H

Tape Diagrams

Opening Exercise

- If you have any tape diagramming experience, try to solve one of these problems using tape diagrams. If not, try to solve it algebraically.

94 children are in a reading club. One-third of the boys and three-sevenths of the girls prefer fiction. If 36 students prefer fiction, how many girls prefer fiction?

Jess spent one-third of her money on a cell phone, and two-fifths of the remainder on accessories. When she got home her parents gave her $350. The ratio of money she had in the end to the money she had before was 4:3. How much money did she have at first?

COMMON CORE

A Story of Ratios

A Coherent Study of
The Major Work of the Grade Band
Grades 6-8

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Session Objectives

1. Develop deep understanding of mathematics of the major work of Grades 6-8.
2. Study the cross-grade coherence of Grade 6-8 to increase our capacity to:
   - Bridge gaps in previous knowledge, and
   - Ensure coherence for development of the mathematics in future grades.

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Key Areas of Focus in Mathematics

K–2	Addition and subtraction - concepts, skills, and problem solving and place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra and linear functions

Cluster Designations – Grade 6

Key: ■ Major Clusters; ◻ Supporting Clusters; ○ Additional Clusters

Ratios and Proportional Reasoning

- ■ Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

- ■ Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- ○ Compute fluently with multi-digit numbers and find common factors and multiples.
- ■ Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

- ■ Apply and extend previous understandings of arithmetic to algebraic expressions.
- ■ Reason about and solve one-variable equations and inequalities.
- ■ Represent and analyze quantitative relationships between dependent and independent variables.

Geometry

- ◻ Solve real-world and mathematical problems involving area, surface area and volume.

Statistics and Probability

- ○ Develop understanding of statistical variability.
- ○ Summarize and describe distributions.

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Cluster Designations – Grade 7

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

Ratios and Proportional Reasoning
- ■ Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System
- ■ Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers.

Expressions and Equations
- ■ Use properties of operations to generate equivalent expressions.
- ■ Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry
- ○ Draw, construct and describe geometrical figures and describe the relationships between them.
- ○ Solve real-life and mathematical problems involving angle measure, area, surface area and volume.

Statistics and Probability
- ◘ Use random sampling to draw inferences about a population.
- ○ Draw informal comparative inferences about two populations.
- ◘ Investigate chance processes and develop, use, and evaluate probability models.

Cluster Designations – Grade 8

Key: ■ Major Clusters; ◘ Supporting Clusters; ○ Additional Clusters

The Number System
- ◘ Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations
- ■ Work with radicals and integer exponents.
- ■ Understand the connections between proportional relationships, lines and linear equations.
- ■ Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions
- ■ Define, evaluate and compare functions.
- ■ Use functions to model relationships between quantities.

Geometry
- ■ Understand congruence and similarity using physical models, transparencies or geometry software.
- ■ Understand and apply the Pythagorean Theorem.
- ○ Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability
- ◘ Investigate patterns of association in bivariate data.

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Agenda

- Ratios & Proportional Relationships
- Geometry
- The Number System
- Expressions and Equations
- Linear Equations and Functions

Tape Diagrams

Using Tape Diagrams

- Promote **perseverance** in reasoning through problems.
- Develop students' independence in asking themselves:
  - "Can I draw something?"
  - "What can I label?"
  - "What do I see?"
  - "What can I learn from my drawing?"

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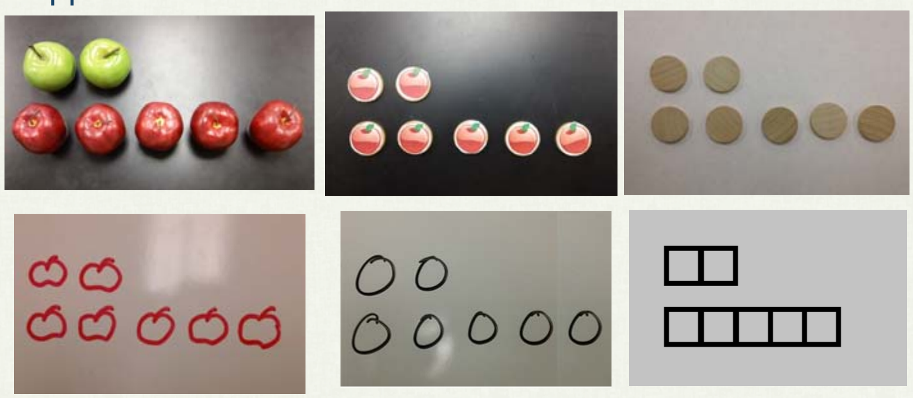
Tape Diagrams

Foundations in PK–1

Sara has 2 apples. Jon has 5 apples. How many apples do they have altogether? How many more apples does Jon have than Sara?



Tape Diagrams

Forms of the Tape Diagram

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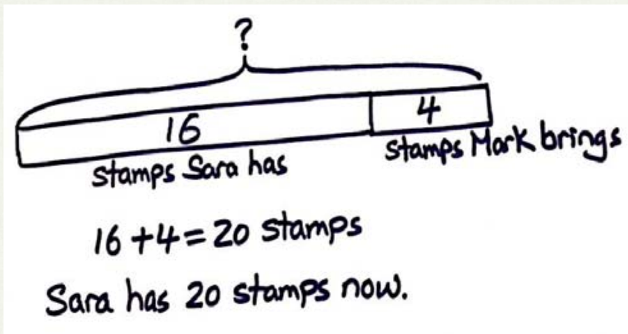
Tape Diagrams

Early Examples

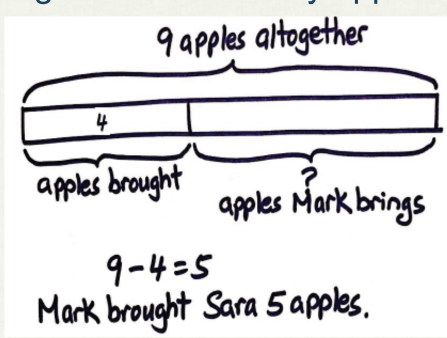
Example A: Sarah has 16 stamps. Mark brings her 4 more stamps. How many stamps does Sara have now?



Tape Diagrams

Early Examples

Example A: Sara brought 4 apples to school. After Mark brings her some more apples, she has 9 apples altogether. How many apples did Mark bring her?



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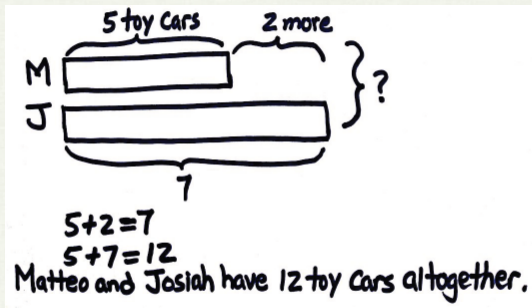
Tape Diagrams

Early Examples

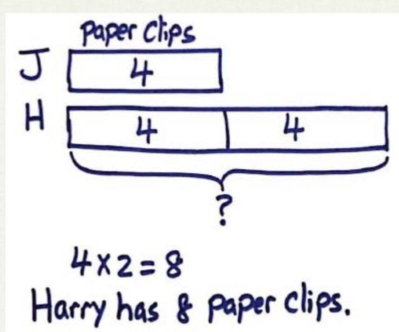
Example B: Matteo has 5 toy cars. Josiah has 2 more than Matteo. How many cars do Matteo and Josiah have altogether?



Tape Diagrams

Early Examples

Example D: Jose has 4 paper clips. Harry has twice as many paper clips as Jose. How many paper clips does Harry have?



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Tape Diagrams

Example 1:

Sam has 7 more stamps than Joe. They have 45 stamps altogether. How many stamps does each boy have?

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Tape Diagrams

Example 2:

Desmond has 5 times as many toy cars as Luke. They have 42 cars altogether. How many cars does each boy have.

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Tape Diagrams

Example 3:

William's weight is 40 kg. He is 4 times as heavy as his youngest brother Sean. What is Sean's weight?

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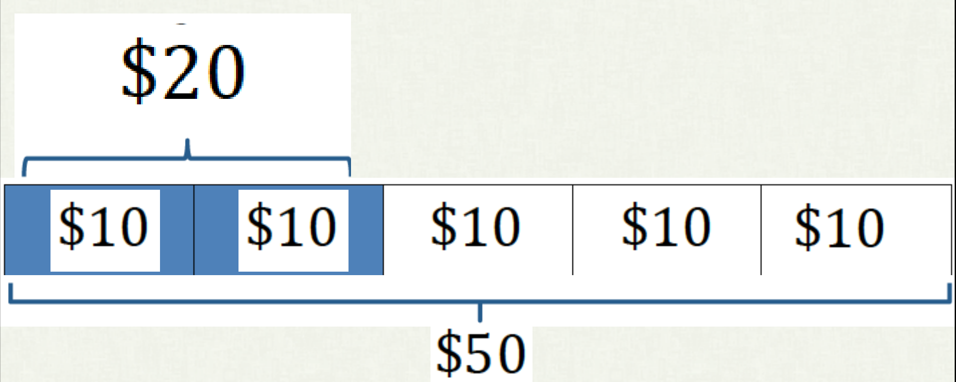
Tape Diagrams

Example 1:

David spent 2/5 of his money on a storybook. The storybook cost $20. How much did he have at first?



