

**COMMON
CORE**

Eureka Math: A Story of Units

Major Work of the Grade Band
3-5: Multiplication and Division with
Whole Numbers and Fractions

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Session Objectives

Discover that multiplication and division are, respectively, composing and decomposing equal units.

Understand how the part to whole and whole to part relationships apply to multiplication and division.

Curriculum Map for A Story of Units

The curriculum map displays standards for Mathematics (3-5) and Science (3-5) across various units. The standards are organized into columns: 3-5 Mathematics, 3-5 Science, and 3-5 Mathematics. The standards are color-coded and include specific mathematical practices and content standards.

Agenda

- **Foundations of Multiplication and Division**
- Grade 3 Progression of Multiplication and Division
- Grade 4 Progression of Multiplication and Division
- Grade 5 Progression of Multiplication and Division
- Continuation of Multiplication and Division in the Middle Grades

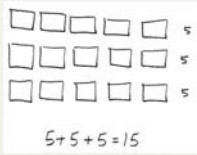

Grade 2 Module 6: Foundations of Multiplication and Division

Equal groups and repeated addition

- Tape diagrams and repeated addition

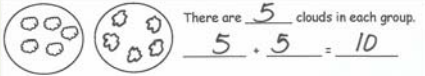

Grade 2 – Module 6 – Overview

Grade 2 Module 6: Foundations of Multiplication and Division

- Arrays and repeated addition
 
- Tiled rectangular arrays
 

Grade 2 – Module 6 – Lessons 7 and 11


Grade 2 Module 6: Foundations of Multiplication and Division

- Doubles
 
- Even and Odd
 

Grade 2 – Module 6 – Lessons 17 and 18


Models Concrete → Pictorial → Abstract

Equal groups pictures



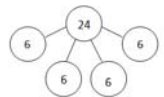
$3 \times 5 = 15$

Array



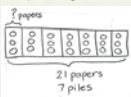
$3 \times 4 = 12$

Number Bond



$4 \times 6 = 24$

Tape diagram

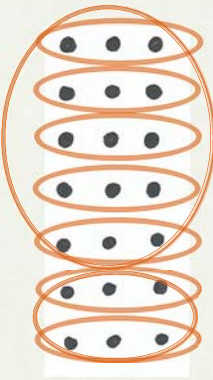


$21 \div 7 = 3$

Strategies

Simple → Complex

Level 1: One object is one unit.
Level 2: A group is a unit.
Level 3: Multiple groups are units.



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Grade 3: Multiplication and Division in A Story of Units

By the end of Grade 3, students will:

Use multiplication and division within 100 to solve word problems. (3.OA.3)

Find the unknown number in a multiplication or division equation. (3.OA.4)

Apply properties of operations as strategies to multiply and divide. (3.OA.5)

Fluently multiply within 100. (3.OA.7)

Identify and explain patterns in the multiplication table. (3.OA.9)

Multiply one-digit whole numbers by multiples of 10. (3.NBT.3)

Relate area to multiplication. (3.MD.7)

Grade 3 Fluency

- Group Counting
 - Use the Commutative Property
 - Find the Common Products
- "Multiply By" Pattern Sheets
 - Multiply by Different Units
- Sprints

A	Multiplication	# Correct
1	$2 \times 4 =$	23 $\times 4 = 40$
2	$3 \times 4 =$	24 $\times 4 = 8$
3	$4 \times 4 =$	25 $\times 4 = 12$
4	$5 \times 4 =$	26 $40 = 4 =$
5	$3 \times 6 =$	27 $20 = 4 =$
6	$8 \times 4 =$	28 $8 = 1 =$
7	$12 \times 4 =$	29 $8 = 4 =$
8	$20 \times 4 =$	30 $12 = 4 =$
9	$4 \times 1 =$	31 $\times 4 = 16$
10	$16 \times 4 =$	32 $\times 4 = 28$
11	$6 \times 4 =$	33 $\times 4 = 36$
12	$7 \times 4 =$	34 $\times 4 = 32$
13	$8 \times 4 =$	35 $28 = 4 =$
14	$9 \times 4 =$	36 $36 = 4 =$
15	$10 \times 4 =$	37 $24 = 4 =$
16	$32 \times 4 =$	38 $32 = 4 =$
17	$28 \times 4 =$	39 $11 \times 4 =$
18	$36 \times 4 =$	40 $44 = 4 =$
19	$24 \times 4 =$	41 $12 = 4 =$
20	$40 \times 4 =$	42 $48 = 4 =$
21	$\times 4 = 20$	43 $14 \times 4 =$
22	$\times 4 = 24$	44 $56 = 4 =$

Multiplication

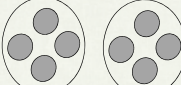
Repeated addition

$$2 + 2 + 2 + 2 + 2 + 2 = 12$$

$$6 \text{ twos} = 12$$

$$6 \times 2 = 12$$

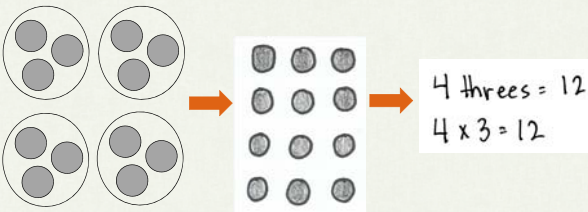
Equal groups pictures



$4 + 4 = 8$
 $2 \text{ fours} = 8$
 $2 \times 4 = 8$

Grade 3 – Module 1 – Lesson 1

Relate Multiplication to the Array Model.

