COMMON CORE

Eureka Math: *A Story of Units*

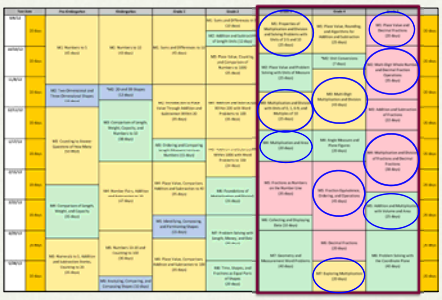
Major Work of the Grade Band
3-5: Multiplication and Division with Whole Numbers and Fractions

Session Objectives

Discover that multiplication and division are, respectively, composing and decomposing equal units.

Understand how the part to whole and whole to part relationships apply to multiplication and division.

Curriculum Map for A Story of Units

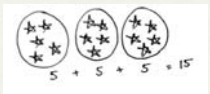


Agenda

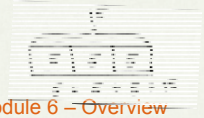
- **Foundations of Multiplication and Division**
- Grade 3 Progression of Multiplication and Division
- Grade 4 Progression of Multiplication and Division
- Grade 5 Progression of Multiplication and Division
- Continuation of Multiplication and Division in the Middle Grades

Grade 2 Module 6: Foundations of Multiplication and Division

Equal groups and repeated addition



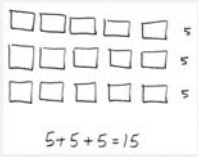
- Tape diagrams and repeated addition



Grade 2 – Module 6 – Overview

Grade 2 Module 6: Foundations of Multiplication and Division

- Arrays and repeated addition

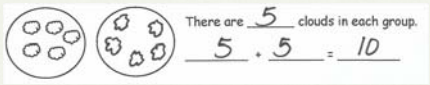


- Tiled rectangular arrays

Grade 2 – Module 6 – Lessons 7 and 11

Grade 2 Module 6: Foundations of Multiplication and Division

- Doubles

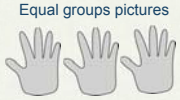


- Even and Odd

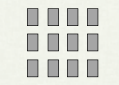
Grade 2 – Module 6 – Lessons 17 and 18

Models Concrete → Pictorial → Abstract

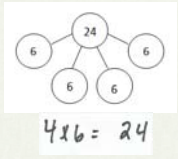
Equal groups pictures



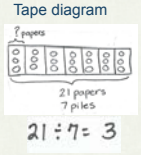
Array



Number Bond



Tape diagram



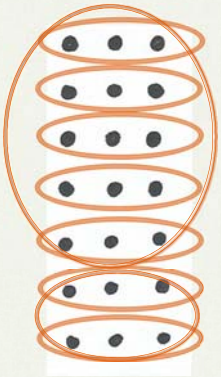
Strategies

Simple → Complex

Level 1: One object is one unit.

Level 2: A group is a unit.

Level 3: Multiple groups are units.



Agenda

- Foundations of Multiplication and Division
- **Grade 3 Progression of Multiplication and Division**
- Grade 4 Progression of Multiplication and Division
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- Continuation of Multiplication and Division in the Middle Grades

Grade 3: Multiplication and Division in A Story of Units

By the end of Grade 3, students will:

Use multiplication and division within 100 to solve word problems. (3.OA.3)

Find the unknown number in a multiplication or division equation. (3.OA.4)

Apply properties of operations as strategies to multiply and divide. (3.OA.5)

Fluently multiply within 100. (3.OA.7)

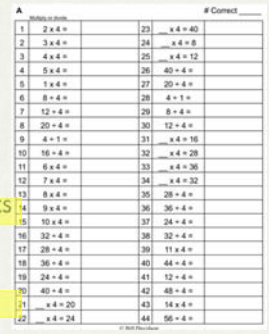
Identify and explain patterns in the multiplication table. (3.OA.9)

Multiply one-digit whole numbers by multiples of 10. (3.NBT.3)

Relate area to multiplication. (3.MD.7)

Grade 3 Fluency

- Group Counting
- Use the Commutative Property
- Find the Common Products
- "Multiply By" Pattern Sheets
- Multiply by Different Units
- Sprints

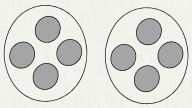


Multiplication

Repeated addition

$2 + 2 + 2 + 2 + 2 + 2 = 12$
6 twos = 12
$6 \times 2 = 12$

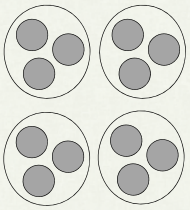
Equal groups pictures



4 + 4 = 8
2 fours = 8
2 × 4 = 8

Grade 3 – Module 1 – Lesson 1

Relate Multiplication to the Array Model.

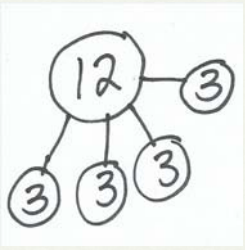


4 threes = 12
4 x 3 = 12

Equal Groups Pictures → Array → Multiplication Number Sentence

Grade 3 – Module 1 – Lesson 2

Relate Multiplication to Number Bonds.



4 threes = 12
4 x 3 = 12

Number Bond

Grade 3 – Module 1 – Lesson 3

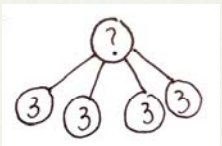
RDW Process

Read, Draw, Write

1. **Read** the problem.
2. **Draw** a picture representing the problem.
3. **Write** a number sentence to solve a problem and a statement answering the problem.

RDW Process

Jordan uses 3 lemons to make 1 pitcher of lemonade. He makes 4 pitchers. How many lemons does he use altogether? Use the RDW process to show your solution.



4 x 3 = 12

Jordan uses 12 lemons altogether.

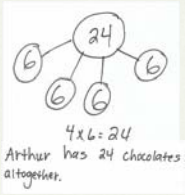
Use the RDW process to solve this problem.

Try Problem # 1.

Arthur has 4 boxes of chocolates. Each box has 6 chocolates inside. How many chocolates does Arthur have altogether?

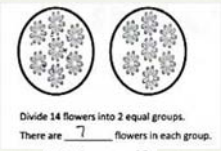
Equal Groups Pictures

Array



Number Bond

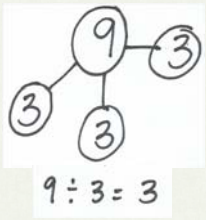
Division



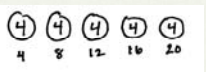
14 ÷ 2 = 7

Equal Groups Pictures

Number Bond



Skip-counting



20 ÷ 4 = 5

Grade 3 – Module 1 – Lessons 4 and 5

Arrays Used to Relate Multiplication and Division

Rick puts 15 tennis balls into cans. Each can has 3 tennis balls. How many cans does Rick use?

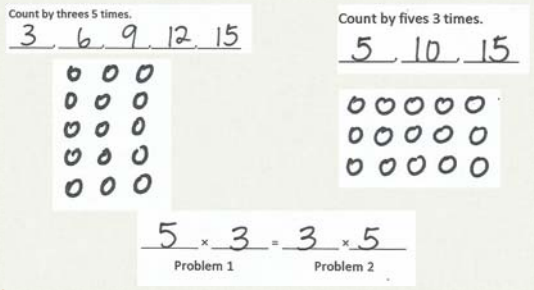
5 x 3 = 15

15 ÷ 3 = 5

The number in the blanks represents the number of groups.

