COMMON CORE

Eureka Math: *A Story of Units*

Major Work of the Grade Band
K-2: Addition and Subtraction

Session Objectives

Examine how addition and subtraction are introduced and developed in A Story of Units.

Study the computation methods used in the primary years to solve addition and subtraction problems.

- Level 1: Counting All
- Level 2: Counting On
- Level 3: Convert to an Easier Problem

Curriculum Map for *A Story of Units*

AGENDA

Kindergarten: Addition and Subtraction, Counting All

Grade 1: Addition and Subtraction, Counting On and Converting to an Easier Problem

Grade 2: Addition and Subtraction, Converting to an Easier Problem

The Beginning of *A Story of Units* in Kindergarten

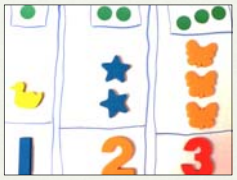
By the end of Kindergarten, students will:

- Fluently add and subtract within 5 (K.OA.5)
- Represent and Solve Addition and Subtraction Word Problems with Totals to 10 (K.OA.1 and 2)
- Decompose numbers less than or equal to 10 (K.OA.3)
- For any number 1-9 find the number that makes 10 (K.OA.4)
- Compose and decompose numbers from 11-19 as 10 ones and some ones (K.NBT.1)

Kindergarten – Foundations to Addition and Subtraction
Level One - Counting All

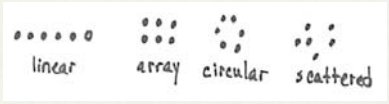
Students need to know:

- Number words in sequence
- Number recognition
- 1:1 correspondence
- Cardinality

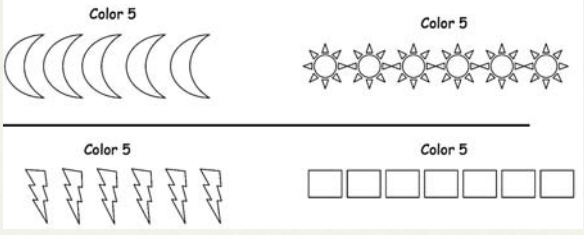


Concrete-pictorial-abstract

Kindergarten – Fluency
Counting All



Kindergarten – Foundations for Counting On



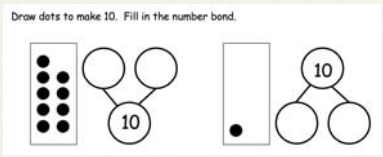
Kindergarten – Module 1 Lesson 19

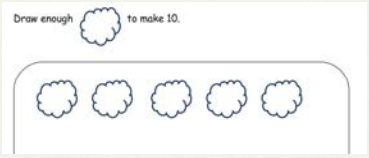
Kindergarten – Foundations for Counting On

1. Counting on the Say Ten Way
2. Counting on the Regular Way

http://www.engageny.org/resource/common-core-video-series-kindergarten-mathematics-double-10-frames

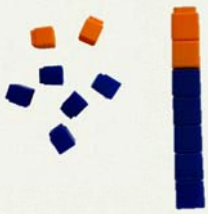
Kindergarten – Make Ten





Kindergarten – Models Number Towers

- Concrete representation
- Often called sticks (e.g., 5-stick) or trains (horizontal)

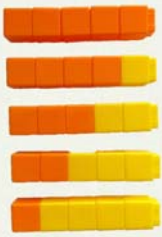


Kindergarten – Models
Number Towers

- Number stairs show pattern of 1 more and 1 less (K.CC.4)
- Color change at 5 (K.OA.5)

Kindergarten – Models
Number Towers

- Decompositions of numbers within 10 (K.OA.3)



5 = 5 + 0
5 = 4 + 1
5 = 3 + 2
5 = 2 + 3
5 = 1 + 4

Kindergarten – Video
Number Towers

http://www.engageny.org/resource/numbers-through-10-number-towers-number-path-number-bond

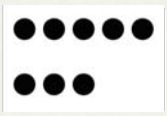
Kindergarten – Models 5-Groups

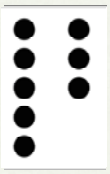
- Represent quantities in relation to 5 and 10.
  - Quickly recognize quantities 6-10 based on 5-patterns (5 + n)
- Support addition and subtraction operations.

