## Opening Exercise

- If you do not have tape diagram experience try to solve the problem on using a diagram or algebra.

94 children are in a reading club. One-third of the boys and three-sevenths of the girls prefer fiction. If 36 students prefer fiction, how many girls prefer fiction?



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## Tape Diagrams

## Session Objectives

- Experience how proficiency in the tape diagram method can be developed in students and colleagues.
- Experience how to support understanding of various types of word problems as outlined in the Progressions document.
- Mathematical Practice 5: *Use appropriate tools strategically* and demonstrate this knowledge by using a tape diagram to solve problems.



## Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Practice Set 1
- Practice Set 2
- Practice Set 3
- Practice Set 4
- Practice Set 5
- Practice Set 6
- Practice Set 7

## What is a Tape Diagram?

A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as strip diagrams, bar model, fraction strip, or length model.

(CCSSM Glossary, p. 87)



## What is a Tape Diagram?

Grade 1: Math Drawings (1.OA.1, 1.OA.2)

Grade 2: Math Drawings (2.OA.1, 2.OA.2, 2.MD.5)

Grade 3: Visual Fraction Model (3.NF.3a-d)

Grade 4: Visual Fraction Model (4.NF.3, 4.NF.4, 4.OA.2)

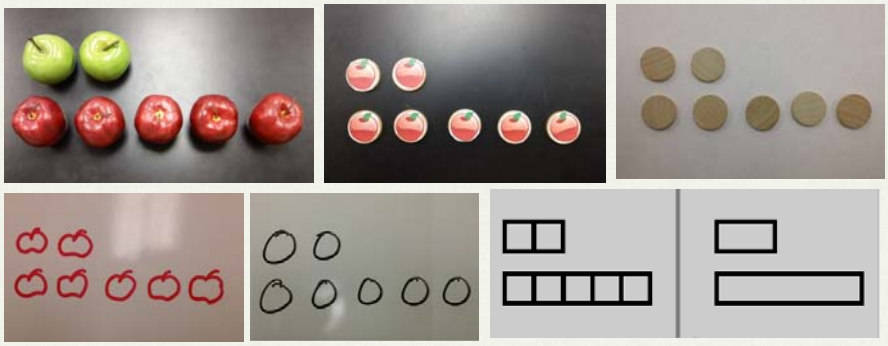
Grade 5: Visual Fraction Model (5.NF.2-4, 6, 7)

Grade 6: Tape Diagrams (6.RP-3) Visual Fraction Model (6.NS-1)

Grade 7: Visual Model for Problem Solving (7RP1-3)

Number Line Diagram (7.NS-1)

## Developing Conceptual Understanding
## Concrete → Pictorial → Visualization → Abstract



2 apples + 5 apples = 7 apples

2 + 5 = 7

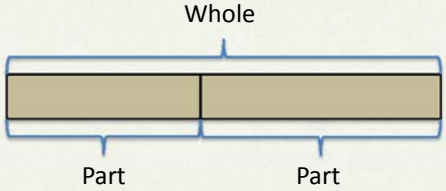
S = J - 3

S + J = 7



## Forms of the Tape Diagram

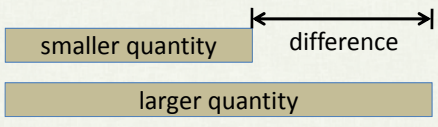
### Part Whole Model




### Fraction Model

3 pieces of size one-third

### Additive Comparison Model




### Models for Ratios & Multiplicative Comparison

4 times as many as; a 1:4 ratio

## Using Tape Diagrams

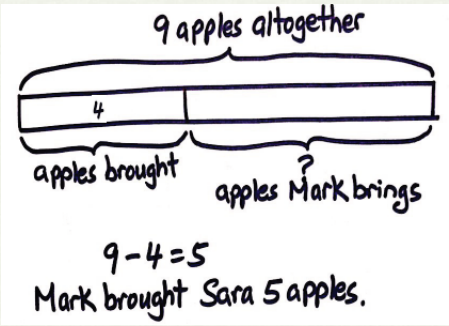
- Promote **perseverance** in reasoning through problems.
- Develop students' independence in asking themselves:
  - "Can I draw something?"
  - "What can I label?"
  - "What do I see?"
  - "What can I learn from my drawing?"



