Lesson 1: An Experience in Relationships as Measuring Rate

Classwork

Example 1: How fast is our class?

Record the results from the paper passing exercise in the table below.

Trial	Number of Papers Passed	Time (in seconds)	Ratio of Number of Papers Passed to Time	Rate	Unit Rate
1					
2					
3					
4					

Key Terms from Grade 6 Ratios and Unit Rates

A ratio is an ordered pair of non-negative numbers, which are not both zero. The ratio is denoted $A: B$ to indicate the order of the numbers: the number A is first and the number B is second.

Two ratios $A: B$ and $C: D$ are **equivalent ratios** if there is a positive number, c, such that $C = cA$ and $D = cB$.

A ratio of two quantities, such as 5 miles per 2 hours, can be written as another quantity called a **rate**.

The numerical part of the rate is called the **unit rate** and is simply the value of the ratio, in this case 2.5. This means that in 1 hour the car travels 2.5 miles. The **unit** for the rate is miles/hour, read miles per hour.

Example 2: Our Class by Gender

	Number of boys	Number of girls	Ratio of boys to girls
Class 1			
Class 2			
Whole 7th grade			

Create a pair of equivalent ratios by making a comparison of quantities discussed in this example.

Exercise 1: Which is the better buy?

Value-Mart is advertising a Back-to-School sale on pencils. A pack of 30 sells for $7.97, whereas a 12-pack of the same brand costs $ 4.77. Which is the better buy? How do you know?

Lesson Summary:

Unit Rate is often a useful means for comparing ratios and their associated rates when measured in different units. The unit rate allows us to compare varying sizes of quantities by examining the number of units of one quantity per one unit of the second quantity. This value of the ratio is the unit rate.

Exit Ticket

Watch the video clip of Tillman the English bulldog, the Guinness World Record holder for Fastest Dog on a Skateboard.

1. At the conclusion of the video, your classmate takes out his or her calculator and says, "Wow that was amazing! That means the dog went about 5 meters in 1 second!" Is your classmate correct and how do you know?

2. After seeing this video, another dog owner trained his dog, Lightning, to try to break Tillman's skateboarding record. Lightning's fastest recorded time was on a 75-meter stretch where it took him 15.5 seconds. Based on this data, did Lightning break Tillman's record for fastest dog on a skateboard? Explain how you know.

Problem Set

1. Find each rate and unit rate.
   a. 420 miles in 7 hours
   b. 360 customers in 30 days
   c. 40 meters in 16 seconds
   d. $\$7.96$ for 5 pounds

2. Write three ratios that are equivalent to the one given: The ratio of right-handed students to left-handed students is $18:4$.

3. Mr. Rowley has 16 homework papers and 14 exit tickets to return. Ms. Rivera has 64 homework papers and 60 exit tickets to return. For each teacher, write a ratio to represent the number of homework papers to number of exit tickets they have to return. Are the ratios equivalent? Explain.

4. Jonathan's parents told him that for every 5 hours of homework or reading he completes, he will be able to play 3 hours of video games. His friend Lucas's parents told their son that he can play 30 minutes for every hour of homework or reading time he completes. If both boys spend the same amount of time on homework and reading this week, which boy gets more time playing video games? How do you know?

5. Of the 30 girls who tried out for the lacrosse team at Euclid Middle School, 12 were selected. Of the 40 boys who tried out, 16 were selected. Are the ratios of the number of students on the team to the number of students trying out the same for both boys and girls? How do you know?

6. Devon is trying to find the unit price on a 6-pack of drinks on sale for $\$2.99$. His sister says that at that price, each drink would cost just over $\$2.00$. Is she correct, and how do you know? If she is not, how would Devon's sister find the correct price?

7. Each year Lizzie's school purchases student agenda books, which are sold in the school store. This year, the school purchased 350 books at a cost of $\$1{,}137.50$. If the school would like to make a profit of $\$1{,}500$ to help pay for field trips and school activities, what is the least amount they can charge for each agenda book? Explain how you found your answer.

Lesson 2: Proportional Relationships

Example 1: Pay by the Ounce Frozen Yogurt!

