Louisiana Department of Education
Mentor Teacher Training

Module 4:
Mathematical Instructional Shifts in Practice

Elementary Cohort
July, 2019

Facilitated by Learning Forward

Mentor Teacher Training

Mentor Training Course Goals

Mentors will:

- Build strong relationships with mentees.
- Diagnose and prioritize mentee's strengths and areas for growth.
- Design and implement a coaching support plan to develop mentee knowledge and skills.
- Assess and deepen mentor content knowledge and content-specific pedagogy.

Module 4 Morning Outcomes:

- Deepen pedagogical content knowledge of the mathematical shifts to increase mentor's ability to coach their mentee's math instruction.
- Experience and analyze a Eureka lesson to identify evidence of the key shifts in practice.
- Plan for interventions to meet the specific needs of a mentee based on observation data.
- Model best practices to support mentee learning.

Module 4 Agenda:

- Welcome & Outcomes
- Explore mathematics shifts in the Eureka curriculum
- Lunch
- Plan for Interventions
- Modeling Best Practices
- Connection to Assessments
- Wrap-up

Mutual Commitments:

Make the learning meaningful

Engage mentally and physically

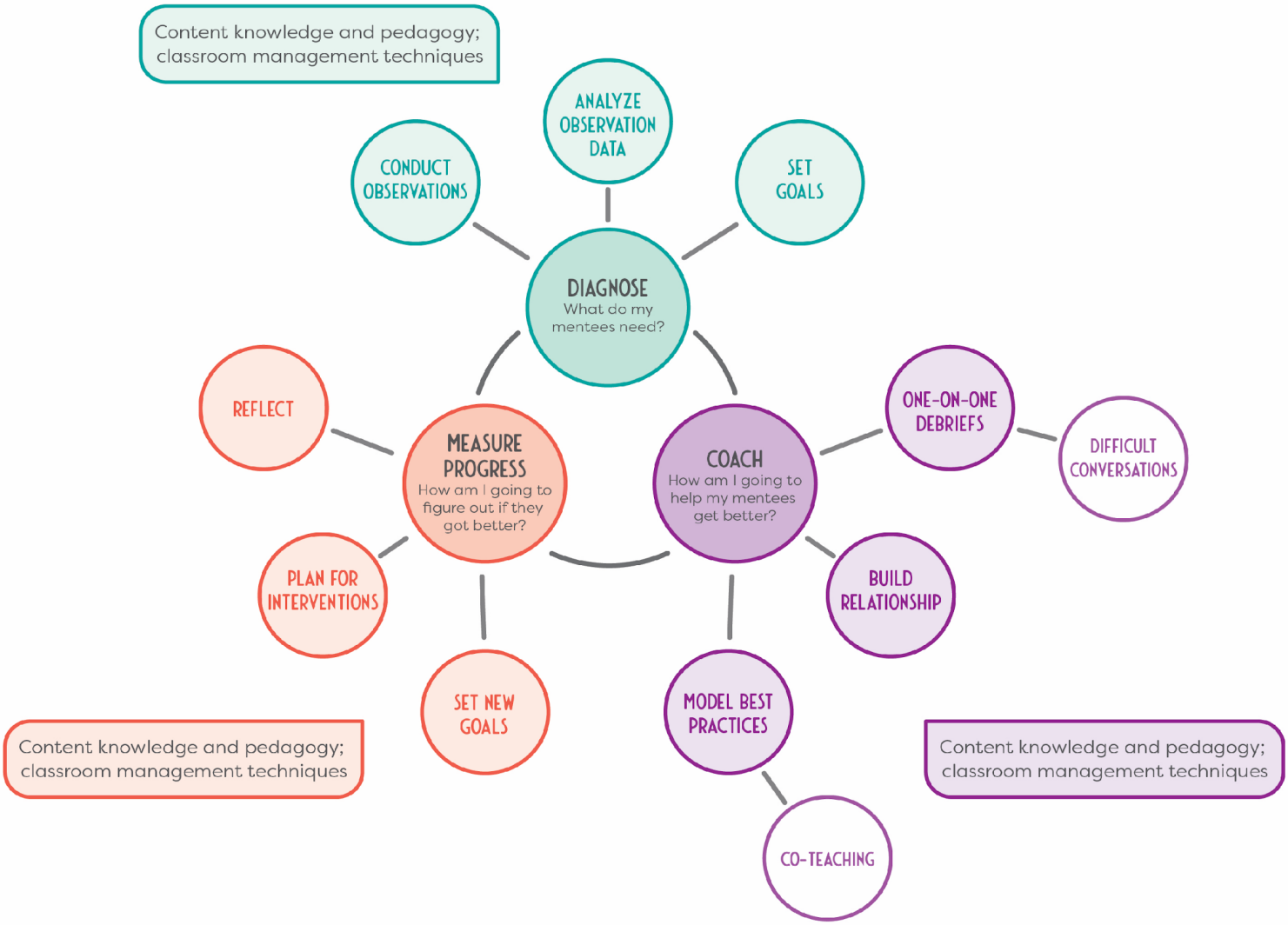
Notice opportunities to support the learning of others

Take responsibility of own learning

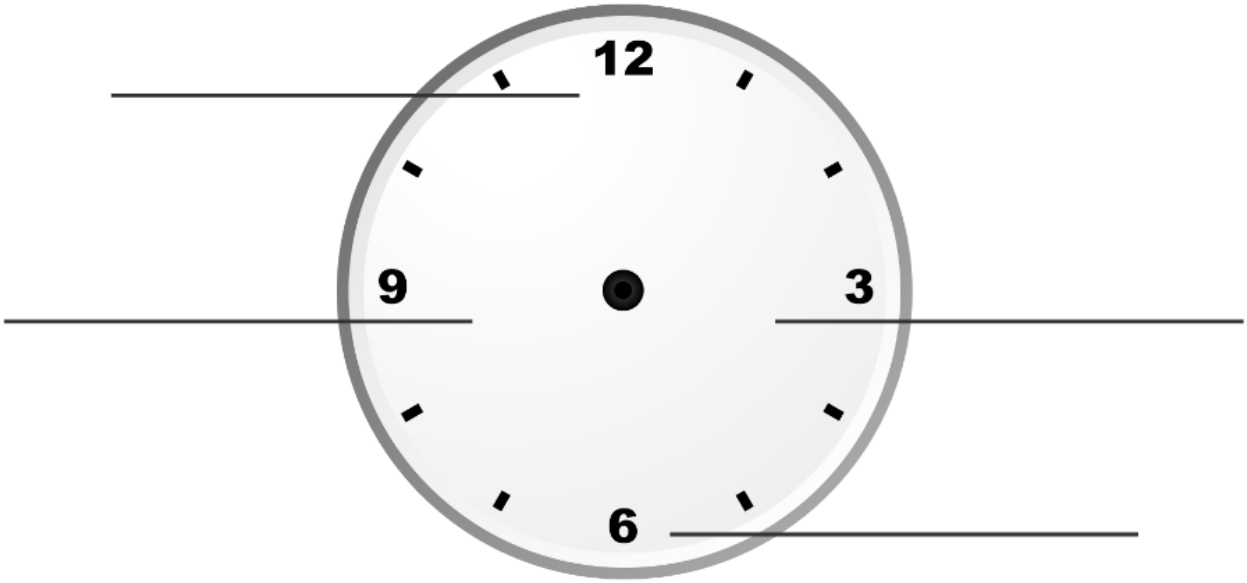
Own the outcomes

Respect the learning environment of self and others

The Mentoring Cycle



Let's Make a Date



Failing by design: How we make teaching too hard for mere mortals

https://edexcellence.net/articles/failing-by-design-how-we-make-teaching-too-hard-for-mere-mortals

Robert Pondiscio
May 10, 2016

If you caught your pediatrician Googling "upset stomach remedies" before deciding how to treat your child and home-brewing medications over an office sink, you might start looking for a new pediatrician. So how would you feel if you learned that Google and Pinterest are where your child's teacher goes to look for instructional materials?

Well, brace yourself, because that's exactly what's happening. And no, your child's teacher is not an exception. A new study from the RAND Corporation finds that nearly every teacher in America—99 percent of elementary teachers, 96 percent of secondary school teachers—draws upon "materials I developed and/or selected myself" in teaching English language arts. And where do they find materials? The most common answer among elementary school teachers is Google (94 percent), followed by Pinterest (87 percent). The numbers are virtually the same for math.

But don't blame teachers. These data, for reasons both good and bad, reveal a dirty little secret about American education. In many districts and schools—maybe even most—the efficacy of the instructional materials put in front of children is an afterthought. For teachers, it makes an already hard job nearly impossible to do well.

Expecting teachers to be expert pedagogues and instructional designers is one of the ways in which we push the job far beyond the abilities of mere mortals. Add the expectation that teachers should differentiate every lesson to meet the needs of each individual student, and the job falls well outside the capacity of nearly all of America's 3.7 million classroom teachers (myself included).

If you're looking for the root causes of America's educational mediocrity, start with how poorly we prepare teachers for one of the most important parts of the job. "Few teachers ever take coursework on instructional design and, therefore, have little knowledge of the role it plays in student learning," notes Marcy Stein, an education professor with expertise in evaluating instructional design at the University of Washington Tacoma. It's like expecting the waiter at your favorite restaurant to serve your meal attentively while simultaneously cooking for twenty-five other people—and doing all the shopping and prepping the night before. You'd be exhausted too.

"Even if teachers were taught about instructional design, they would likely not have the time to prepare instructional materials, field test those materials to determine if they are effective, and modify the materials before using them to teach students. An iterative process is crucial for the development of effective materials," Stein points out.

There is good evidence to suggest that we are making a serious mistake by not paying more attention to curriculum, classroom materials, and instructional design. A 2012 Brookings study by Russ Whitehurst and Matt Chingos demonstrated that the "effect size" of choosing a better second-grade math curriculum was larger than replacing a fiftieth-percentile teacher with a seventy-fifth-percentile teacher. This is a powerful result, especially considering that it's relatively easy to give all children a better curriculum but extremely difficult to dramatically increase the effectiveness of their teachers. It's cost-neutral too: A Center for American Progress report by Ulrich Boser and Chingos showed virtually no difference in price between effective and ineffective curricula.