Tape Diagrams

## Early Examples

Sara brought 4 apples to school. After Mark brings her some more apples, she has 9 apples altogether. How many apples did Mark bring her?

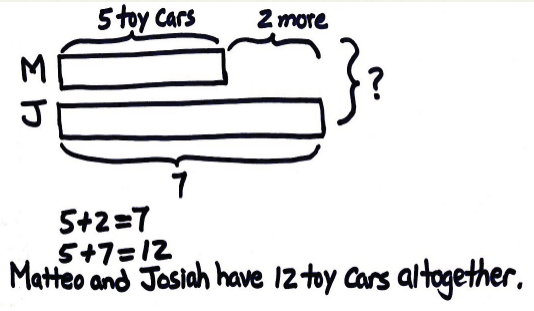



Add to: Change Unknown

Tape Diagrams

## Early Examples

Matteo has 5 toy cars. Josiah has 2 more than Matteo. How many cars do Matteo and Josiah have altogether?




Compare: Bigger Unknown



## Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Practice Set 1
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- Practice Set 6

Example A:

Rose has a vase with 13 flowers. She puts 7 more flowers in the vase. How many flowers are in the vase?

G1 M4 L19



Example B:

Nine dogs were playing at the park. Some more dogs came to the park. Then there were 11 dogs. How many more dogs came to the park?

G1 M4 L20

Example C:

Ben and Peter caught 17 tadpoles. They gave some to Anton. They have 4 tadpoles left. How many tadpoles did they give to Anton?

G1 M4 L21



Example D:

Some yellow beads were on Tamra's bracelet. After she put 14 purple beads on the bracelet, there were 18 beads. How many yellow beads did Tamra's bracelet have at first?

G1 M4 L20

Example E:

Kiana found some shells at the beach. She gave 8 shells to her brother. Now she has 9 shells left. How many shells did Kiana find at the beach?

G1 M4 L21



Example F:

The students were playing with 7 balls on the playground. They accidentally kicked some of the balls into a big puddle and now some are muddy! What is one way the balls might look?

GK M4 L18

## Stop to Reflect:

What did you notice about the problems in Problem Set 1?

- They were all addition or subtraction problems, and were all conducive to use of the *Part-Whole* model.



## Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  Addition and Subtraction Models – Part Whole
- Problem Set 2
- Problem Set 3
- Problem Set 4
- Problem Set 5
- Problem Set 6
- Problem Set 7

Example A:

Rose wrote 8 letters. Nikii wrote 12 letters. How many more letters did Nikii write than Rose?

G1 M6 L1

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Example B:

Ben played 9 songs on his banjo. Lee played 3 more songs than Ben. How many songs did Lee play?

G1 M6 L1

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Example C:

Rose saw 14 monkeys at the zoo. She saw 5 fewer monkeys than foxes. How many foxes did Rose see?

G1 M6 L17

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Example D:

Emi planted 12 flowers. Rose planted 3 fewer flowers than Emi. How many flowers did Rose plant?

G1 M6 L2

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Example E:

Peter has 8 more green crayons than yellow crayons. Peter has 10 green crayons. How many yellow crayons does Peter have?

G1 M6 L23



## Stop to Reflect:

What did you notice about the problems in Problem Set 2?

- They were all additive comparison problems and were all conducive to use of the *Comparison* model.