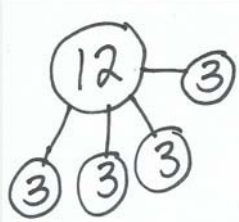


Equal Groups Pictures \rightarrow Array \rightarrow Multiplication Number Sentence

$4 \text{ threes} = 12$
 $4 \times 3 = 12$

Grade 3 – Module 1 – Lesson 2

Relate Multiplication to Number Bonds.



4 threes = 12
 $4 \times 3 = 12$

Number Bond

Grade 3 – Module 1 – Lesson 3

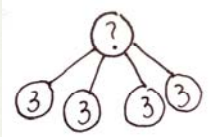
RDW Process

Read, Draw, Write

1. **Read** the problem.
2. **Draw** a picture representing the problem.
3. **Write** a number sentence to solve a problem and a statement answering the problem.

RDW Process

Jordan uses 3 lemons to make 1 pitcher of lemonade. He makes 4 pitchers. How many lemons does he use altogether? **Use the RDW process to show your solution.**



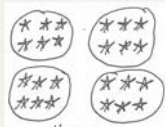
$4 \times 3 = 12$

Jordan uses 12 lemons altogether.

Use the RDW process to solve this problem.


Try Problem # 1.

Arthur has 4 boxes of chocolates. Each box has 6 chocolates inside. How many chocolates does Arthur have altogether?



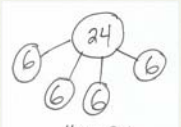
$4 \times 6 = 24$
Arthur has 24 chocolates altogether.

Equal Groups Pictures



$4 \times 6 = 24$
Arthur has 24 chocolates altogether.


Array



$4 \times 6 = 24$
Arthur has 24 chocolates altogether.

Number Bond

Division

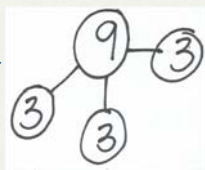


Divide 14 flowers into 2 equal groups.
There are 7 flowers in each group.

$14 \div 2 = 7$

Equal Groups Pictures

Number Bond



$9 \div 3 = 3$

Skip-counting


4	4	4	4	4
4	8	12	16	20

$20 \div 4 = 5$

Grade 3 – Module 1 – Lessons 4 and 5

Arrays Used to Relate Multiplication and Division

Rick puts 15 tennis balls into cans. Each can has 3 tennis balls. How many cans does Rick use?




$5 \times 3 = 15$ $15 \div 3 = 5$

The number in the blanks represents the number of groups.


Grade 3 – Module 1 – Lesson 6

Arrays and the Commutative Property

Count by threes 5 times.
3 6 9 12 15



Count by fives 3 times.
5 10 15

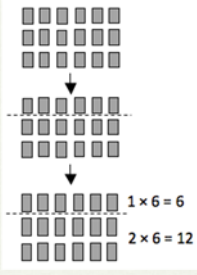


$5 \times 3 = 3 \times 5$

Problem 1 Problem 2

Grade 3 – Module 1 – Lesson 8

Arrays and the Distributive Property



3 sixes = 1 six + 2 sixes

$$3 \times 6 = (1 \times 6) + (2 \times 6)$$

$$= 6 + 12$$

$$= 18$$

Grade 3 – Module 1 – Lesson 10

Try Problems # 2 and # 3.

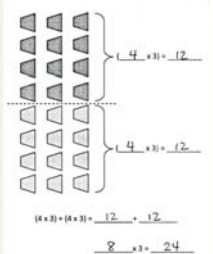
Hint: Draw 2 arrays to model Erin's and Vittesh's thinking.

Erin's Array: $2 \times 9 = 18$

Vittesh's Array: $4 \times 3 = 12$

Erin and Vittesh are both correct. It just depends on how you are looking at the array. If you turn Erin's array, it looks like Vittesh's array. They are the same.

2. $8 \times 3 = (4 \times 3) + (4 \times 3) = 24$



Grade 3 – Module 1 – Lessons 8 and 10

Types of Division: Measurement

$12 \div 4 = ?$

Types of Division: Partitive

$12 \div 4 = ?$

Tape Diagrams and Types of Division

2 students equally share 8 crackers. How many crackers does each student get?

Partitive Division: number of groups is known

$8 \div 2 = 4$

Grade 3 – Module 1 – Lesson 12

Tape Diagrams and Types of Division

There are 8 crackers, but this time each student gets 2. How many students get crackers?

Measurement Division: size of each group is known

Grade 3 – Module 1 – Lesson 12

$$8 \div 2 = 4$$

Use a Tape Diagram to Model and Solve Multiplication.

Grade 3 – Module 1 – Lessons 14 and 15

Try Problems # 4 and # 5.

How are the tape diagrams similar? Different?

Grade 3 – Module 1 – Lessons 12 and 15

Model the 5 + n Pattern as a Strategy for Multiplying.

$(5 \times 4) = 20$

$(3 \times 4) = 12$

8 fours = 5 fours + 3 fours
 $8 \times 4 = (5 \times 4) + (3 \times 4)$
 $= (5 + 3) \times 4$

Grade 3 – Module 1 – Lesson 16

Apply the Distributive Property to Decompose Units to Divide.