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Tape Diagrams

Example 2:

Max spent 3/5 of his money in a shop and 1/4 of the remainder in another shop. What fraction of his money was left? If he had $90 left, how much did he have at first?

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Tape Diagrams

Example 6:

Three-fifths of Jan's money is twice as much as Lena's money. What fraction of Jan's money is Lena's money?

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Agenda

- Ratios & Proportional Relationships
- Geometry
- The Number System
- Expressions and Equations
- Linear Equations and Linear Functions

Ratios and Proportional Relationships

The transition into ratios and proportional relationships:

- Why study ratios?
  - Make sense of proportional relationships
    - Definition and study of ratio, value of the ratio, equivalent ratios, unit rate of A:B, unit rate of B:A, constant of proportionality
    - Percent
  - Geometry
    - Scale Factor and Scale Drawings
    - Similarity
      - Properties of Similar Figures
      - Corresponding Side Lengths Equal in Ratio

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Ratios & Proportional Relationships

When do we really use ratios? Jimmy's allowance is twice as much as Johnny's and Jeremy's is 4/5 of Johnny's. How much allowance does Jimmy get?

- List as many real-world applications of ratios and proportional relationships as you can think of.
- Choose one of them to make up your own ratio application problem that seems especially real-world.

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Ratios & Proportional Relationships

Transitioning into ratio problems

- Students have solved problems of multiplicative comparison.

  There were 4 times as many boys as girls at the party. If there were 30 kids at the party, how many were boys?

- What would motivate a student to want to use ratios to describe this type of situation?
- Read through G6-M1, Lesson 1, Examples 1-2

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Ratios & Proportional Relationships

Definitions and Descriptions

Ratio

A pair of non-negative numbers, A:B, which are not both zero. They are used to indicate that there is a relationship between two quantities such that when there are A units of one quantity, there are B units of the second quantity.

Value of a Ratio

For the ratio A:B, the value of the ratio is the quotient A/B as long as B is not zero. Likewise, for the ratio B:A, the value of the ratio is the quotient B/A as long as A is not zero.

Equivalent Ratios

Two ratios A:B and C:D are *equivalent* if there is a positive number, *k*, such that *C=kA* and *D=kB.* They are ratios that have the same value.

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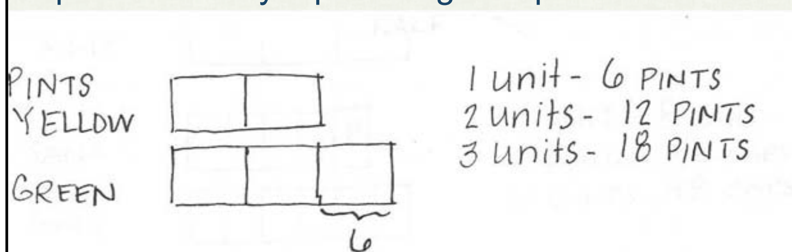
Tape Diagrams

Example 3:

Ingrid is mixing yellow and green paint together for a large art project. She uses a ratio of 2 pints of yellow paint for every 3 pints of green paint.



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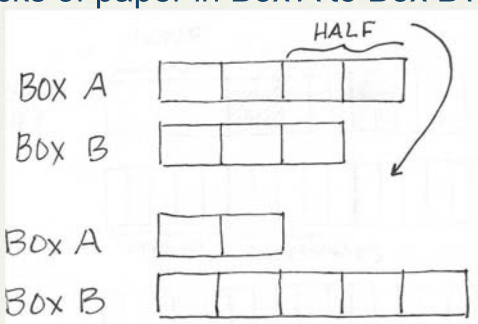
Tape Diagrams

Example 4:

Lena finds two boxes of printer paper in the teacher supply room. The ratio of the packs of paper in Box A to the packs of paper in Box B is 4:3. If half of the paper in Box A is moved to Box B, what is the new ratio of packs of paper in Box A to Box B?



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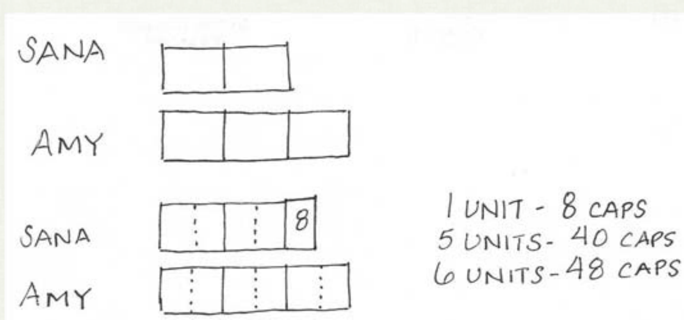
Tape Diagrams

Example 5:

Sana and Amy collect bottle caps. The ratio of the number of bottle caps Sana has to the number Amy has is 2 : 3. The ratio became 5 : 6 when Sana added 8 more bottle caps to her collection. How many bottle caps does Amy have?



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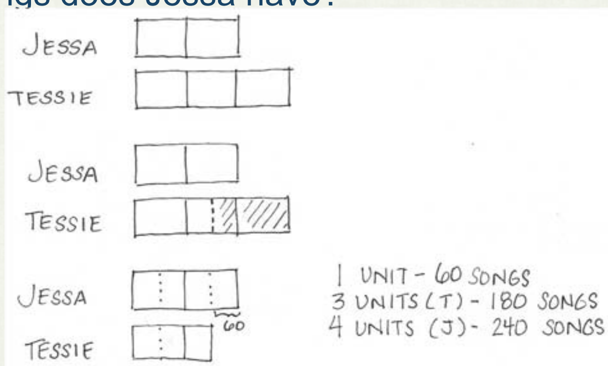
Tape Diagrams

Example 6:

The ratio of songs on Jessa's phone to songs on Tessie's phone is 2 to 3. Tessie deletes half of her songs and now has 60 fewer songs than Jessa. How many songs does Jessa have?



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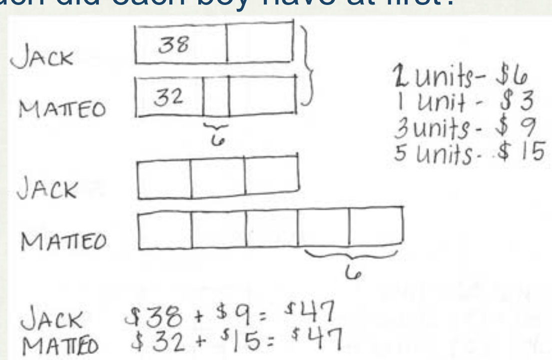
Tape Diagrams

Example 7:

Jack and Matteo had an equal amount of money each. After Jack spent $38 and Matteo spent $32, the ratio of Jack's money to Matteo's money was 3:5. How much did each boy have at first?



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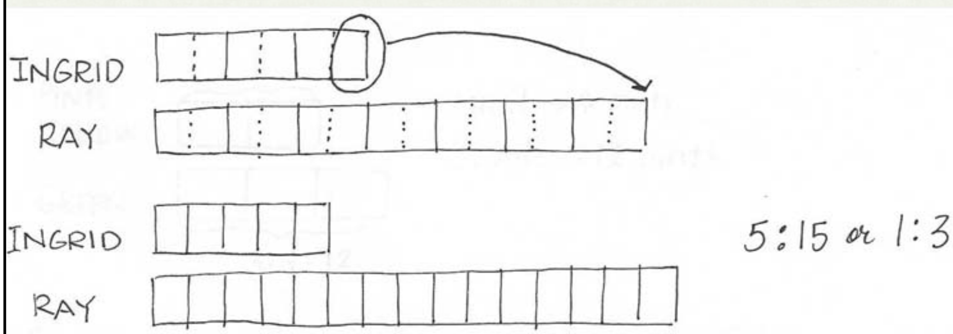
Tape Diagrams

Example 8:

The ratio of the number of Ingrid's stamps to the number of Ray's stamps is 3 : 7. If Ingrid gives one-sixth of her stamps to Ray, what will be the new ratio of the number of Ingrid's stamps to the number of Ray's stamps?



Tape Diagrams

Example 8:

The ratio of the Gavin's money to Manuel's was 6 : 7. After Gavin spent two-thirds of his money and Manuel spent $39 Manuel had twice as much money as Gavin. How much money did Gavin have at first?

