COMMON CORE

Eureka Math
A Story of Functions

Grade 9 Overview

Opening Exercise

Complete the following exercise:

Opening Exercise

1. Identify the type of function the each table represents (e.g., quadratic, linear, exponential, square root, etc.).
2. Explain how you were able to identify the function.
3. Find the symbolic representation of the function.
4. Plot the graphs of your data.

A

x	y
1	5
2	7
3	9
4	11
5	13

B

x	y
1	6
2	9
3	13.5
4	20.25
5	30.375

C

x	y
1	3
2	12
3	27
4	48
5	75

Participant Poll

- Classroom teacher
- Math trainer
- Principal or school leader
- District representative / leader
- Other

Session Objectives

- Experience the instructional approaches to teaching the major work of Algebra I.
- Relate the instructional approaches to the Mathematical Practice Standards
- Make coherent connections within Algebra 1, and to content from prior / subsequent grade levels.

Major Work of Algebra I

Review the PARCC Model Content Framework's definition of the 'Major Work' of Algebra I

Flow of Algebra I

Module 1: Relationships Between Quantities and Their Graphs

Module 2: Descriptive Statistics

Module 3: Linear and Exponential Functions

Module 4: Polynomial and Quadratic Expressions, Equations and Functions

Module 5: A Synthesis of Modeling with Equations and Functions

Relationships… Quantities… Graphs

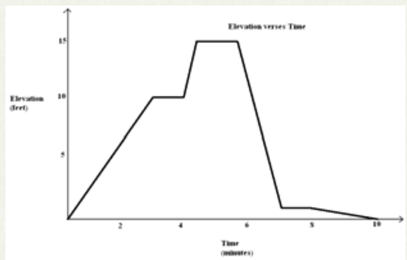
- http://mrmeyer.com/graphingstories1/graphingstories2.mov

  (watch only the first 1:08 minutes)
- Describe the motion of the man in the video

G9-M1 Lesson 1 Example 1

Relationships… Quantities… Graphs

- Here is an elevation vs. time graph of a person's motion, can we describe what the person might have been doing?



G9-M1 Lesson 1 Example 2

Relationships… Quantities… Graphs

- G9-M1 Mid-Module Assessment – Question #1
- G9-M1 Lesson 3 Example 2

  https://www.youtube.com/watch?v=gEwzDydciWc

- Descriptive Statistics –

  Modeling Relationships with a Line

  G9-M2 Lesson 14

From Base 10 to Base X

- Why do humans have a predilection for the number 10?
- What do you suppose our number system would look like if humans had only one hand, only 5 fingers?

G9-M1 Lesson 8

From Base 10 to Base X

- How would we write the number 113 in base 5?

From Base 10 to Base X

- Let's be as general as possible and write a number in base ***x***

$$\mathbf{1} \times \boldsymbol{x}^{\mathbf{3}} \quad + \quad \mathbf{2} \times \boldsymbol{x}^{\mathbf{2}} \quad + \quad \mathbf{7} \times \boldsymbol{x} \quad + \quad \mathbf{3} \times \mathbf{1}$$

- A variable is a placeholder for a number.

  If x is 10, what is the number?

  If x is 5, what is the number?

- Is it acceptable that we have a coefficient of 7 if we are in base 5?

Polynomial Expressions

Polynomial Expression – a **recursive** definition

POLYNOMIAL EXPRESSION. A polynomial expression is either

(1) a numerical expression or a variable symbol, or

(2) the result of placing two previously generated polynomial expressions into the blanks of the addition operator (__+__) or the multiplication operator (__×__).

Polynomial Expressions

Some Vocabulary and Analogies

Numerical expression:

$(3 + 2)(3 \times 3)$

Number:

45

Polynomial expression:

$(x^2 + x)(x - 3) + x^2$

Polynomial:

$x^3 - x^2 - 3x$

Pictorial Models of Operations and Their Properties

- Consider that:

  $1 + 2 + 3 = 1(2)(3)$
- Use pictorial models to show that, in general:

  $a + b + c \neq a(b)(c)$
- Draw a picture to represent:

  $(a + b) \times (c + d) \times (e + f + g)$
- Excerpts from the Lesson 6 Problem Set
- Excerpts from Lessons 7-8

G9-M1 Lesson 6

Foundations in Expressions

- What is an expression?
- What is a variable?
- Why use variables and expressions?
- What does it mean for two expressions to be equivalent?
- When would it be useful to set one algebraic expression equal to another?
- What capacities do students need with expressions to be successful in Algebra 1?