$28 \div 4 = \underline{\quad}$

$(20 \div 4) = 5$

$(8 \div 4) = 2$

$28 \div 4$
 $20 \div 4$ $8 \div 4$

$(28 \div 4) = (20 \div 4) + (8 \div 4)$
 $= 5 + 2$
 $= 7$

Grade 3 – Module 1 – Lesson 19

Try Problem # 6.

$32 \div 4 = \underline{8}$

$(20 \div 4) = 5$

$(12 \div 4) = 3$

$32 \div 4 = (20 \div 4) + (12 \div 4)$
 $= 5 + 3$
 $= 8$

Why is the array decomposed this way?

Grade 3 – Module 1 – Lesson 19

**Multiply and Divide with Familiar Facts
Using a Letter to Represent the Unknown.**

Twenty-four people line up to use the canoes at the park. Three people are assigned to each canoe. How many canoes are used for all 24 people?

$3 \times 7 = 24$
 $24 \div 3 = ?$
 $? = 8$
 The people use 8 canoes.

$3 \times c = 24$
 $24 \div 3 = c$
 $c = 8$
 The people use 8 canoes.

Grade 3 – Module 3 – Lesson 3

Count by Units of 6 and 7 to Multiply and Divide.

6, 12, 18, 24, 30, 36, 42

$7 \times 6 = 42$

$42 \div 6 = 7$

Grade 3 – Module 3 – Lesson 4

**Model the Distributive Property with
Tape Diagrams and Number Bonds.**

Tape Diagram

5×7 4×7
 $9 \times 7 =$
 $(5 \times 7) + (4 \times 7) =$
 $35 + 28 = 63$
 $9 \times 7 = 63$

Number Bond

$48 \div 6$
 $30 \div 6$ $18 \div 6$
 $48 \div 6 = (30 \div 6) + (18 \div 6)$
 $= 5 + 3$
 $= 8$

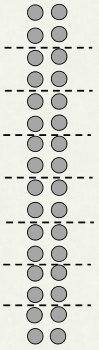
Grade 3 – Module 3 – Lesson 6

Model the Associative Property as a Strategy to Multiply.

$$16 \times 2 = (8 \times 2) \times 2$$


$$= 8 \times (2 \times 2)$$

$$= 8 \times 4$$


$$= 32$$


Grade 3 – Module 3 – Lesson 9

Try Problem # 7.



a) $3 \times 12 = 36$




b) $(3 \times 3) \times 4$
 $= 9 \times 4$
 $= 36$

Is there a different way to use the associative property to solve this problem?

Grade 3 – Module 3 – Lesson 9

Apply $9 = 10 - 1$ as a Strategy to Multiply by 9.

$$9 = 10 - 1$$

$$9 \times 4$$



9 fours = 10 fours - 1 four

$$40 - 4 = 36$$

Grade 3 – Module 3 – Lesson 12


Identify and Use Arithmetic Patterns to Multiply.

$3 \times 9 = \underline{\quad}$



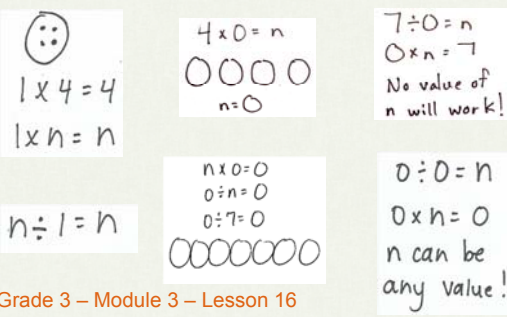
$9 = 10 - 1$
3 nines = 3 tens - 3 ones

$3 \times 9 = 30 - 3$
 $3 \times 9 = 27$



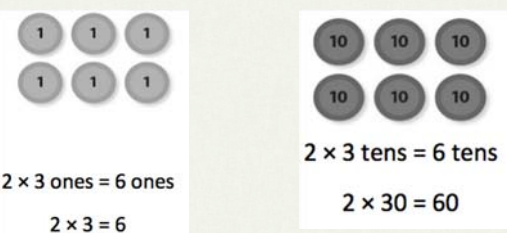
Grade 3 – Module 3 – Lesson 14

Multiply and Divide Using Units of 1 and 0.



Grade 3 – Module 3 – Lesson 16

Multiply by Multiples of 10 Using Place Value Disks.

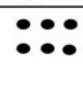



$2 \times 3 \text{ ones} = 6 \text{ ones}$
 $2 \times 3 = 6$

$2 \times 3 \text{ tens} = 6 \text{ tens}$
 $2 \times 30 = 60$



Grade 3 – Module 3 – Lesson 19

Multiply by Multiples of 10 Using a Place Value Chart.

tens	ones	tens	ones
$2 \times 3 \text{ ones} = \underline{\quad} \text{ ones}$ $2 \times 3 = \underline{\quad}$		$2 \times 3 \text{ tens} = \underline{\quad} \text{ tens}$ $2 \times 30 = \underline{\quad}$	