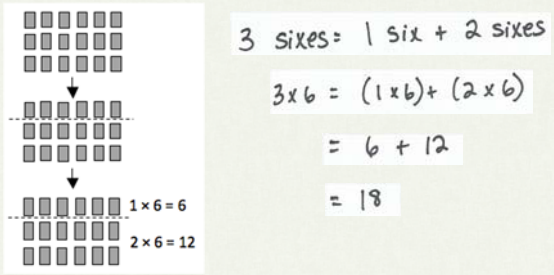
Grade 3 – Module 1 – Lesson 6

Arrays and the Commutative Property



Grade 3 – Module 1 – Lesson 8

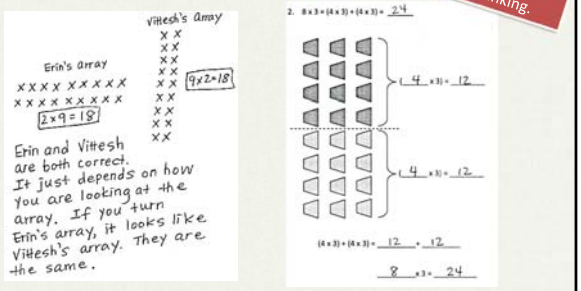
Arrays and the Distributive Property



Grade 3 – Module 1 – Lesson 10

Try Problems # 2 and # 3.

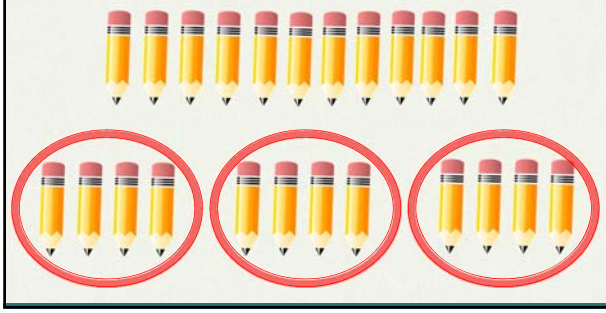
Hint: Draw 2 arrays to model Erin's and Vittesh's thinking.



Grade 3 – Module 1 – Lessons 8 and 10

Types of Division: Measurement

12 ÷ 4 = ?

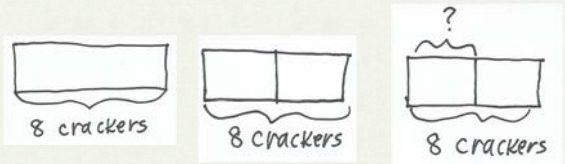


Types of Division: Partitive

12 ÷ 4 = ?

Tape Diagrams and Types of Division

2 students equally share 8 crackers. How many crackers does each student get?



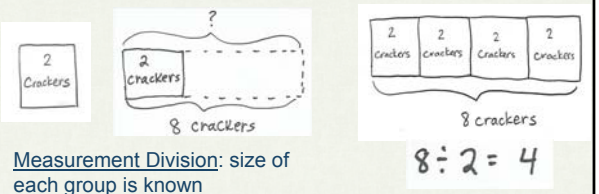
Partitive Division: number of groups is known

8 ÷ 2 = 4

Grade 3 – Module 1 – Lesson 12

Tape Diagrams and Types of Division

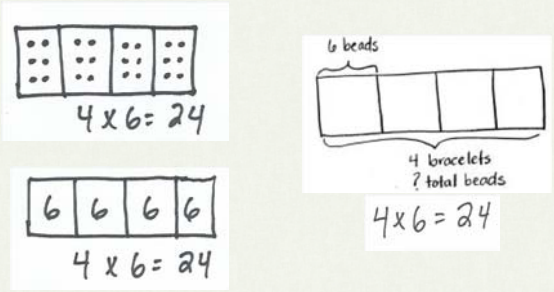
There are 8 crackers, but this time each student gets 2. How many students get crackers?



Measurement Division: size of each group is known

Grade 3 – Module 1 – Lesson 12

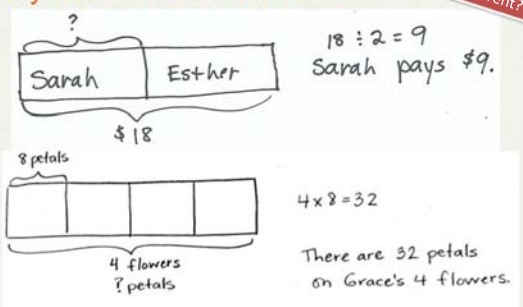
Use a Tape Diagram to Model and Solve Multiplication.



Grade 3 – Module 1 – Lessons 14 and 15

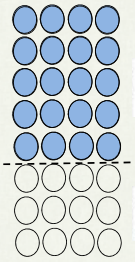
Try Problems # 4 and # 5.

How are the tape diagrams similar? Different?



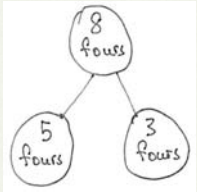
Grade 3 – Module 1 – Lessons 12 and 15

Model the 5 + n Pattern as a Strategy for Multiplying.



(5 x 4) = 20

(3 x 4) = 12

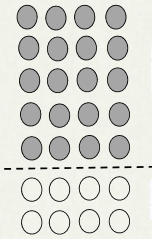


8 fours = 5 fours + 3 fours
8 x 4 = (5 x 4) + (3 x 4)
= (5 + 3) x 4

Grade 3 – Module 1 – Lesson 16

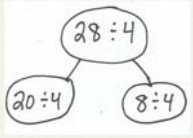
Apply the Distributive Property to Decompose Units to Divide.

28 ÷ 4 = ____



(20 ÷ 4) = 5

(8 ÷ 4) = 2



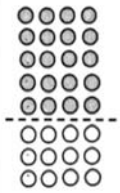
(28 ÷ 4) = (20 ÷ 4) + (8 ÷ 4)
= 5 + 2
= 7

Grade 3 – Module 1 – Lesson 19

Try Problem # 6.

Why is the array decomposed this way?

32 ÷ 4 = 8



(20 ÷ 4) = 5

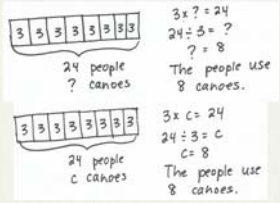
(12 ÷ 4) = 3

(32 ÷ 4) = (20 ÷ 4) + (12 ÷ 4)
= 5 + 3
= 8

Grade 3 – Module 1 – Lesson 19

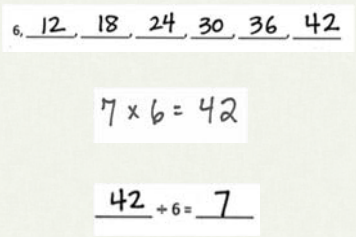
Multiply and Divide with Familiar Facts Using a Letter to Represent the Unknown.

Twenty-four people line up to use the canoes at the park. Three people are assigned to each canoe. How many canoes are used for all 24 people?



Grade 3 – Module 3 – Lesson 3

Count by Units of 6 and 7 to Multiply and Divide.