To be clear, there are perfectly good reasons why even the best teachers would be hitting the Internet for lesson planning—to find supplemental materials for individual students, for example, or adaptations for special needs kids. And teachers report using books and materials from myriad sources, including those selected by their schools and districts. But the RAND study offers a window into a phenomenon that is rarely discussed in American education: What children learn in school varies wildly from state to state, within districts, and even within grades in the same school.

If we're serious about raising the output of our K–12 system at large—not by a little, but a lot—here are some of the questions we should be asking: What exactly is the teacher's job, and what is the best use of her limited time? Is it deciding what to teach, or how to teach it? Is the soul of the work instructional design or instructional delivery? Do you want your child's teacher to have the time to analyze student work and develop a keen eye for diagnosing mistakes and misunderstandings? Do you want her to give your child rich and meaningful feedback on assignments and homework? How about developing warm and productive relationships with your child and your family?

Now ask how you expect her to do all those things at a high level while spending precious hours every week creating curricula from scratch. Nearly half of teachers in the RAND study reported spending more than four hours per week developing or selecting their own instructional materials. Newer teachers almost certainly spend the most, hampering their ability to develop their craft.

To be sure, there are master teachers to whom we should eagerly grant nearly complete classroom autonomy, including over curriculum. You wouldn't tell Prince, "Just work on your guitar playing.

Someone else will write the songs." But it's simply unrealistic to assume that every teacher is a Prince-level virtuoso and polymath—let alone to base the job description on that assumption. No one would accuse Yo-Yo Ma of being a second-rate talent because he merely plays notes written by Bach.

Without question, we want our best teachers to play a significant role in instructional design so that more children and teachers can benefit from their expertise. But it is equally certain that twelve-plus years of a well-designed and sequenced curriculum would lead to better outcomes for children than the occasional year with a great yet isolated teacher. It would also let teachers focus more time on the art of teaching—that is, more time with student work and less time on Pinterest on Sunday night with an empty plan book at their elbow.

Great teachers need great instructional materials. It's time we got serious about providing them.

Robert Pondiscio is a Senior Fellow and the Vice President for External Affairs at the Thomas B. Fordham Institute.

Pondiscio, Robert. "Failing by design: How we make teaching too hard for mere mortals." Fordham Institute, 10 May. 2016, https://edexcellence.net/articles/failing-by-design-how-we-make-teaching-too-hard-for-mere-mortals.

Looking for Evidence of Student Engagement in the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ Students access prior learning from major work in the grade in order to support new learning.	
Rigor	
Conceptual Understanding ☐ Students access concepts and ideas from a variety of perspectives. ☐ Students explain mathematical ideas behind a particular concept in a variety of ways. ☐ Students use examples and counterexamples to make and support conjectures applied to one problem to multiple situations. ☐ Students create and use a variety of models to analyze relationships. ☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations. **Procedural Skills and Fluency** ☐ Students select tools (e.g. physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem. ☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways. ☐ Students look for patterns, generalizations, and shortcuts. ☐ Students are flexible in their use of procedures and skills to solve problems. **Application** ☐ Students decontextualize and contextualize quantities in problem situations. ☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

Note: To help educators look for evidence of grade-level-appropriate student engagement in mathematical tasks, these narrative descriptors are adapted from Illustrative Mathematics. (2014, February 12). *Standards for Mathematical Practice: Commentary and Elaborations for K–5 and 6-8*. Tucson, AZ. Available at http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards

Lesson 4

Objective: Make a ten to add within 20.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(8 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

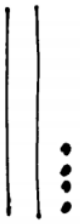
Fluency Practice (12 minutes)

- Draw Tens and Ones **2.OA.2** (3 minutes)
- Make Ten **2.OA.2** (3 minutes)
- Make the Next Ten Within 100 **2.OA.2** (4 minutes)
- Take Out One **2.OA.2** (2 minutes)

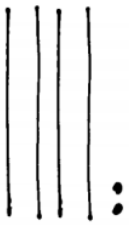
Draw Tens and Ones (3 minutes)

Materials: (T) Linking cubes with ten-sticks and extra cubes, place value chart (S) Personal white board

Note: This fluency activity reviews place value as students analyze two representations of two-digit numbers.



T: Draw the number of cubes I show with quick tens and ones.
T: (Show 2 linking cube ten-sticks and 4 ones.)
S: (Draw as pictured to the right.)
T: Show me your boards. Tell me the number.
S: 24.
T: Draw the number I show with quick tens and ones.
T: (Write the number 42 on the place value chart.)
S: (Draw as pictured to the right.)
T: Tell me the number.
S: 42.



For the next minute, represent 18 and 81, 37 and 73, 29 and 92, alternating between showing the smaller number of each pair with cubes and the larger number with the place value chart.

Make Ten (3 minutes)

Materials: (S) Personal white board

Note: This is a foundational skill for mastery of sums and differences to 20.

T: I'll say a number, and you say how many more to make ten.
T: 9. Get ready.
S: 1.
T: Write the addition sentence. (Pause.) Show me your boards.
S: (Show 9 + 1 = 10.)
T: (Scan each board, and accept 1 + 9 = 10, 10 = 9 + 1, etc.)
T: 8. (Pause as students write.) Get ready.
S: 2.
T: Write the addition sentence. (Pause.) Show me your boards.
S: (Show 8 + 2 = 10.)

Continue with the following possible sequence: 2, 5, 6, 4, 7, and 3.

Make the Next Ten Within 100 (4 minutes)

Materials: (T) Rekenrek (S) Personal white board

Note: In this fluency activity, students apply their knowledge of partners to ten to find analogous partners to 20, 30, and 40 to prepare for today's lesson. Keep them motivated to use the patterns by removing the Rekenrek at times.

T: (Show 19.) Say the number.
S: 19.
T: Write the number sentence, starting with 19, to get to or make the next ten on your personal white board.
S: (Write 19 + 1 = 20.)
T: (Scan the boards.) Tell me the addition sentence.
S: 19 + 1 = 20.
T: (Move 1 bead to make 20 as students answer.)
T: (Show 39.) Say the number.
S: 39.
T: Write the number sentence, starting with 39, to make the next ten on your personal white board.
S: (Write 39 + 1 = 40.)
T: (Scan the boards.) Tell me the addition sentence.
S: 39 + 1 = 40.
T: (Move 1 bead to make 40 as students answer.)

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

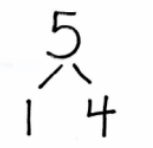
Once the Rekenrek is removed, encourage students who need support to visualize the beads (ten-frames or 5-groups), or guide them to use fingers to model the number of ones in order to determine how many more make ten.

Continue with the following possible sequence: 15, 35, 85; 18, 48, 68; 12, 52, and 92.

Take Out One (2 minutes)

Materials: (S) Personal white board

Note: In the lesson, students add 9 and 6 by adding 9 and 1 and 5. They "take out 1" from 5.



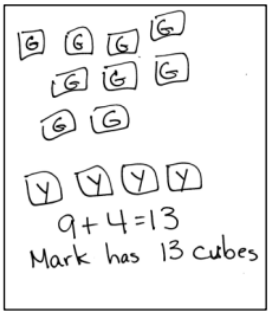
T: Let's take out 1 from each number. I say 5. You write the number bond and say the two parts, 1 and 4.
T: 5.
S: (Draw number bond.) 1 and 4.

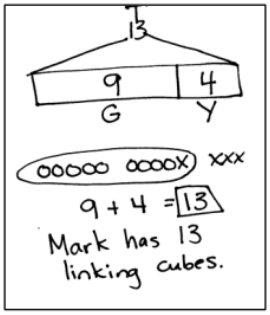
Continue with the following possible sequence: 3, 10, 4, 7, 9, 8, and 6.

Application Problem (8 minutes)

Mark had a stick of 9 green linking cubes. His friend gave him 4 yellow linking cubes. How many linking cubes does Mark have now?

MP.2





Note: This *add to result unknown* problem's tape diagram can be compared to that of Lesson 3 when a part was subtracted.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

"Mark's Linking Cubes" bridges into today's Concept Development of making a ten to add. Rather than teach the make ten strategy during the Application Problem, notice what strategies students are independently using, and integrate these observations into the Concept Development. During the Student Debrief, consider coming back to the Application Problem, and invite students to apply today's learning to show another way to solve the problem.

Concept Development (30 minutes)

Materials: (S) Personal white board

Part 1: Making ten from an addend of 9, 8, or 7.

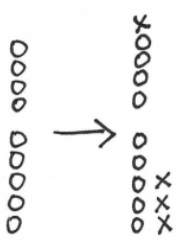
Note: In Grade 1, students used ten-sticks and quick ten drawings extensively when making ten. Now in Grade 2, the objective is to work at the numerical level as soon as possible.

T: (Write 9 + 4 on the board.)
T: Let's draw to solve 9 + 4 using circles and Xs.

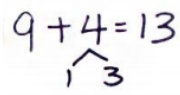
NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

If time or precision is a factor, create templates of pre-drawn circles to model addends of 9, 8, and 7. Then, students can attend to drawing Xs to complete the ten and model the remainder of the problem.

T: (Quickly draw and count aloud 9 circles in a 5-group column as seen in the first image.)



T: How many Xs will we add?
S: 4 Xs.
T: (Using the X symbol, complete the ten and draw the other 3 Xs to the right as seen in the second image.)
T: Did we make a ten?
S: Yes!
T: Our 9 + 4 is now a ten-plus fact. What fact can you see in the drawing?
S: 10 + 3 = 13.
T: 10 + 3 equals?
S: 13.
T: So, 9 + 4 equals?
S: 13. (Write the solution.)
T: What did we take out of 4 so that we could make 10?
S: 1.
T: (Draw the number bond under 4 as shown to the right.)



T: (Write 9 + 5.)
T: Solve using a number bond. (If students want or need to draw, allow them to.)

Continue with the following possible sequence: 9 + 6, 9 + 7, 8 + 9, 8 + 3, 8 + 4, 8 + 7, and 7 + 5. Have students explain their work to a partner.

Part 2: Observing patterns.