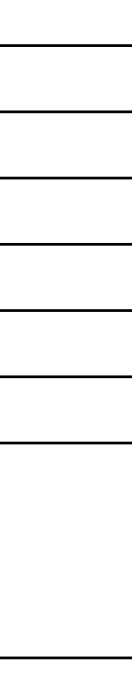
Grade 3 – Module 3 – Lesson 19

Use Place Value Strategies & the Associative Property to Multiply Multiples of 10.

tens	ones	tens	ones
		c) $(3 \times 5) \times 10$ $= (15 \text{ ones}) \times 10$ $= 150$	d) $3 \times (5 \times 10)$ $= 3 \times (5 \text{ tens})$ $= 150$

Grade 3 – Module 3 – Lesson 20

Grade 3 Module 4: Multiplication and Area



Module 1 and Module 3 Module 4

Array \rightarrow Tiled Rectangular Array \rightarrow Area Model

Grade 3 – Module 4 – Overview

Multiplying and Dividing With the Area Model

a.

7 ft

4 ft Area: 28 sq ft

$4 \times 7 = 28$

6 in

A = 24 sq in

$6 \times ? = 24$
 $6 \times 4 = 24$
 $24 \div 6 = 4$

Grade 3 – Module 4 – Lesson 8

Apply the Distributive Property as a Strategy to Find Area.

6

5 5×6

3 3×6

$(5 \times 6) + (3 \times 6)$
 $= 30 + 18$
 $= 48$ square units

Distributive Property

Grade 3 – Module 4 – Lessons 10 and 11

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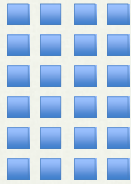
A Continuation of a Story of Units in Grade 4

By the end of Grade 4 students will:

- Multiply up to a four-digit number by a one-digit number. (4.NBT.5)
- Multiply two two-digit numbers. (4.NBT.5)
- Multiply a fraction by a whole number (4.NF.4)
- Divide up to four-digit numbers by one digit divisors finding whole number quotients with remainders. (4.NBT.6)
- Interpret multiplication as comparison. (4.OA.1)
- Solve word problems using multiplication and division. (4.OA.2,3)

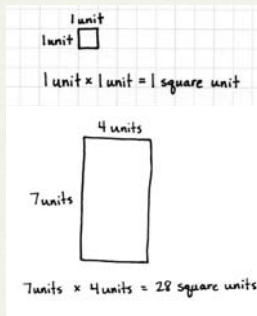
Multiplication Models

Array → Area Model

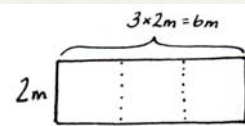


$$A = l \times w$$

Grade 4 – Module 3 – Lesson 1



Multiplicative Comparison Word Problems



6 is 3 times as many as 2.
 $6 = 3 \times 2!$

$$2m \times 6m = 12 \text{ square } m$$

Area Model

Grade 4 – Module 3 – Lesson 2

Multiplication by 10, 100, and 1000

Place Value Chart

Number Disks

Unit Language

3 ones $\times 10 = 3$ tens

3 tens $\times 10 = 3$ hundreds

3 hundreds $\times 10 = 3$ thousands

Grade 4 – Module 3 – Topic B

4 ones $\times 3 = 12$ ones

4 tens $\times 3 = 12$ tens

4 hundreds $\times 3 = 12$ hundreds

Multiplication by 10, 100, and 1000

$4 \times 20 \Rightarrow 4 \times 2 \text{ tens} \Rightarrow (4 \times 2) \times 10 \Rightarrow 8 \times 10$

Grade 4 – Module 3 – Topic B

$8 \times 2 \text{ ones} = 16 \text{ ones}$
 $8 \times 2 \text{ tens} = 16 \text{ tens}$
 $8 \times 2 \text{ hundreds} = 16 \text{ hundreds}$
 $8 \times 2 \text{ thousands} = 16 \text{ thousands}$

Multiplication by 10, 100, and 1000

40×20

Grade 4 – Module 3 – Topic B

4 tens $\times 2$ tens = 8 hundreds

Multiplication of Up to 4 Digits by Single-Digit Whole Numbers

8 tens + 12 ones
9 tens + 2 ones

$$\begin{array}{r} 23 \\ \times 4 \\ \hline 12 \leftarrow 4 \times 3 \text{ ones} \\ + 80 \leftarrow 4 \times 2 \text{ tens} \\ \hline 92 \end{array}$$

Place Value Chart & Number Disks → Partial Products

Grade 4 – Module 3 – Lesson 7

Multiplication of Up to 4 Digits by Single-Digit Whole Numbers

Place Value Chart
Number Disks
Algorithm

$$\begin{array}{r} 1423 \\ \times 3 \\ \hline 4269 \end{array}$$

Grade 4 – Module 3 – Lessons 8-10

Multiplication of Up to 4 Digits by Single-Digit Whole Numbers

$$\begin{array}{r} 2,374 \\ \times 5 \\ \hline 11,870 \end{array}$$

Algorithm

Grade 4 – Module 3 – Lessons 9-10

Multiplication of Up to 4 Digits by Single-Digit Whole Numbers

Area Model

Distributive Property

Partial Products

Grade 4 – Module 3 – Lesson 11

Try Problem # 8.

Use RDW to solve.

The Turner family uses 548 liters of water each day. The Hill family uses 3 times as much water as the Turner family per day. How much water does the Hill family use each week?