Number Sentences and Equations

Exercise 1: Answer the following for each sentence:

- Is the statement a grammatically correct sentence?
- What is the subject of the sentence?
- What is the verb in the sentence?
- What is the object of the sentence?
- Is the sentence true?

  a. **"The President of the United States is a United States citizen."**

  b. **"The President of France is a United States citizen."**

  c. **"2 + 3 = 1 + 4."**

  d. **"2 + 3 = 9 + 4."**

G9-M1 Lesson 10

Number Sentences and Equations

A ***number sentence*** is a statement of equality between two numerical expressions.

A number sentence is said to be ***true*** if both numerical expressions are equivalent (that is, both evaluate to the same number). It is said to be ***false*** otherwise. True and false are called ***truth values***.

Complete Lesson 10 Exercise 2.

G9-M1 Lesson 10

Number Sentences and Equations

An ***algebraic equation*** is a statement of equality between two expressions.

Algebraic equations can be number sentences (when both expressions are numerical), but often they contain symbols whose values have not been determined.

Solving Equations

When algebraic equations contain a symbol whose value has not yet been named, we use analysis to determine whether:

- The equation is true for all the possible values of the variable(s), or
- The equation is true for a certain set of the possible value(s) of the variable(s), or
- The equation is never true for any of the possible values of the variable(s).

Complete G9-M1 Lesson 10 Exercise 5

An Analogy:

Julie is 300 feet away from her friend's front porch and observes, **"Someone is sitting on the porch."**

- Given that she didn't specify otherwise, we would assume that the 'someone' Julie thinks she sees is a human.
- We can't guarantee that Julie's observatory statement is true. It could be that Julie's friend has something on the porch that merely looks like a human from far away.
- Julie assumes she is correct, and moves closer to see if she can figure out who it is. As she nears the porch she declares, "Ah, it is our friend, John Berry."

Solving Equations

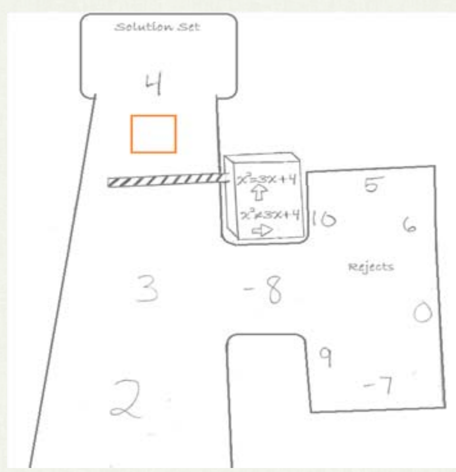
- In the sentence $w^2 = 4$, w can represent any real number we care to choose (its domain).
- If we choose to let w be 5, then the number sentence is false.
- If we let $w = 2$, then the sentence is true.
- Is there another value for w that would also make the sentence true?

Solution Sets

Consider: $x^2 = 3x + 4$

The ***solution set*** of an equation written with only one variable is the set of all values one can assign to that variable to make the equation a true statement. Any one of those values is said to be **a *solution to the equation***.

To "**solve an equation**" means to "find the solution set" for that equation.



G9-M1 Lesson 11

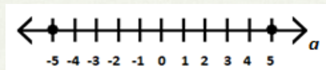
Solution Sets

Example 3: Solve $a^2 = 25$.

IN WORDS: $a^2 = 25$ has solutions 5 and -5.

IN SET NOTATION: $\{-5, 5\}$

IN A GRAPHICAL REPRESENTATION ON A NUMBER LINE:



Exercise 1: Solve for a: $a^2 = -25$.

Exercise 2: Solve for p: $7 + p = 12$.

Exercise 3: Solve for x: $\frac{x}{x} = 1$

Exercise 4: Solve for α: $\alpha + \alpha^2 = \alpha(\alpha + 1)$

Solving Equations

Why should the equations:

$(x - 1)(x + 3) = 17 + x$ and $(x + 3)(x - 1) = 17 + x$

have the same solution set?

- If x is a solution to an equation, then it will also be a solution to any new equation we make by applying the commutative and associative properties to the expression in that equation.
- Does this reasoning apply to the distributive property as well?
- To the properties of rational exponents?

