = 42°$.

Find the measure of angle $\angle A$.

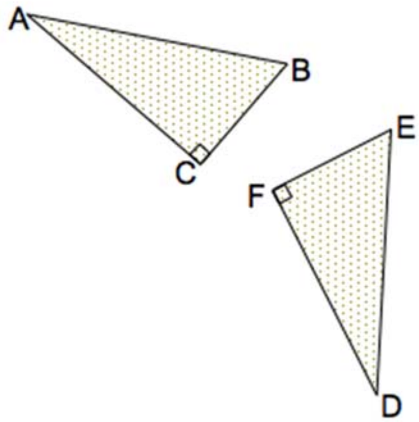


3. In the figure below, $\overline{AD}$ is the angle bisector of $\angle BAC$. $\overline{BAP}$ and $\overline{BDC}$ are straight lines, and $\overline{AD} \parallel \overline{PC}$.

Prove that $AP = AC$.



4. The triangles ΔABC and ΔDEF in the figure below are such that $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$.

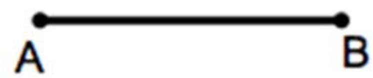


a. Which criteria for triangle congruence (ASA, SAS, SSS) implies that $\Delta ABC \cong \Delta DEF$?

b. Describe a sequence of rigid transformations that shows $\Delta ABC \cong \Delta DEF$.

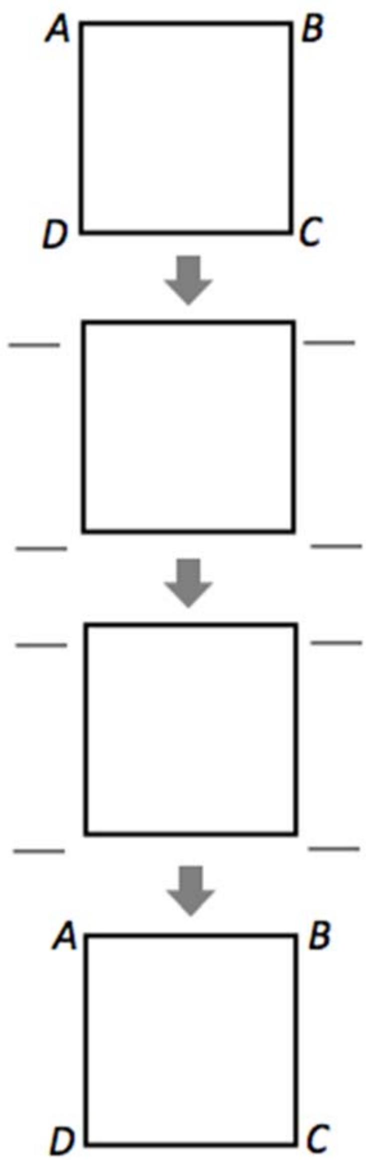
5.

a. Construct a square $ABCD$ with side $\overline{AB}$. List the steps of the construction.



b. Three rigid motions are to be performed on square $ABCD$. The first rigid motion is the reflection through line $\overline{BD}$. The second rigid motion is a $90°$ clockwise rotation around the center of the square.

Describe the third rigid motion that will ultimately map $ABCD$ back to its original position. Label the image of each rigid motion A, B, C, and D in the blanks provided.



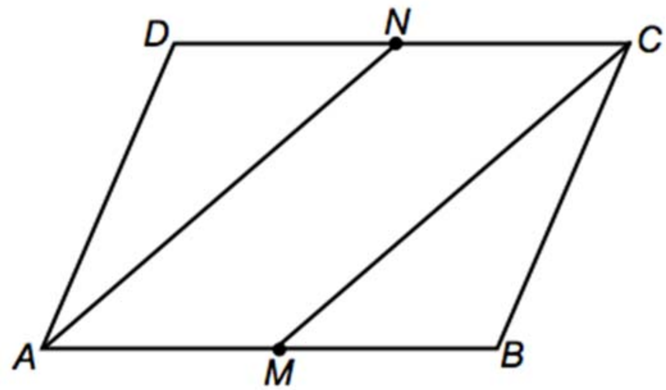
Rigid Motion 1 Description: Reflection through line $\overline{BD}$

Rigid Motion 2 Description: $90°$ clockwise rotation around the center of the square.

Rigid Motion 3

Description: ____________________________

6. Suppose that $ABCD$ is a parallelogram and that M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$, respectively. Prove that $AMCN$ is a parallelogram.



Common Core Mathematics High School Acronyms

- Number & Quantity
  - N-RN – The Real Number System
  - N-Q – Quantities
  - N-CN – The Complex Number System
  - N-VM – Vector and Matrix Quantities
- Algebra
  - A-SSE – Seeing Structure in Equations
  - A-APR – Arithmetic with Polynomials and Rational Expressions
  - A-CED – Creating Equations
  - A-REI – Reasoning with Equations and Inequalities
- Functions
  - F-IF – Interpreting Functions
  - F-BF – Building Functions
  - F-LE – Linear, Quadratic and Exponential Models
  - F- TF – Trigonometric Functions
- Geometry
  - G-CO - Congruence
  - G-SRT – Similarity, Right Triangles, & Trigonometry
  - G-C – Circles
  - G-GPE – Expressing Geometric Properties with Equations
  - G-GMD – Geometric Measurement & Dimension
  - G-MG – Modeling with Geometry
- Statistics and Probability
  - S-ID – Interpreting Categorical & Quantitative Data
  - S-IC – Making Inferences & Justifying Conclusions
  - S-CP – Conditional Probability and Rules of Probability
  - S-MD – Using Probability to Make Decisions

Mathematics - High School Geometry: Introduction

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations.

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Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Geometry Overview

Congruence

• Experiment with transformations in the plane

• Understand congruence in terms of rigid motions

• Prove geometric theorems

• Make geometric constructions

Similarity, Right Triangles, and Trigonometry

• Understand similarity in terms of similarity transformations

• Prove theorems involving similarity

• Define trigonometric ratios and solve problems involving right triangles

• Apply trigonometry to general triangles

Expressing Geometric Properties with Equations

• Translate between the geometric description and the equation for a conic section

• Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension

• Explain volume formulas and use them to solve problems

• Visualize relationships between twodimensional and three-dimensional objects

Modeling with Geometry

• Apply geometric concepts in modeling situations

Circles

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Congruence G-CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, & Trigonometry G-SRT

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:

   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Apply trigonometry to general triangles

9. (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles G-C

Understand and apply theorems about circles

1. Prove that all circles are similar.

2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

2. Derive the equation of a parabola given a focus and directrix.

3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

Geometric Measurement & Dimension G-GMD

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

Visualize relationships between two-dimensional and three-dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry G-MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

Mathematics: Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.