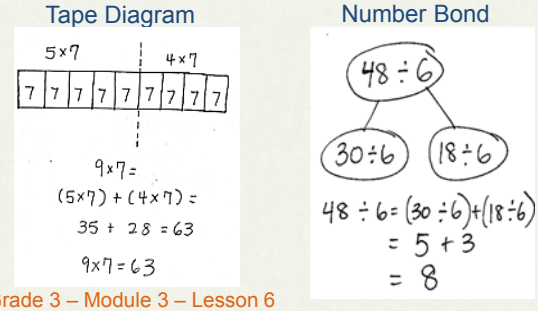


Grade 3 – Module 3 – Lesson 4

Model the Distributive Property with Tape Diagrams and Number Bonds.

Tape Diagram

Number Bond



Grade 3 – Module 3 – Lesson 6

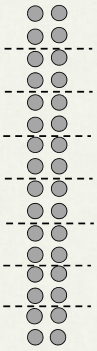
Model the Associative Property as a Strategy to Multiply.

$16 \times 2 = (8 \times 2) \times 2$

$= 8 \times (2 \times 2)$

$= 8 \times 4$

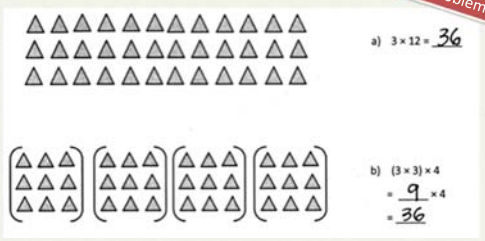
$= 32$



Grade 3 – Module 3 – Lesson 9

Try Problem # 7.

Is there a different way to use the associative property to solve this problem?

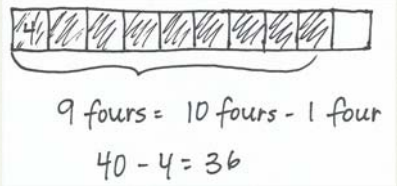


Grade 3 – Module 3 – Lesson 9

Apply 9 = 10 – 1 as a Strategy to Multiply by 9.

9 = 10 - 1

9 x 4



Grade 3 – Module 3 – Lesson 12

Identify and Use Arithmetic Patterns to Multiply.

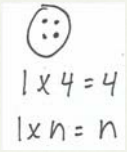
3 x 9 = ____

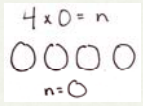
9 = 10 – 1
3 nines = 3 tens – 3 ones

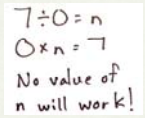
3 x 9 = 30 – 3
3 x 9 = 27

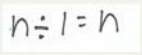
Grade 3 – Module 3 – Lesson 14

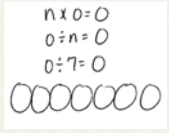
Multiply and Divide Using Units of 1 and 0.

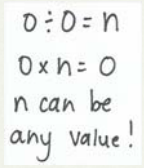






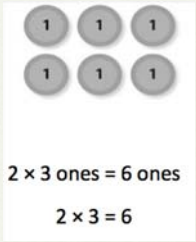


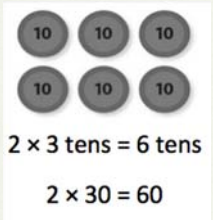




Grade 3 – Module 3 – Lesson 16

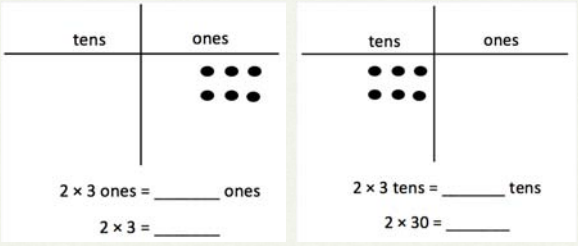
Multiply by Multiples of 10 Using Place Value Disks.





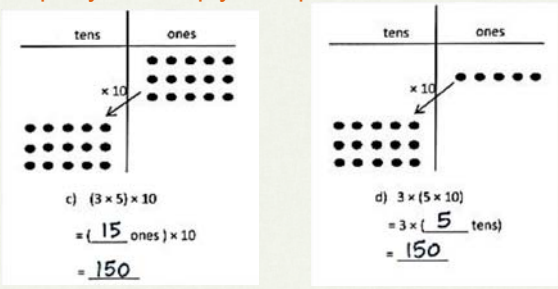
Grade 3 – Module 3 – Lesson 19

Multiply by Multiples of 10 Using a Place Value Chart.



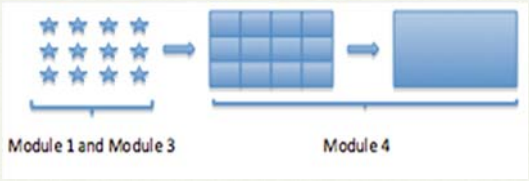
Grade 3 – Module 3 – Lesson 19

Use Place Value Strategies & the Associative Property to Multiply Multiples of 10.



Grade 3 – Module 3 – Lesson 20

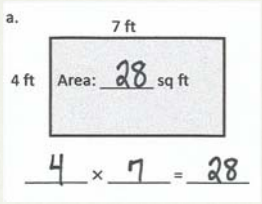
Grade 3 Module 4: Multiplication and Area

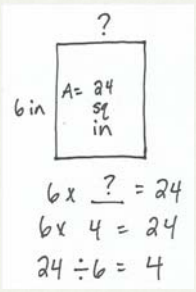


Array → Tiled Rectangular Array → Area Model

Grade 3 – Module 4 – Overview

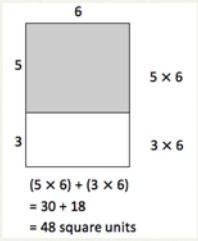
Multiplying and Dividing With the Area Model





Grade 3 – Module 4 – Lesson 8

Apply the Distributive Property as a Strategy to Find Area.



Distributive Property

Grade 3 – Module 4 – Lessons 10 and 11

Agenda

- Foundations of Multiplication and Division
- Grade 3 Progression of Multiplication and Division
- **Grade 4 Progression of Multiplication and Division**
- Grade 5 Progression of Multiplication and Division
- Continuation of Multiplication and Division in the Middle Grades

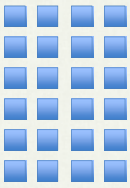
A Continuation of a Story of Units in Grade 4

By the end of Grade 4 students will:

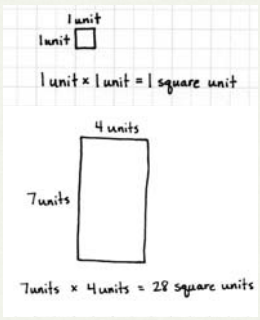
- Multiply up to a four-digit number by a one-digit number. (4.NBT.5)
- Multiply two two-digit numbers. (4.NBT.5)
- Multiply a fraction by a whole number (4.NF.4)
- Divide up to four-digit numbers by one digit divisors finding whole number quotients with remainders. (4.NBT.6)
- Interpret multiplication as comparison. (4.OA.1)
- Solve word problems using multiplication and division. (4.OA.2,3)