Horizontal 5-groups

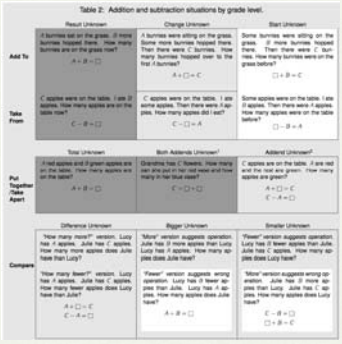
Vertical 5-groups

Kindergarten – Fluency 5 + n pattern

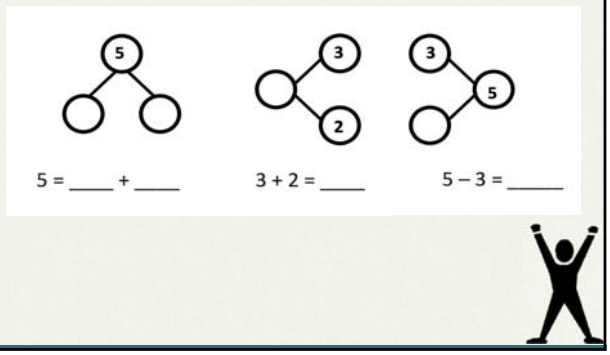




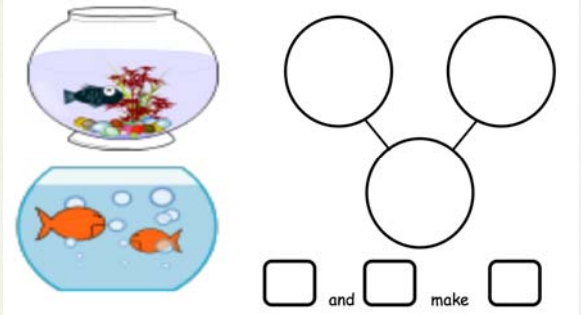
Story Problem Types



Kindergarten
Problem Type Number Bonds



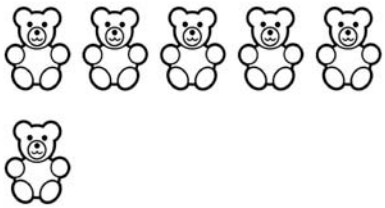
Kindergarten – Story Problems



Kindergarten – Module 4 – Lesson 1

Kindergarten – Story Problems

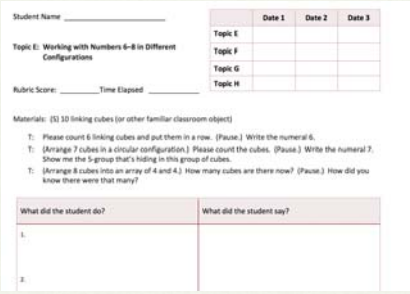
Cross out the bears to match the number sentence.



6 – 1 = 5

Kindergarten – Module 4 – Lesson 20

Kindergarten – Assessment



Student Name _____

	Date 1	Date 2	Date 3
Topic E			
Topic F			
Topic G			
Topic H			

Topic E: Working with Numbers 6–8 in Different Configurations

Rubric Score: _____ Time Elapsed _____

Materials: (S) 10 linking cubes (or other familiar classroom object)

T: Please count 6 linking cubes and put them in a row. (Pause.) Write the numeral 6.
T: (Arrange 7 cubes in a circular configuration.) Please count the cubes. (Pause.) Write the numeral 7. Show me the 5-group that's hiding in this group of cubes.
T: (Arrange 8 cubes into an array of 4 and 4.) How many cubes are there now? (Pause.) How did you know there were that many?

What did the student do?	What did the student say?
1.	
2.	

Kindergarten – Module 1 – Topic E

A Continuation of A Story of Units in Grade 1

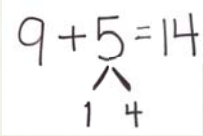
By the end of Grade 1, students will:

- Fluently add and subtract within 10 (1.OA.6)
- Add and subtract within 20 (1.OA.6)
- Use addition and subtraction within 20 to solve word problems (1.OA.1)
- Add within 100 (1.NBT.4)
- Subtract multiples of 10 in the range of 10 through 90 (1.NBT.6)

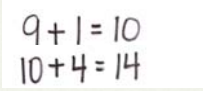
Foundational Skills for Grade 1

1. Partners to ten (K.OA.4)

1. Decompositions for all numbers within 10 (K.OA.3)



1. Representations of teen numbers as 10 + n (K.NBT.1 and 1.NBT.2b)



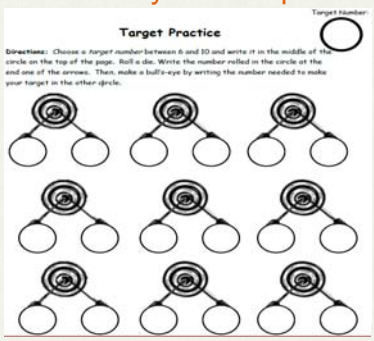
Grade 1 – Level 2: Counting On

- Use embedded numbers.
- Find decompositions of 6, 7, 8, 9, and 10.
- Solve addend unknown/change unknown problems.
- Understand the meaning of subtraction as it relates to addition.