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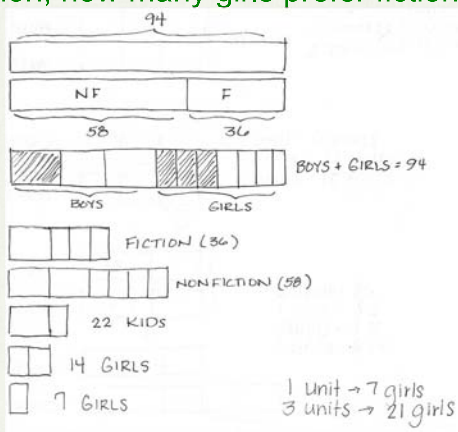
Tape Diagrams

Revisiting the Opening Exercise

94 children are in a reading club. One-third of the boys and three-sevenths of the girls prefer fiction. If 36 students prefer fiction, how many girls prefer fiction?



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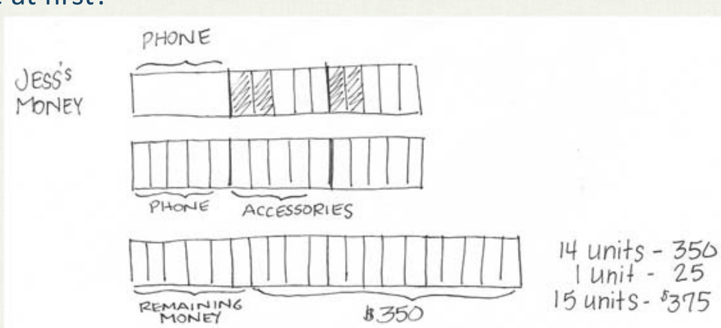
Tape Diagrams

Revisiting the Opening Exercise

Jess spent one-third of her money on a cell phone, and two-fifths of the remainder on accessories. When she got home her parents gave her $350. The ratio of money she had in the end to the money she had before is 4:3. How much money did she have at first?



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Double Number Line Diagrams

Example: Rate Problems

A photocopier can print 12 copies in 36 seconds. At this rate, how many copies can it print in 1 minute?

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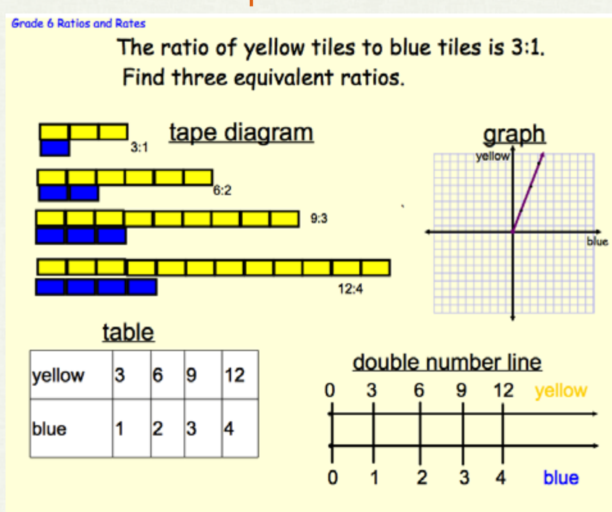
Ratios & Proportional Relationships

Representations of Equivalent Ratios



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Ratios & Proportional Relationships

Equivalent Ratios – Tape Diagrams and Double Number Line Diagrams

Represent each situation using a tape diagram or double number line diagram.

- Monique walks 3 miles in 25 minutes.
- Sean spends 5 minutes watching television for every 2 minutes he spends on homework.

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Ratios & Proportional Relationships

Rate and Unit Rate

Rate: If I traveled 180 miles in 3 hours; my average speed is 60 mph. The quantity, 60 mph, is an example of a rate.

Unit Rate: The numeric value of the rate, e.g. in the rate 60 mph, the unit rate is 60.

Rate's Unit: The unit of measurement for the rate, e.g. mph.

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Ratios & Proportional Relationships

Ratio & Proportions Exercise 1

In Jasmine's favorite fruit salad, the ratio of the number of cups of grapes to number of cups of peaches is 5 to 2.

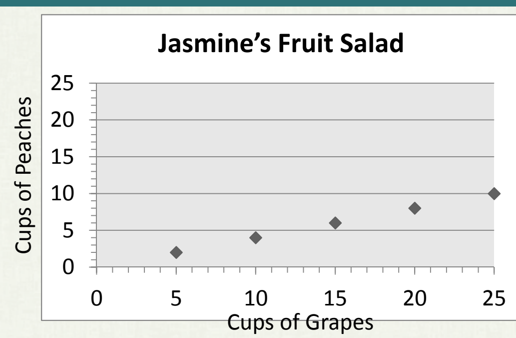
a. How many cups of peaches will be used if 25 cups of grapes are used?

b. Create both a ratio table and a graph of the proportional relationship to depict the relationship and find your answer.

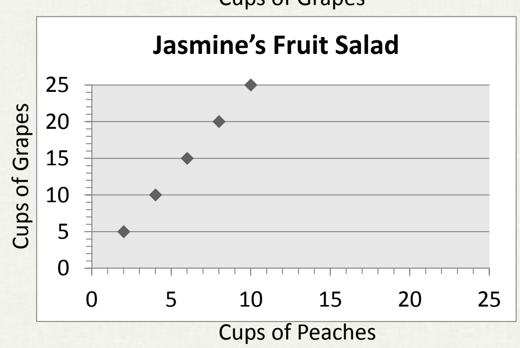
c. What is the constant of proportionality?

d. Write the equation of the line depicted in the graph.

Cups of Grapes	Cups of Peaches
5	2
10	4
15	6
20	8
25	10



Cups of Peaches	Cups of Grapes
2	5
4	10
6	15
8	20
10	25



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Ratios & Proportional Relationships

Ratio & Proportions Exercise 2

Jack is taking a hike through a forested park. He moves at a constant rate, covering **5 miles every 2 hours**.

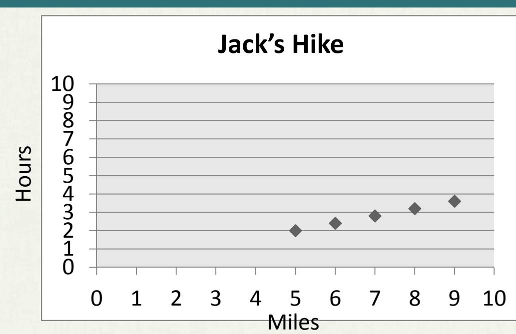
a. How much time will have passed when he has hiked 9 miles?

b. Create both a ratio table and a graph of the proportional relationship to depict the relationship and find your answer.

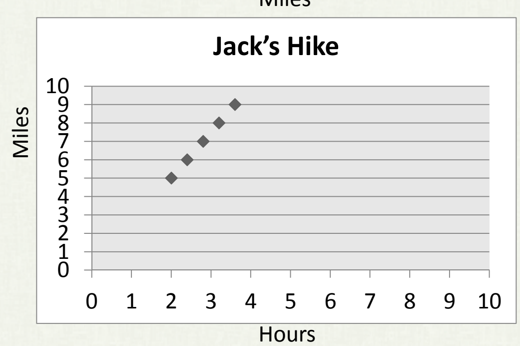
c. What is the constant of proportionality?

d. Write the equation of the line depicted in the graph.

Miles	Hours
5	2
6	2.4
7	2.8
8	3.2
9	3.6



Hours	Miles
2	5
2.4	6
2.8	7
3.2	8
3.6	9



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Ratios & Proportional Relationships

Constant of Proportionality

Given a ratio A : B, and given that one places the quantity associated with A on the x-axis and the quantity associated with B on the y-axis, then:

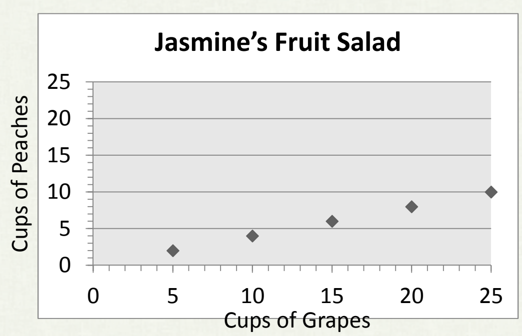
<u>The constant of proportionality for the ratio A : B is the unit rate of the ratio B : A</u> (the value of the ratio B:A).

Ratios & Proportional Relationships
Constant of Proportionality

And the equation of the line depicted will be:

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the ***constant of proportionality***.; e.g., If the ratio of x to y is *5* to *2*, then the constant of proportionality represents the ratio of y to x and is *2/5*, and $y = (2/5)\,x$.