Multiplication Models

Array → Area Model

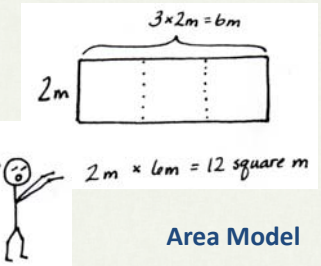


$A = l \times w$



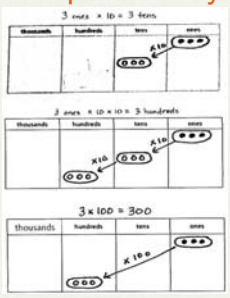
Grade 4 – Module 3 – Lesson 1

Multiplicative Comparison Word Problems



Grade 4 – Module 3 – Lesson 2

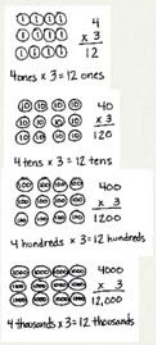
Multiplication by 10, 100, and 1000



Place Value Chart

Number Disks

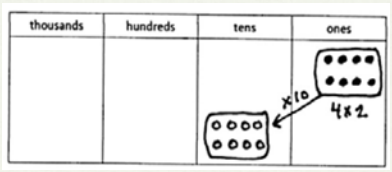
Unit Language



Grade 4 – Module 3 – Topic B

Multiplication by 10, 100, and 1000

4×20 ➡ 4×2 tens ➡ $(4 \times 2) \times 10$ ➡ 8×10

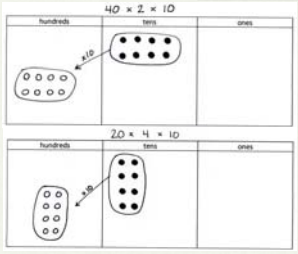


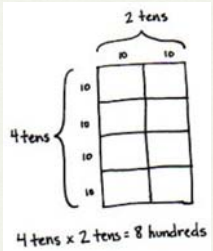
Grade 4 – Module 3 – Topic B

8 × 2 ones = 16 ones
8 × 2 tens = 16 tens
8 × 2 hundreds = 16 hundreds
8 × 2 thousands = 16 thousands

Multiplication by 10, 100, and 1000

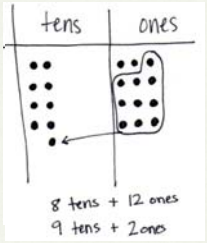
40×20

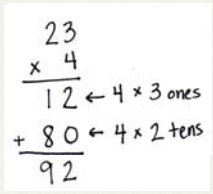




Grade 4 – Module 3 – Topic B

Multiplication of Up to 4 Digits by Single-Digit Whole Numbers

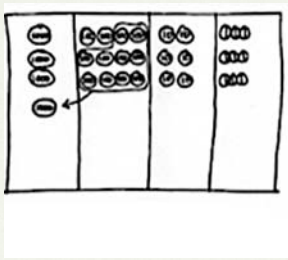


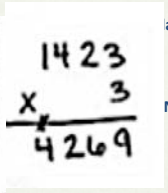


Place Value Chart & Number Disks → Partial Products

Grade 4 – Module 3 – Lesson 7

Multiplication of Up to 4 Digits by Single-Digit Whole Numbers





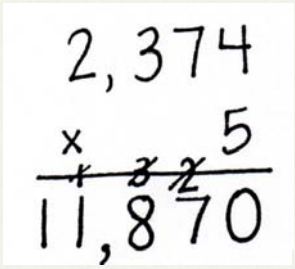
ace Value Chart

Number Disks

Algorithm

Grade 4 – Module 3 – Lessons 8-10

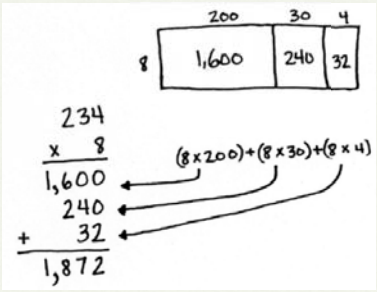
Multiplication of Up to 4 Digits by Single-Digit Whole Numbers



Algorithm

Grade 4 – Module 3 – Lessons 9-10

Multiplication of Up to 4 Digits by Single-Digit Whole Numbers



Area Model

Distributive Property

Partial Products

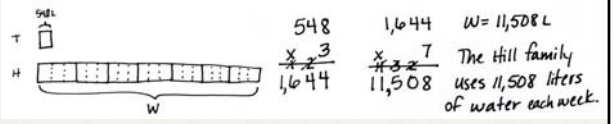
Grade 4 – Module 3 – Lesson 11

Try Problem # 8.

Hint: This is a 2-step problem. Try solving using a variety of methods: algorithm, area model, distributive property, partial products, or number disks.

Use RDW to solve.

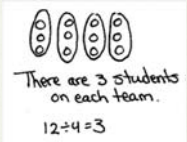
The Turner family uses 548 liters of water each day. The Hill family uses 3 times as much water as the Turner family per day. How much water does the Hill family use each week?



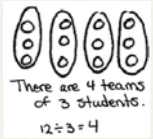
Grade 4 – Module 3 – Lesson 12

Division of Tens and Ones with Successive Remainders

There are 12 students in PE class separated into 4 teams. How many students are on each team?



There are 12 students in PE class. The teacher makes teams of 3 students each. How many teams are there?

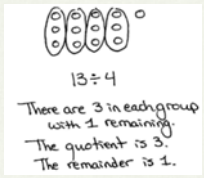


Array

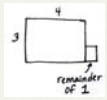
Grade 4 – Module 3 – Lesson 14

Division of Tens and Ones with Successive Remainders

There are 13 students in PE class separated into 4 teams. How many students are on each team?



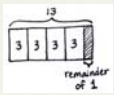
Array → Area Model



~~13 ÷ 4 = 3R1~~

13 ÷ 4 ≠ 7 ÷ 2

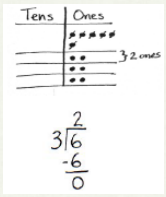
Tape Diagram



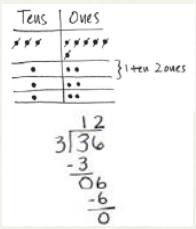
Grade 4 – Module 3 – Lesson 14

Division of Tens and Ones with Successive Remainders

6 ÷ 3



36 ÷ 3



Place Value Chart

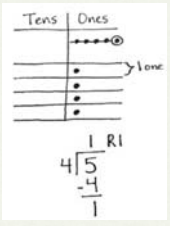
Number Disks

Algorithm

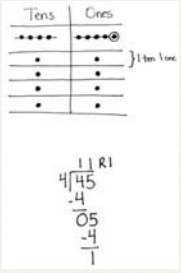
Grade 4 – Module 3 – Lesson 16

Division of Tens and Ones with Successive Remainders

5 ÷ 4



45 ÷ 4



Place Value Chart

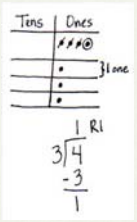
Number Disks

Algorithm

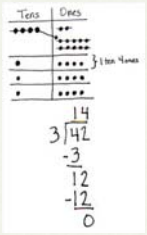
Grade 4 – Module 3 – Lesson 16

Division of Tens and Ones with Successive Remainders

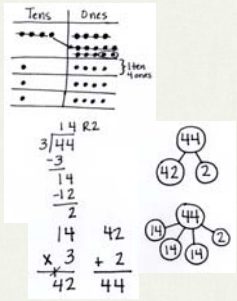
4 ÷ 3



42 ÷ 3



44 ÷ 3



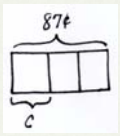
Grade 4 – Module 3 – Lesson 17

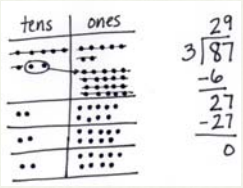
Try Problem # 9.