T: Look at our list of problems where one part, or addend, is 9. Tell your partner what you notice about adding to 9.
S: You get 1 out! → The answer is 10 and 1 less than the other addend.
T: Look at the problems with 8 as an addend. Tell your partner what you notice.
S: You get 2 out! → You always take 2 out of the other addend.
T: How is solving 9 + 4 and 8 + 4 different?
S: We used 2 to make 10 when we added to 8 and 1 to make 10 when we added to 9. → We used a different number bond.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Make a ten to add within 20.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

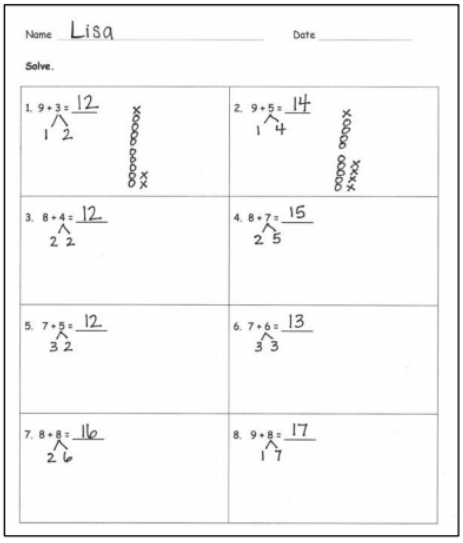
Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Let's look at Problems 11–14. How are the problems the same and different?
- Do you notice a pattern that will help you memorize your 9-plus facts? What other patterns do you notice?
- Explain the strategy we reviewed today. Can you think of another problem that the make ten strategy will help us solve?
- Can you figure out the math goal of today's lesson? What name would you give this lesson?

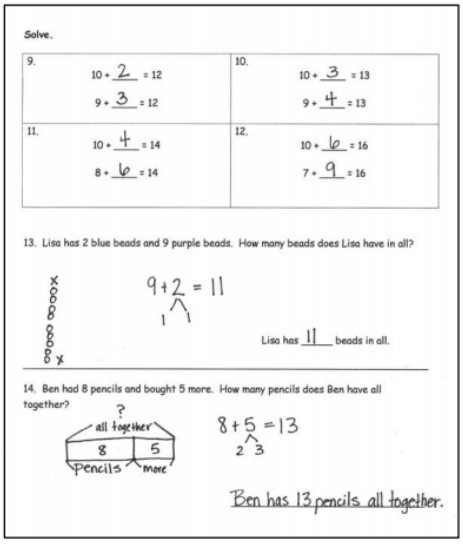
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.


Name Lisa Date

Solve.

1. 9 + 3 = 12 (1, 2)
2. 9 + 5 = 14 (1, 4)
3. 8 + 4 = 12 (2, 2)
4. 8 + 7 = 15 (2, 5)
5. 7 + 5 = 12 (3, 2)
6. 7 + 6 = 13 (3, 3)
7. 8 + 8 = 16 (2, 6)
8. 9 + 8 = 17 (1, 7)


Solve.

9. 10 + 2 = 12; 9 + 3 = 12
10. 10 + 3 = 13; 9 + 4 = 13
11. 10 + 4 = 14; 8 + 6 = 14
12. 10 + 6 = 16; 7 + 9 = 16

13. Lisa has 2 blue beads and 9 purple beads. How many beads does Lisa have in all?

9 + 2 = 11 (1, 1)

Lisa has 11 beads in all.

14. Ben had 8 pencils and bought 5 more. How many pencils does Ben have all together?

? all together; 8 Pencils; 5 more

8 + 5 = 13 (2, 3)

Ben has 13 pencils all together.

Name _____ Date _____

Solve.

1. 9 + 3 = ____	2. 9 + 5 = ____
3. 8 + 4 = _____	4. 8 + 7 = _____
5. 7 + 5 = _____	6. 7 + 6 = _____
7. 8 + 8 = _____	8. 9 + 8 = _____

Solve.

9. 10 + _____ = 12 9 + _____ = 12	10. 10 + _____ = 13 9 + _____ = 13
11. 10 + _____ = 14 8 + _____ = 14	12. 10 + _____ = 16 7 + _____ = 16

13. Lisa has 2 blue beads and 9 purple beads. How many beads does Lisa have in all?

Lisa has _____ beads in all.

14. Ben had 8 pencils and bought 5 more. How many pencils does Ben have altogether?

Name _____ Date _____

Solve.

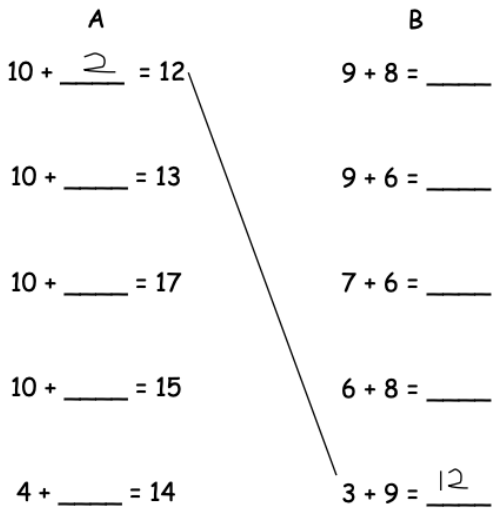
1. 9 + 6 = ____	2. 8 + 5 = ____

Name _____ Date _____

Solve.

1. 8 + 4 = ____ / \ 2 2 8 + 2 = 10 10 + 2 = 12	2. 9 + 7 = ____
3. 9 + 3 = _____	4. 8 + 6 = _____
5. 7 + 6 = _____	6. 7 + 8 = _____
7. 8 + 8 = _____	8. 8 + 9 = _____

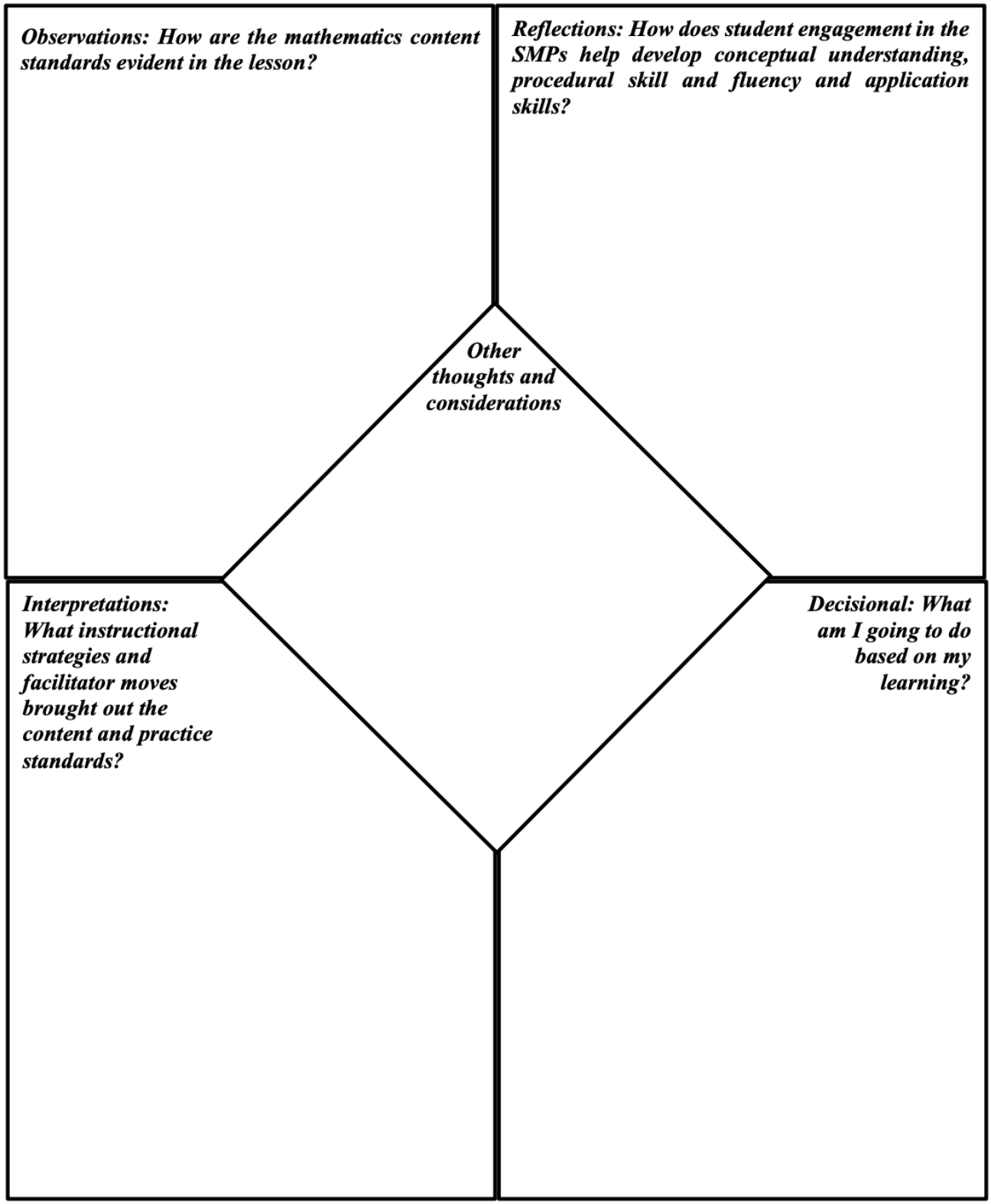
9. Solve and match.



10. Ronnie uses 5 brown bricks and 8 red bricks to build a fort. How many bricks does Ronnie use in all?

Ronnie uses _____ bricks.