Cups of Grapes	Cups of Peaches
5	2
10	4
15	6
20	8
25	10



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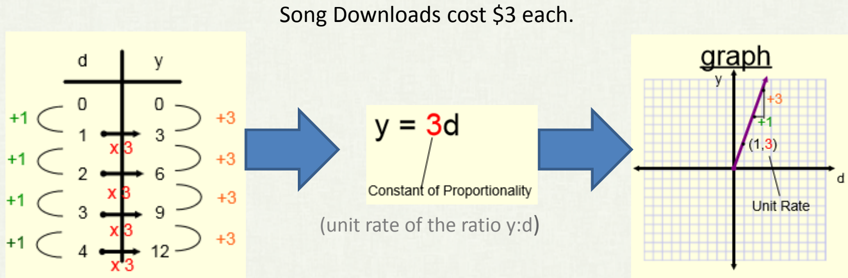
Ratios & Proportional Relationships

Constant of Proportionality

Where and how is the constant of proportionality represented?

Song Downloads cost $3 each.



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Ratios & Proportional Relationships

Average Rate vs. Constant Rate

AVERAGE RATE. Let a time interval of *t* hours be given. Suppose that an object travels a total distance of *d* miles during this time interval, *t*. The object's *average rate in the given time interval* is *d/t* miles per hour.

CONSTANT RATE. For any positive real number *v*, an object travels at a *constant rate of v mph* over a given time interval, *t*, if the average rate is always equal to *v* mph for every fixed time interval.

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Ratios & Proportional Relationships

Testing for Proportional Relationships

- G7-M1 Lesson 3 Exercises 1-3
- Now create graphs of the data in each table. What do you notice?
- Recall the lesson from G8-M4 Lesson 10:
- Calculating an average rate does not dictate that there was a constant rate on that interval, nor does it guarantee that the rate will continue at that same average rate.

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Ratios & Proportional Relationships

Scale Factor

- G7-M1 End of Module Assessment, Item 2
- Scale factor is a unit rate, and is therefore unit-less. To calculate the true scale factor, one must compare using the same unit of measure in each quantity.
- Students work informally to know that the area of the scaled drawing is altered by a factor of $(\text{scale factor})^2$
- Does creating a scale drawing ever effect the measure of the angles in the drawing?

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Ratios & Proportional Relationships

Percent is a rate per 100

Suppose the ratio of girls to boys in a class is 1:4.

What are some associated ratios?

The ratio of boys to girls is 4:1,

The ratio of girls to total students in the class is 1:5

The ratio of boys to total students in the class is 4:5.

If the total number of students in the class were 50, how many girls are there?

Ratios & Proportional Relationships

Percent is a rate per 100

- To give a percentage is to define a ratio relationship between two quantities by telling how many/much of A per 100 of B.
- When I ask, “What is the percentage of girls in the class?” I am asking, “How many girls for every 100 students in the class?”
- How can you answer that question?
- The same way you answer any equivalent ratio problem.

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Ratios & Proportional Relationships

Percent is a rate per 100

Convert each example of a percent statement into a ratio statement.

The sale price of the computer is 80% of the original price.

- The ratio of the sales price to the original price is 80:100.

35% of the kids at my school play an instrument.

- The ratio of kids who play an instrument to the total number of kids is 35:100.

Ratios & Proportional Relationships

Percent, Fraction, Decimal

Convert each statement into a ratio statement and a percent statement.

2/5 of the cars in the parking lot were white.

- The ratio of cars that were white to cars in the parking lot was 2:5.
- 40% of the cars in the parking lot were white.

Six-tenths (0.6) of the boxes that arrived were damaged.

- The ratio of boxes that arrived damaged to total boxes that arrived was 6:10.
- 60% of the boxes that arrived were damaged.

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Ratios & Proportional Relationships

Percent of a Quantity

40% of my crayons are broken.

- Rewrite this using ratio language.

If I have 150 crayons, how many are broken?

- Is this problem asking the same thing as, "How many is 40% of the 150 crayons?"

Of the 25 girls on the soccer team, 40% also play on a travel team. How many of the 25 girls play on a travel team?

Ratios & Proportional Relationships

Percent of a Quantity

What is 40% of 60?

- Could this problem be viewed as a ratio problem?
- If so, what information are we being given?
- Create a context for the problem.

Work on fluency of these computations before moving on.

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Ratios & Proportional Relationships

Solving Percent Problems

Jane paid \$40 for an item after she received a 20% discount. Jane's friend says this means that the original price of the item was \$48.

a. How do you think Jane's friend arrived at this amount?

b. Is her friend correct? Why or why not?

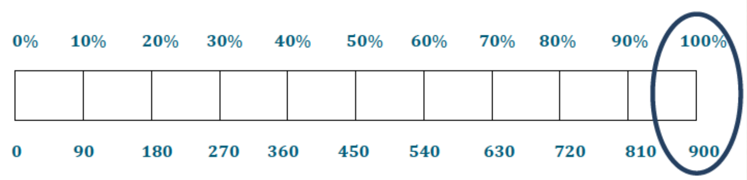
Purchasing a TV that is 20% off will save \$180.

a. Name the different parts with the words: PART, WHOLE, PERCENT.

PERCENT	*PART*	*WHOLE*
20% off	\$180	Original Price

b. What was the original price of the TV? Show two methods for finding your answer.

Method 1:



Method 2:

$20\% = \frac{20}{100}$

$\frac{20 \times 9 \rightarrow}{100 \times 9 \rightarrow} = \frac{180}{900}$

The original price was \$900.

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Example 3: An Algebraic Approach to Finding a Part, Given a Percent of the Whole

A bag of candy contains 300 pieces of which 28% are red. How many pieces are red?

Which quantity represents the whole?

The total number of candies in the bag, 300, is the whole because the number of red candies is being compared to it.

Which of the terms in the percent equation is unknown? Define a letter (variable) to represent the unknown quantity.

We do not know the part, the number of red candies in the bag. Let r represent the number of red candies in the bag.

Write an expression using the percent and the whole to represent the number of pieces of red candy.

$\frac{28}{100}\cdot(300)$ ***or*** $0.28\cdot(300)$ ***is the amount of red candy since the number of red candies is 28% of the 300 pieces of candy in the bag.***

Write and solve an equation to find the unknown quantity.

$Part = Percent \times Whole$

$r = \frac{28}{100}\cdot(300)$

$r = 28\cdot 3$

$r = 84$

There are 84 red pieces of candy in the bag.

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Ratios & Proportional Relationships

Percent & Proportional Relationships

- G7-M4 Lesson 3 Exit Ticket, Number 1

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H

Ratios & Proportional Relationships

Fluency

- Rapid white board exchanges
- Sprints

H

Ratios & Proportional Relationships

Percent Increase and Decrease

G7-M4 Lesson 4 Opening Exercise

G7-M4 Lesson 4 Exit Ticket

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Agenda

- Ratios & Proportional Relationships
- Geometry
- The Number System
- Expressions and Equations
- Linear Equations and Linear Functions

Geometry

- Congruence (G8-M2)
- Similarity (G8-M3)

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Geometry

Why move things around?

- To avoid direct measurement…

  … while answering questions, making conjectures.

  - Work through: G8-M2 Lesson 1 Exploratory Challenge
- Students describe motions intuitively, working towards comfort with formal concepts and language.

Geometry

Transformation of the Plane

Think of a plane as a sheet of overhead projector transparency, or a sheet of paper.

Consider an aerial photo of a portion of a city. Map each point on the street to a point on your paper (the map) in a way that intuitively "preserves the shape" (foreshadowing similarity).

Another way of mapping, we can project (using a light source) from one sheet to another.

A ***transformation of the plane***, to be denoted by *F*, is a rule that associates (or assigns) to each point *P* of the plane to a unique point which will be denoted by *F*(*P*).

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Geometry

Three Rigid Transformations

Understand and perform:

- Translation:
  a transformation along a vector.
- Reflection:
  a transformation across a line.
- Rotation:
  a transformation about a point for a given angle measure

H

Geometry

Three Rigid Transformations

- Translations, reflections, and rotations have **three basic properties**:
  - Map lines to lines, rays to rays, segments to segments, and angles to angles.
  - Preserve the lengths of segments.
  - Preserve the measures of angles.
- Students verify these properties informally.
  - Work through:
    G8 M2 Lesson 2
    G8 M2 Lesson 4
    G8 M2 Lesson 5

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Geometry

Congruence

- Rigid transformations can be sequenced. Does order matter?
- Can sequences of rigid transformations be reversed?
  - Work through:

    G8 M2 L8

    G8 M2 L9

H

Geometry

Congruence

- A congruence is a sequence of basic rigid motions (translations, reflections, or rotations) that maps one figure onto another.
  - Work through:

    G8 M2 L10
- A two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.
  - Work through:

    G8 M2 L11

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Geometry

Congruence

- Are these properties true for a congruence?
  - Map lines to lines, rays to rays, segments to segments, and angles to angles.
  - Preserve the lengths of segments.
  - Preserve the measures of angles.
- A two-dimensional figure is ***congruent*** to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.

Geometry

What does “same shape” mean?