Table 1. Common addition and subtraction situations.[6]

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| **Add to** | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?<br>2 + 3 = ? | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?<br>2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?<br>? + 3 = 5 |
| **Take from** | Five apples were on the table. I ate two apples. How many apples are on the table now?<br>5 – 2 = ? | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?<br>5 – ? = 3 | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?<br>? – 2 = 3 |
| | **Total Unknown** | **Addend Unknown** | **Both Addends Unknown[1]** |
| **Put Together/ Take Apart[2]** | Three red apples and two green apples are on the table. How many apples are on the table?<br>3 + 2 = ? | Five apples are on the table. Three are red and the rest are green. How many apples are green?<br>3 + ? = 5, 5 – 3 = ? | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?<br>5 = 0 + 5, 5 = 5 + 0<br>5 = 1 + 4, 5 = 4 + 1<br>5 = 2 + 3, 5 = 3 + 2 |



# Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  Addition and Subtraction Models: Part Whole
- Problem Set 2: Addition Comparison Problems
- Problem Set 3
- Problem Set 4
- Problem Set 5
- Problem Set 6
- Problem Set 7

Example A:

A parking structure has 10 levels. There are 3 cars parked on each level. How many cars are parked in the structure?

G3 M6 L18



Example B:

The total weight of a football and 10 tennis balls is 1 kg. If the weight of each tennis ball is 60 g, find the weight of the football.

## Stop to Reflect:

What did you notice about the problems in Problem Set 3?

- The problems were all multiplication or division problems that were conducive to using the *Part-Whole* model.



## Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
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- Problem Set 3:
  Multiplication and Division Models: Part Whole
- Problem Set 4
- Problem Set 5
- Problem Set 6

Example A:

There are 400 children at Park Elementary School. Park High School has 4 times as many students. How many students in all attend both schools?

G4 M3

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Example B:

35 students ordered hamburgers. That is 5 times as many as the number of students who ordered a salad. How many students ordered a salad?

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Example C:

Tiffany spent $\frac{1}{7}$ of her money on a teddy bear. If the teddy bear costs $28, how much money did she have at first?

G5 M4 L7



Example D:

Sarah is 9 years old. Sarah's grandfather is 90 years old. Sarah's grandfather is how many times as old as Sarah?

G4 M1 L1

# Stop to Reflect:

What did you notice about the problems in Problem Set 4?

- The problems were all multiplicative comparison problems that were conducive to using the *Comparison* model.

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# Multiplication Problem Chart

| | Unknown Product<br>3 × 6 = ? | Group Size Unknown ("How many in each group?" Division)<br>3 × ? = 18, and 18 ÷ 3 = ? | Number of Groups Unknown ("How many groups?" Division)<br>? × 6 = 18, and 18 ÷ 6 = ? |
|---|---|---|---|
| **Equal Groups** | There are 3 bags with 6 plums in each bag. How many plums are there in all?<br>*Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?<br>*Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed?<br>*Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| **Arrays,[4] Area[5]** | There are 3 rows of apples with 6 apples in each row. How many apples are there?<br>*Area example.* What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row?<br>*Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?<br>*Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| **Compare** | A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?<br>*Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 | A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?<br>*Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How | A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?<br>*Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as |

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## Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  Addition and Subtraction Models: Part Whole
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- Problem Set 4: Multiplication Comparison Problems
- Problem Set 5
- Problem Set 6
- Problem Set 7



Example A:

David spent 2/5 of his money on a storybook. The storybook cost $20. How much did he have at first?

Example B:

Max spent 3/5 of his money in a shop and 1/4 of the remainder in another shop. What fraction of his money was left? If he had $90 left, how much did he have at first?



Example C:

Alex bought some chairs. One third of them were red and one fourth of them were blue. The remaining chairs were yellow. What fraction of the chairs were yellow?

Example D:

Jim had 360 stamps. He sold 1/3 of them on Monday and 1/4 of the remainder on Tuesday. How many stamps did he sell on Tuesday?



Example E:

Three-fifths of Jan's money is twice as much as Lena's money. What fraction of Jan's money is Lena's money?