Hint: This is a 2-step problem. Try solving using a variety of methods: algorithm, area model, distributive property, partial products, or number disks.

Grade 4 – Module 3 – Lesson 12

Division of Tens and Ones with Successive Remainders

There are 12 students in PE class separated into 4 teams. How many students are on each team?

There are 12 students in PE class. The teacher makes teams of 3 students each. How many teams are there?

Array

Grade 4 – Module 3 – Lesson 14

Division of Tens and Ones with Successive Remainders

$4 \div 3$

Tens	Ones
1	1
0	1
0	1
0	1

1 R1
$$\begin{array}{r} 3 \overline{)4} \\ -3 \\ \hline 1 \end{array}$$

$42 \div 3$

Tens	Ones
4	2
0	2
0	2
0	2

14
$$\begin{array}{r} 3 \overline{)42} \\ -3 \\ \hline 12 \\ -12 \\ \hline 0 \end{array}$$

$44 \div 3$

Tens	Ones
4	4
0	4
0	4
0	4

14 R2
$$\begin{array}{r} 3 \overline{)44} \\ -3 \\ \hline 14 \\ -12 \\ \hline 2 \end{array}$$

14 42
$$\begin{array}{r} 14 \\ \times 3 \\ \hline 42 \end{array}$$

14 44
$$\begin{array}{r} 14 \\ \times 3 \\ \hline 42 \\ + 2 \\ \hline 44 \end{array}$$

Grade 4 – Module 3 – Lesson 17

Try Problem # 9.

Use RDW to solve:

The Grand Market sells 3 pounds of oranges for 87 cents. How much does 1 pound of oranges cost at Grand Market?

Try modeling with number disks to support the algorithm.

tens	ones
8	7
0	7
0	7
0	7

29
$$\begin{array}{r} 3 \overline{)87} \\ -6 \\ \hline 27 \\ -27 \\ \hline 0 \end{array}$$

c = 29¢
1 pound of oranges costs 29 cents.

Grade 4 – Module 3 – Lesson 18

Division of Thousands, Hundreds, Tens, and Ones

Thousands	hundreds	tens	ones
4	3	2	5
0	3	2	5
0	3	2	5
0	3	2	5

Number Disks

1441 R2
$$\begin{array}{r} 3 \overline{)4325} \\ -3 \\ \hline 13 \\ -12 \\ \hline 12 \\ -12 \\ \hline 05 \\ -3 \\ \hline 2 \end{array}$$

Algorithm

1441 4323
$$\begin{array}{r} 1441 \\ \times 3 \\ \hline 4323 \end{array}$$

Checking Work with Multiplication

Grade 4- Module 3 – Lesson 29

Try Problem #10.

There are twice as many cows as goats on a farm. All the cows and goats have a total of 1,116 legs. How many goats are there?

Try modeling with number disks to support the algorithm.

Solution A

$$\begin{array}{r} 372 \\ 3 \overline{)1116} \\ \underline{-9} \\ 21 \\ \underline{-21} \\ 00 \\ \underline{-0} \\ 0 \end{array}$$

1 unit: 372 legs = 93 goats

Solution B

$$\begin{array}{r} 279 \\ 4 \overline{)1116} \\ \underline{-8} \\ 31 \\ \underline{-31} \\ 00 \\ \underline{-0} \\ 0 \end{array}$$

3 units: 279 animals
1 unit: 93 animals

Tape Diagram

Algorithm

Grade 4 – Module 3 – Lesson 29

Try Problem # 11.

Solve $1,584 \div 2$:

a) Using number disks. b) Use the algorithm.

Try solving using number disks and the standard algorithm at the same time.

Thousands	Hundreds	Tens	Ones
1	5	8	4
0	7	9	2

7 hundreds
9 tens
2 ones

$$\begin{array}{r} 792 \\ 2 \overline{)1,584} \\ \underline{-14} \\ 18 \\ \underline{-18} \\ 04 \\ \underline{-4} \\ 0 \end{array}$$

Grade 4 – Module 3 – Lesson 33

The Area Model for Division

$1,344 \div 6$

$A = l \times w$
 $A \div w = l$

$$\begin{array}{l} 1344 \\ \circlearrowleft 1200 \\ \circlearrowleft 120 \\ \circlearrowleft 24 \\ (1200 \div 6) + (120 \div 6) + (24 \div 6) \\ = 200 + 20 + 4 \\ = 224 \end{array}$$

Grade 4 – Module 3 – Lesson 33

Multiplication of 2-Digit by 2-Digit Numbers

Area Model

Grade 4 – Module 3 – Lesson 35

Multiplication of 2-Digit by 2-Digit Numbers

Area Model

Grade 4 – Module 3 – Lesson 35

Multiplication of 2-Digit by 2-Digit Numbers

Area Model
Distributive Property
4 Partial Products

Grade 4 – Module 3 – Lesson 36

Multiplication of 2-Digit by 2-Digit Numbers

Area Model
Distributive Property
4 Partial Products

Grade 4 – Module 3 – Lesson 36

Multiplication of 2-Digit by 2-Digit Numbers

Grade 4 – Module 3 – Lesson 37

Multiplication of 2-Digit by 2-Digit Numbers

Algorithm

Grade 4 – Module 3 – Lesson 38

Try Problem #12.
Solve: 84×73 .

Try using an area model, 4 partial products, and the standard algorithm to solve.

