Louisiana Department of Education
Mentor Teacher Training

Module 4:
Mathematical Instructional Shifts in Practice

Secondary Math Cohort
July, 2019

Facilitated by Learning Forward

Mentor Teacher Training

Mentor Training Course Goals

Mentors will:

- Build strong relationships with mentees.
- Diagnose and prioritize mentee's strengths and areas for growth.
- Design and implement a coaching support plan to develop mentee knowledge and skills.
- Assess and deepen mentor content knowledge and content-specific pedagogy.

Module 4 Morning Outcomes:

- Deepen pedagogical content knowledge of the mathematical shifts to increase mentor's ability to coach their mentee's math instruction.
- Experience and analyze a Eureka lesson to identify evidence of the key shifts in practice.
- Plan for interventions to meet the specific needs of a mentee based on observation data.
- Model best practices to support mentee learning.

Module 4 Agenda:

- Welcome & Outcomes
- Explore mathematics shifts in the Eureka curriculum
- Lunch
- Plan for Interventions
- Modeling Best Practices
- Connection to Assessments
- Wrap-up

Mutual Commitments:

Make the learning meaningful

Engage mentally and physically

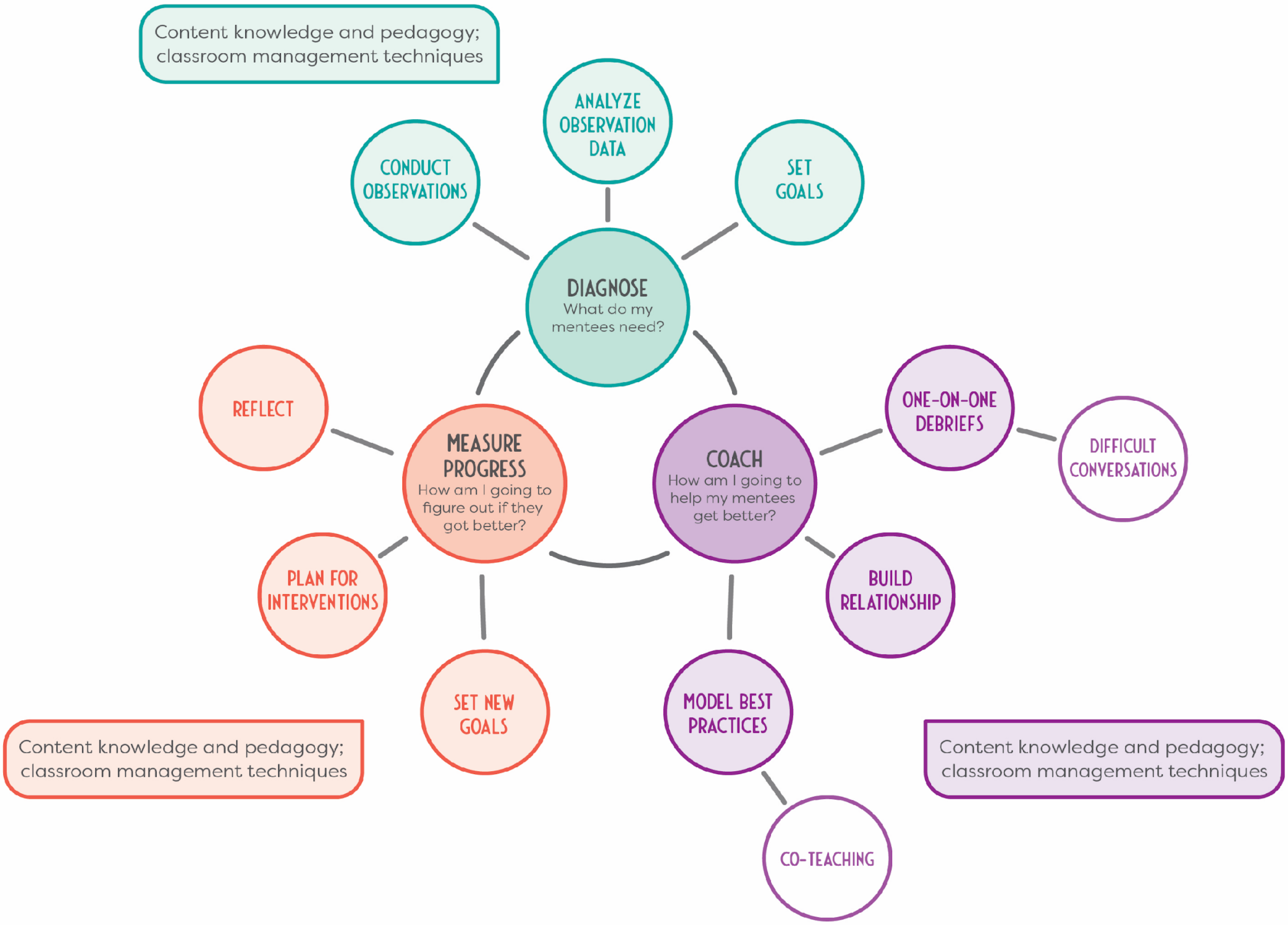
Notice opportunities to support the learning of others