Diamond Reflection
Unpacking the EngageNY Lesson



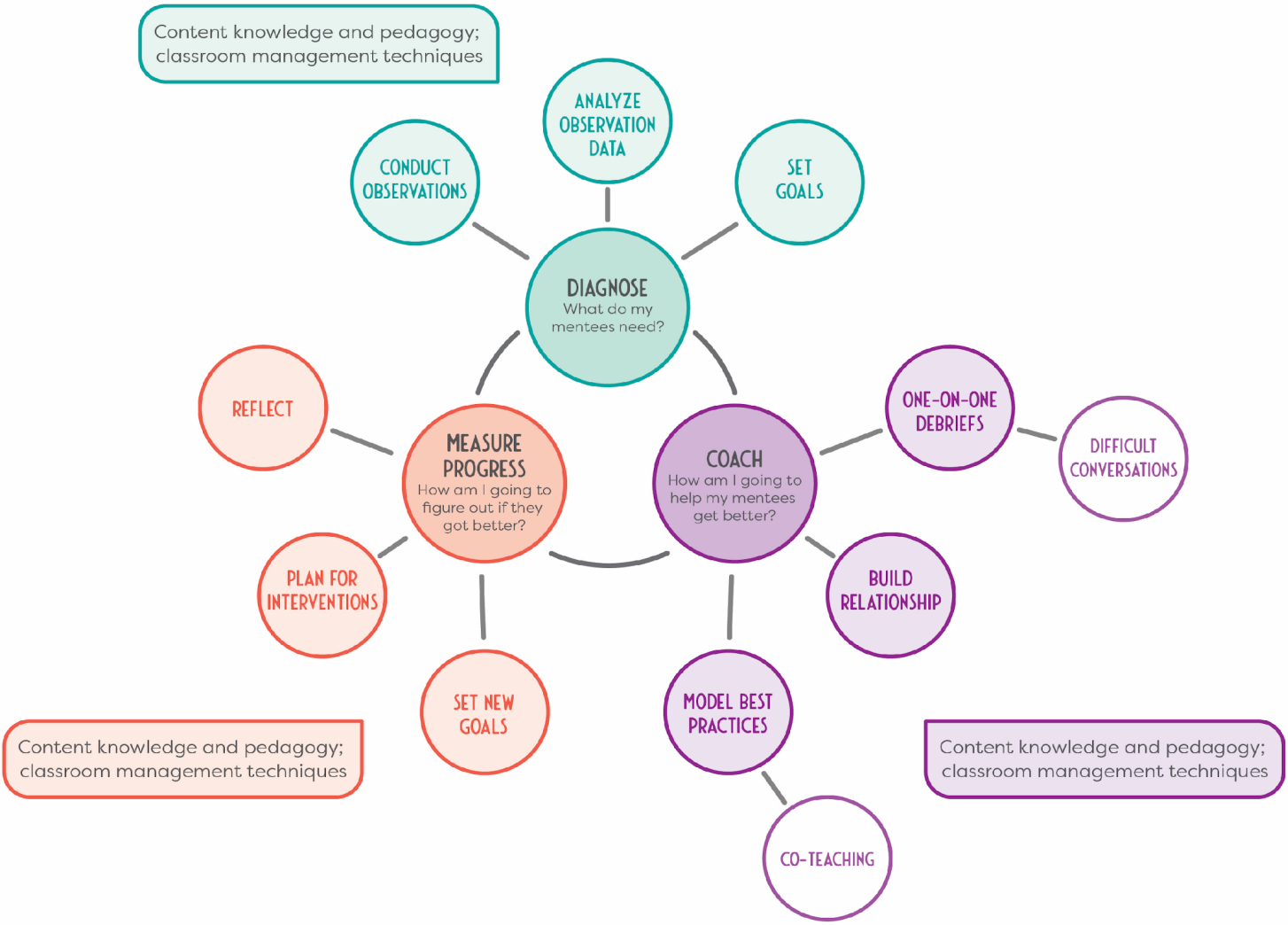
Key Takeaway:
Using a Tier 1 Curriculum ensures that all key shifts in mathematics are being implemented.

Key Takeaway:
Having a strong understanding of the instructional shifts in math increases the mentor's ability to coach their mentee's math instruction.

Module 4 Afternoon Outcomes

- Plan for interventions to meet the specific needs of a mentee based on observation data.
- Model best practices to support mentee learning.

The Mentor Cycle



Plan for Interventions: 3 Key Components

- Clarify the new learning
- Align the intervention method
- Write a coaching plan

Clarify the new learning

Content	Practice
What does my mentee need to understand?	What do I lean on in my teaching practice in order to do this?
What does the Tier 1 resource recommend?	What does my mentee need to be able to do?
How could my mentee gain this knowledge?	How could my mentee gain this skill?

Sample SMART Goal 1

The teacher will effectively facilitate application tasks during math instruction in unit 4 so students can better plan and choose a solution pathway when applying their mathematical knowledge to different situations as measured by at least 90% of students achieving passing scores on application tasks on the post assessment.	**What does the mentee need to learn?**

Sample SMART Goal 2

Students will effectively select tools that are relevant and useful for the task or problem given when focusing on procedural skills and fluency activities over the next 5 lessons as measured by teacher observations of student choice of appropriate tools.	**What does the mentee need to learn?**

Summarize: Model vs. Co-Teach - When do we use each method?

Which intervention?

Scenario	SMART Goal	Intervention
The teacher has been sharing with their mentor about how students really seem to get the concept she is teaching when they are working on procedural type tasks, but when she puts the content into a new situation, like an application task, they can't seem to find or choose a solution pathway to solve the problem. The teacher would like to learn how to facilitate an application task to support students in choosing a method for attacking a task.	*The teacher will effectively facilitate application tasks during math instruction in unit 4 so students can better plan and choose a solution pathway when applying their mathematical knowledge to different situations as measured by at least 90% of students achieving passing scores on application tasks on the post assessment.*	
Your mentee shares with you that when students are working on procedural skill/fluency tasks that it seems to be a "hit or miss" in the classroom in that some students are selecting the right tools and others not. They are pretty inconsistent. The teacher needs some support in being little more deliberate on helping students to select appropriate tools. He is starting to do it but just needs a little more help/guidance.	*Students will effectively select tools that are relevant and useful for the task or problem given when focusing on procedural skills and fluency activities over the next 5 lessons as measured by teacher observations of student choice of appropriate tools.*	

Align the intervention: Overcoming Barriers

Location:	**Time:**
Lesson "bite size":	**Group size:**

Mentor Coaching Plan

Mentee SMART goal(s)
The teacher will effectively facilitate application tasks during math instruction in unit 4 so students can better plan and choose a solution pathway when applying their mathematical knowledge to different situations as measured by at least 90% of students achieving passing scores on application tasks on the post assessment.

What activities and resources will mentor and mentee engage in to achieve goal(s)?

Specific Activity or Resource	How is it aligned to the goal(s)?	Why will it be effective?	How will you integrate relationship building?	Projected timeline

How will you monitor your mentee's progress toward the identified goals?

Mentor Coaching Plan

Mentee SMART goal(s)
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Mentor Coaching Plan

Mentee SMART goal(s)

What activities and resources will mentor and mentee engage in to achieve goal(s)?

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Plan for Interventions: One Sentence Summary

Key Takeaway:
Coaching plans keep mentor and mentee on track to achieve SMART goals.

Model Best Practices: 3 Key Components

- Co-plan instruction
- Model for demonstration
- Debrief

Co-Plan Instruction

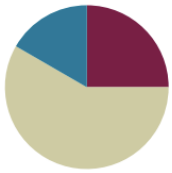
- Revisit agreements
- Confirm the purpose/goal and connection to SMART goal
- Confirm that you're modeling
- Make thinking visible as you co-plan the lesson or activity
- Create a "look-fors" checklist based on the goal of the model lesson or activity

Lesson 2

Objective: Solve word problems in varied contexts using a letter to represent the unknown.

Suggested Lesson Structure

Fluency Practice	(15 minutes)
Concept Development	(35 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



A NOTE ON STANDARDS ALIGNMENT:

Problems 2 and 5 on the Problem Set, the Exit Ticket, and Problems 1 and 5 on the Homework are two-step word problems involving milliliters and grams. The masses and volumes are given in the same units in each problem. Standard **3.MD.2** specifically states that students "solve one-step problems involving masses or volumes that are given in the same units." However, these problems look ahead to **4.MD.2**. Students working above grade level might enjoy the challenge of solving these two-step word problems involving milliliters and grams. To make these problems accessible to students working below grade level, modify the problems so they can be solved with one step.

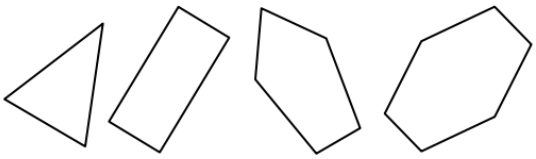
Fluency Practice (15 minutes)

- Name the Shape **2.G.1** (3 minutes)
- Multiply by 3 **3.OA.7** (8 minutes)
- Equivalent Counting with Units of 4 **3.OA.7** (4 minutes)

Name the Shape (3 minutes)

Note: This activity reviews Grade 2 geometry concepts in preparation for Topic B.

T: (Project the triangle.) What's the name of the shape?
S: Triangle.
T: (Project the rectangle.) What's one name for this shape?
S: Rectangle (or parallelogram or quadrilateral).
T: How many sides does a rectangle have?
S: Four.
T: How many right angles does a rectangle have?
S: Four!
T: What's the name for all four-sided figures?
S: Quadrilateral.



Continue with the following possible shapes: pentagon and hexagon.

Multiply by 3 (8 minutes)

Materials: (S) Multiply by 3 (6–10) Pattern Sheet

Note: This activity builds fluency with multiplication facts using units of 3. It works toward students knowing from memory all products of two one-digit numbers. See Lesson 1 for the directions for administration of a Multiply-By Pattern Sheet.

T: (Write 7 × 3 = _____.) Let's skip-count up by threes. I'll raise a finger for each three. (Raise a finger for each number to track the count.)
S: 3, 6, 9, 12, 15, 18, 21.
T: Let's skip-count by threes starting at 15. Why is 15 a good place to start?
S: It's a fact we already know, so we can use it to figure out a fact we don't know.
T: (Track with fingers as students say the numbers.)
S: 15 (5 fingers), 18 (6 fingers), 21 (7 fingers).
T: Let's see how we can skip-count down to find the answer, too. Start at 30 with 10 fingers, 1 for each three. (Count down with fingers as students say the numbers.)
S: 30 (10 fingers), 27 (9 fingers), 24 (8 fingers), 21 (7 fingers).

Continue with the following possible sequence: 9 × 3, 6 × 3, and 8 × 3.

T: (Distribute the Multiply by 3 Pattern Sheet.) Let's practice multiplying by 3. Be sure to work left to right across the page.