A new self-serve frozen yogurt store opened this summer that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed their dish and this is what they found.

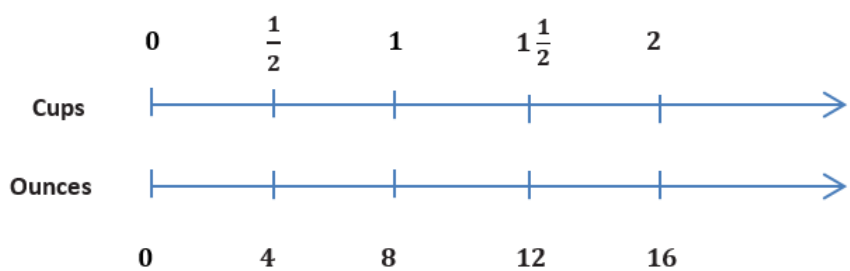
Determine if the cost is proportional to the weight.

Weight (ounces)	12.5	10	5	8
Cost ($)	5	4	2	3.20

The cost ____ the weight.

Example 2: A Cooking Cheat Sheet!

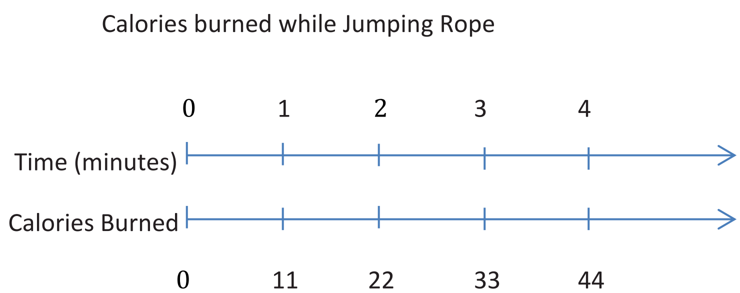
In the back of a recipe book, a diagram provides easy conversions to use while cooking.



The ounces ____the cups.

Exercise 1

During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting how many calories (on average) would be burned by completing the activity.



a. Is the number of Calories burned proportional to time? How do you know?

b. If Jose jumped rope for 6.5 minutes, how many calories would he expect to burn?

Lesson 3: Identifying Proportional and Non-Proportional Relationships in Tables

Example

You have been hired by your neighbors to babysit their children Friday night. You are paid $8 per hour. Complete the table relating your pay to the number of hours you worked.

Hours Worked	Pay
1	
2	
3	
4	
$4\frac{1}{2}$	
5	
6	
6.5	

Based on the table above, is the pay proportional to the hours worked? How do you know?

Exercises

For the following exercises, determine if y is proportional to x. Justify your answer.

1. The table below represents the relationship of the amount of snowfall (in inches) in 5 counties to the amount of time (in hours) of a recent winter storm.

x Snow Fall (in)	y Time (hrs)
2	10
6	12
8	16
2.5	5
7	14

2. The table below shows the relationship between cost of renting a movie (in dollars) to the number of days the movie is rented.

x Number of Days	y Cost
6	2
9	3
24	8
3	1

Lesson 4: Identifying Proportional and Non-Proportional Relationships in Tables

Exercise 2

Mark recently moved to a new state. During the first month he visited five state parks. Each month after he visited two more. Complete the table below and use the results to determine if the number of parks visited is proportional to the number of months.

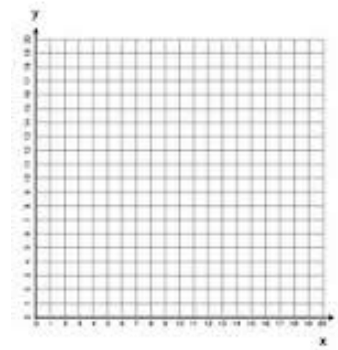
Number of Months	Number of State Parks
1	
2	
3	
	23

Lesson 5: Identifying Proportional and Non-Proportional Relationships in Graphs

Example 1: From a table to graph

Use the ratio provided, create a table that shows money received is proportional to the number of candy bars sold. Plot the points in your table and on the grid.

x	y
2	3



Lesson 7: Unit Rate as the Constant of Proportionality

Example 2: You Need WHAT???

Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needs 3 cookies for each of the 96 students in 7th grade. Unfortunately, he needs the cookies the very next day! Brandon and his mother determined that they can fit 36 cookies on two cookie sheets.

a. Is the number of cookies proportional to the number of cookie sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies baked.

Table:

The unit rate of $\frac{y}{x}$ is __________.

Constant of Proportionality:

Explain the meaning of the constant of proportionality in this problem:

b. It takes 2 hours to bake 8 sheets of cookies. If Brandon and his mother begin baking at $4:00$ p.m., when will they finish baking the cookies?

Exit Ticket

Susan and John are buying cold drinks for a neighborhood picnic. Each person is expected to drink one can of soda. Susan says that if you multiply the unit price for a can of soda by the number of people attending the picnic, you will be able to determine the total cost of the soda. John says that if you divide the cost of a 12-pack of soda by the number of sodas, you will be able to determine the total cost of the sodas. Who is right and why?

Lesson 8: Representing Proportional Relationships with Equations

Classwork:

Example 1: Do We have Enough Gas to Make it to the Gas Station?

Your mother has accelerated onto the interstate beginning a long road trip and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 26 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Mother's Gas Record

Gallons	Miles driven
8	224
10	280
4	112

a. Find the constant of proportionality and explain what it represents in this situation.

b. Write equation(s) that will relate the miles driven to the number of gallons of gas.

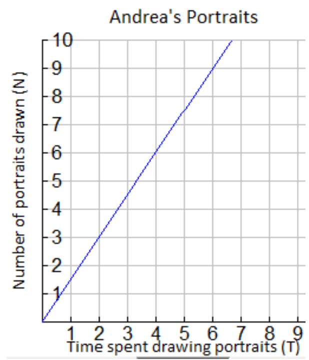
c. Knowing that there is a half-gallon left in the gas tank when the light comes on, will she make it to the nearest gas station? Explain why or why not.

d. Using the equation found in part (b), determine how far your mother can travel on 18 gallons of gas. Solve the problem in two ways: once using the constant of proportionality and once using an equation.

e. Using the constant of proportionality, and then using the equation found in part (b), determine how many gallons of gas would be needed to travel 750 miles.

Example 2: Andrea's Portraits

Andrea is a street artist in New Orleans. She draws caricatures (cartoon-like portraits) of tourists. People have their portrait drawn and then come back later to pick it up from her. The graph below shows the relationship between the number of portraits she draws and the amount of time in hours she needs to draw the portraits.



a. Write several ordered pairs from the graph and explain what each ordered pair means in the context of this graph.

b. Write several equations that would relate the number of portraits drawn to the time spent drawing the portraits.

c. Determine the constant of proportionality and explain what it means in this situation.

Problem Set

Write an equation that will model the proportional relationship given in each real-world situation.

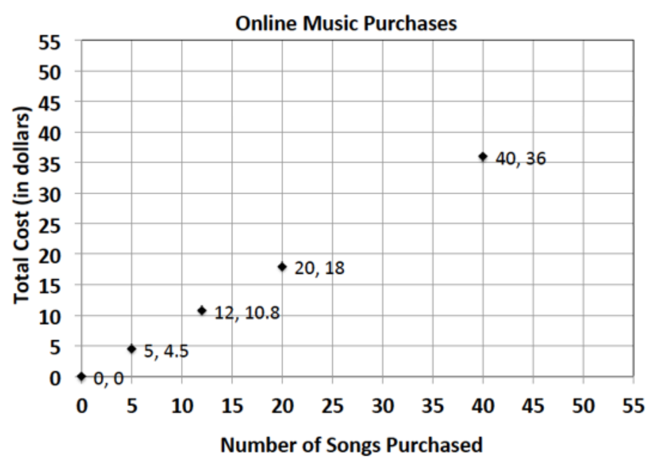
1. There are 3 cans that store 9 tennis balls. Consider the number of balls per can.
   a. Find the constant of proportionality for this situation.
   b. Write an equation to represent the relationship.

2. In 25 minutes Li can run 10 laps around the track. Determine the number of laps she can run per minute.
   a. Find the constant of proportionality in this situation.
   b. Write an equation to represent the relationship.