Grade 4 – Module 3 – Lesson 38

Multiplication of Fractions

1 fourth + 1 fourth + 1 fourth = 3 fourths

$3 \times (1 \text{ fourth}) = 3 \text{ fourths}$

$(3 \times 1) \text{ fourths} = 3 \text{ fourths}$

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$$3 \times \frac{1}{4} = \frac{3}{4}$$

$$\frac{3 \times 1}{4} = \frac{3}{4}$$

Grade 4 – Module 5 – Topic A

Multiplication of Fractions

Tape Diagram

$\frac{5}{3} = 5 \times \frac{1}{3}$

$\frac{5}{3} = (3 \times \frac{1}{3}) + (2 \times \frac{1}{3})$

$$\frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5} \frac{1}{5}$$

3 fifths 3 fifths 3 fifths 3 fifths

$$4 \times (3 \text{ fifths}) = (4 \times 3) \text{ fifths}$$

$$= 12 \text{ fifths}$$

$$4 \times 3 \text{ fifths} = 12 \text{ fifths}$$

$$4 \times \frac{3}{5} = \frac{12}{5}$$

Grade 4 – Module 5 – Topic E

Multiplication of Fractions

Number Line

$6 \times 2 = 12$
 $6 \times \frac{1}{2} = \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{1}{2}\right)$
 $6 \times \frac{1}{2} = 3 \times \frac{2}{2}$

Grade 4 – Module 5 – Topic E

Multiplication of Fractions

Tape Diagram Distributive Property

$2 \times 3\frac{1}{5} = (2 \times 3) + (2 \times \frac{1}{5})$
 $= 6 + \frac{2}{5}$
 $= 6\frac{2}{5}$

Grade 4 – Module 5 – Lessons 37 & 38

Try Problem #13.

Use RDW to solve.

A bricklayer places 12 bricks along an outside wall of a shed. Each brick is $\frac{3}{4}$ foot long. How many feet long is that wall of the shed?

$12 \times \frac{3}{4} = \frac{12 \times 3}{4} = \frac{36}{4} = 9$
 $B = 9$ feet
 The wall of the shed is 9 feet long.

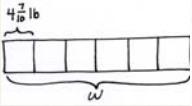
Try using the associative property to solve.

Grade 4 – Module 5 – Lesson 36

Try Problem #14.

Use RDW to solve.

Kelly's new puppy weighed $4\frac{7}{10}$ pounds when she brought him home. Now the puppy weighs six times as much as he did when he came home. How much does the dog weigh now?



$6 \times 4\frac{7}{10} = (6 \times 4) + (6 \times \frac{7}{10})$
 $= 24 + \frac{42}{10}$
 $= 24 + 4\frac{2}{10}$
 $= 28\frac{2}{10}$

$w = 28\frac{2}{10}$ lb
The dog weighs $28\frac{2}{10}$ pounds now.

Grade 4 – Module 5 – Lesson 37

Try using the distributive property to solve.

Agenda

- Foundations of Multiplication and Division
- Grade 3 Progression of Multiplication and Division
- Grade 4 Progression of Multiplication and Division
- **Grade 5 Progression of Multiplication and Division**
- Continuation of Multiplication and Division in the Middle Grades

A Continuation of a Story of Units in Grade 5

By the end of Grade 5 students will:

- Use whole-number exponents to denote power of 10. (5.NBT.2)
- Fluently multiply multi-digit whole numbers. (5.NBT.5)
- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. (5.NBT.6)
- Multiply and divide decimals to hundredths. (5.NBT.7)
- Interpret a fraction as division of the numerator by the denominator. (5.NF.3)
- Multiply a fraction or whole number by a fraction. (5.NF.4)
- Solve real world problems involving multiplication of fractions and mixed numbers. (5.NF.6)
- Divide unit fractions by whole numbers and whole numbers by unit fractions. (5.NF.7)

Patterns on the Place Value Chart: Multiplication

Place Value Chart
 $2.16 \times 10^3 =$
 $2.16 \times 10 \times 10 \times 10 =$
 2,160.

Unit Language
 $2.16 \times 10^3 =$
 216 hundredths $\times 10^3 =$
 216,000 hundredths =
 2,160.

Grade 5 – Module 1 – Lessons 5 & 6

Patterns on the Place Value Chart: Division

Place Value Chart
 $754 \div 10^3 =$
 $754 \div 10 \div 10 \div 10 =$
 .754

Grade 5 – Module 1 – Lessons 5 & 6

Decimal Multiplication

ones | tenths | hundredths | thousandths

1 | 6 | 9 | 2

0.423
 $\times 4$
 \hline
 1.692

Grade 5 – Module 1 - Lesson 11

Decimal Multiplication

4 tenths + 2 hundredths + 3 thousandths			
4	16 tenths	8 hundredths	12 thousandths
	1.6	+ 0.08	+ 0.012 = 1.692

Area Model & Unit Language

Grade 5 – Module 1 - Lesson 11

Dividing Decimals

b. $1.324 \div 2 =$ _____

Ones	Tenths	Hundredths	Thousandths
1	3	2	4
0	0	0	0
0	0	0	0

$$\begin{array}{r} 662 \\ 2 \overline{) 1.324} \\ \underline{-12} \\ 12 \\ \underline{-12} \\ 4 \\ \underline{-4} \\ 0 \end{array}$$

Place Value Chart → Standard Algorithm

Grade 5 – Module 1 - Lesson 14

Try Problem # 15.