Grade 1 – Embedded Numbers and Decomposition Bridge to Counting On

Grade 1 – Module 1 – Lesson 6

Grade 1 – Fluency: Decompositions



Grade 1 – Counting On: Add to with Change Unknown

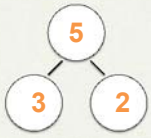
- Counting on to solve add to with change unknown math stories.

Once upon a time, 3 little bears went to play tag in the forest. Then some more bears came over. In the end, there were 5 little bears playing tag in the woods altogether. How many bears came to play?

Grade 1 – Module 1 – Lesson 11

Grade 1 – Counting On: Relate Subtraction to Addition

- Understanding the meaning of subtraction as it relates to addition

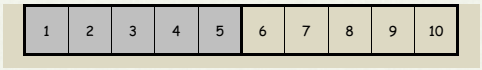


3 + ___ = 5

5 – 3 = ___

Grade 1 – Module 1 – Lesson 25

Grade 1 – Models Number Path



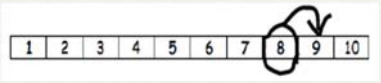
- 7 + _____ = 8 8 – 7 = _____
- 2 + _____ = 9 9 – 2 = _____
- 8 + _____ = 9 9 – 8 = _____

Grade 1 – Module 1 – Lessons 26 and 27

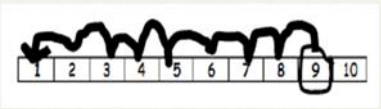
Grade 1 – Models Number Path

When solving 9 – 8 = ___ on the number path, which strategy is more efficient? Counting on or counting back?

8 + ___ = 9

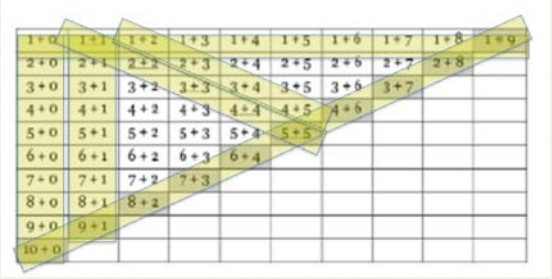


9 – 8 = ___

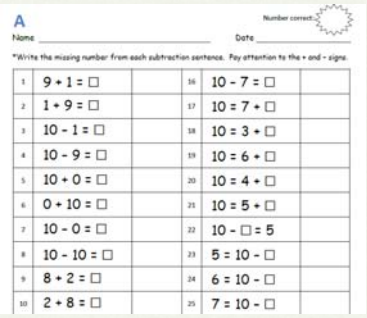


Grade 1 – Module 1 – Lessons 26 and 27

Grade 1 – Addition Chart



Grade 1 – Addition and Subtract within 10



A

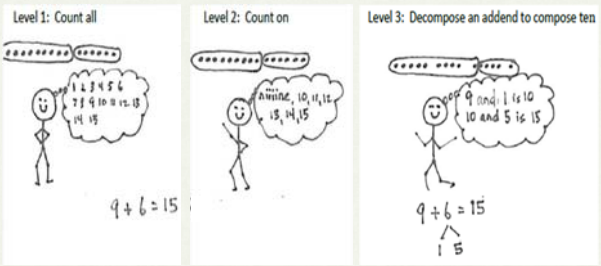
Name _____ Date _____

Number correct: _____

*Write the missing number from each subtraction sentence. Pay attention to the + and – signs.

1	9 + 1 = □		16	10 - 7 = □	
2	1 + 9 = □		17	10 = 7 + □	
3	10 - 1 = □		18	10 = 3 + □	
4	10 - 9 = □		19	10 = 6 + □	
5	10 + 0 = □		20	10 = 4 + □	
6	0 + 10 = □		21	10 = 5 + □	
7	10 - 0 = □		22	10 - □ = 5	
8	10 - 10 = □		23	5 = 10 - □	
9	8 + 2 = □		24	6 = 10 - □	
10	2 + 8 = □		25	7 = 10 - □	