Try modeling with number disks to support the algorithm.

Use RDW to solve:

The Grand Market sells 3 pounds of oranges for 87 cents. How much does 1 pound of oranges cost at Grand Market?



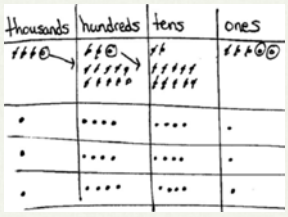


c = 29¢

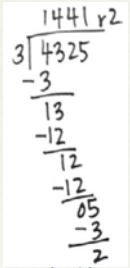
1 pound of oranges costs 29 cents.

Grade 4 – Module 3 – Lesson 18

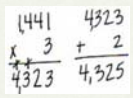
Division of Thousands, Hundreds, Tens, and Ones



Number Disks



Algorithm



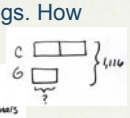
Checking Work with Multiplication

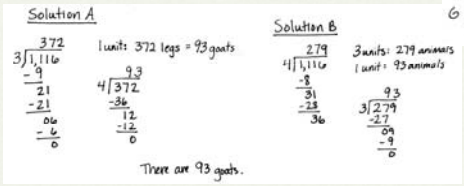
Grade 4- Module 3 – Lesson 29

Try modeling with number disks to support the algorithm.

Try Problem #10.

There are twice as many cows as goats on a farm. All the cows and goats have a total of 1,116 legs. How many goats are there?





Tape Diagram

Algorithm

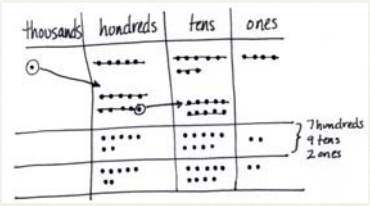
Grade 4 – Module 3 – Lesson 29

Try solving using number disks and the standard algorithm at the same time.

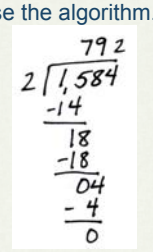
Try Problem # 11.

Solve 1,584 ÷ 2:

a) Using number disks.



b) Use the algorithm.



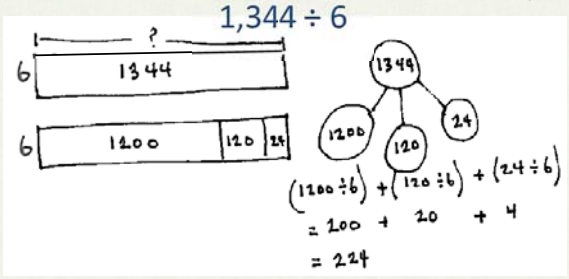
Grade 4 – Module 3 – Lesson 33

The Area Model for Division

$A = l \times w$

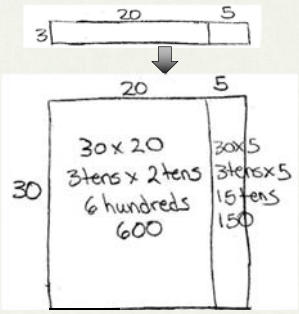
$A \div w = l$

1,344 ÷ 6



Grade 4 – Module 3 – Lesson 33

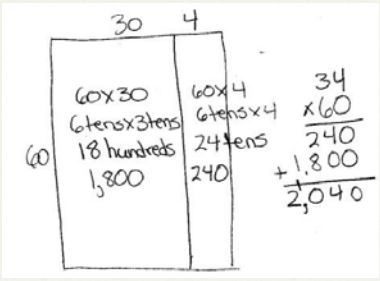
Multiplication of 2-Digit by 2-Digit Numbers



Area Model

Grade 4 – Module 3 – Lesson 35

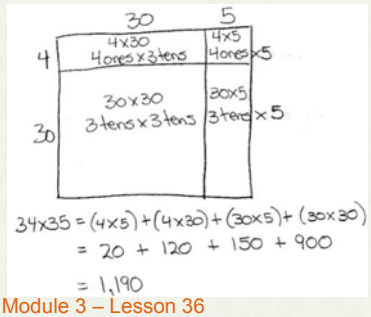
Multiplication of 2-Digit by 2-Digit Numbers



Area Model

Grade 4 – Module 3 – Lesson 35

Multiplication of 2-Digit by 2-Digit Numbers



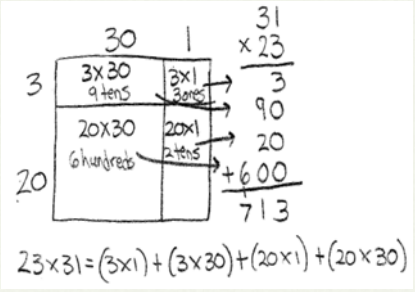
Area Model

Distributive Property

4 Partial Products

Grade 4 – Module 3 – Lesson 36

Multiplication of 2-Digit by 2-Digit Numbers



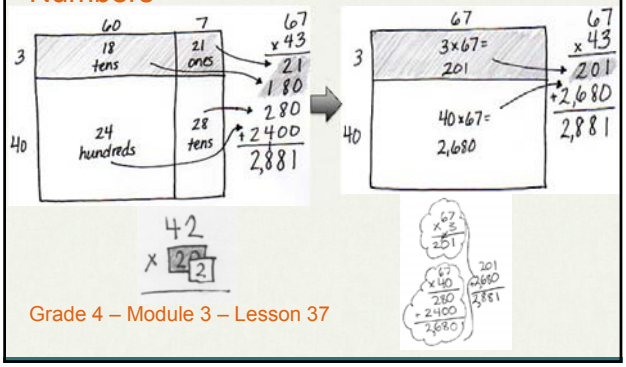
Area Model

Distributive Property

4 Partial Products

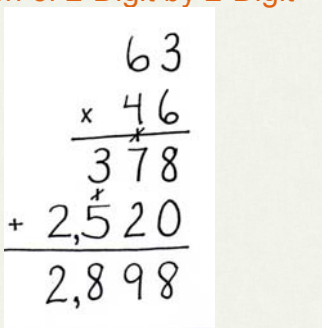
Grade 4 – Module 3 – Lesson 36

Multiplication of 2-Digit by 2-Digit Numbers



Grade 4 – Module 3 – Lesson 37

Multiplication of 2-Digit by 2-Digit Numbers



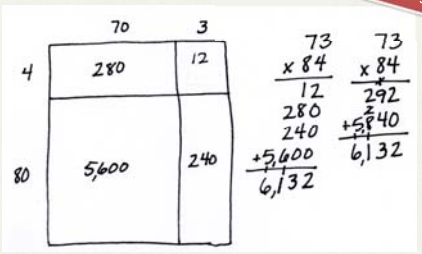
Algorithm

Grade 4 – Module 3 – Lesson 38

Try Problem #12.

Try using an area model, 4 partial products, and the standard algorithm to solve.

Solve: 84 × 73.