Example F:

Henry bought 280 blue and red paper cups. He used 1/3 of the blue ones and 1/2 of the red ones at a party. If he had an equal number of blue cups and red cups left, how many cups did he use altogether?



## Stop to Reflect:

What did you notice about the problems in Problem Set 5?

- The problems were all fraction problems that were conducive to using the *Part Whole* model.

## Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  Addition and Subtraction Models: Part Whole
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- Problem Set 3:
  Multiplication and Division Models: Part Whole
- Problem Set 4: Multiplication Comparison Problems
- Problem Set 5: Fraction Models: Part Whole
- Problem Set 6:
- Problem Set 7:



Example A:

The ratio of the length of Tom's rope to the length of Jan's rope was 3:1. The ratio of the length of Maxwell's rope to the length of Jan's rope was 4:1. If Tom, Maxwell and Jan have 80 feet of rope altogether, how many feet of rope does Tom have?

Example B:

The Business Direct Hotel caters to people who travel for different types of business trips. On Saturday night there is not a lot of business travel, so the ratio of the number of occupied rooms to the number of unoccupied rooms is 2:5. On Sunday night, the ratio changes to 6:1. If the Business Direct Hotel has 432 occupied rooms on Sunday night, how many unoccupied rooms does it have on Saturday night?

G6 M1 L6



Example C:

Lena finds two boxes of printer paper in the teacher supply room. The ratio of the packs of paper in Box A to the packs of paper in Box B is 4:3. If half of the paper in Box A is moved to Box B, what is the new ratio of packs of paper in Box A to Box B?

Example D:

Sana and Amy collect bottle caps. The ratio of the number of bottle caps Sana has to the number Amy has is 2 : 3. The ratio became 5 : 6 when Sana added 8 more bottle caps to her collection. How many bottle caps does Amy have?



Example E:

The ratio of songs on Jessa's phone to songs on Tessie's phone is 2 to 3. Tessie deletes half of her songs and now has 60 fewer songs than Jessa. How many songs does Jessa have?

Example F:

Jack and Matteo had an equal amount of money each. After Jack spent $38 and Matteo spent $32, the ratio of Jack's money to Matteo's money was 3 : 5. How much did each boy have at first?



Example G:

The ratio of the Gavin's money to Manuel's was 6 : 7. After Gavin spent two-thirds of his money and Manuel spent $39 Manuel had twice as much money as Gavin. How much money did Gavin have at first?

## Stop to Reflect:

What did you notice about the problems in Problem Set 6?

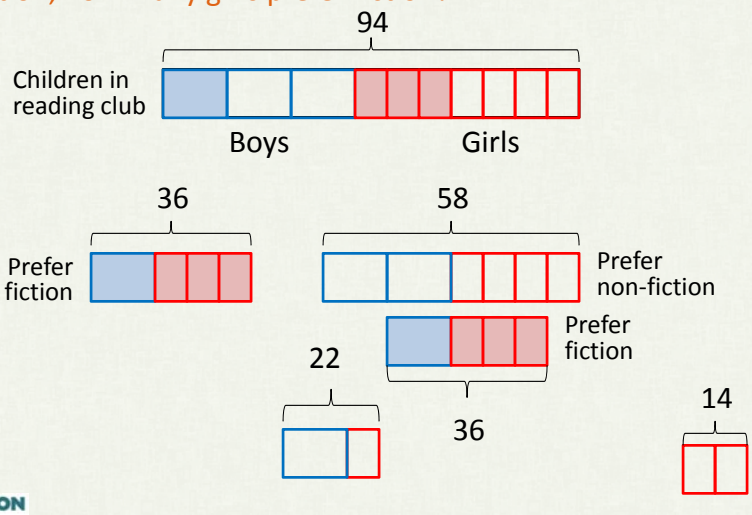
- The problems were all ratio problems that were conducive to using the *Comparison* model.



Opening Exercise:

94 children are in a reading club. One-third of the boys and three-sevenths of the girls prefer fiction. If 36 students prefer fiction, how many girls prefer fiction?