Grade 1 – Level 3: Convert to an Easier Problem

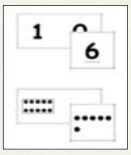


Grade 1 – Fluency Focus on 10 + n

http://www.engageny.org/resource/nti-november-2012-rigor-breakdown-counting-exercises-for-k-1

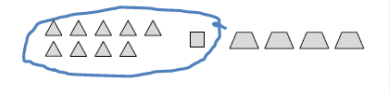
- Happy Counting: the Say Ten Way
- Tens and Ones
- Hide Zero Number Sentences

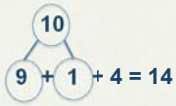

Rekenrek

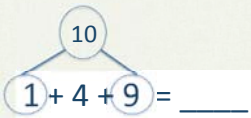


Hide Zero Cards

Grade 1 – Convert to an Easier Problem Make Ten

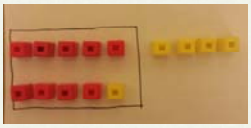


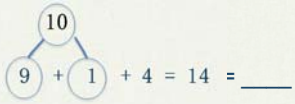




Grade 1 – Module 2 – Lessons 1 and 2

Grade 1 – Convert to an Easier Problem Make Ten





Grade 1 – Module 2 – Lessons 3-10

Grade 1 – Convert to an Easier Problem Make Ten

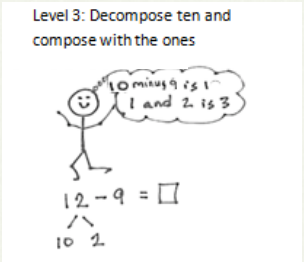
Now You Try!

9 + 3 = ____ 5 + 9 = ____

8 + 3 = ____ 7 + 8 = ____

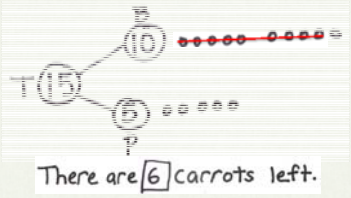
Grade 1 – Module 2 – Lessons 3-10

Grade 1 – Convert to an Easier Problem Take from Ten



Grade 1 – Convert to an Easier Problem Take from Ten

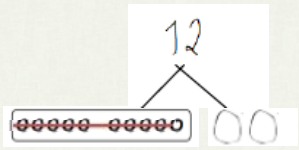
Bailey Bunny has 15 carrots. 10 are in the basket and 5 are on the plate. She ate 9 carrots from the basket. How many carrots were left?



Grade 1 – Module 2 – Lesson 12

Grade 1 – Convert to an Easier Problem Take from Ten

12 – 9 = _____

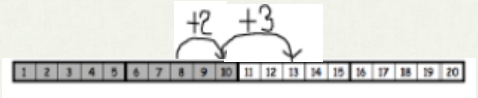


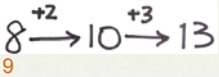
Grade 1 – Module 2 – Lesson 13

Grade 1 – Convert to an Easier Problem: Make Ten to Subtract

13 – 8 = _____

8 + _____ = 13





Grade 1 – Module 2 – Lesson 19

Grade 1 – Convert to an Easier Problem
Take from Ten

14 – 8 = _____

- Number path
- 5-group row drawing
- Number bond

How do these models relate to one another?

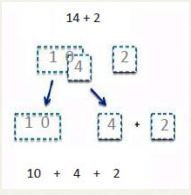
Grade 1 – Convert to an Easier Problem
Take from Ten

Now you try!

13 – 9 = _____ 15 – 9 = _____

15 – 8 = _____ 13 – 7 = _____

Grade 1 – Models
Hide Zero Cards and Magic Counting Sticks



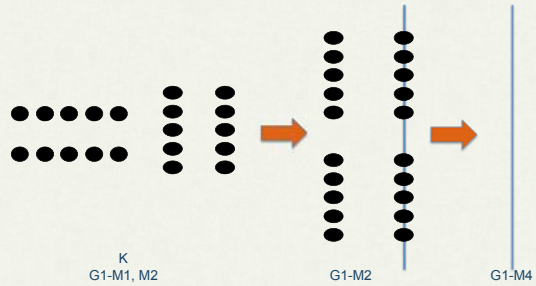
Hide Zero Cards

Magic Counting Sticks

Grade 1 – Module 2 – Lesson 27

Grade 1 – Models
5-Groups

Evolution of 5-group formations:



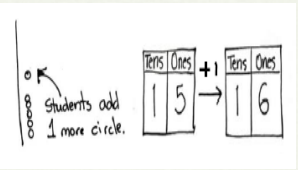
K
G1-M1, M2

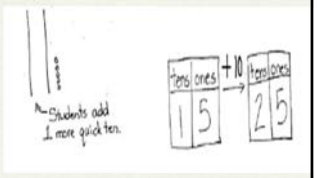
G1-M2

G1-M4

Grade 1 – Models
Quick Ten Drawing and Place Value Chart

Adding ones to ones, tens to tens





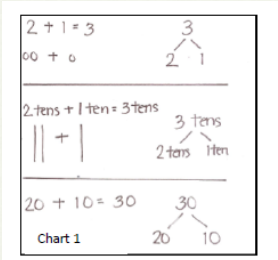
Grade 1 – Module 4 – Lesson 5

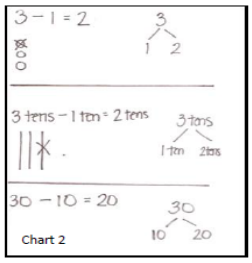
Grade 1 – Models
Dimes and Pennies

Adding ones to ones, tens to tens

Grade 1 – Module 4 – Lesson 6

Grade 1 – Addition and Subtraction of Tens



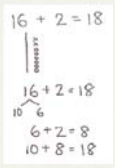


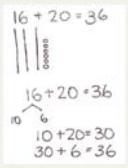
Grade 1 – Module 4 – Lesson 11

Grade 1 – Adding Ones to Ones, Tens to Tens

Do we add to the ones or to the tens?

16 + 2 = _____ 16 + 20 = _____

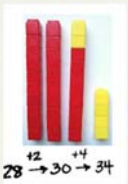


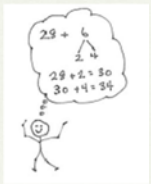


Grade 1 – Module 4 – Lesson 16

Grade 1 – Convert to an Easier Problem Making the Next Ten

28 + 6 = _____

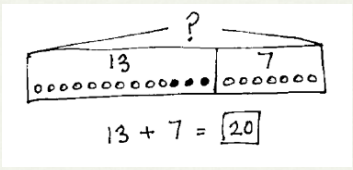




Grade 1 – Module 4 – Lesson 14

Grade 1 – Tape Diagrams

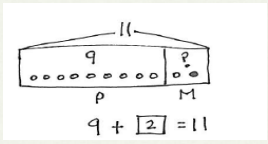
Rose has a vase with 13 flowers. She puts 7 more flowers in the vase. How many flowers are in the vase?



Grade 1 – Module 4 – Lessons 19-22

Grade 1 – Tape Diagrams

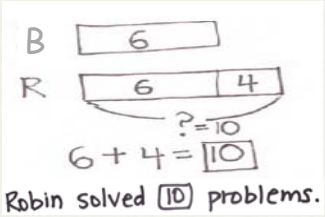
Nine dogs were playing at the park. Some more dogs came to the park. Then there were 11 dogs. How many more dogs came to the park?



Grade 1 – Module 4 – Lessons 19-22

Grade 1 – Tape Diagrams

Ben solved 6 math problems. Robin solved 4 more problems than Ben. How many problems did Robin solve?



Grade 1 – Module 6 – Lesson 2

Grade 1 – End-of-Module Assessment

Use place value understanding and properties of operations to add and subtract.

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete modules or drawings and strategies based on place value, properties of operations, and/or relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Grade 1 – End-of-Module Assessment

Solve for each unknown number. Use the space provided to draw quick tens, a number bond, or the arrow way to show your work. You may use your kit of ten-sticks if needed.

d. 100 - ____ = 40	e. 78 + 6 = ____	f. 47 + 40 = ____
g. 65 + 34 = ____	h. 75 + 25 = ____	i. 47 + 36 = ____

A Continuation of A Story of Units in Grade 2

By the end of Grade 2, students will:

- Fluently add and subtract within 20 (2.OA.2)
- Fluently add and subtract within 100 (2.NBT.5)
- Add and subtract within 1,000

Grade 2 – Considerations

Meeting students where they are:

- Counting All
- Counting On
- Convert to an Easier Problem

The significance of understanding how addition and subtraction are introduced and developed across the primary grade span.