Grade 4 – Module 3 – Lesson 38

Multiplication of Fractions

= 3 apples

= 3 ones

= 3 fourths

1 fourth + 1 fourth + 1 fourth = 3 fourths → $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$

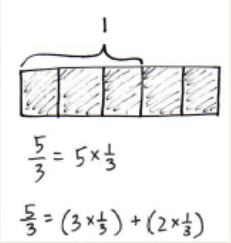
3 × (1 fourth) = 3 fourths → $3 \times \frac{1}{4}=\frac{3}{4}$

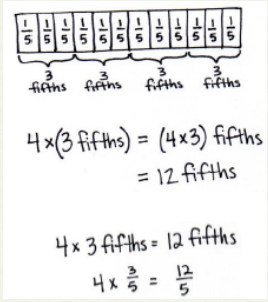
(3 × 1) fourths = 3 fourths → $\frac{3 \times 1}{4}=\frac{3}{4}$

Grade 4 – Module 5 – Topic A

Multiplication of Fractions

Tape Diagram

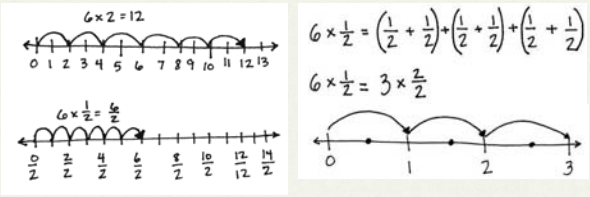




Grade 4 – Module 5 – Topic E

Multiplication of Fractions

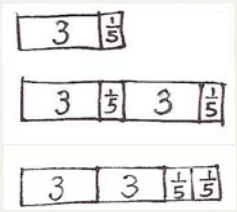
Number Line



Grade 4 – Module 5 – Topic E

Multiplication of Fractions

Tape Diagram



Distributive Property

$$2 \times 3\frac{1}{5} = (2 \times 3) + (2 \times \tfrac{1}{5})$$
$$= 6 + \tfrac{2}{5}$$
$$= 6\tfrac{2}{5}$$

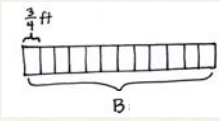
Grade 4 – Module 5 – Lessons 37 & 38

Try Problem #13.

Try using the associative property to solve.

Use RDW to solve.

A bricklayer places 12 bricks along an outside wall of a shed. Each brick is ¾ foot long. How many feet long is that wall of the shed?



$12 \times \frac{3}{4} = \frac{12 \times 3}{4} = \frac{36}{4} = 9$

B = 9 feet

The wall of the shed is 9 feet long.

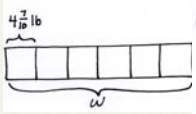
Grade 4 – Module 5 – Lesson 36

Try Problem #14.

Try using the distributive property to solve.

Use RDW to solve.

Kelly's new puppy weighed 4 7/10 pounds when she brought him home. Now the puppy weighs six times as much as he did when he came home. How much does the dog weigh now?



$6 \times 4\frac{7}{10} = (6 \times 4) + (6 \times \frac{7}{10})$
$= 24 + \frac{42}{10}$
$= 24 + 4\frac{2}{10}$
$= 28\frac{2}{10}$

$w = 28\frac{2}{10}$ lb
The dog weighs $28\frac{2}{10}$ pounds now.

Grade 4 – Module 5 – Lesson 37

Agenda

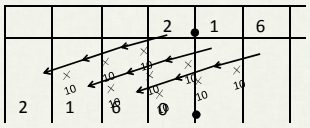
- Foundations of Multiplication and Division
- Grade 3 Progression of Multiplication and Division
- Grade 4 Progression of Multiplication and Division
- **Grade 5 Progression of Multiplication and Division**
- Continuation of Multiplication and Division in the Middle Grades

A Continuation of a Story of Units in Grade 5

By the end of Grade 5 students will:

- Use whole-number exponents to denote power of 10. (5.NBT.2)
- Fluently multiply multi-digit whole numbers. (5.NBT.5)
- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. (5.NBT.6)
- Multiply and divide decimals to hundredths. (5.NBT.7)
- Interpret a fraction as division of the numerator by the denominator. (5.NF.3)
- Multiply a fraction or whole number by a fraction. (5.NF.4)
- Solve real world problems involving multiplication of fractions and mixed numbers. (5.NF.6)
- Divide unit fractions by whole numbers and whole numbers by unit fractions. (5.NF.7)

Patterns on the Place Value Chart: Multiplication



Place Value Chart

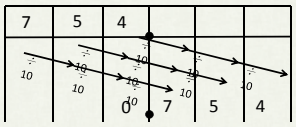
$2.16 \times 10^3 =$
$2.16 \times 10 \times 10 \times 10 =$
2,160.

Unit Language

$2.16 \times 10^3 =$
216 hundredths $\times 10^3 =$
216,000 hundredths =
2,160.

Grade 5 – Module 1 – Lessons 5 & 6

Patterns on the Place Value Chart: Division

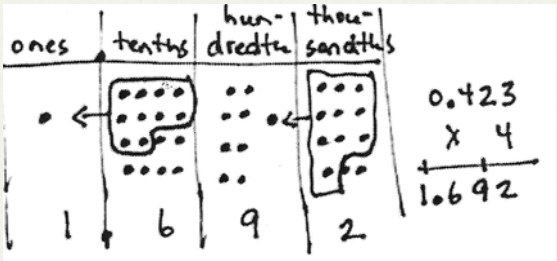


Place Value Chart

$754 \div 10^3 =$
$754 \div 10 \div 10 \div 10 =$
.754

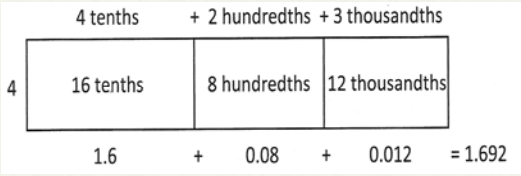
Grade 5 – Module 1 – Lessons 5 & 6

Decimal Multiplication



Grade 5 – Module 1 - Lesson 11

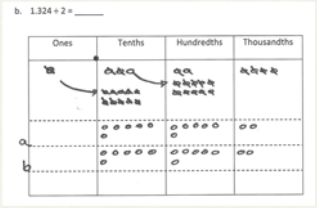
Decimal Multiplication

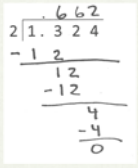


Area Model & Unit Language

Grade 5 – Module 1 - Lesson 11

Dividing Decimals





Place Value Chart → **Standard Algorithm**

Grade 5 – Module 1 - Lesson 14

Try Problem # 15.

Hint: When completing the division algorithm, use language that connects it to the work shown on the place value chart.

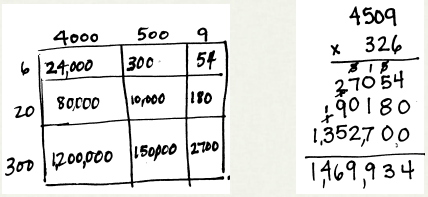
The total weight of 5 pieces of butter is 3.445 kg. What is the weight of each piece of butter?

Use the place value chart and algorithm to solve.