- Similarity is often referred to as “same shape” (but not necessarily same size).
- But what does “same shape” mean? Which if any of these properties would still apply to a ‘Similarity Transformation’?
  - Map lines to lines, rays to rays, segments to segments, and angles to angles.
  - Preserve the lengths of segments.
  - Preserve the measures of angles.

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Geometry

Dilations

- A dilation is a transformation of the plane with center *O* and scale factor *r*, that assigns to each point *P* of the plane a point *Dilation*(*P*) so that:

  Dilation(*O*) = *O and*

  If $P \neq O$, then *Dilation*(*P*), denoted as *P'*, is the point on the ray *OP* so that $|OP'| = r\,|OP|$.
- When $r = 1$, then the figures are congruent.
- When $0 < r < 1$, then the dilated figure is smaller than the original.
- When $r > 1$, then the dilated figure is larger than the original.

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Geometry

Dilations

- Do dilations:
  - Map lines to lines, rays to rays, segments to segments, and angles to angles?
  - Preserve the lengths of segments?
  - Preserve the measures of angles?

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Activity – Part 1

- On a lined piece of paper:
  - Choose a point near the top of the page **on a line** to use as our center of dilation. Label the point *O*.
  - Draw a ray, from point *O*, through a whole number of lines, e.g., through 2 lines, or 5 lines, etc. Label this point *P*.
  - Draw another ray, from point *O*, through the same number of lines. Label this point *Q*.
  - Along ray (you may need to extend it), find another point, *P'*, a whole number of lines away from *O.*
  - Along ray (you may need to extend it), find another point, *Q'*, the same whole number of lines away from *O.*
  - Connect points *P* and *Q*. Connect points *P'* and *Q'*.

A

Activity – Part 2

- What do you notice about lines *PQ* and *P'Q'*?
  - Lines *PQ* and *P'Q'* are parallel, i.e., they never intersect.
- Write the scale factor that represents the dilation from *O* to *P* and *O* to *P'*, e.g., if you went 2 lines for *P* and 5 lines for *P'*, then scale factor $r = 5/2$.

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Activity – Part 3

- Measure the distance from to , label your diagram. Measure and label and .
- Compare the length of the dilated segment to the original: to , and to . (Use values rounded to nearest tenths place.) What do you notice?

  That the length of the dilated segment *OP'* divided by the original segment *OP* is equal to *OQ'* divided by *OQ.*

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Activity – Part 4

- Measure the lengths *P'Q'* and *PQ*. Compare the lengths as before. What do you notice?
- Now compare the ratio of the lengths to the scale factor. What do you notice?

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Geometry

Fundamental Theorem of Similarity

The activity leads to an understanding of FTS:

Theorem:
Given a dilation with center *O* and scale factor *r*, then for any two points *P, Q* in the plane (when points *O, P, Q* are not collinear), the lines *PQ* and *P'Q'* are parallel, where *P'* = *Dilation*(*P*) and *Q'* = *Dilation*(*Q*), and furthermore, $|P'Q'| = r\,|PQ|$.

Geometry

Dilations

- Dilations:
  - DO map lines to lines, rays to rays, segments to segments, and angles to angles.
  - DO NOT preserve the lengths of segments.
  - DO preserve the measures of angles.

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Geometry

Similarity

- Two figures are said to be ***similar*** if you can map one onto another by a dilation followed by a congruence.
- Show figures are similar by describing the sequence of the dilation and congruence.

H

Geometry

Similarity – The AA Criterion

- G8-M3 Lesson 10

  Two triangles are said to be similar if they have two pairs of corresponding angles that are equal.
- G8-M3 Lesson 11

  If two triangles have one pair of equal corresponding angles and the ratio of corresponding sides (along each side of the given angle) are equal, then the triangles are similar.

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Agenda

- Ratios & Proportional Relationships
- Geometry
- The Number System
- Expressions and Equations
- Linear Equations and Linear Functions

The Number System

- Dividing a fraction by a fraction (G6-M2)
- Introducing negative numbers (G6-M3)
- Operating with negative numbers: (G7-M2)
  Why should a negative x a negative = a positive?

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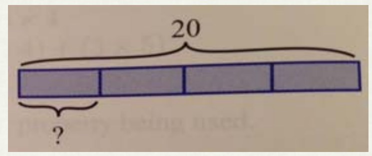
The Number System

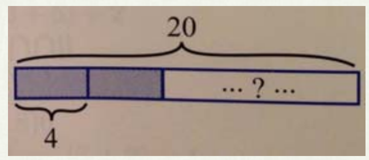
Fractions ÷ Fraction: Division Basics

There are two models of division:

Partitive:
20 is 4 groups of how many?



Measurement:
How many groups of 4 in 20?



Which model does this problem fit into:

- 12 is 2/3 of what number?

The Number System

Fractions ÷ Fraction: G5-M4-L25

Whole number ÷ unit fraction

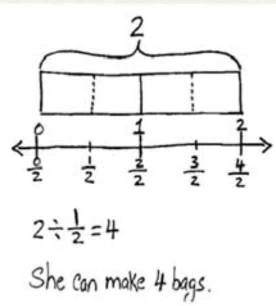
Problem 1

Jenny buys 2 pounds of pecans.

a. If Jenny puts 2 pounds in each bag, how many bags can she make?

b. If she puts 1 pound in each bag, how many bags can she make?

c. If she puts $\frac{1}{2}$ pound in each bag, how many bags can she make?



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The Number System

Fractions ÷ Fraction: G5-M4-L26

Unit fraction ÷ whole number

Problem 3

If Melanie pours $\frac{1}{2}$ liter of water into 4 bottles, putting an equal amount in each, how many liters of water will be in each bottle?

T: (Post Problem 3 on the board, and read it together with the class.) How many liters of water does Melanie have?

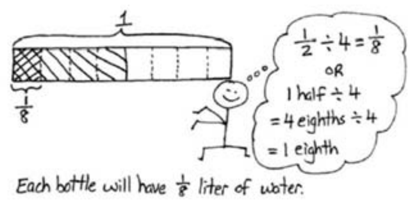
S: Half a liter.

T: Half of liter is being poured into how many bottles?

S: 4 bottles.

T: How do you solve this problem? Turn and discuss.

S: We have to divide. → The division sentence is $\frac{1}{2} \div 4$. → I need to divide the dividend 1 half by the divisor, 4. → I can draw 1 half, and cut it into 4 equal parts. → I can think of this as $\frac{1}{2} = 4 \times$ ____.



The Number System

A Progression of the Measurement Model

A Progression for students: Use tape diagram to demonstrate the answer to the following:

- How many ½'s are in 6?
- How many 1/3's are in 6?
- How many 1/3's are in 1?
- How many 1/3's are in 2/3?
- How many 1/3's are in ½?
- How many 5/2's are in 2/3?

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The Number System

Fractions ÷ Fraction

Grade 6 – Module 2: Lesson 1

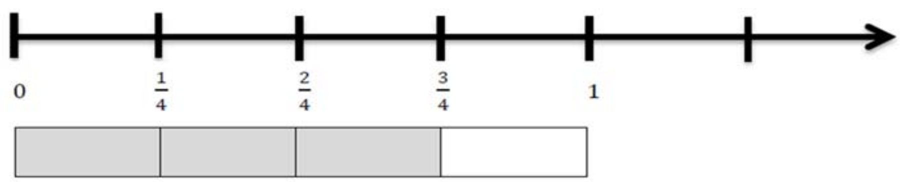
Interpreting Division of a Fraction by a Whole Number—Visual Models (Partitive)

Example 1

Maria has $\frac{3}{4}$ lb. of trail mix. She needs to share it equally among 6 friends. How much will each friend be given? What is this question asking us to do?

We are being asked to divide the trail mix into six equal portions. So we need to divide three-fourths by six.

How can this question be modeled?



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The Number System

Fractions ÷ Fraction

Grade 6 – Module 2: Lesson 2

Interpreting Division of Whole Number by a Fraction—Visual Models (Measurement)

Example 1 (15 minutes)

At the beginning of class, break students into groups. Each group will need to answer the question it has been assigned and draw a model to represent its answer. Multiple groups could have the same question.

Group 1: How many half-miles are in 12 miles? $12 \div \frac{1}{2} = 24$

Group 2: How many quarter hours are in 5 hours? $5 \div \frac{1}{4} = 20$

Group 3: How many one-third cups are in 9 cups? $9 \div \frac{1}{3} = 27$

MP.1 & MP.2

Group 4: How many one-eighth pizzas are in 4 pizzas? $4 \div \frac{1}{8} = 32$

Group 5: How many one-fifths are in 7 wholes? $7 \div \frac{1}{5} = 35$

Models will vary but could include fraction bars, number lines, or area models (arrays).