Grade 2 – Considerations

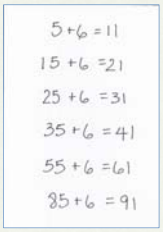
Meeting students where they are:

- Use of coherent models and strategies across grade levels
- Emphasis on the ten-structure
  - Identify a unit of ten
  - Make a unit of ten
  - Take from a unit of ten

Grade 2 – Considerations

Meeting students where they are:

- Filling gaps
  - Concrete representations
  - Multiple representations
  - Simple to complex



The Importance of Part–Whole Thinking

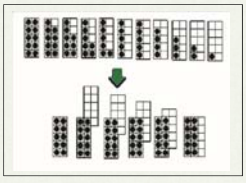
- Building number sense/seeing relationships
- Composition and decomposition require 2 key understandings
  - Seeing embedded numbers
    - A unit can always be decomposed into smaller units
  - Properties of operations
    - Commutative property (3 + 9 is the same as 9 + 3)
    - Associative property (85 + 6 = 85 + 5 + 1)

Grade 2 – Foundations

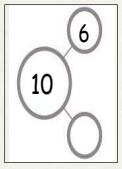
Sums and Differences to 20

2.OA.2 Fluently add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers.

Grade 2 – Convert to an Easier Problem Make a Ten



Ten-frame cards



Number bond

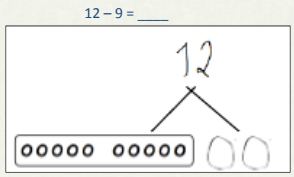
Grade 2 – Module 1 – Lesson 1

Grade 2 – Fluency

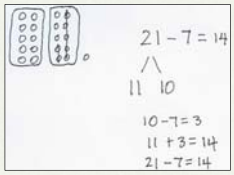
- Get the One Out
- Get the Two Out
- Partners to 10

Grade 2 – Convert to an Easier Problem Take from Ten

Decompose to subtract from a ten when subtracting within 100.



G1

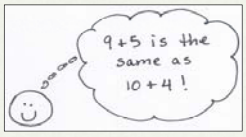


G2

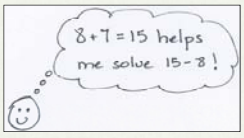
Grade 2 – Module 1 – Lesson 8

Grade 2 – Fluency

- Think 10 to Add 9



- Related Facts Within 20



Grade 2 – Measurement

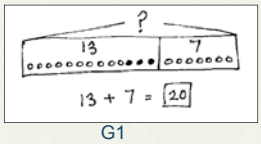
Addition and Subtraction of Length Units

- Metric units lead into place value units in Module 3 (10 tens inside 100 cm, 10 tens inside 1 hundred).
- The unit is central to the addition and subtraction algorithms of Modules 4 and 5.
- Rich context for word problems.

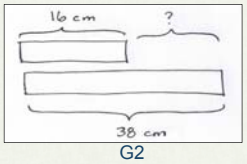
Grade 2 – Models
Tape Diagram

Relate Addition and Subtraction to Length.

- Create tape diagrams to represent and compare lengths.
- Apply conceptual understanding of measurement by solving two-step word problems.



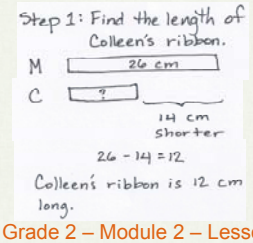
G1

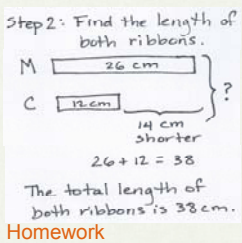


G2

Grade 2 – Models
Tape Diagram

Maura's ribbon is 26 cm long. Colleen's ribbon is 14 cm shorter than Maura's ribbon. What is the total length of both ribbons?

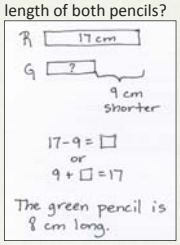


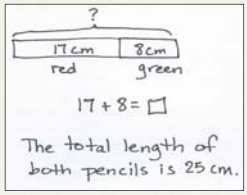


Grade 2 – Module 2 – Lesson 10 Homework

Grade 2 – Models
Tape Diagram

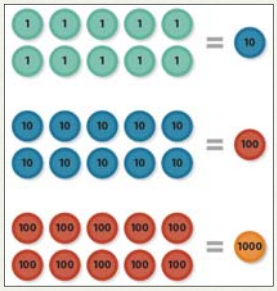
The red colored pencil is 17 centimeters long. The green colored pencil is 9 centimeters shorter than the red colored pencil. What is the total length of both pencils?