3. Jennifer is shopping with her mom. They pay $\$2$ per pound for tomatoes at the vegetable stand.
   a. Find the constant of proportionality in this situation.
   b. Write an equation to represent the relationship.

4. It cost $\$5$ to send 6 packages through a certain shipping company. Consider the number of packages per dollar.
   a. Find the constant of proportionality for this situation.
   b. Write an equation to represent the relationship.

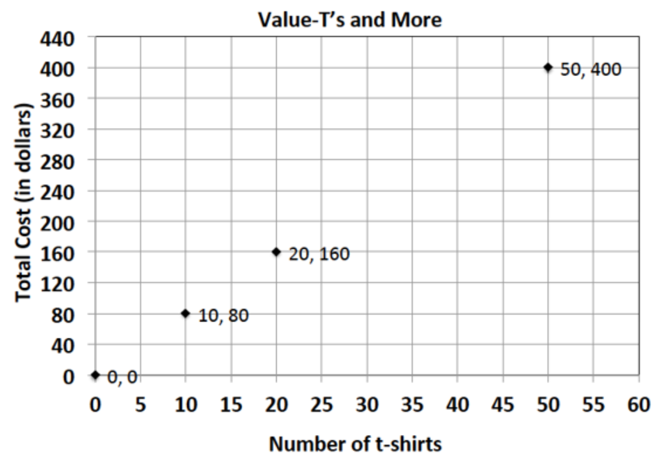
5. On average, Susan downloads 60 songs per month. An online music vendor sells package prices for songs that can be downloaded on to personal digital devices. The graph below shows the package prices for the most popular promotions. Susan wants to know if she should buy her music from this company or pay a flat fee of $\$58.00$ for the month offered by another company. Which is the better buy?
   a. Find the constant of proportionality for this situation.
   b. Write an equation to represent the relationship.
   c. Use your equation to find the answer to Susan's question above. Justify your answer with mathematical evidence and a written explanation.



6. Allison's middle school team has designed t-shirts containing their team name and color. Allison and her friend Nicole have volunteered to call local stores to get an estimate on the total cost of purchasing t-shirts. Print-o-Rama charges a set-up fee, as well as a fixed amount for each shirt ordered. The total cost is shown below for the given number of shirts. Value T's and More charges $8 per shirt. Which company should they use?

Print-o-Rama

# shirts	Total cost
10	95
25	
50	375
75	
100	



a. Does either pricing model represent a proportional relationship between quantity of t-shirts and total cost? Explain.

b. Write an equation relating cost and shirts for Value T's and More.

c. What is the constant of proportionality ValueTt's and More? What does it represent?

d. How much is Print-o-Rama's set up fee?

e. Write a proposal to your teacher indicating which company the team should use. Be sure to support your choice. Determine the number of shirts that you need for your team.

Lesson 9: Representing Proportional Relationships with Equations

Exit Ticket

Oscar and Maria each wrote an equation that they felt represented the proportional relationship between distance in km and distance in miles. One entry in the table paired 150 km with 95 miles. If k represents the number of kilometers and m represents the number of miles, who wrote the correct equation that would relate miles to kilometers. Explain why.

Oscar wrote the equation $k = 1.6m$, and he said that the rate $\frac{1.6}{1}$ represents kilometers per mile.

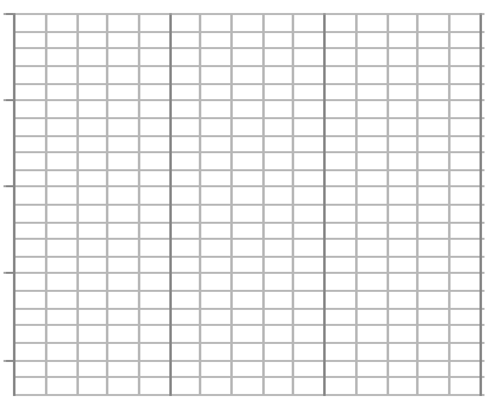
Maria wrote the equation $k = 0.625m$ as her equation, and she said that 0.625 represents kilometers per mile.