Equivalent Counting with Units of 4 (4 minutes)

Note: This activity builds fluency with multiplication facts using units of 4. The progression builds in complexity. Work students up to the highest level of complexity where they can confidently participate.

T: Count to 10. (Write as students count. See the chart below.)
S: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
T: (Write 1 four beneath the 1.) Count to 10 fours. (Write as students count.)

1	2	3	4	5	6	7	8	9	10
1 four	2 fours	3 fours	4 fours	5 fours	6 fours	7 fours	8 fours	9 fours	10 fours
4	8	12	16	20	24	28	32	36	40
1 four	8	3 fours	16	5 fours	24	7 fours	32	9 fours	40
4	2 fours	12	4 fours	20	6 fours	28	8 fours	36	10 fours

S: 1 four, 2 fours, 3 fours, 4 fours, 5 fours, 6 fours, 7 fours, 8 fours, 9 fours, 10 fours.
T: Count by fours to 40. (Write as students count.)
S: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40.

T: (Write 1 four beneath the 4. Write 8 beneath the 8.) I'm going to give you a challenge. Let's alternate between saying the units of four and the number. (Write as students count.)

S: 1 four, 8, 3 fours, 16, 5 fours, 24, 7 fours, 32, 9 fours, 40.

T: (Write 4 beneath 1 four and 2 fours beneath the 8.) Let's alternate again. (Write as students count.)

S: 4, 2 fours, 12, 4 fours, 20, 6 fours, 28, 8 fours, 36, 10 fours.

Concept Development (35 minutes)

Materials: (S) Problem Set, 1 piece of chart paper per pair or triad, 1 different color marker per student in each group

Part 1: Work cooperatively to identify multiple solution paths.

Note: Sample talking points and questions to guide student explanations and class participation are listed in Part 2 of this lesson. Use them as a resource in Part 1.

Create groups of two or three students. Distribute the Problem Set, chart paper, and markers to students.

T: Today, we're going to work in groups to solve Problem 6. Let's prepare our chart paper by folding it into three equal parts. (Model for students, and allow them time to fold.) With your group, read Problem 6 now.

S: The total amount of rain that fell in New York City in two years was 282 centimeters. In the first year, 185 centimeters of rain fell. How many more centimeters of rain fell in the first year than in the second year?

T: Take a quiet moment to visualize the problem. (Give students about 15 seconds to visualize.) Describe the problem to your group.

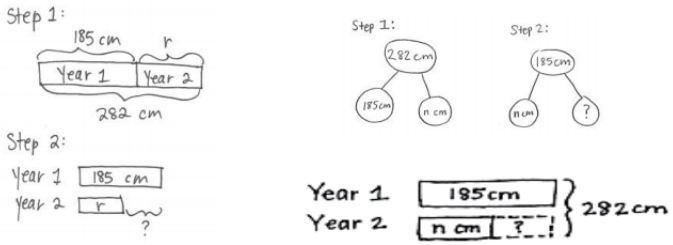
S: It's a problem about rain, and someone measured it. → Maybe with a graduated cylinder. → That would be a huge cylinder! Imagine how tall 282 centimeters is! → They probably measured the rain each day or week and then added to find the total. → We're talking about a lot of rain.

T: Think about our Read-Draw-Write process. At the signal, say the question we should be asking ourselves. (Signal.)

S: What can I draw?

T: Work with your group to draw at least two different ways to represent the problem. Make the drawings on the top third of your paper. Each of you has a different color marker so that your participation shows on your poster. Make sure each member of your group contributes.

S: (Discuss and draw. Some possible drawings are shown below.)



T: As you drew, what did you notice about the problem that will help you solve?

S: We noticed it's a two-step problem. → We know the total and the amount of rain in Year 1. → We have to find out how much rain there was in Year 2. → That doesn't answer the question, though. We have to know how much *more* rain there was in Year 1. That's subtracting two times!

T: You have more than one drawing on your paper. As a group, discuss which one represents the problem most clearly. Circle it, and be ready to talk about your choice.

S: (Discuss and circle a model.)

Select two or three groups to share their thinking with the rest of the class. Choose groups strategically to spark discussion and push learning in terms of both modeling and oral explanation. Selections could include a group with an exemplary choice, a group with an unusual choice, or a group with an excellent explanation.

S: (Listen to groups share, ask questions, and compare work.)

T: Is your thinking about your work or the problem different after listening to your friends? Take a moment to check in with your group. Adjust your drawing or thinking based on what you saw and heard.

S: (Discuss and possibly make modifications to work.)

T: Think about the Read-Draw-Write process. What is our next step?

S: To write equations and solve!

T: Work with your group to write equations and solve the problem. Use your drawing. Record your work in the middle third of your chart paper, and be ready to talk about your steps.

S: The first step is just subtraction. We can do 282 cm – 185 cm to find the amount of rain in Year 2. → It's not that easy with mental math. Let's use the algorithm. → Actually, you can think of 282 as 285. Then, I can subtract 185 easily to get 100. Since I added 3 to 282 to get 285, I have to subtract 3 from the answer, so it's 97. → Now, I think we should subtract again. We can do 185 – 97 to find out how much more rain there was. → Let's solve that one with the algorithm. 185 – 97 = 88. So, the answer is 88 centimeters. → I don't have to use the algorithm. I can break apart 185 as 100 and 85. That's 3 + 85 because I took the 97 from 100. The answer is 88 cm.

Select a few groups to share their thinking with the rest of the class. Again, choose groups strategically. Allow students time to listen to the groups, share, and ask questions.

T: Take a moment to compare your work with what you saw and heard, and maybe make adjustments.

S: (Briefly discuss comparison within groups and possibly modify work.)

T: Work with your group to finish the problem. What is our final step?

S: To write a sentence that answers the question.

T: Record your sentence on the bottom third of your paper.

S: (Write a sentence with words to answer the question. Possible responses: 88 more centimeters of rain fell in the first year than in the second. → There were 88 more centimeters of rain in Year 1.)

Select a few groups to share their work with the rest of the class. Notice which students may not have reread the question before writing. If necessary, guide students to adjust their sentences so that their answers more closely align with the question asked.

Part 2: Work independently to solve and present problems using multiple solution paths.

Assign each student two problems from the Problem Set. Challenge them to record more than one way to draw for each problem they solve. Ask students to share their work with the members of their groups from Part 1. When sharing, students should include answers to the following questions:

- How does your drawing represent the problem clearly?
- How did your drawing help you decide on a way to solve?
- Why does the equation that you used to model make sense with your drawing and with the problem?
- How do you know you answered the question?

MP.3

Have students share their work in groups of three or four. Encourage group members to practice asking questions of the presenter. They might ask some of the questions listed below.

- I'm not sure what you mean. Can you say more about that?
- Why did you decide _____?
- What do you think about _____ instead?
- Which other way did you try to draw the problem?

One way to close this process is to have students write a compliment to another presenter. If time allows, students may solve problems on the Problem Set that they have not already completed on their own before the Student Debrief.

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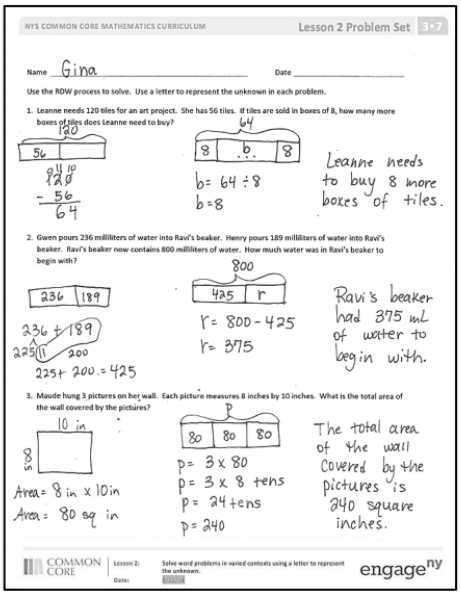

NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 2 Problem Set 3•7

Name Gina Date

Use the RDW process to solve. Use a letter to represent the unknown in each problem.

1. Leanne needs 120 tiles for an art project. She has 56 tiles. If tiles are sold in boxes of 8, how many more boxes of tiles does Leanne need to buy?

120 − 56 = 64
64 ÷ 8 = 8
Leanne needs to buy 8 more boxes of tiles.

2. Gwen pours 236 milliliters of water into Ravi's beaker. Henry pours 189 milliliters of water into Ravi's beaker. Ravi's beaker now contains 800 milliliters of water. How much water was in Ravi's beaker to begin with?

236 + 189 = 225 + 200 = 425
800 − 425 = 375
Ravi's beaker had 375 mL of water to begin with.

3. Maude hung 3 pictures on her wall. Each picture measures 8 inches by 10 inches. What is the total area of the wall covered by the pictures?

Area = 8 in × 10 in
Area = 80 sq in
p = 3 × 80
p = 3 × 8 tens
p = 24 tens
p = 240
The total area of the wall covered by the pictures is 240 square inches.

COMMON CORE Lesson 2: Solve word problems in varied contexts using a letter to represent the unknown. Date: engage^ny

Student Debrief (10 minutes)

Lesson Objective: Solve word problems in varied contexts using a letter to represent the unknown.

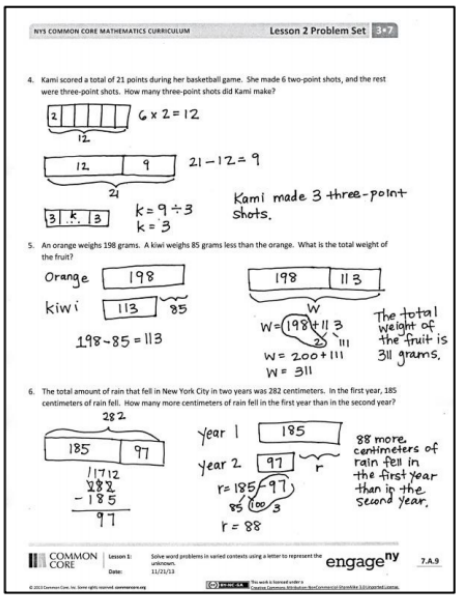
The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion, depending on how the students were asked to solve the Problem Set.