Students will draw models on blank paper, construction paper, or chart paper. Hang up only student models, and have students travel around the room answering the following:

1. Write the division question that was answered with each model.
2. What multiplication question could this model also answer?
3. Rewrite the question given to each group as a multiplication question.

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The Number System

Fractions ÷ Fraction

Grade 6 – Module 2: Lesson 2

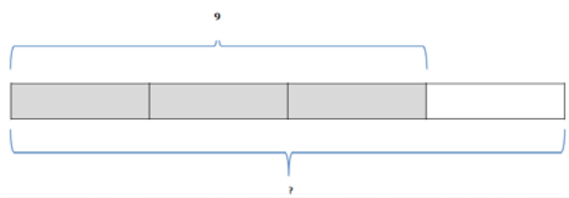
Interpreting Division of Whole Number by a Fraction—Visual Models (Partitive)

Example 2

Molly used 9 cups of flour to bake bread. If this was $\frac{3}{4}$ of the total amount of flour she started with, what was the original amount of flour?

- What is different about this question from the measurement questions?
  - *In this example, we are not trying to figure out how many three-fourths are in 9. We know that 9 cups is a part of the entire amount of flour needed. Instead, we need to determine three-fourths of what number is 9.*

a. Create a model to represent what the question is asking for.



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The Number System

Fractions ÷ Fraction

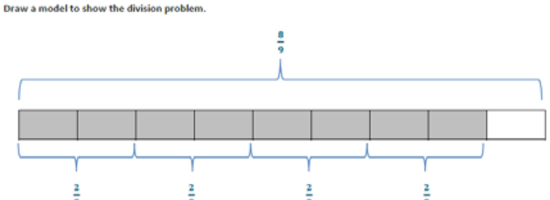
G6M2: Lesson 3 - Interpreting and Computing Division of a Fraction by a Fraction

- What is $\frac{8}{9} \div \frac{2}{9}$? Take a moment to use what you know about division to create a model to represent this division problem. Give students a chance to explore this question and draw models without giving them the answer first. After three minutes or so, ask for students to share the models that they have created and to discuss what conclusions they have made about dividing fractions with the same denominator.
  - *One way to interpret the question is to say how many $\frac{2}{9}$ are in $\frac{8}{9}$. From the model, I can see that there are 4 groups of $\frac{2}{9}$ in $\frac{8}{9}$. This would give the same solution as dividing 8 by 2 to get 4.*

Example 1

$\frac{8}{9} \div \frac{2}{9}$

Draw a model to show the division problem.



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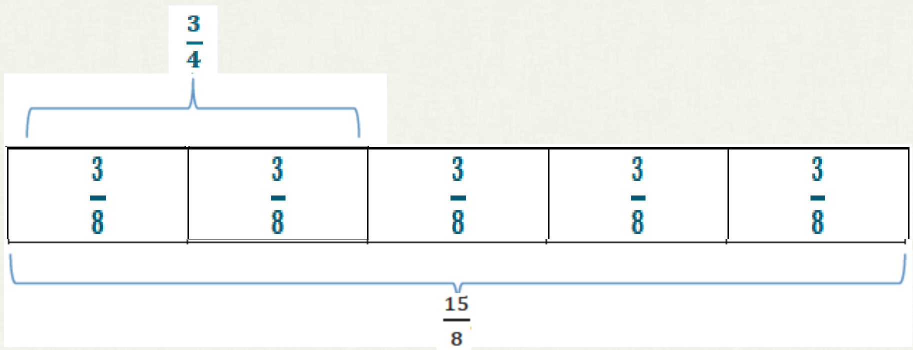
The Number System

Fractions ÷ Fraction

$\frac{3}{4} \div \frac{2}{5}$

$\frac{2}{5}$ of what number is $\frac{3}{4}$?



Fraction ÷ Fraction

Invert and Multiply

Using the Any-Order Property, we can say the following:

$$\frac{1}{2} \cdot \frac{3}{4} \cdot 5 = \frac{3}{4} \cdot \left(\frac{1}{2} \cdot 5\right) = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}.$$

So $\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}$ and $\frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$.

Therefore, $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2}$.

This method for dividing a fraction by a fraction is called "invert and multiply". Dividing by a fraction is the same as multiplying by its inverse. It is important to invert the second fraction (the divisor) and ***not*** the first fraction (the dividend).

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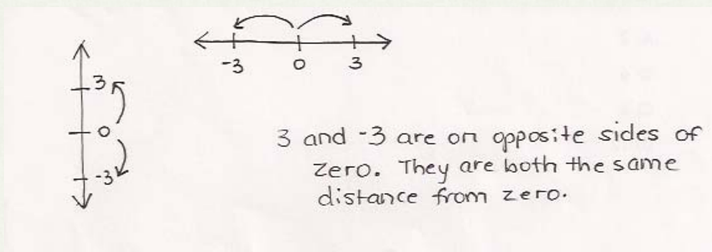
The Number System

Introducing Negative Numbers

- Have you ever heard of a negative number? Who can give an example of a when someone would use a negative number?



Example 1: Explain and show how to find the opposite of the number 5 on a number line.

The Number System

Introducing Negative Numbers

Example 2:

Write an integer to represent each of the following situations:

A business loses $250,800 in 2012. __________

You receive $15 for babysitting. __________

A fish swims at 49 feet below sea level. __________

The temperature is 4 degrees below zero. __________

Mount McKinley is 20,320 feet in elevation. __________

Example 3:

Describe a situation that can be modeled by the integer -5. Explain what zero represents in the situation.

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The Number System

Magnitude and Absolute Value

G6-M3 Lesson 11

Example 2: Using Absolute Value to Find Magnitude

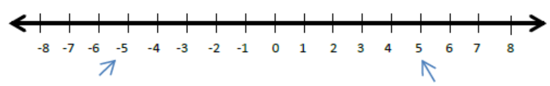
Mrs. Owens received a call from her bank because she had a checkbook balance of -45 dollars. What was the magnitude of the amount overdrawn?

$|-45| = 45$ *Mrs. Owens overdrew her checking account by $45.*

The magnitude of a quantity is found by taking the absolute value of its numerical part

17. Use a number line to show why a number and its opposite have the same absolute value.

A number and its opposite are the same distance from zero, but on opposite sides. An example is 5 and -5. These numbers are both 5 units from zero. Their distance is the same, so they have the same absolute value, 5.



The Number System

Distance Between Two Rational Numbers

G7-M2 Lesson 6

Exercise 1

Use the number line to answer each of the following:

Person A	Person B
What is the distance between -4 and 5? *9*	What is the distance between 5 and -4? *9*
What is the distance between -5 and -3? *2*	What is the distance between -3 and -5? *2*
What is the distance between 7 and -1? *8*	What is the distance between -1 and 7? *8*

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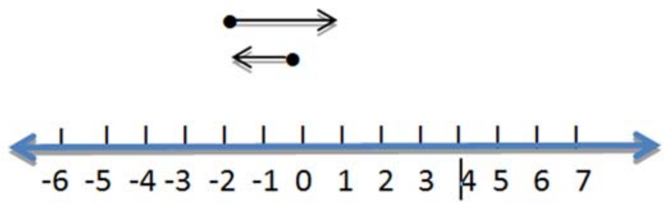
The Number System

Adding Using the Number Line Model

<u>Example 1</u>. Model -2 + 3 on the number line.



-2 + 3 = 1

The Number System

Subtracting a Negative is Like Adding

If I had these 3 cards,
the total would be
4 + (-1 + (-2))
= 4 + -3
= 1

But if you took away the two negative cards, my score would go up, because I would have 4 (instead of 1). So my total went up by positive 3. That shows that subtracting -3 is the same as adding 3.

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The Number System

Operations with Negatives

Why is (5)(-3) negative?

Why is (-5)(3) negative?

Why should a negative x a negative = a positive?

- Using properties of operations

  -5(0)=0

  -5(3+(-3))=0

  -5(3)+-5(-3)=0

  -15 + ? = 0

  -15 + 15 = 0, therefore -5(-3) must be 15.

The Number System

Operations with Negatives

Why is (5)(-3) negative?

Why is (-5)(3) negative?

Why should a negative x a negative = a positive?

- Using properties of operations
- Using extension of the area model
- Using the context of the card game

  Repeated (5 times) laying down (-) of card whose value is -3; what is the affect on your score?