Lesson 10: Interpreting Graphs of Proportional Relationships

Example 1

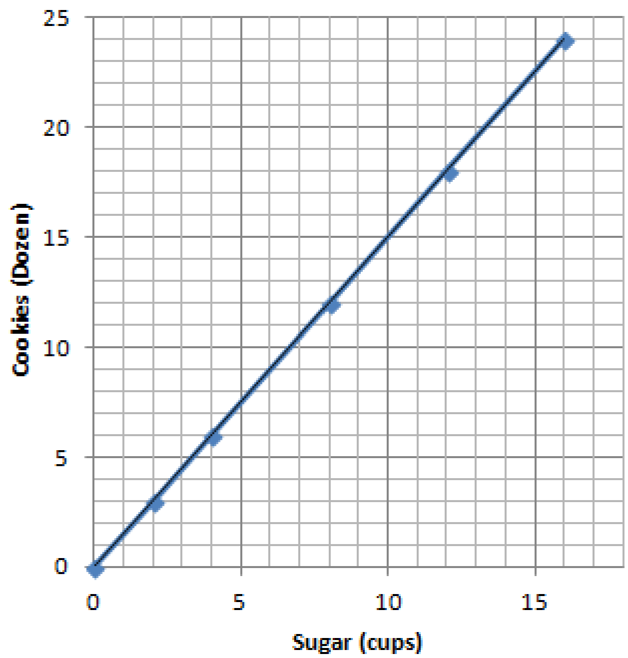
Grandma's Special Chocolate Chip Cookie recipe, which yields 4 dozen cookies, calls for 3 cups of flour.

Using this information, complete the chart:

Create a table comparing the amount of flour used to the amount of cookies.	Is the number of cookies proportional to the amount of flour used? Explain why or why not.	What is the unit rate of cookies to flour $(\frac{y}{x})$ and what is the meaning in the context of the problem?
Model the relationship on a graph. 	Does the graph show the two quantities being proportional to each other? Explain.	Write an equation that can be used to represent the relationship.

Example 2

Below is a graph modeling the amount of sugar required to make Grandma's Chocolate Chip Cookies.



a. Record the coordinates from the graph in a table. What do these ordered pairs represent?

b. Grandma has 1 remaining cup of sugar. How many dozen cookies will she be able to make? Plot the point on the graph above.

c. How many dozen cookies can grandma make if she has no sugar? Can you graph this on the coordinate plane provided above? What do we call this point?

Lesson 11: Ratios of Fractions and Their Unit Rates

Example 1: Who is Faster?

During their last workout, Izzy ran $2\frac{1}{4}$ miles in 15 minutes and her friend Julia ran $3\frac{3}{4}$ miles in 25 minutes. Each girl thought she was the faster runner. Based on their last run, which girl is correct? Use any approach to find the solution.

Lesson 12: Ratios of Fractions and Their Unit Rates

Example 1: Time to Remodel

You have decided to remodel your bathroom and install a tile floor. The bathroom is in the shape of a rectangle and the floor measures 14 feet, 8 inches long by 5 feet, 6 inches wide. The tiles you want to use cost $\$5$ each, and each tile covers $4\frac{2}{3}$ square feet. If you have $\$100$ to spend, do you have enough money to complete the project?

Make a Plan: Complete the chart to identify the necessary steps in the plan and find a solution.

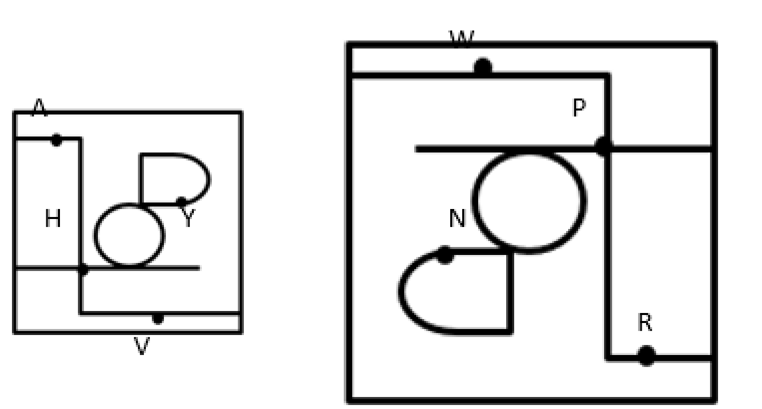
What I Know	What I Want to Find	How to Find it

Compare your plan with a partner. Using your plans, work together to determine how much money you will need to complete the project and if you have enough money.