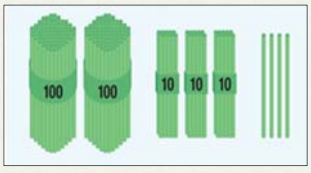
Grade 2 – Module 2 – Lesson 10

Grade 2 – Place Value Understanding



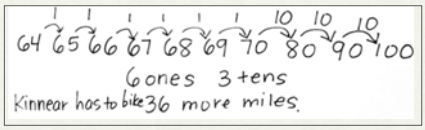
Grade 2 – Models
Place Value Manipulatives

Bundles



Grade 2 – Level 2: Counting On Benchmark Numbers

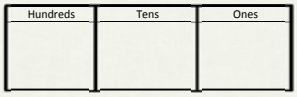
Kinnear decided that he would bike 100 miles this year. If he has biked 64 miles so far, how much farther does he have to bike?



Grade 2 – Module 3 – Lesson 3

Grade 2 – Models Place Value Chart

- The Place Value Chart



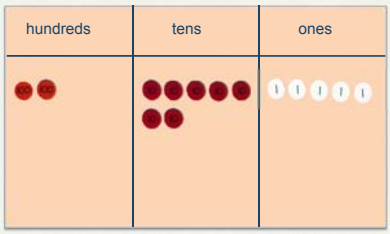
- Hide Zero Cards

Terminology

- Standard Form … 576
- Expanded Form … 500 + 70 + 6
- Unit Form … 5 hundreds 7 tens 6 ones
  57 tens 6 ones
  5 hundreds 76 ones
  576 ones
- Word Form … five hundred seventy-six

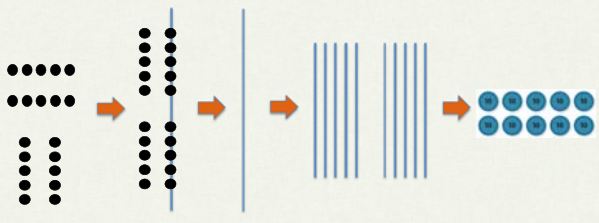
Grade 2 – Models
Money

Grade 2 – Models
Place Value Disks



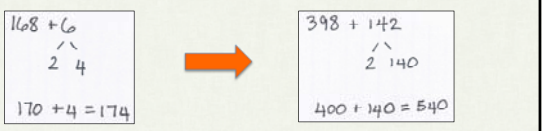
Building Towards the Algorithm:
Base-Ten Structure

Conceptual understanding leading up to Modules 4 and 5



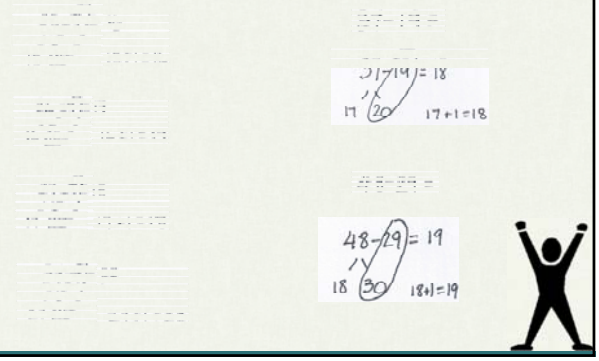
Building Towards the Algorithm: Base-Ten Structure

Conceptual understanding leading up to Modules 4 and 5



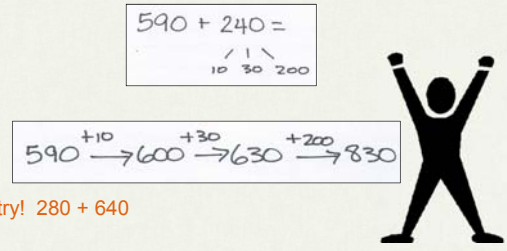
G2-M4 G2-M5

Grade 2 – Simplifying Strategies: Take from Ten



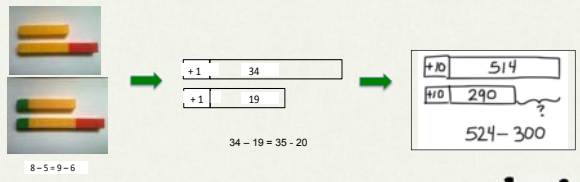
Grade 2 – Simplifying Strategies: Extend Make a Ten to Make a Hundred

590 + 240 =



You try! 280 + 640

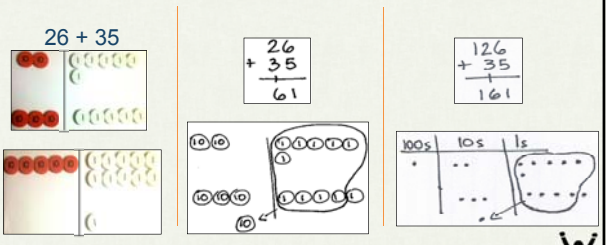
Grade 2 – Simplifying Strategies: Compensation