Take responsibility of own learning

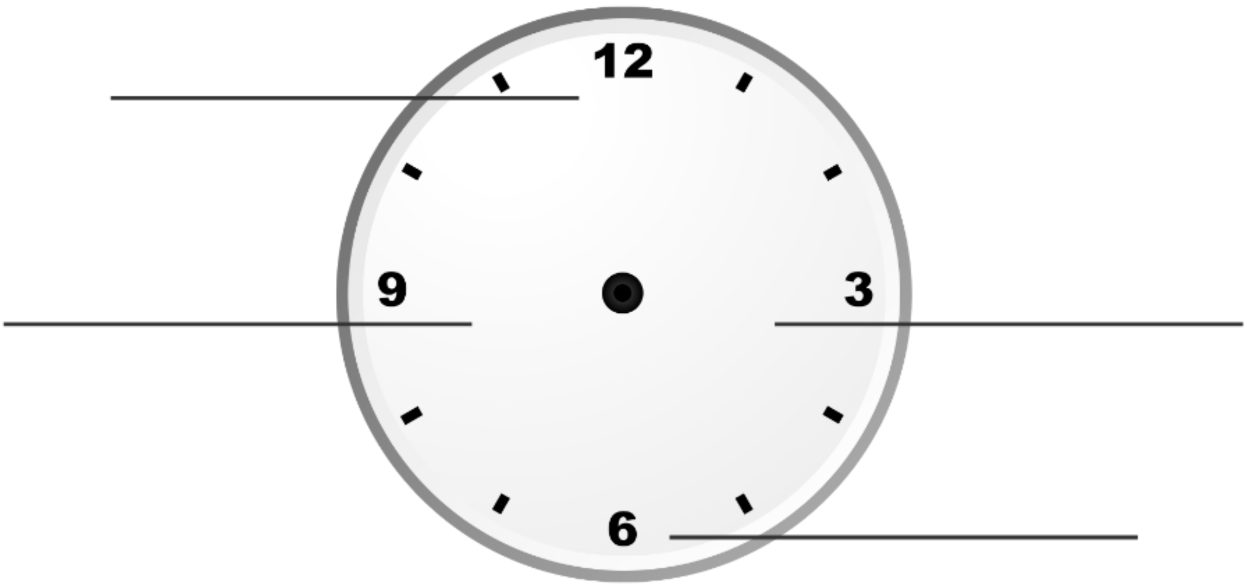
Own the outcomes

Respect the learning environment of self and others

The Mentoring Cycle



Let's Make a Date



Failing by design: How we make teaching too hard for mere mortals

https://edexcellence.net/articles/failing-by-design-how-we-make-teaching-too-hard-for-mere-mortals


Getty Images/ALLVISIONN

Robert Pondiscio
May 10, 2016

If you caught your pediatrician Googling "upset stomach remedies" before deciding how to treat your child and home-brewing medications over an office sink, you might start looking for a new pediatrician. So how would you feel if you learned that Google and Pinterest are where your child's teacher goes to look for instructional materials?

Well, brace yourself, because that's exactly what's happening. And no, your child's teacher is not an exception. A new study from the RAND Corporation finds that nearly every teacher in America—99 percent of elementary teachers, 96 percent of secondary school teachers—draws upon "materials I developed and/or selected myself" in teaching English language arts. And where do they find materials? The most common answer among elementary school teachers is Google (94 percent), followed by Pinterest (87 percent). The numbers are virtually the same for math.

But don't blame teachers. These data, for reasons both good and bad, reveal a dirty little secret about American education. In many districts and schools—maybe even most—the efficacy of the instructional materials put in front of children is an afterthought. For teachers, it makes an already hard job nearly impossible to do well.

Expecting teachers to be expert pedagogues and instructional designers is one of the ways in which we push the job far beyond the abilities of mere mortals. Add the expectation that teachers should differentiate every lesson to meet the needs of each individual student, and the job falls well outside the capacity of nearly all of America's 3.7 million classroom teachers (myself included).