The total weight of 5 pieces of butter is 3.445 kg. What is the weight of each piece of butter?
Use the place value chart and algorithm to solve.

Hint: When completing the division algorithm, use language that connects it to the work shown on the place value chart.

Grade 5 – Module 1 – Lesson 14

Multi-Digit Whole Number Multiplication

4509 x 326

	4000	500	9
6	24,000	3000	54
20	80,000	10,000	180
300	1,200,000	1,500,000	2,700

Area Model

$$\begin{array}{r}
 4509 \\
 \times 326 \\
 \hline
 27054 \\
 90180 \\
 1352700 \\
 \hline
 1469934
 \end{array}$$

Standard Algorithm

Area Model → Standard Algorithm

Grade 5 - Module 2 - Lesson 7

Multi-Digit Multiplication with Decimals

41 x 7.38

	700	30	8
1	28,000	1,500	320

Area Model

$$\begin{array}{r}
 738 \\
 \times 41 \\
 \hline
 738 \\
 + 29520 \\
 \hline
 30258
 \end{array}$$

Algorithm

Area Model → Algorithm

Compensation Strategy

$41 \times 7.38 \rightarrow 4100 \times 738 \rightarrow 3025800 \rightarrow 30258$

Grade 5 - Module 2 - Lesson 11

Division with 2-digit Divisors

590 divided by 17

estimates

60 tens ÷ 20 = 3 tens
80 ones ÷ 20 = 4 ones

$$\begin{array}{r}
 34 \\
 17 \overline{) 590} \\
 \underline{-51} \\
 80 \\
 \underline{-68} \\
 12
 \end{array}$$

check

$$\begin{array}{r}
 34 \quad 578 \\
 \times 17 \quad + 12 \\
 \hline
 238 \quad 590 \\
 + 340 \\
 \hline
 578
 \end{array}$$

Unit Language for Estimation → Standard Algorithms

Grade 5 - Module 2 - Lesson 22

Multi-Digit Decimal Division

Decompose the divisor.

Grade 5 – Module 2 – Lesson 24

Try Problem # 16.

$54 \div 900$ $5.4 \div 900$

Hint: Decompose the divisor so these problems become "mental math".

Grade 5 – Module 2 – Lesson 24

Fractions as Division

Grade 5 – Module 4 – Lesson 2

Fractions as Division

$3 \div 2 = 1\frac{1}{2}$

$3 \div 2 = 6 \text{ halves} \div 2 = 3 \text{ halves} = \frac{3}{2}$

Grade 5 – Module 4 – Lesson 2

Fractions as Division

$8 \div 3 = 2\frac{2}{3}$
24 thirds \div 3 = 8 thirds

$\frac{2\frac{2}{3}}{3} \frac{2\frac{2}{3}}{3}$ Check: $3 \times 2\frac{2}{3}$
 $\frac{2\frac{2}{3}}{3} \frac{2\frac{2}{3}}{3}$ $= 2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3}$
 $\frac{2\frac{2}{3}}{3} \frac{2\frac{2}{3}}{3}$ $= 6 + \frac{6}{3}$
 $\frac{2\frac{2}{3}}{3} \frac{2\frac{2}{3}}{3}$ $= 6 + 2$
 $\frac{2\frac{2}{3}}{3} \frac{2\frac{2}{3}}{3}$ $= 8$

Grade 5 – Module 3 – Lesson 3

Fractions as Division

$3 \div 4 = \frac{3}{4}$

Tape Diagrams

4 units = 3
1 unit = $3 \div 4$
 $= \frac{3}{4}$

Grade 5 – Module 4 – Lesson 4

Fraction of a Set

$\frac{1}{4}$ of 12 =

Array
(Similar to Area Model)

Tape Diagram

Grade 5 – Module 4 – Lessons 6 & 7

Fraction x Whole Number

2 x 6

- $6 + 6 \rightarrow 2 + 2 + 2 + 2 + 2 + 2$
- 6 copies of 2
- 6 times as much as 2

Grade 5 – Module 4 – Lessons 6 & 7

Fraction x Whole Number

$\frac{2}{3}$ of 6

Fraction of a Set:
2-thirds of 6

$\frac{2}{3} \times 6$

Repeated Addition 6 copies of 2/3

Grade 5 – Module 4 – Lesson 8

Fraction x Whole Number

Simplification Strategy

Grade 5 – Module 4 – Lesson 8 – Concept Development

Fraction x Fraction: Pictorial

$\frac{1}{3}$ of $\frac{1}{3}$
 $= \frac{1}{3} \times \frac{1}{3}$
 $= \frac{1}{9}$

Unit Fraction x Unit Fraction

$\frac{2}{3}$ of $\frac{2}{3}$
 $= \frac{2}{3} \times \frac{2}{3}$
 $= \frac{4}{9}$

Non-unit Fraction x Non-unit Fraction

Area Models

Grade 5 – Module 4 – Lessons 13 & 15

Try Problem # 17.

$\frac{1}{2}$ of $\frac{1}{3}$

$\frac{2}{3} \times \frac{3}{4}$

Hint: Draw an area model to solve. As you solve, use the language you'd use in your classroom while modeling.