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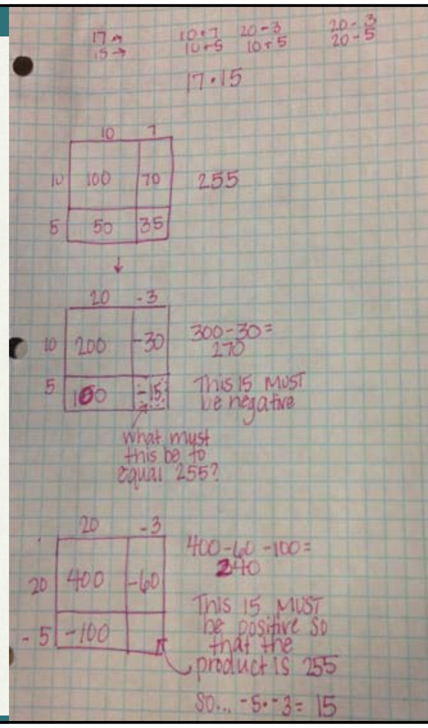

Agenda

- Ratios & Proportional Relationships
- Geometry
- The Number System
- Expressions and Equations
- Linear Equations and Linear Functions

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Expressions and Equations

- The what, the why, and the how of it all
- Categories of student capacities
- A progression of each capacity

Expressions and Equations

What is an expression?

An expression expresses something – using more concise and more powerful representations than words.

How can I express the total cost if I know that it costs a flat fee of $5 plus 4 cents for every page?

Numerical expressions have only numbers, no variables for which the value hasn't been stated.

Algebraic expression suggests that there is at least one variable.

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Expressions and Equations

What is a variable?

How do we define / describe what a variable is?

- A *variable* is a letter standing for a number (the EE progression)
- A *variable* is a placeholder for a number (Eureka math curriculum)

When we describe what our variable represents

it must be a number of something or the amount of something or the unit rate in a rate.

Expressions and Equations

Why use variables?

The same reason we humans have studied math for centuries…

… to make generalizations and answer questions and curiosities!

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Expressions and Equations

What we do with expressions

The big picture:

- Create an algebraic expression, thereby decontextualizing it, and treating it as an object in its own right
- Make full use of the properties of numbers, (that they've come to understand in K-5) to manipulate that expression in USEFUL ways
- Re-contextualize it **to answer the original questions or curiosities**

Expressions and Equations

From expressions to equations

<u>Often, it is USEFUL to make a statement of equality about two expressions</u>.

- A ***number sentence*** is a statement of equality between two numerical expressions.
- It is said to be ***true*** if both numerical expressions are equivalent (that is, both evaluate to the same number). It is said to be ***false*** otherwise.

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Expressions and Equations

From expressions to equations

- An ***algebraic equation*** is a statement of equality between two expressions.
- Algebraic equations can be number sentences (when both expressions are numerical), but often they contain symbols whose values have not been named.

Expressions and Equations

Categories of Student Capacities

1. Replace numbers in numerical expressions and number sentences with variables to express generalizations.
2. Write algebraic expressions to represent a verbal expression,
   a. appropriately describing the number that the variable(s) represents.

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Expressions and Equations

Equivalent Expressions

How do we define / describe what it means for two algebraic expressions to be equivalent?

Grades 6 and 7:

Replacing the variable(s) with a number computes to the same number for both expressions no matter which number we choose.

Grade 8:

Using the commutative, associative, and distributive properties, one expression can be converted to the other expression.

Expressions and Equations

An analogy to solving equations:

Julie is 300 feet away from her friend's front porch and observes, **"Someone is sitting on the porch."**

- Given that she didn't specify otherwise, we would assume that the 'someone' Julie thinks she sees is a human.
- We can't guarantee that Julie's observatory statement is true. It could be that Julie's friend has something on the porch that merely looks like a human from far away.
- Julie assumes she is correct, and moves closer to see if she can figure out who it is. As she nears the porch she declares, "Ah, it is our friend, John Berry."

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Equations and Expressions

A Progression of Each Capacity

- Read through the page in the handout to learn about the progression through grades 6-8 of equations and expressions.
- Next, we will take a closer look at lessons in each grade.

H

Equations and Expressions

A Progression of Each Capacity

- In Grade 6, students work with concrete numbers and symbols to relate addition and subtraction.
  - Work through Grade 6, Module 4, Lesson 1
  - Work through Grade 6, Module 4, Lesson 9
- In Grade 8, students are expected to know how to write equations using symbols. Note the increased rigor.
  - Work through Grade 8, Module 4, Lesson 1

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Equations and Expressions

A Progression of Each Capacity

- In Grade 6, students continue work with writing expressions that contains numbers and variables.
  - Work through Grade 6, Module 4, Lesson 10
- In Grades 6-7, students generate equivalent expressions and verify equivalence by evaluating expressions.
  - Work through Grade 6, Module 4, Lesson 20
  - Work through Grade 7, Module 3, Lesson 1

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Equations and Expressions

A Progression of Each Capacity

1. Replace numbers in numerical expressions and number sentences with variables to express generalizations.

G6-M4 Lessons 1-4, 8

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Equations and Expressions

A Progression of Each Capacity

2. Write algebraic expressions to represent a verbal expression (or statement of equality or inequality) (and vice versa),
    a. appropriately describing the number that the variable(s) represents.

G6-M4 Lessons 9-10, 13-22, 34

G7-M2 Lessons 18 and19

G8-M4 Lesson 1

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Equations and Expressions

A Progression of Each Capacity

3. Manipulate expressions using the properties of numbers and properties of operations
    a. Be certain about whether two expressions are algebraically equivalent
    b. Develop an intuition about what manipulations might be useful in a given situation

G6-M4 Lessons 5-6, 9-12

G7-M3 Lessons 1-6

G8: we make use of equivalent expressions to explain how to use the substitution method for solving a linear system. When two expressions are equal to the same number, then the expressions are equal to one other.

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Equations and Expressions

A Progression of Each Capacity

4. Evaluate an expression by replacing the variable(s) with a single number.

G6-M4 Lessons 7, 18-22

G7-M3 Lessons 16-26

Equations and Expressions

A Progression of Each Capacity

5. Solving equations (or inequalities), that is, finding the value(s) of a variable that creates a true number sentence, given a statement of equality between two expressions.

   Using the properties of operations and numbers on either expression, and

   a. Using the properties of equality

G6-M4 Lessons 23-34

G7-M3 Lessons 7-15, some of 16-26

G8-M4 Lessons 3-8, 10-14, 24-31

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Expressions and Equations

A Progression of Each Capacity Solving Equations – Grade 6

- Use tape diagrams paired with algebraic solutions that makes use of the properties developed in G6-M4 Topic A such as *a* + *b* – *b* = *a*.
- Check it by substituting into both equations:

$$a + 2 = 8 \qquad 6 + 2 = 8$$
$$a + 2 - 2 = 8 - 2 \qquad 6 + 2 - 2 = 8 - 2$$
$$a = 6 \qquad 6 = 6$$

- Work through:
  G6-M4 Lesson 26 Exercise 1
  G6-M4 Lesson 27 Exercise 3, 17-19
  G6-M4 Lesson 30 Exercise 3-5

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Expressions and Equations

A Progression of Each Capacity Solving Equations – Grade 7

- Includes rational numbers
- Includes using the commutative and associative and distributive properties; promotes efficiency in understanding that they lead to an ***'any order, any grouping'*** consequence for addition and multiplication.
- Introduces the ***properties of equality*** and inequality, and refers to them as ***'if-then' moves***. If then If then .
  - Read through:
    G7-M3 Lesson 7 Example 1 and Exercise
    G7-M3 Lesson 9 Example 1 and Problem Set 6
    G7-M3 Lesson 10 Exercise 1, Example 2

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Expressions and Equations

A Progression of Each Capacity Solving Equations – Grade 8

- Includes using the distributive property to expand and to combine like terms.
- Asks students to generalize when a linear equation in 1 variable will have one solution, infinite solutions or no solutions.
- Emphasizes there are multiple ways to solve.
- Emphasizes that answers aren't always integers.
  - Work through:
    G8-M4 Lesson 4 Exercises 1-5
    G8-M4 Lesson 5 Examples 1-2
    G8-M4 Lesson 28 Examples 1-2

Expressions and Equations

Solving Equations

In Grades 6-8: Find which value makes the equation true, recognize that there are some equations that are always true: identities and equations in one variable that simplify down to the identity a = a.

In Grade 9:

- The solution set is the set of all values that make the equation or inequality true.
- Applying the properties of equality is guaranteed to preserve the solution set.
- Applying the Distributive, Associative, and Commutative Properties or the properties of rational exponents to either side is guaranteed to preserve the solution set.

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Expressions and Equations

An Example from Grade 9

Consider the equation $x - 3 = 5$.

Multiply both sides of the equation by a constant and show that the solution set did not change.

Now, multiply both sides by x.

$$x(x - 3) = 5x$$

Show that $x = 8$ is still a solution to the new equation.

Show that $x = 0$ is also a solution to the new equation.

Why did my solution set get altered? What did I do?

Expressions and Equations

The Graph of an Equation in 2 Variables

Grade 6-7:

- Plot points of proportional relationships.
- The graph itself is only a ray (or a line) if the context of the situation suggests that all the points along the ray (or line) are possible.

Grade 8:

- All the possible solutions to the equation.