94 children are in a reading club. One-third of the boys and three-sevenths of the girls prefer fiction. If 36 students prefer fiction, how many girls prefer fiction?






# Agenda – Tape Diagrams

- Introduction to Tape Diagrams
- Problem Set 1:
  Addition and Subtraction Models: Part Whole
- Problem Set 2: Addition Comparison Problems
- Problem Set 3:
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- Problem Set 4: Multiplication Comparison Problems
- Problem Set 5: Fraction Models: Part Whole
- Problem Set 6: Ratio Models
- Problem Set 7:

## Double Number Line Diagrams

Example A: Percentage Problems

Mia's weekly salary is $928. This is 80% of Sana's weekly salary. Find Sana's weekly salary.



## Double Number Line Diagrams

Example B: Rate Problems

A photocopier can print 12 copies in 36 seconds. At this rate, how many copies can it print in 1 minute?

Example C:

A club had 600 members. 60% of them were males. When 200 new members joined the club, the percentage of male members was reduced to 50%. How many of the new members were males?



## Using Variables with Tape Diagrams

Example D:

Mary had \$460. She bought 6 beach towels at \$$x$ each. Express the amount of money she had left in terms of $x$. If $x = 17$, how much money did she have left?

## Using Variables with Tape Diagrams

Example E:

Max had $x$ brownies. He ate 4 brownies and shared the remaining brownies among his 6 friends equally. How many brownies did each friend receive? Express your answer in terms of $x$. If $x = 34$, how many brownies did each friend receive?



## Grade 7 – Module 3 – Lesson 7

Solve the problem first with a tape diagram, then an equation.

The ages of three sisters are consecutive integers. The sum of their ages is 45. Find their ages.

## Grade 7 – Module 3 – Lesson 8

Solve the problem first with a tape diagram, then an equation.

Julia, Keller, and Israel are volunteer firefighters. On Saturday the volunteer fire department held its annual coin drop fundraiser at a streetlight. After one hour, Keller had collected $42.50 more than Julia, and Israel had collected $15 less than Keller. Altogether, the three firefighters collected $125.95. How much did each person collect?



## Grade 7 – Module 4 – Lesson 2

Solve the problem first with a tape diagram, then an equation.

In Ty's art lass, 12% of the Flag Day art projects received a perfect score. There were 25 art projects turned in by Ty's class. How many of the art projects earned a perfect score?

## Grade 7 – Module 6 – Lesson 1

Solve the problem first with a tape diagram, then an equation.

The measures of two supplementary angles are in the ratio of 2:3. Find the measures of the two angles.



## Grade 7 – Module 6 – Lesson 1

Solve the problem first with a tape diagram, then an equation.

In a pair of complementary angles, the measurement of the larger angle is three times that of the smaller angle. Find the measures of the two angles.

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## Algebra I – Module 1 – Lesson 25

Solve the problem first with a tape diagram, then an equation.

In a school choir, one-half of the members were girls. At the end of the year, 3 boys left the choir, and the ratio of boys to girls became 3:4. How many boys remained in the choir?



## Key Points

- When building proficiency in tape diagraming skills start with simple accessible situations and add complexities one at a time.
- Develop habits of mind in students to reflect on the size of bars relative to one another.
- Part-whole models are more helpful in modeling situations where:_____
- Compare to models are more helpful in modeling situations where: _____

## Session Objectives

- Experience how proficiency in the tape diagram method can be developed in students and colleagues.
- Experience how to support understanding of various types of word problems as outlined in the Progressions document.
- Mathematical Practice 5: *Use appropriate tools strategically* and demonstrate this knowledge by using a tape diagram to solve problems.

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## Feedback

- Now that you've experienced either one, two or three days, comment on what you learned each day.
- Comment on the order of the sessions (Day 1: Major Works 3-5, Day 2: Major Works 6-8, Day 3: Tape Diagrams).
- We welcome any other comments.

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