Lesson 16: Relating Scale Drawings to Ratios and Rates

Example 2

Derek's family took a day trip to a modern public garden. Derek looked at his map of the park that was a reduction of the map located at the garden entrance. The dots represent the placement of rare plants. The diagram below is the top-view as Derek held his map while looking at the posted map.

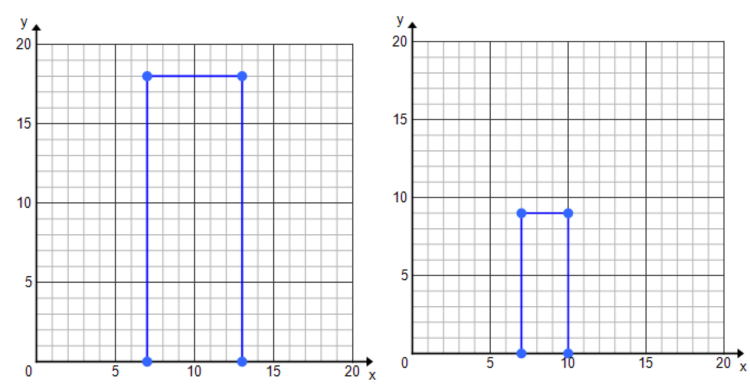


What are the corresponding points of the scale drawings of the maps?

Point A to ________ Point V to __________ Point H to __________ Point Y to __________

Example 3

Celeste drew an outline of a building for a diagram she was making and then drew a second one mimicking her original drawing. Label the vertices and fill in the table.



Lengths of the original drawing		
Lengths of the second drawing		

Notes:

Lesson 17: The Scale Factor for a Scale Drawing

Example 2

Use a Scale Factor of 3 to create a scale drawing of the picture below.

Picture of the Flag of________________**:** Sketch and notes:

Exercise 2

Scale Factor: $\frac{1}{2}$

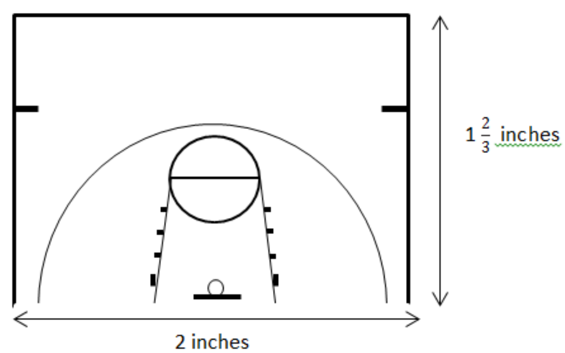
Picture of the Flag of_____________: Sketch and notes:

Lesson 18: Computing Actual Length from a Scale Drawing

Example 1: Basketball at Recess?

Vincent proposes an idea to the Student Government to install a basketball hoop along with a court marked with all the shooting lines and boundary lines at his school for students to use at recess. He presents a plan to install a half-court design as shown below. After checking with school administration, he is told it will be approved if it will fit on the empty lot that measures 25 feet by 75 feet on the school property. Will the lot be big enough for the court he planned? Explain.