- How are your models related to your equations in Problem 1?

- Invite students to share different equations that can be used to solve Problem 3.
- What operations are used to solve Problem 4? In what order? How did you figure that out?
- Invite students to articulate their thought processes for preparing to present their work.
- How did it feel to present your work to friends?
- What did you learn about yourself or your work by presenting?
- What was it like to be an audience member to a friend who was presenting?
- Did you find it easy or difficult to ask your friends questions about their work? Why?


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 2 Problem Set 3•7

4. Kami scored a total of 21 points during her basketball game. She made 6 two-point shots, and the rest were three-point shots. How many three-point shots did Kami make?

6 x 2 = 12
21 − 12 = 9
k = 9 ÷ 3
k = 3
Kami made 3 three-point shots.

5. An orange weighs 198 grams. A kiwi weighs 85 grams less than the orange. What is the total weight of the fruit?

Orange 198
kiwi 113 85
198 − 85 = 113
W = 198 + 113
W = 200 + 111
W = 311
The total weight of the fruit is 311 grams.

6. The total amount of rain that fell in New York City in two years was 282 centimeters. In the first year, 185 centimeters of rain fell. How many more centimeters of rain fell in the first year than in the second year?

282
185 97
282 − 185 = 97
Year 1 185
Year 2 97 r
r = 185 − 97
r = 88
88 more centimeters of rain fell in the first year than in the second year.

COMMON CORE Lesson 1: Solve word problems in varied contexts using a letter to represent the unknown. Date: 11/21/13 engage^ny 7.A.9

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____ Date _____

Use the RDW process to solve. Use a letter to represent the unknown in each problem.

1. Leanne needs 120 tiles for an art project. She has 56 tiles. If tiles are sold in boxes of 8, how many more boxes of tiles does Leanne need to buy?

2. Gwen pours 236 milliliters of water into Ravi's beaker. Henry pours 189 milliliters of water into Ravi's beaker. Ravi's beaker now contains 800 milliliters of water. How much water was in Ravi's beaker to begin with?

3. Maude hung 3 pictures on her wall. Each picture measures 8 inches by 10 inches. What is the total area of the wall covered by the pictures?

4. Kami scored a total of 21 points during her basketball game. She made 6 two-point shots, and the rest were three-point shots. How many three-point shots did Kami make?

5. An orange weighs 198 grams. A kiwi weighs 85 grams less than the orange. What is the total weight of the fruit?

6. The total amount of rain that fell in New York City in two years was 282 centimeters. In the first year, 185 centimeters of rain fell. How many more centimeters of rain fell in the first year than in the second year?

Name _____ Date _____

Use the RDW process to solve the problem below. Use a letter to represent the unknown.

Jaden's bottle contains 750 milliliters of water. He drinks 520 milliliters at practice and then another 190 milliliters on his way home. How many milliliters of water are left in Jaden's bottle when he gets home?

Modeling Best Practices: Co-Planning Conversation Transcript

Mentor - *Glad we got to meet this morning to talk about how I can best support you in meeting the SMART goal we came up when we met last time. Again, I really appreciated getting to observe in your classroom and debrief with you. I've been doing some thinking about your goal, and as you saw in the coaching plan, I think one of the ways I can best support you in reaching it is to model a portion of an upcoming Eureka math lesson that focuses on solving word problems in a variety of contexts.*

Mentee - *Yes, my students always seem to get the concept when we are practicing together. Like they know how to add, subtract, multiply and divide, but when it comes to choosing which operation to do and choosing that solution pathway they just really struggle. Sometimes it seems like they won't even try to figure out what to do, they are just waiting on me to tell them the way to solve even though we talk all the time about how there's more than one way to solve math problems. I just wish they would take a risk and try. I think I'd like to see how you get students to do this in action.*

Mentor - *Great, I'm really glad. This will be my first time modeling with someone else's kiddos but I'm excited to how you what it can look like - just know that it won't be perfect, by any means, but it'll be a good learning experience for both of us.*

Mentee - *Yeah I'm excited to see you try it with them.*

Mentor - *So I was looking at what you have coming up in your lesson plans and it looks like you are doing Lesson 2-Solve word problems in varied contexts using a letter to represent the unknown - from Eureka Math tomorrow, is that right?*

Mentee - *Yes.*

Mentor - *So I was thinking I could come into your classroom after you've gotten the lesson started. You could have already taken them through the warm-up, fluency practice part of the lesson and then I'll jump in when it gets to the concept development portion of the lesson.*

Mentee - *So I would get the lesson started and then you would take over the second part? I think that sounds good. But could you be in there for the part that I am teaching too in case it isn't going well?*

Mentor - *Sure, I think I can work that with my schedule. Let's confirm the timing, what time does math start?"*

[Mentor and Mentee finish confirming the logistics]

Mentor - *This is the lesson that is coming up tomorrow, It's from Eureka Math's Module 7, Topic A, Lesson 2 and I think it will work perfectly in alignment with your SMART goal. Students will have the opportunity to work in cooperative groups to identify multiple solution paths in application word problems. So as I look through this lesson plan I want really hone in on the concept development section and really think through how I am going to facilitate this part of the lesson so students increase their ability to choose a solution path. So on this part on page 28 the teacher asks the students to work with their group to solve problem 6. The teacher asks the students to take a quiet moment to visualize the problem, first independently and then share with their group members. I am going to make a note here that I want to model how to visualize a problem for the students. I'll use a think aloud process so students can get a better understanding of what visualizing the problem means and how it can help them choose a solution path.*

[Mentor continues reading through this section of the lesson plan and makes annotating type notes, thinking aloud for the mentee on how the mentor plans to implement this part of the activity. The mentor is focusing on making their thinking visible as they go through the activity together and jotting down their own notes for the model piece.]

Mentor - *The last thing we need to discuss is what you'll be doing while I am modeling. You should definitely be observing both me and the students, but I want us to come up with a specific look-fors checklist for you to complete while you observe me teaching.*

Mentee - *Okay, that sounds good. I know one thing I really want to watch out for is how you get students to actually use what they know to try out a solution pathway without having to give them too many hints.*

Mentor - *That's a great thing to put on the checklist. One thing I really want you to take note of when I am modeling it the type of questioning I use to scaffold the learning.* **[Mentor begins filling out a checklist using the"Look-Fors" checklist template.]** *This type of questioning is very open-ended and doesn't baby-step the students to the solution. Another thing I want to add to this look-fors checklist is having students visualize the problem prior to choosing a solution path. This gets students really thinking about what the problem is describing or asking them to do before they choose a solution pathway.*

[The mentor and mentee continue adding "look-fors" to the checklist.]

Mentor - *Well, I look forward to seeing you tomorrow at 10 o'clock for this lesson. When is a good time for you to meet after tomorrow to debrief?*

Mentee - *I could do Friday during our lunch time - will that work?*

Mentor - *Sounds great - I will see you tomorrow!*

Model Best Practices: "Look-Fors" Checklist

Sample checklist from co-planning conversation transcript

Look-Fors	Observation Notes
• How to get students to use what they know to try out a solution pathway without giving them too many "hints"	
• Types of questioning used that scaffolds student thinking/learning	
• How students visualize the problem	
• The wait time provided for processing & discussing	

TRY IT OUT: Model Best Practices: "Look-For's" Checklist

Look-For's	Observation Notes

Look-For's Checklist

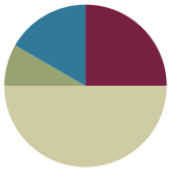
Look-For's	Observation Notes

Lesson 1

Objective: Study commutativity to find known facts of 6, 7, 8, and 9.

Suggested Lesson Structure

Fluency Practice	(15 minutes)
Application Problem	(5 minutes)
Concept Development	(30 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (15 minutes)

- Sprint: Mixed Multiplication **3.OA.7** (9 minutes)
- Group Counting **3.OA.1** (3 minutes)
- Commutative Property of Multiplication **3.OA.5** (3 minutes)

Sprint: Mixed Multiplication (9 minutes)

Materials: (S) Mixed Multiplication Sprint

Note: This Sprint reviews familiar multiplication facts from Module 1 and prepares students for today's lesson on using commutativity with known facts to find unknown facts.

Group Counting (3 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes, sevens, eights, and nines in this activity anticipates multiplication using those units later in the module.

Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Group Counting in Module 3 no longer explicitly includes twos, threes, fours, and fives. However, consider including those units if the class has not yet mastered those facts.

Whisper/talking, hum/talking, or think/talking by threes and fours can also work as a scaffold to build fluency with sixes and eights.

Commutative Property of Multiplication (3 minutes)

Materials: (S) Personal white board

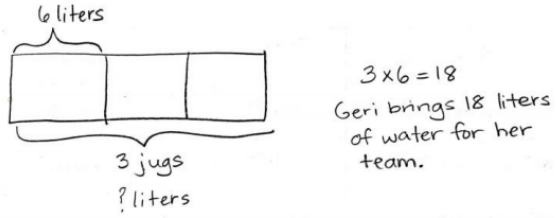
Note: This activity reviews the commutative property from Module 1 and anticipates its use in today's lesson.

T: (Project array with 3 groups of 2 circles.) Write two multiplication sentences and two division sentences for this array.
S: (Write 3 × 2 = 6, 2 × 3 = 6, 6 ÷ 2 = 3, and 6 ÷ 3 = 2.)

Continue with the following suggested sequence: 2 groups of 9, 3 groups of 7, and 5 groups of 8.

Application Problem (5 minutes)

Geri brings 3 water jugs to her soccer game to share with teammates. Each jug contains 6 liters of water. How many liters of water does Geri bring?



NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Extend for students working above grade level with a related word problem with larger factors.

For example, "Kelly drinks 3 liters of water each day. How many liters of water does she drink in a week?"

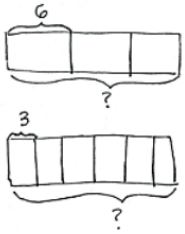
Note: This problem reviews multiplication using units of three. It leads into the discussion of commutativity in the Concept Development.

Concept Development (30 minutes)

Materials: (S) Personal white board, Problem Set

Part 1: Explore commutativity as it relates to multiplication.

Draw or project the tape diagrams shown to the right.