Solving Equations

Do you think the equations:

$(x - 1)(x + 3) = 17 + x$ and $(x - 1)(x + 3) + 500 = 517 + x$

should have the same solution set? Why?

Whenever $a = b$, is true, then $a + c = b + c$ will also be true for all real numbers c.

Whenever $a = b$, is false, then $a + c = b + c$ will also be false for all real numbers c.

Whenever $a = b$, is true, then $ac = bc$ will also be true, (and when $a = b$, is false, then $ac = bc$ will also be false) for all non-zero, real numbers c.

G9-M1 Lesson 12

Solving Equations

Consider the equation: $|x| + 5 = 2$

Is it true, then, that $|x| + 5 - 5 = 2 - 5$?

Create another equation for which it is not obvious at the onset that it has no possible solution.

G9-M1 Lesson 12

Solving Equations

Applying the properties of Equality is guaranteed to preserve the solution set.

Applying the Distributive, Associative, and Commutative Properties or the properties of rational exponents to either side is also guaranteed to preserve the solution set.

Some Potential Dangers

When Solving Equations

Consider the equation:

$$\frac{x}{12} = \frac{1}{3}$$

You can do anything that's useful, but it is not guaranteed to preserve your solution set!

Some Potential Dangers

When Solving Equations

Exercise 7:

Consider the equation $x - 3 = 5$.

Multiply both sides of the equation by a constant and show that the solution set did not change.

Now, multiply both sides by x.

$$x(x - 3) = 5x$$

Show that $x = 8$ is still a solution to the new equation.

Show that $x = 0$ is also a solution to the new equation.

Why did my solution set get altered? What did I do?

G9-M1 Lesson 13

Integer Sequences: Should You Believe in Patterns?

- What is the next number in the sequence?

  2, 4, 6, 8, …

- Is it 17?

G9-M3 Lesson 1

Terms, Term Numbers, and "the n^{th} term"

- What do I mean by "the n^{th} term"?
- Create a table of the terms of the sequence.

G9-M3 Lesson 1

Introducing the $f(n)$ notation

- I'd like to have a formula that works like this:

  I pick <u>any</u> term number I want and plug it into the formula, and it will give me the value of that term.
- In this case:

  A formula for the n^{th} term $= 2^{n-1}$
- Would it be ok if I wrote $f(n)$ to stand for "a formula for the n^{th} term"?

$$f(n) = 2^{n-1}$$

G9-M3 Lesson 1

Consolidating Understanding

Closing:

- Why is it important to have a formula to represent a sequence?
- Can one sequence have two different formulas?
- What does f(n) represent? How is it read aloud?

Lesson Summary:

- A sequence can be thought of as an ordered list of elements. To define the pattern of the sequence, an explicit formula is often given, and unless specified otherwise, the first term is found by substituting 1 into the formula.

G9-M3 Lesson 1

Arithmetic Sequences

Term 1: 5

Term 2: 8 = 5 + 3

Term 3: 11 = 5 + 3 + 3

Term 4: 14 = 5 + 3 + 3 + 3

Term 5: 17 = 5 + 3 + 3 + 3 + 3

...

Term n:

G9-M3 Lesson 2 Example 1

Recursive Formulas for Sequences

- When Johnny saw Akeila's sequence he wrote the following:

  $A(n+1) = A(n) + 3$ for $n \geq 1$ and $A(1) = 5$
- Why do you suppose he would write that? Can you make sense of what he is trying to convey?
- What does the $A(n+1)$ part mean?

G9-M3 Lesson 2

Geometric Sequences

- Look at the sequence: $1, 3, 9, 27$ ...
- How is this sequence different from the one we just analyzed?
- Write a recursive formula for the sequence.
- Write an explicit formula for the sequence.
- Plot the terms of the sequence on a coordinate plane.

G9-M3 Lesson 3 Example 1

Arithmetic and Geometric Sequences

1. $-2, 2, 6, 10, \ldots$
2. $2, 4, 8, 16, \ldots$
3. $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$
4. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$
5. $10, 1, 0.1, 0.01, 0.001, \ldots$
6. $4, -1, -6, -11, \ldots$

G9-M3 Lesson 3 Exercise 1

Arithmetic and Geometric Sequences

Describe as precisely as possible what an arithmetic sequence is.