Scale Drawing: 1 inch on drawing corresponds to 15 feet of actual length



Exit Ticket

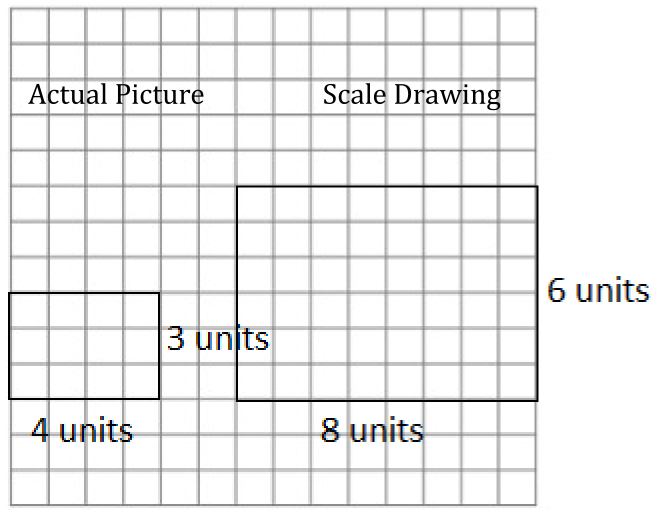
A drawing of a surfboard in a catalog shows its length as $8\frac{4}{9}$ inches. Find the actual length of the surfboard if $\frac{1}{2}$ inch length on the drawing corresponds to $\frac{3}{8}$ foot of actual length.

Lesson 19: Computing Actual Areas from a Scale Drawing

Examples 1–3: Exploring Area Relationships

Use the diagrams below to find the scale factor and then find the area of each figure.

Example 1



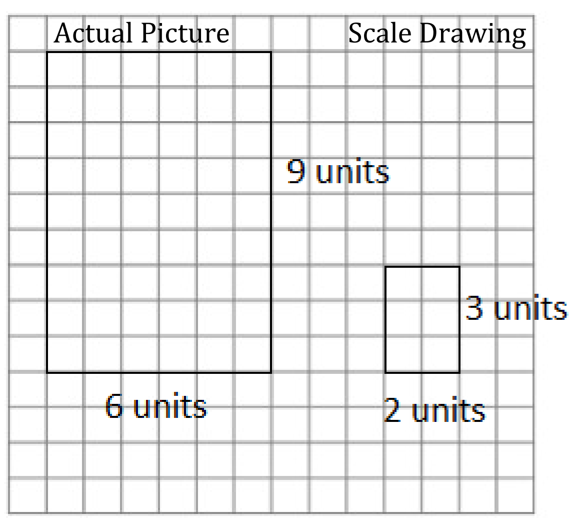
Scale factor: _________

Actual Area = _______________

Scale Drawing Area = _________________

Value of the Ratio of the Scale Drawing Area to the Actual Area: _________

Example 2



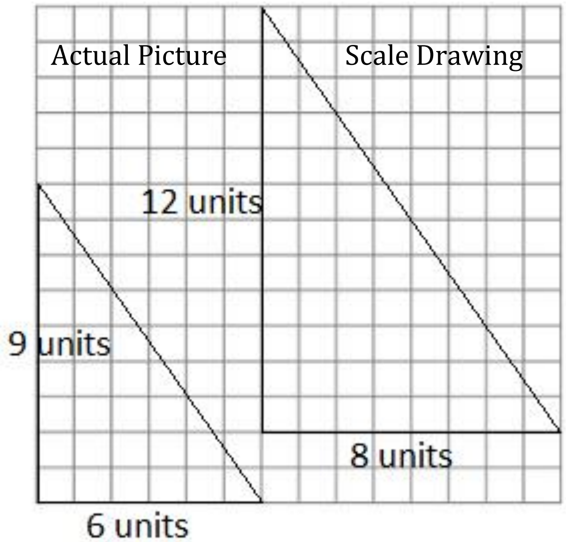
Scale factor: _________

Actual Area = _______________

Scale Drawing Area = _________________

Value of the Ratio of the Scale Drawing Area to the Actual Area: _________

Example 3



Scale factor: _________

Actual Area = ______________

Scale Drawing Area = ________________

Value of the Ratio of the Scale Drawing Area to the Actual Area: _________

Results: What do you notice about the ratio of the areas in Examples 1-3? Complete the statements below.

When the scale factor of the sides was 2, then the value of the ratio of the areas was _______________.

When the scale factor of the sides was $\frac{1}{3}$, then the value of the ratio of the areas was _____________.

When the scale factor of the sides was $\frac{4}{3}$, then the value of the ratio of the areas was _____________.

Based on these observations, what conclusion can you draw about scale factor and area?

If the scale factor of the sides is r, then the ratio of areas is ___________________.