T: Talk to your partner. Which tape diagram represents the Application Problem? How do you know? (Allow time for discussion.)
T: Draw both tape diagrams on your personal white board. Write a multiplication sentence for each. (Allow time for students to work and finish.)
T: How are the multiplication sentences related?
S: They use the same numbers. → Both have a product of 18. → They use the same factors but in a different order. The product is the same.

T: This is an example of the commutative property that we studied in Module 1. What does this property tell us about the product and its factors?

S: Even if the order of the factors changes, the product stays the same!

MP.7

T: Earlier in the year, we learned our threes, including 3 × 6. If we know 3 × 6, what other fact do we know?

S: 6 × 3.

T: What is the product of both 3 × 6 and 6 × 3?

S: 18.

T: To show that 3 × 6 and 6 × 3 equal the same amount, we can write 3 × 6 = 6 × 3. (Model.)

T: Using commutativity as a strategy, we know many more facts than just the ones we've practiced!

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Review the commutative property by exploring arrays—concrete or pictorial. Review 3 twos is 2 threes, for example, by 6 students standing in 2 rows of 3, and then 3 rows of 2.

When drawing the array, use color to differentiate 6 threes from 3 sixes.

Continue with the following suggested sequence:

- 2 × 7 = 7 × 2
- 5 eights = 8 fives
- 4 nines = 9 fours

Part 2: Use the multiplication chart to find known facts through commutativity.

T: Problem 1(a) on your Problem Set shows a multiplication chart. The shaded numbers along the left column and the top are factors. The numbers inside the chart are products. Each un-shaded box represents the product of one multiplication fact. Find the total number of facts on your multiplication chart. (Allow time for students to count.) How many facts are on the chart?

S: 100 facts.

T: Let's use the chart to locate the product of 3 and 6. Put your finger on the row labeled 3, and slide it to the right until it's also in the column labeled 6. The number in the square where the row and column meet is the product, which has been done for you. Using the chart, what is the product of 3 and 6?

S: 18.

T: Let's now locate the product of 6 and 3. Find the square where the row for 6 and the column for 3 meet. Use commutativity to write the product of 6 and 3 in that square on your chart.

S: (Write 18.)

T: We can use commutativity to solve many new facts and fill in the products on the chart. On the chart, write the products for all the facts that we've already studied. Then, fill in those you can solve using commutativity. (Allow time for students to work.)

T: Shade in the facts you completed. (Allow time for students to work.) How many are left to learn?

S: 16.

T: Look carefully at those 16 facts. Are there any that you will be able to solve using the commutative property once you know one?

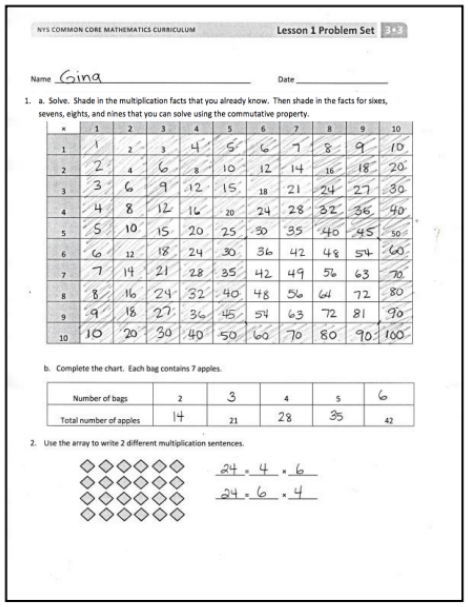
S: Yes! There are 12 facts that we can use the commutative property to solve. That means we only need to know 6 of them.

T: Really, there are only 10 new facts to learn before you know all the facts up to 10 × 10.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the purposeful sequencing of the Problem Set guide selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Consider assigning incomplete problems for homework or at another time during the day.


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 1 Problem Set 3•3

Name Gina Date

1. a. Solve. Shade in the multiplication facts that you already know. Then shade in the facts for sixes, sevens, eights, and nines that you can solve using the commutative property.

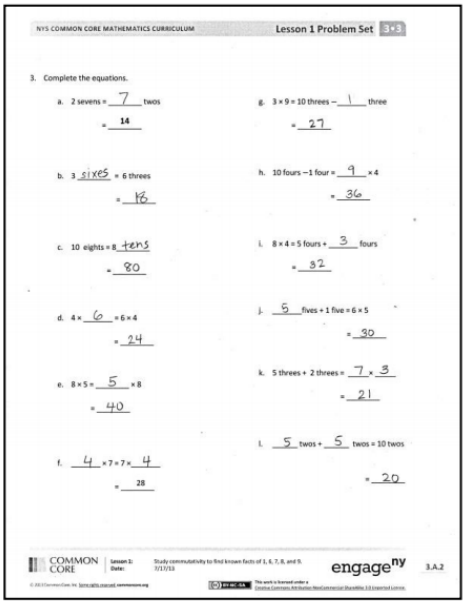
×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

b. Complete the chart. Each bag contains 7 apples.

Number of bags	2	3	4	5	6
Total number of apples	14	21	28	35	42

2. Use the array to write 2 different multiplication sentences.

24 = 4 × 6
24 = 6 × 4


NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 1 Problem Set 3•3

3. Complete the equations.

a. 2 sevens = 7 twos = 14
b. 3 sixes = 6 threes = 18
c. 10 eights = 8 tens = 80
d. 4 × 6 = 6 × 4 = 24
e. 8 × 5 = 5 × 8 = 40
f. 4 × 7 = 7 × 4 = 28
g. 3 × 9 = 10 threes − 1 three = 27
h. 10 fours − 1 four = 9 × 4 = 36
i. 8 × 4 = 5 fours + 3 fours = 32
j. 5 fives + 1 five = 6 × 5 = 30
k. 5 threes + 2 threes = 7 × 3 = 21
l. 5 twos + 5 twos = 10 twos = 20

COMMON CORE Lesson 1: Date: Study commutativity to find known facts of 1, 6, 7, 8, and 9. engage^ny 3.A.2

Student Debrief (10 minutes)

Lesson Objective: Study commutativity to find known facts of 6, 7, 8, and 9.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How did commutativity help you solve more facts than you thought you knew in Problem 1(a)?
- Invite students to share their processes for finding the multiplication facts for the array in Problem 2.

- In Problems 3(a), 3(b), and 3(c), what do you notice about the words and numbers on each side of the equal sign? How are they related?
- How did you know to subtract 1 three in Problem 3(g)? What would that problem look like rewritten as an equation?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:

English language learners and others benefit from reviewing *commutative property* and *commutativity* during the Debrief. Allow students to explain the property to a partner in their first language, and/or record the term with an example in a personal math dictionary.

Number Correct: _____

A

Mixed Multiplication

1.	2 × 1 =		23.	2 × 7 =	
2.	2 × 2 =		24.	5 × 5 =	
3.	2 × 3 =		25.	5 × 6 =	
4.	4 × 1 =		26.	5 × 7 =	
5.	4 × 2 =		27.	4 × 5 =	
6.	4 × 3 =		28.	4 × 6 =	
7.	1 × 6 =		29.	4 × 7 =	
8.	2 × 6 =		30.	3 × 5 =	
9.	1 × 8 =		31.	3 × 6 =	
10.	2 × 8 =		32.	3 × 7 =	
11.	3 × 1 =		33.	2 × 7 =	
12.	3 × 2 =		34.	2 × 8 =	
13.	3 × 3 =		35.	2 × 9 =	
14.	5 × 1 =		36.	5 × 7 =	
15.	5 × 2 =		37.	5 × 8 =	
16.	5 × 3 =		38.	5 × 9 =	
17.	1 × 7 =		39.	4 × 7 =	
18.	2 × 7 =		40.	4 × 8 =	
19.	1 × 9 =		41.	4 × 9 =	
20.	2 × 9 =		42.	3 × 7 =	
21.	2 × 5 =		43.	3 × 8 =	
22.	2 × 6 =		44.	3 × 9 =	

B

Number Correct: _____

Improvement: _____

Mixed Multiplication

1.	5 × 1 =		23.	5 × 7 =	
2.	5 × 2 =		24.	2 × 5 =	
3.	5 × 3 =		25.	2 × 6 =	
4.	3 × 1 =		26.	2 × 7 =	
5.	3 × 2 =		27.	3 × 5 =	
6.	3 × 3 =		28.	3 × 6 =	
7.	1 × 7 =		29.	3 × 7 =	
8.	2 × 7 =		30.	4 × 5 =	
9.	1 × 9 =		31.	4 × 6 =	
10.	2 × 9 =		32.	4 × 7 =	
11.	2 × 1 =		33.	5 × 7 =	
12.	2 × 2 =		34.	5 × 8 =	
13.	2 × 3 =		35.	5 × 9 =	
14.	4 × 1 =		36.	2 × 7 =	
15.	4 × 2 =		37.	2 × 8 =	
16.	4 × 3 =		38.	2 × 9 =	
17.	1 × 6 =		39.	3 × 7 =	
18.	2 × 6 =		40.	3 × 8 =	
19.	1 × 8 =		41.	3 × 9 =	
20.	2 × 8 =		42.	4 × 7 =	
21.	5 × 5 =		43.	4 × 8 =	
22.	5 × 6 =		44.	4 × 9 =	

Name _____ Date _____

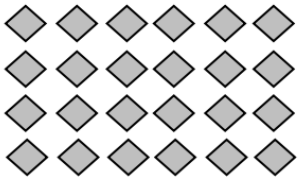
1. a. Solve. Shade in the multiplication facts that you already know. Then, shade in the facts for sixes, sevens, eights, and nines that you can solve using the commutative property.

×	1	2	3	4	5	6	7	8	9	10
1		2	3							
2		4		8				16		
3						18				
4					20					
5										50
6		12								
7										
8										
9										
10										

b. Complete the chart. Each bag contains 7 apples.