ARITHMETIC SEQUENCE - described as follows: A sequence is called *arithmetic* if there is a real number d such that each term in the sequence is the sum of the previous term and d.

GEOMETRIC SEQUENCE - described as follows: A sequence is called *geometric* if there is a real number r such that each term in the sequence is a product of the previous term and r.

Exponential Functions

Which is better?

- Getting paid $33,333.34 every day for 30 days (for a total of just over $ 1 million dollars), ***OR***
- Getting paid $0.01 today and getting paid double the previous day's pay for the 29 days that follow?
- Why does the 2nd option turn out to be better?
- What if the experiment only went on for 15 days?

Exponential Growth vs. Linear Growth

- **Is it fair to say that the values of the geometric sequence grow faster than the values of the arithmetic sequence?**
- Review G9-M3 Lesson 5 Opening Exercise and Examples 1 and 2

Why stay with whole numbers?

- Why are square numbers called square numbers?

  If $S(n)$ denotes the n^{th} square number, what is a formula for $S(n)$?
- In this context what would be the meaning of

  $S(0), S(\pi), S(-1)$?
- Exercises 5-8: Suppose we extend our thinking to consider squares of side-length x cm… Create a formula for the area, $A(x)$ cm^2 of a square of side length x cm.
- Review Exercises 9-12, taking time to do #10 and #12
- Do Exercises 13-14

G9-M3 Lesson 8

Transformations

- Foundations from Grade 8 Modules 2 and 3
- Transformations of the plane observed by figures in the plane
- Grade 8 Module 2 Lesson 1:

  Students describe motions intuitively, working towards comfort with formal concepts and language.

Transformations of the Plane

- Think of a plane as a sheet of overhead projector transparency, or a sheet of paper.
- The notion of mapping. Map each point on the street to a point on your paper (the map) in a way that intuitively "preserves the shape".
- Projection, using a light source, from one sheet to another.
- A ***transformation of the plane***, to be denoted by F, is a rule that associates (or assigns) to each point P of the plane to a unique point which will be denoted by $F(P)$.

G8-M2 Lesson 1

Three Rigid Transformations

Understand and perform:

- Translation:
  - a transformation along a vector.
- Reflection:
  - a transformation across a line.
- Rotation:
  - a transformation about a point for a given angle measure

Rigid Transformations

- Do translations, reflections and rotations:
  - Map lines to lines, rays to rays, segments to segments, and angles to angles?
  - Preserve the lengths of segments?
  - Preserve the measures of angles.?
- Students verify these properties informally.

Congruence

- Rigid transformations can be sequenced. Does order matter?
- Can sequences of rigid transformations be reversed?
- A congruence is a sequence of basic rigid motions (translations, reflections, or rotations) that maps one figure onto another.
- A two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.

Dilations

- A dilation is a transformation of the plane with center *O* and scale factor *r*, that assigns to each point *P* of the plane a point *Dilation*(*P*) so that:

  Dilation(*O*) = *O and*

  If $P \neq O$, then *Dilation*(*P*), denoted as *P'*, is the point on the ray *OP* so that $|OP'| = r\,|OP|$.
- Do dilations:
  - Map lines to lines, rays to rays, segments to segments, and angles to angles?
  - Preserve the lengths of segments?
  - Preserve the measures of angles?

Similarity

- Two figures are said to be ***similar*** if you can map one onto another by a dilation followed by a congruence.

Transformations of Functions

- Student progression:
  - o Transformations of the plane observed by figures
  - o Transformations of the plate observed by graphs of functions
  - o Transformations of functions
- Selected excerpts from G9-M3 Lessons 17-20

Challenge

Discuss with a neighbor – which of the following phrases contain incorrect usage of language or symbols?

- The graph of $f(x)$ has an average rate of change of 3 on the interval $(0, 1)$.
- The function $g(x) = x^2 + 3$ is increasing on the interval
  (0, ∞).
- The function $g(x)$ defined above is the function $f(x) = x^2$ shifted to the left 3 units.
- The terms of the arithmetic sequence, $f(n) = 2 + 3n$, are a straight line.

Differences and Diagonals

What is the next number in the sequence?