Grade 5 – Module 1 – Lesson 14

Multi-Digit Whole Number Multiplication

4509 x 326

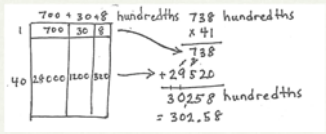


Area Model → Standard Algorithm

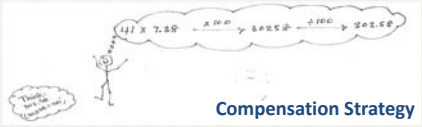
Grade 5 - Module 2 – Lesson 7

Multi-Digit Multiplication with Decimals

41 x 7.38



Area Model → Algorithm

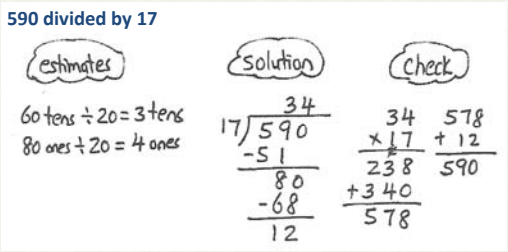


Compensation Strategy

Grade 5 – Module 2 – Lesson 11

Division with 2-digit Divisors

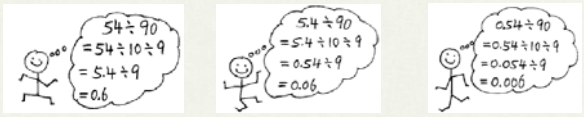
590 divided by 17



Unit Language for Estimation → Standard Algorithms

Grade 5 – Module 2 – Lesson 22

Multi-Digit Decimal Division



Decompose the divisor.

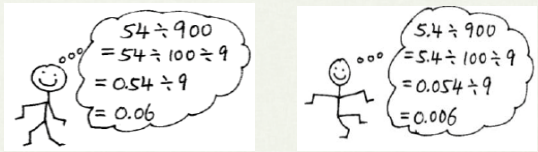
Grade 5 – Module 2 – Lesson 24

Try Problem # 16.

Hint: Decompose the divisor so these problems become "mental math".

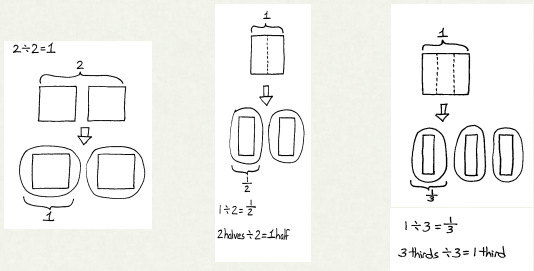
54 ÷ 900

5.4 ÷ 900



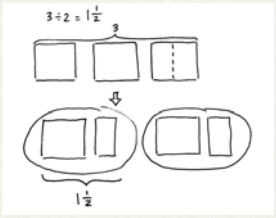
Grade 5 – Module 2 – Lesson 24

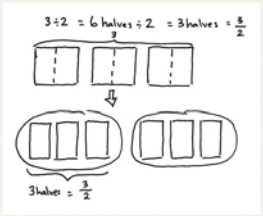
Fractions as Division



Grade 5 – Module 4 – Lesson 2

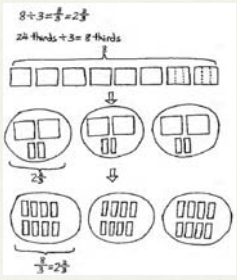
Fractions as Division

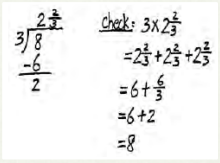




Grade 5 – Module 4 – Lesson 2

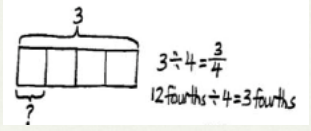
Fractions as Division



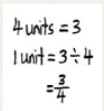


Grade 5 – Module 3 – Lesson 3

Fractions as Division



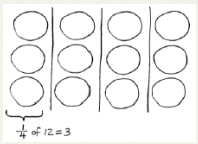
Tape Diagrams



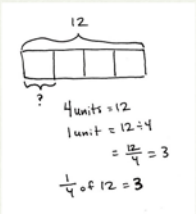
Grade 5 – Module 4 – Lesson 4

Fraction of a Set

$\frac{1}{4}$ of 12 =



Array
(Similar to Area Model)



Tape Diagram

Grade 5 – Module 4 – Lessons 6 & 7

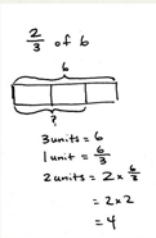
Fraction x Whole Number

2 x 6

- **6 + 6 → 2 + 2 + 2 + 2 + 2 + 2**
- **6 copies of 2**
- **6 times as much as 2**

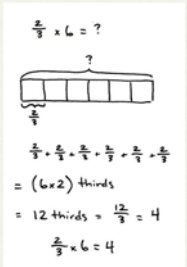
Grade 5 – Module 4 – Lessons 6 & 7

Fraction x Whole Number



Fraction of a Set:
2-thirds of 6

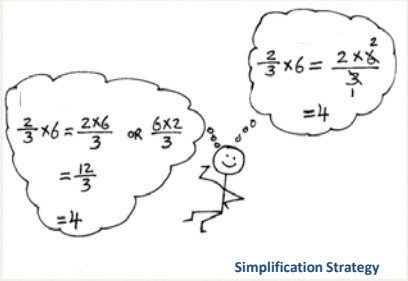
$\frac{2}{3} \times 6$



Repeated Addition 6 copies of 2/3

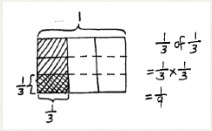
Grade 5 – Module 4 – Lesson 8

Fraction x Whole Number

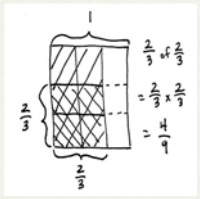


Grade 5 – Module 4 – Lesson 8 – Concept Development

Fraction x Fraction: Pictorial



Unit Fraction x Unit Fraction



Non-unit Fraction x Non-unit Fraction

Grade 5 – Module 4 – Lessons 13 & 15

Try Problem # 17.

Hint: Draw an area model to solve. As you solve, use the language you'd use in your classroom while modeling.

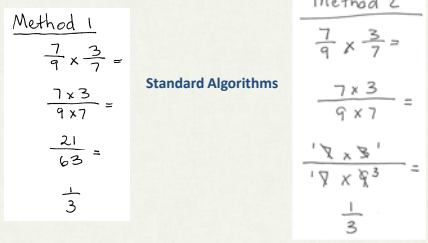
1/2 of 1/3

2/3 x 3/4

Grade 5 – Module 2 – Lesson 24

Fraction x Fraction

7/9 x 3/7

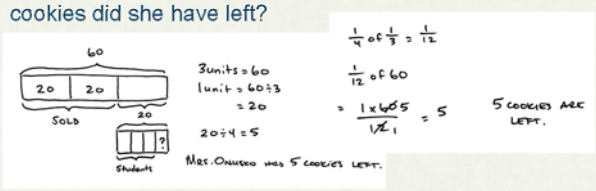


Standard Algorithms

Grade 5 – Module 4 – Lesson 15 – Concept Development

Fraction Multiplication

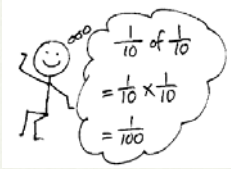
Mrs. Onusko made 60 cookies for a bake sale. She sold 2/3 of them and gave ¾ of the remaining cookies to the students working at the table. How many cookies did she have left?