Number of Bags	2		4	5	
Total Number of Apples		21			42

2. Use the array to write two different multiplication sentences.



_____ = _____ × _____

_____ = _____ × _____

3. Complete the equations.

a. 2 sevens = _____ twos

= 14

b. 3 _____ = 6 threes

= _____

c. 10 eights = 8 _____

= _____

d. 4 × _____ = 6 × 4

= _____

e. 8 × 5 = _____ × 8

= _____

f. _____ × 7 = 7 × _____

= 28

g. 3 × 9 = 10 threes – _____ three

= _____

h. 10 fours – 1 four = _____ × 4

= _____

i. 8 × 4 = 5 fours + _____ fours

= _____

j. _____ fives + 1 five = 6 × 5

= _____

k. 5 threes + 2 threes = _____ × _____

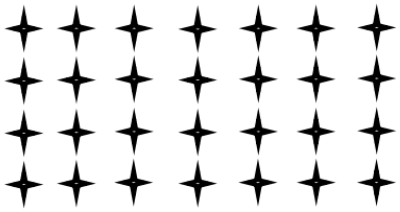
= _____

l. _____ twos + _____ twos = 10 twos

= _____

Name _____ Date _____

1. Use the array to write two different multiplication facts.



_____ = _____ × _____

_____ = _____ × _____

2. Karen says, "If I know 3 × 8 = 24, then I know the answer to 8 × 3." Explain why this is true.

Name _____ Date _____

1. Complete the charts below.

a. A tricycle has 3 wheels.

Number of Tricycles	3		5		7
Total Number of Wheels		12		18	

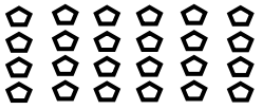
b. A tiger has 4 legs.

Number of Tigers			7	8	9
Total Number of Legs	20	24			

c. A package has 5 erasers.

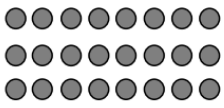
Number of Packages	6				10
Total Number of Erasers		35	40	45	

2. Write two multiplication facts for each array.



_____ = _____ × _____

_____ = _____ × _____



_____ = _____ × _____

_____ = _____ × _____

3. Match the expressions.

3 × 6	7 threes
3 sevens	2 × 10
2 eights	9 × 5
5 × 9	8 × 2
10 twos	6 × 3

4. Complete the equations.

a. 2 sixes = _____ twos
= 12

b. _____ × 6 = 6 threes
= _____

c. 4 × 8 = _____ × 4
= _____

d. 4 × _____ = _____ × 4
= 28

e. 5 twos + 2 twos = _____ × _____
= _____

f. _____ fives + 1 five = 6 × 5
= _____

Model for Demonstration

- Share with students about this growth opportunity
- Make your thinking and decision making visible
- Step in and out of the teacher role vs. mentor role
- Encourage mentee to watch how students respond to the instruction
- The mentee should be actively engaged using the checklist
- Remember you don't have to model the ENTIRE lesson - keep it focused!

Sentence Starters for Stepping In and Out of Modeling

- Did you notice how I just _____?
- I am about to try _____, so watch how I do that.
- When I did _____, what did you notice about students reactions?
- I was hoping _____ would occur, but then I had to adjust by _____.
- That strategy did not seem to work, so now I am going to try _____ and see if the results are different.

Sample Modeling For Demonstration Transcript

Mentee - *Good morning class! Remember how I told you all yesterday that I was going to have a friend come by our classroom today and help us work on some things? Well here she is! This is Mrs. Carlson - Can you all say hi? Mrs. Carlson is such an awesome teacher and she has agreed to help me with a personal goal that I am working on. So today she is going to be your guest teacher and I am going to be watching very closely as she works on our Eureka math lesson with you all.*

Mentor - *That's right! I am so happy to be here and am looking forward to teaching you all today. I also want to let you all in on a little secret - we are all learners in this classroom. All of you, your teacher, and me - today we are all going to be learners. So while I am teaching you today, your teacher and I are also learning. So sometimes I'll be talking to all of you, and sometimes us teachers will be talking to each other. There might be something really specific I want to point out or tell your teacher so I might pause the lesson a few times and ask you to talk to a shoulder partner, or to think silently for a minute or two so I can go chat with your teacher and point out some things about our lesson today that are working or maybe even not working. Can you all help me with that today? Awesome!*

[Mentee teaches the Fluency Practice part of the lesson plan and afterwards goes and sits down ready to observe the mentor begin the Concept Development part of the lesson plan with checklist in hand.]

Mentor - *Today we're going to work in groups to solve Problem 6. Let's prepare our chart paper by folding it into three equal parts.* ***(Students follow directions to fold chart paper)*** *With your group, read Problem 6 now.* ***(Students read the problem)*** *So as I think about this problem I am going to try to picture it in my brain. I know that there was a lot of rain that fell and someone measured it because they said in two years there was 282 cm worth of rain. Wow! That's a lot of rain. They had to measure every time it rained for two whole years! So I also know they combined what they measured during year 1 and combined it with year 2 to get 282 cm of rain. Turn to a partner in your group and share what you are thinking/visualizing about this problem.*

(Mentor steps over to mentee and has quick discussion)

Mentor (to mentee) - When I modeled visualizing the problem for students, what did you notice about their reactions?

Mentee (to mentor) - I saw lots of heads nodding and they seem to be following your line of thinking.

Mentor (to mentee) - Let's see how they do when we get to the read-draw-write process and see how they do.

(Mentor steps back over to students and gathers their attention)

Mentor **-** *Now that you have a picture in your mind about what is happening in this problem, work with your group to draw at least two different ways to represent the problem. Make the drawings on the top third of your paper. Each of you has a different color marker so that your participation shows on your poster. Make sure each member of your group contributes.*

[Mentor begins to circulate as groups get started and notices one group having a hard time putting marker to paper. She approaches that group. The mentee moves closer to hear their conversation.]

Mentor - *I see you all are having a little trouble getting started. What is this problem about?*

Julian (student) - *It's about the amount of rain that fell in New York City in 2 years.*

Mentor - *That's right! And what do you need to find out in order to solve the problem?*

Suzanne (student) - *Well we know that the total of rain was 282 centimeters and in one of those years it rained 185 centimeters, so I think we need to find out how much it rained in the second year.*

Mentor - *Great! So discuss with your group some different models that you've learned about that can help you make sense of the problem.*

[Mentor walks away from the group and over to the mentee.]

Mentor - *Did you notice anything in particular about my line of questioning to support that group in getting started?*

Mentee - *I noticed the questions were not leading questions, but very open-ended to really get them to do the thinking and connect to their prior knowledge of drawing models of a word problem.*

Mentor - *Yes! They knew what to do, but just needed a tiny push in the right direction to reassure their thinking was on the right path and build their confidence. I am going to bring the class back together to have them share their models now. Make sure to take note on the questions I ask when I facilitate this discussion.*

[The mentor finishes the model lesson continuing to step in and out to discuss key points with the mentee.]

Debrief Model Teaching

- Mentee reflects on what they observed using the checklist
- Mentee identifies the reasons, processes, and/or strategies that made the teaching successful or not successful
- Mentee makes a plan for applying the new learning into their practice

The purpose of modeling is LEARNING. Amplify learning in the debriefing.

Model Best Practices: Debrief the lesson

Suggested Guiding Questions for Discussion	Debrief Meeting Notes
Primary Questions	
How did this model lesson or activity help you?	
What did you see that was effective? (Encourage mentee to use their checklist from the observation)	
What did you see that was ineffective? (Encourage mentee to use their checklist from the observation)	
Application Questions	
What will you integrate into your teaching? How will you do that?	
What would you change/modify if you were teaching this lesson and why?	
Clarifying Questions	
What parts of what I was modeling during this lesson or activity still need further clarification?	
Closing Questions	
What is/are the top learnings you are taking away from the model lesson or activity?	
How can I support you as you begin to integrate what you are learning?	

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How can I support you as you begin to integrate what you are learning?	

Sample Debriefing a Model Lesson Transcript

Mentor - *Thanks for taking the time to meet with me. I had a great time modeling in your classroom and now just want to take some time to debrief about what you observed and hopefully some new learning that occurred for you during this process.*

Mentee - *Yeah, thanks for coming in. It was fun watching you with my kids.*

Mentor - *So just thinking about the model lesson overall, how do you think it helped you with regards to your SMART goal?"*

Mentee - *Well, I enjoyed getting to see someone else facilitate an application task with my students. A lot of times people have just told me ideas to try or read this blog for new ideas and while that is great, it was much better to see these new ideas live in person with my own students.*

Mentor - *That's great to hear. It sounds like seeing some new strategies used live in action had a powerful impact on you. So tell me, what were some things you observed using your checklist that were effective in the lesson?*

Mentee - *Well starting off it was helpful to be part of the planning process because something I noticed you did was as you went through the Eureka lesson, you made note of where students were going to have to engage in problem solving and choose a solution path. This showed me how you already knew when and where in the lesson students may struggle, and you were prepared ahead of time because of making those notes as we went through the lesson plan. During the lesson I noticed when students struggled to get started you used really good questions to scaffold their process. The questions you asked were very open ended and put the work and thinking back on the students. You gave them the confidence to use what they know to attack the problem.*

Mentor - *I'm so glad you noticed that. Often times we jump to solving the problem for a solution rather than visualizing the problem and really understanding what it is stating to know which models are best to use to choose a solution pathway.*

[Mentor adds a few comments on additional things she hoped the mentee would pick up on but didn't mention earlier in the conversation such as the think aloud to model visualizing the problem and the impact that had on students.]

Mentor - *What will you integrate into your teaching as a result of what you saw during the model?*

Mentee - *I plan to take the time prior to the lesson or activity to look through the lesson plan and note where students will have to engage in an application task. This way I can better prepare levels of questioning that will support students' processes and thinking when they get*

to that part of the lesson. I also have some new tools in my toolbox to help students when they are

struggling like providing more wait time and time to discuss with their group members and modeling and engaging in visualization of the problem.

Mentor - *That's great - I am so glad to hear that. So how can I continue to support you as you integrate this new learning into your practice?*

Mentee - *I think it would be helpful for you to come observe me teaching another lesson that is focused on facilitating an application task and provide me with some feedback on how I utilize these new strategies with my students. I would love some help in determining if my new learning is truly having the impact on student learning that I need it to.*

Mentor - I can definitely do that. When would you like me to come?"

[The conversation continues and results in the mentor and mentee setting up an additional observation and feedback session.]

Key Takeaway:
Mentors use model teaching to demonstrate practices they expect to see mentees use to address their SMART goals.