- 4, 7, 10, 13, 16, …
- 4, 5, 8, 13, 20, 29, …
- 0, 2, 20, 72, 176, 350, 612, …
- 1, 2, 4, 8, 16, 32, 64, 128, 256, …
- 1, 4, 9, 16, 25, 36, 49, 64, 81, …
- 1, 8, 27, 64, 125, 216, 343, …

Differences and Diagonals

- If I only gave you the leading diagonal (and it eventually led to a row of zero's) could you find every number in the sequence?
- What does the leading diagonal look like for each of the following:

  1: 1, 1, 1, 1, 1, ….

  n: 1, 2, 3, 4, 5, 6, …

  n^2: 1, 4, 9, 16, 25, 36, …

  n^3: 1, 8, 27, 64, 125, 216, …

Differences and Diagonals

- What do you suppose the leading diagonal would look like for $n^2 + n$?
- If I gave you only the leading diagonal could you find an explicit formula for the sequence?

Objects in Motion

- Suppose I wish to test the speed and acceleration of a golf ball dropped from the top of a building that is 100 meters tall.
- How would I set up my experiment?
- What data would I need to collect?
- How could I estimate the speed of the ball?
- How could I estimate the acceleration of the ball?

Solving Quadratic Equations

- A progression from simple to complex

The Quadratic Formula

- Deriving the quadratic formula using the geometric model of the square
- Deriving the quadratic formula algebraically

Graphing Quadratic Functions

- Graphing the basic quadratic parent function: $f(x) = x^2$
- Describe what you see.
- Symmetry!
- Graph transformations of the parent function
- Graph f and g on the same coordinate plane:

$$f(x) = x^2 \ \ and \ \ g(x) = 3x + 1$$

- Graph the equation $y = f(x) + g(x)$

The Role of Factoring

- Using geometric models to aid in factoring
- What is the geometric significance of the factors of a quadratic?
- What does it mean to factor completely?
- The zero factor property
- Factoring as an aid to graphing quadratics

G9-M4 Lessons 1-6

Meeting the Standards

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Meeting the Standards

- Completing the square for an expression
- Completing the square to go from standard form to vertex form

G9-M4 Lessons 11 and 17

U-Shaped Symmetric Graphs

Can a hanging chain be modeled precisely by a quadratic function?

1. Tape the ends of the chain so that the lowest part of the chain falls right at the origin.
2. Mark off evenly spaced x-intervals
3. Find the y-value for each x-value marked
4. Does the data support the idea that the curve is quadratic

Bridging Gaps in Prerequisite Knowledge

- Conceptual understanding comes first!
- Fluency – quick and accurate:
  - Rapid White Board Exchange
  - Sprint

A Synthesis of Modeling with Equations and Functions

Topic A: Elements of Modeling

- Lesson 1: Analyzing a Graph
  - Example 1 and Exercises 1-6
  - Problem Set
- Lesson 2: Analyzing a Data Set
  - Opening
  - Exercise 2
  - Problem Set #4
- Lesson 3: Analyzing a Verbal Description
  - Exercises 1-4

A Synthesis of Modeling with Equations and Functions

Topic B: Completing the Modeling Cycle

- Lesson 4: Modeling a Context from a Graph
  - Example 1
  - Exercise 2
  - Exit Ticket
- Lesson 5: Modeling from a Sequence
  - Opening
  - Problem Set – Problems 2-4

A Synthesis of Modeling with Equations and Functions

Topic B: Completing the Modeling Cycle

- Lesson 6-7: Modeling a Context from Data
  - L6: Opening Exercise
  - L6: Example 1
  - L6: Exercises 1-3
  - L6: Exit Ticket
- Lesson 8-9: Modeling a Context from a Verbal Description
  - L8: Examples 1-2
  - L8: Problem Set # 1-2

Key Points

- Lessons emphasize a freedom to ask questions, experiment, observe, look for structure, reason and communicate.
- Timing of lessons cannot possibly meet the needs of all student populations. Teachers should preview the lesson and make conscious choices about how much time to devote to each portion.
- While many exercises support the mathematical practices in and of themselves, the **discussions and dialog points** are often critical for both their content and for enacting the mathematical practice standards.

Key Points

- Reinforce the notion that we are **modeling** with functions –
  - Push students to recognize when they are or are not given enough information to be sure that the function will be an accurate representation of the data or information described.
  - Use precision when speaking and encourage students to do the same by questioning them for accuracy and precision.
  - Remain aware of the discrete vs. continuous domain in the context of the situation.
- Approach Lesson Summaries as opportunities for students to consolidate their knowledge – ask and scaffold, don't tell.