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Expressions and Equations

The Graph of an Equation in 2 Variables

Grade 6:

- Students associate ratios with ordered pairs and plot the collected data as points on the coordinate plane.
- Students represent discrete data as points on a graph (not a line or ray).
- Students represent continuous data as points on a graph that can be connected by a line or ray.
- Students are not expected to know the terms collected data, discrete or continuous.

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Expressions and Equations

The Graph of an Equation in 2 Variables

Grade 8:

- Students graph equations in two variables with and without context.
  - The context, when given, dictates whether to connect the points or not.
- Students are expected to know the terms discrete rate and continuous rate.
- Allusions are made to domain and range, but students are not expected to know these terms.
  - Read: G8 M5 L2 Excerpt

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Agenda

- Ratios & Proportional Relationships
- Geometry
- The Number System
- Expressions and Equations
- Linear Equations and Functions

Linear Equations and Functions

Slope of a Line

- Choose any 2 pairs of points on the line. Use the points to create right triangles.
- The AA criterion say these triangles are similar.
- Thus, the proportion of the vertical leg to the horizontal leg is the same for each triangle.
- Thus for any two points on a non-vertical line the ratio the vertical distance: horizontal distance between the two points is equal.

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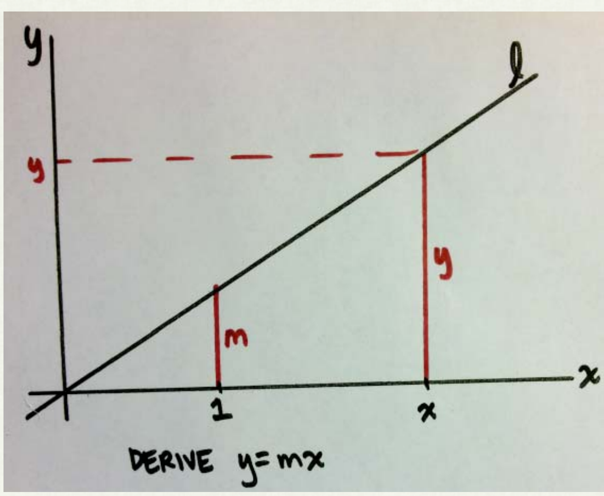
Linear Equations and Functions

- Deriving the equation of a line
- The 'why' and 'what' of functions?
- Linear functions and rate of change
- Solving systems of linear equations

Linear Equations and Functions

Deriving the Equation for a Line

For a line going through the origin:



$$\frac{y}{m} = \frac{x}{1}$$

$$\boxed{y = mx}$$

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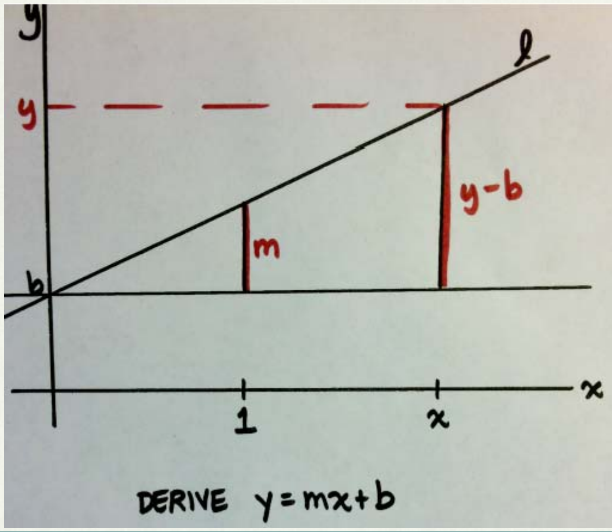
Linear Equations and Functions

Deriving the Equation for a Line

For a line passing through point *b* on the *y*-axis:



$$\frac{y-b}{m} = \frac{x}{1}$$

$$y-b = mx$$

$$\boxed{y = mx+b}$$

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Linear Equations and Functions

Why study functions?

- Functions allow us to
  - make predictions,
  - classify the data in our environment.
- G8-M5 Lesson 1 Examples 1 & 2

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Linear Equations and Functions

What is a function?

- Grade 8: A ***function*** is a rule that assigns to each input exactly one output.
- Grade 9: A ***function*** is a correspondence between two sets, X and Y, in which each element of X is matched to one and only one element of Y. The set X is called the domain; the set Y is called the range.
- The ***graph of a function*** is the set of ordered pairs consisting of an input and the corresponding output.

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Linear Equations and Functions

Building on Concepts from EE

- An expression in 1 variable defines a general calculation in which the variable can represent / can be replaced with a single number (chosen from a set of acceptable inputs).
- It's useful to relate a function to an input-output machine with a variable representing the input, and an expression representing the output.
  - Review G8 M5 L2 Problem Set

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Linear Equations and Functions

Building on Concepts from EE

- If we then choose a different variable to represent the output, we have an equation in two variables.
- Plotting points gives a visual representation of the relationship between the two variables.
  - Work through:

    G8 M5 L5 Exercise 4

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Linear Equations and Functions

Linear functions and rate of change

- Grade 8 Module 4 defines ***slope*** as a number that describes "steepness" or "slant" of a line. It is the ***constant rate of change***.
  - Review G8 M5 Lesson 7 Exercise 4

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Linear Equations and Functions

Linear functions and rate of change

- G8-M7 Lesson 22 introduces the concept of ***average rate of change***.
- Grade 9 Module 3 defines average rate of change:

  Given a function whose domain includes the interval and whose range is real numbers, the *average rate of change on the interval* is

Linear Equations and Functions – Systems of Equations

Solving Systems of Equations

Consider the following question:

For all questions, $R =$ All Real Numbers and $i = \sqrt{-1}$.

Precalculus:

1. In how many distinct points will the graph of $x^2 + xy + y^2 = 3$ intersect the graph of $x + xy + y + 1 = 0$?

   a. 0
   b. 1
   c. 2
   d. 3 or more
   e. None of the Above

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Linear Equations and Functions – Systems of Equations

Solving Systems of Equations

Here is the solution according to the answer key for the test:

For all questions, R = All Real Numbers and $i = \sqrt{-1}$.

1. **D**

$$xy = 3 - x^2 - y^2$$
$$xy = -1 - x - y$$
$$3 - x^2 - y^2 = -1 - x - y$$
$$4 = x^2 + y^2 - x - y$$

A circle has infinitely many distinct points.

Linear Equations and Functions – Systems of Equations

Solving Systems of Equations

A-REI.5

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

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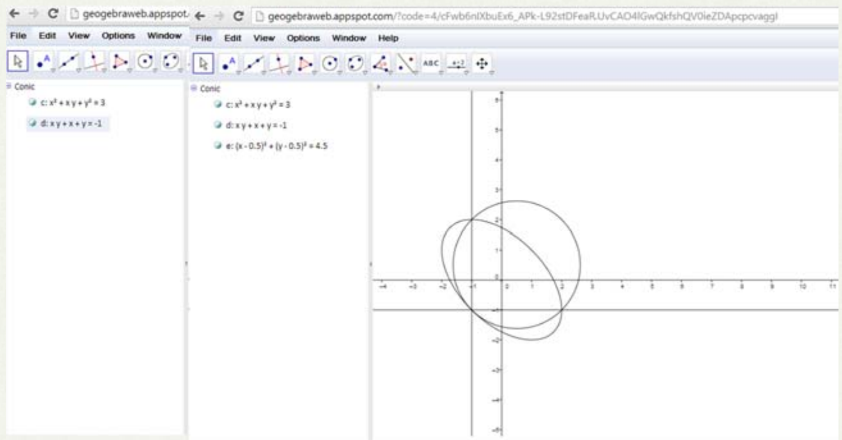
Linear Equations and Functions – Systems of Equations

Solving Systems of Equations

Here is a graph of the two equations:



Linear Equations and Functions – Systems of Equations

Correcting the Misconception

Sketch the graph of each equation in the following system:

$$\begin{cases} 3x - y = -6 \\ x + 2y = 5 \end{cases}$$

Replace one equation with the sum of the first equation and the second equation and the sketch a graph the two equations.

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Linear Equations and Functions – Systems of Equations

Correcting the Misconception

Addressed in G8-M4 re: 8.EE.8

"The graph of the new equation will also pass through (or contain) the intersection point (the solution point).

Suppose the new equation is $x = 3$. The graph of that equation passes through the solution point, therefore the solution point must have an x-coordinate of 3."

Key Points

- The progression of ratios from Grades 6 – 8: ratio relationship, ratio, value of a ratio, unit rate of A:B, unit rate of B:A (associated ratios), constant of proportionality, scale factor, similarity, and slope.
- The progression of the number system from Grades 6 – 8: division of fractions, long division including division of decimals, integers, operations with integers, irrational numbers.
- The progression of expressions and equations from Grades 6 – 8: the concrete representation in the form of diagrams and graphs to the abstract with variables using properties of operations and properties of equality.

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