Grade 5 – Module 2 – Lesson 24

Fraction x Fraction

$\frac{7}{9} \times \frac{3}{7}$

Method 1

$$\frac{7}{9} \times \frac{3}{7} =$$

$$\frac{7 \times 3}{9 \times 7} =$$

$$\frac{21}{63} =$$

$$\frac{1}{3}$$

Standard Algorithms

Method 2

$$\frac{7}{9} \times \frac{3}{7} =$$

$$\frac{7 \times 3}{9 \times 7} =$$

$$\frac{\overset{1}{\cancel{7}} \times \overset{3}{\cancel{7}}}{9 \times \overset{1}{\cancel{7}}^3} =$$

$$\frac{1}{3}$$

Grade 5 – Module 4 – Lesson 15 – Concept Development

Fraction Multiplication

Mrs. Onusko made 60 cookies for a bake sale. She sold $\frac{2}{3}$ of them and gave $\frac{3}{4}$ of the remaining cookies to the students working at the table. How many cookies did she have left?

60

20	20	20
----	----	----

Sold

20

Students

3 units = 60
1 unit = $60 \div 3 = 20$
 $20 \div 4 = 5$

$\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$
 $\frac{1}{12}$ of 60
 $= \frac{1 \times 60}{12} = 5$

5 COOKIES ARE LEFT.

Mrs. Onusko has 5 cookies left.

Grade 5 – Module 4 – Lesson 16 – Concept Development

Fraction x Fraction with Decimal Notation

Fraction Notation →

Decimal Notation

$0.1 \times 0.1 = 0.01$

Grade 5 – Module 4 – Lesson 17

Fraction x Fraction with Decimal Notation

Fraction Notation → Decimal Notation

Grade 5 – Module 4 – Lesson 17

Fraction x Fraction with Decimal Notation

2.3 x 1.8

Fraction Notation

$$2.3 \times 1.8 = \frac{23}{10} \times \frac{18}{10}$$

$$= \frac{23 \times 18}{100}$$

$$= \frac{414}{100}$$

Decimal Notation with Unit Language

$$\begin{array}{r} 23 \text{ (tenths)} \\ \times 18 \text{ (tenths)} \\ \hline 184 \\ + 230 \\ \hline 414 \text{ (hundredths)} \\ = 4.14 \end{array}$$

Grade 5 – Module 4 – Lesson 18

Area with Fractional Side Lengths

Tiling with Patty Paper

Grade 5 – Module 5 – Lesson 12

Area with Fractional Side Lengths

Tiling with Patty Paper
Grade 5 – Module 5 – Lesson 12

Area with Fractional Side Lengths

Tiling with Patty Paper
Grade 5 – Module 5 – Lesson 13

Area with Fractional Side Lengths

Grade 5 – Module 5 – Lesson 13

Whole Number ÷ Unit Fraction

Jenny has 2 pounds of pecans. If she puts $\frac{1}{2}$ pound in each bag, how many bags can she make?

Measurement Division & Tape Diagrams

$2 \div \frac{1}{2} = 4$

She can make 4 bags.

Grade 5 – Module 4 – Lesson 25

Whole Number ÷ Unit Fraction

Jenny has 2 pounds of pecans. If this is $\frac{1}{2}$ the number she needs to make pecan pies, how many pounds will she need?

Partitive Division & Number Line

Grade 5 – Module 4 – Lesson 25

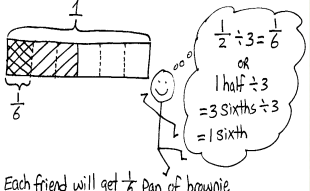
Try Problem # 18.

Hint: Use both a tape diagram and a number line to support your answer to each question.

If 2 is $\frac{1}{4}$, the whole is 8. 1 whole. es. 2 wholes.

Unit Fraction ÷ Whole Number

Nolan gives some pans of brownies to his 3 friends to share equally. If he has $\frac{1}{2}$ a pan of brownies, how much of a pan will each friend get?



Each friend will get $\frac{1}{6}$ pan of brownie.

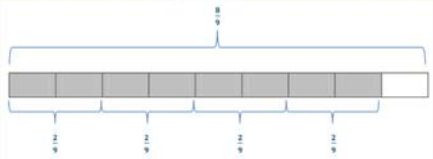
Grade 5 – Module 4 – Lesson 26

Agenda

- Foundations of Multiplication and Division
- Grade 3 Progression of Multiplication and Division
- Grade 4 Progression of Multiplication and Division
- Grade 5 Progression of Multiplication and Division
- **Continuation of Multiplication and Division in the Middle Grades**

Continuation of Multiplication and Division in the Middle Grades

The students will complete their work with fraction operations in Grade 6 by dividing fractions by fractions. (6.NS.1)

$$\frac{8}{9} \div \frac{2}{9}$$


Grade 6 – Module 2 – Lesson 4

Continuation of Multiplication and Division in the Middle Grades

$\frac{3}{4} \div \frac{1}{4}$

Draw a model to support your answer.

There are 3 one-fourths in three-fourths.

Grade 6 – Module 2 – Lesson 5

Key Points

- Extends the work of K-2.
- Use of unit language.
- Familiar models support understanding.
- Fractions operate as whole numbers.
- Supports “Story of Ratios”.