You try! 440 – 280

Grade 2 – Models
Place Value Disks and Drawings

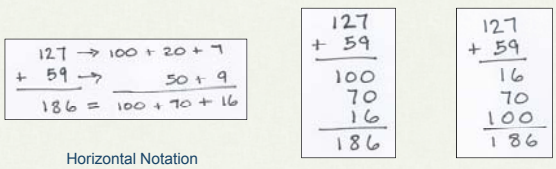
Addition: Strategies for Composing Tens and Hundreds



Progression of understanding towards the addition algorithm

Grade 2 – Addition Strategies

Totals Below



Horizontal Notation

Left to Right

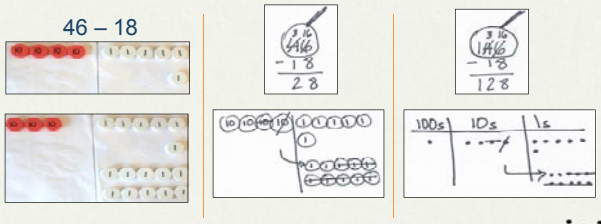
Right to Left

You try! 125 + 75

Grade 2 – Models
Place Value Disks and Drawings

Subtraction: Strategies for Decomposing Tens and Hundreds

46 – 18



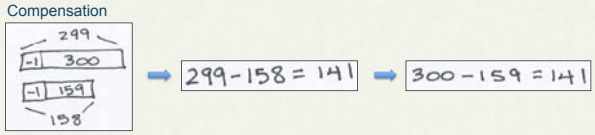
Progression of understanding towards the subtraction algorithm

Grade 2 – Simplifying Strategies

Subtraction: Subtract from Multiples of 100

300 – 159

Compensation



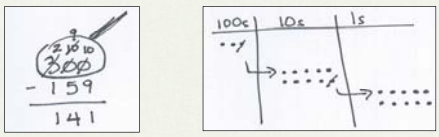
Arrow Notation

Grade 2 – Models
Place Value Disks and Drawings

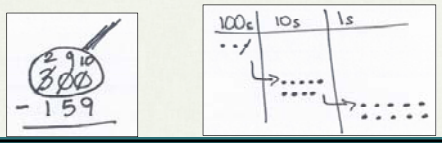
Subtraction: Subtract from Multiples of 100

300 – 159

2 Step



1 Step



Grade 2 – Assessment

End-of-Module Assessment Task Standard Addressed	Topics A–D
Use Place Value Understanding and Properties of Operations to Add and Subtract	
2.NBT.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relates the strategy to a written method. Understand that in adding and subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
2.NBT.8	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
2.NBT.9	Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

Grade 2 – Module 5 – End-of-Module Assessment

Grade 2 – Assessment

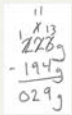
1. Solve each problem with a written strategy such as a tape diagram, a number bond, the arrow way, the vertical method, or chips on a place value chart.

a. 460 + 200 = _____	b. _____ = 865 – 300	c. _____ + 400 = 598
d. 240 – 190 = _____	e. _____ = 760 – 280	f. 330 – 170 = _____

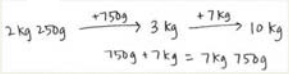
Grade 2 – Module 5 – End-of-Module Assessment

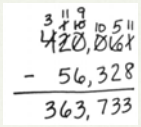
The Story Continues

Grade 3

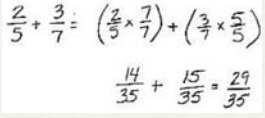


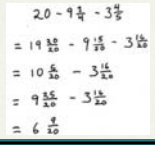
Grade 4





Grade 5





Reflection

- What simple ideas have you learned about addition and subtraction?
- Can you now manipulate units better than before?
- What new strategies did you learn that use the structure of 10 to add and subtract?
- Can you now help students see embedded numbers?

Key Points for Addition and Subtraction

- Addition and subtraction are about the manipulation of units.
- Addition and subtraction are about seeing part/whole relationships.
- Addition is putting together -- the composing of units to make a larger unit.
- Subtraction is taking apart -- the decomposing of larger units into smaller units.
- Subtraction is also the comparison of units.
- Knowledge of bonds within 10, bonds that make 10, and teen numbers as 10 and some ones are key understandings necessary for the manipulation of units.
- Conceptual understanding is developed and supported by learning, sharing, and reasoning about strategies based on place value, properties of operations, and the relationship between addition and subtraction.
- Specifically chosen models and fluency activities support learning.