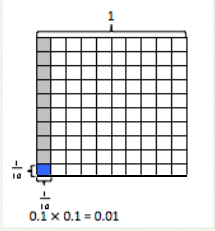


Grade 5 – Module 4 – Lesson 16 – Concept Development

Fraction x Fraction with Decimal Notation

Fraction Notation → Decimal Notation

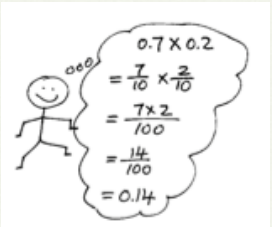


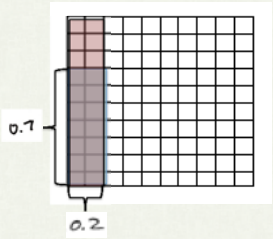


Grade 5 – Module 4 – Lesson 17

Fraction x Fraction with Decimal Notation

Fraction Notation → Decimal Notation





Grade 5 – Module 4 – Lesson 17

Fraction x Fraction with Decimal Notation

2.3 x 1.8

$2.3 \times 1.8 = \frac{23}{10} \times \frac{18}{10}$

$= \frac{23 \times 18}{100}$

$= \frac{414}{100}$

Fraction Notation

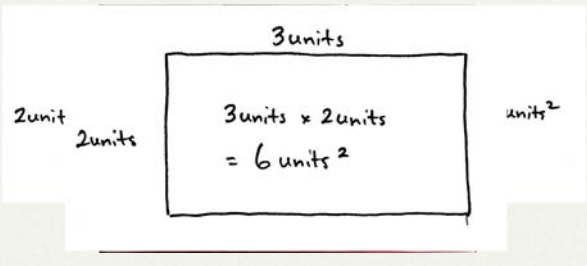
2 3 (tenths)
× 1 8 (tenths)
1 8 4
+ 2 3 0
4 1 4 (hundredths)

= 4.14

Decimal Notation with Unit Language

Grade 5 – Module 4 – Lesson 18

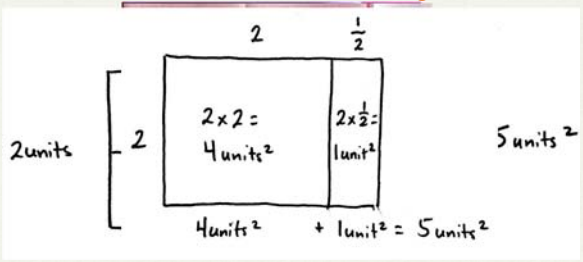
Area with Fractional Side Lengths



Tiling with Patty Paper

Grade 5 – Module 5 – Lesson 12

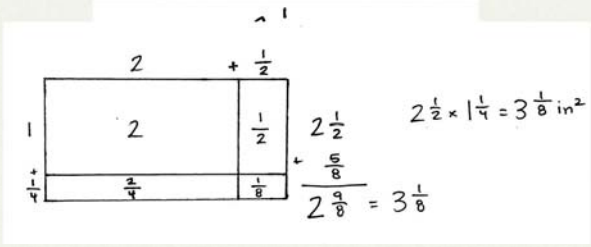
Area with Fractional Side Lengths



Tiling with Patty Paper

Grade 5 – Module 5 – Lesson 12

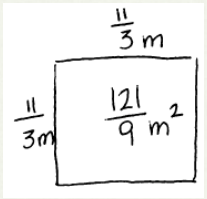
Area with Fractional Side Lengths

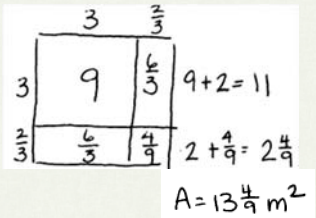


Tiling with Patty Paper

Grade 5 – Module 5 – Lesson 13

Area with Fractional Side Lengths

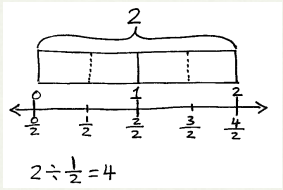




Grade 5 – Module 5 – Lesson 13

Whole Number ÷ Unit Fraction

Jenny has 2 pounds of pecans. If she puts ½ pound in each bag, how many bags can she make?



$2 \div \frac{1}{2} = 4$

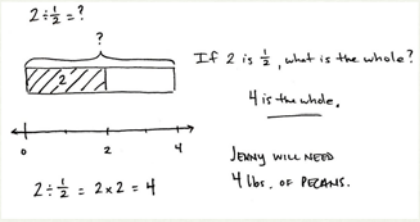
She can make 4 bags.

Measurement Division & Tape Diagrams

Grade 5 – Module 4 – Lesson 25

Whole Number ÷ Unit Fraction

Jenny has 2 pounds of pecans. If this is ½ the number she needs to make pecan pies, how many pounds will she need?

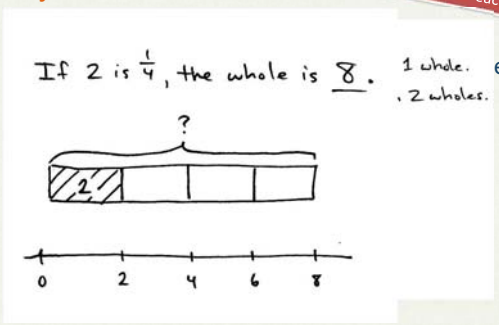


Partitive Division & Number Line

Grade 5 – Module 4 – Lesson 25

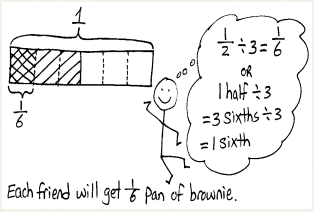
Try Problem # 18.

Hint: Use both a tape diagram and a number line to support your answer to each question.



Unit Fraction ÷ Whole Number

Nolan gives some pans of brownies to his 3 friends to share equally. If he has ½ a pan of brownies, how much of a pan will each friend get?



Grade 5 – Module 4 – Lesson 26

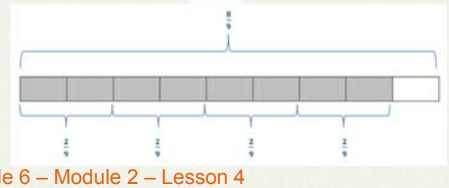
Agenda

- Foundations of Multiplication and Division
- Grade 3 Progression of Multiplication and Division
- Grade 4 Progression of Multiplication and Division
- Grade 5 Progression of Multiplication and Division
- **Continuation of Multiplication and Division in the Middle Grades**

Continuation of Multiplication and Division in the Middle Grades

The students will complete their work with fraction operations in Grade 6 by dividing fractions by fractions. (6.NS.1)

$\frac{8}{9} \div \frac{2}{9}$

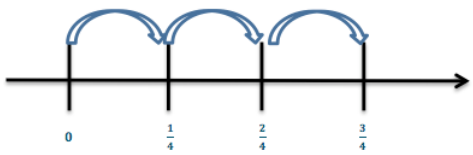


Grade 6 – Module 2 – Lesson 4

Continuation of Multiplication and Division in the Middle Grades

$\frac{3}{4} \div \frac{1}{4}$

Draw a model to support your answer.



There are 3 one-fourths in three-fourths.

Grade 6 – Module 2 – Lesson 5

Key Points

- Extends the work of K-2.
- Use of unit language.
- Familiar models support understanding.
- Fractions operate as whole numbers.
- Supports "Story of Ratios".