If you're looking for the root causes of America's educational mediocrity, start with how poorly we prepare teachers for one of the most important parts of the job. "Few teachers ever take coursework on instructional design and, therefore, have little knowledge of the role it plays in student learning," notes Marcy Stein, an education professor with expertise in evaluating instructional design at the University of Washington Tacoma. It's like expecting the waiter at your favorite restaurant to serve your meal attentively while simultaneously cooking for twenty-five other people—and doing all the shopping and prepping the night before. You'd be exhausted too.

"Even if teachers were taught about instructional design, they would likely not have the time to prepare instructional materials, field test those materials to determine if they are effective, and modify the materials before using them to teach students. An iterative process is crucial for the development of effective materials," Stein points out.

There is good evidence to suggest that we are making a serious mistake by not paying more attention to curriculum, classroom materials, and instructional design. A 2012 Brookings study by Russ Whitehurst and Matt Chingos demonstrated that the "effect size" of choosing a better second-grade math curriculum was larger than replacing a fiftieth-percentile teacher with a seventy-fifth-percentile teacher. This is a powerful result, especially considering that it's relatively easy to give all children a better curriculum but extremely difficult to dramatically increase the effectiveness of their teachers. It's cost-neutral too: A Center for American Progress report by Ulrich Boser and Chingos showed virtually no difference in price between effective and ineffective curricula.

To be clear, there are perfectly good reasons why even the best teachers would be hitting the Internet for lesson planning—to find supplemental materials for individual students, for example, or adaptations for special needs kids. And teachers report using books and materials from myriad sources, including those selected by their schools and districts. But the RAND study offers a window into a phenomenon that is rarely discussed in American education: What children learn in school varies wildly from state to state, within districts, and even within grades in the same school.

If we're serious about raising the output of our K–12 system at large—not by a little, but a lot—here are some of the questions we should be asking: What exactly is the teacher's job, and what is the best use of her limited time? Is it deciding what to teach, or how to teach it? Is the soul of the work instructional design or instructional delivery? Do you want your child's teacher to have the time to analyze student work and develop a keen eye for diagnosing mistakes and misunderstandings? Do you want her to give your child rich and meaningful feedback on assignments and homework? How about developing warm and productive relationships with your child and your family?

Now ask how you expect her to do all those things at a high level while spending precious hours every week creating curricula from scratch. Nearly half of teachers in the RAND study reported spending more than four hours per week developing or selecting their own instructional materials. Newer teachers almost certainly spend the most, hampering their ability to develop their craft.

To be sure, there are master teachers to whom we should eagerly grant nearly complete classroom autonomy, including over curriculum. You wouldn't tell Prince, "Just work on your guitar playing.

Someone else will write the songs." But it's simply unrealistic to assume that every teacher is a Prince-level virtuoso and polymath—let alone to base the job description on that assumption. No one would accuse Yo-Yo Ma of being a second-rate talent because he merely plays notes written by Bach.

Without question, we want our best teachers to play a significant role in instructional design so that more children and teachers can benefit from their expertise. But it is equally certain that twelve-plus years of a well-designed and sequenced curriculum would lead to better outcomes for children than the occasional year with a great yet isolated teacher. It would also let teachers focus more time on the art of teaching—that is, more time with student work and less time on Pinterest on Sunday night with an empty plan book at their elbow.

Great teachers need great instructional materials. It's time we got serious about providing them.

Robert Pondiscio is a Senior Fellow and the Vice President for External Affairs at the Thomas B. Fordham Institute.

Pondiscio, Robert. "Failing by design: How we make teaching too hard for mere mortals." Fordham Institute, 10 May. 2016, https://edexcellence.net/articles/failing-by-design-how-we-make-teaching-too-hard-for-mere-mortals.

Looking for Evidence of Student Engagement in the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ Students access prior learning from major work in the grade in order to support new learning.	
Rigor	
Conceptual Understanding ☐ Students access concepts and ideas from a variety of perspectives. ☐ Students explain mathematical ideas behind a particular concept in a variety of ways. ☐ Students use examples and counterexamples to make and support conjectures applied to one problem to multiple situations. ☐ Students create and use a variety of models to analyze relationships. ☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations. **Procedural Skills and Fluency** ☐ Students select tools (e.g. physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem. ☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways. ☐ Students look for patterns, generalizations, and shortcuts. ☐ Students are flexible in their use of procedures and skills to solve problems. **Application** ☐ Students decontextualize and contextualize quantities in problem situations. ☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

Note: To help educators look for evidence of grade-level-appropriate student engagement in mathematical tasks, these narrative descriptors are adapted from Illustrative Mathematics. (2014, February 12). *Standards for Mathematical Practice: Commentary and Elaborations for K–5 and 6-8*. Tucson, AZ. Available at http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards

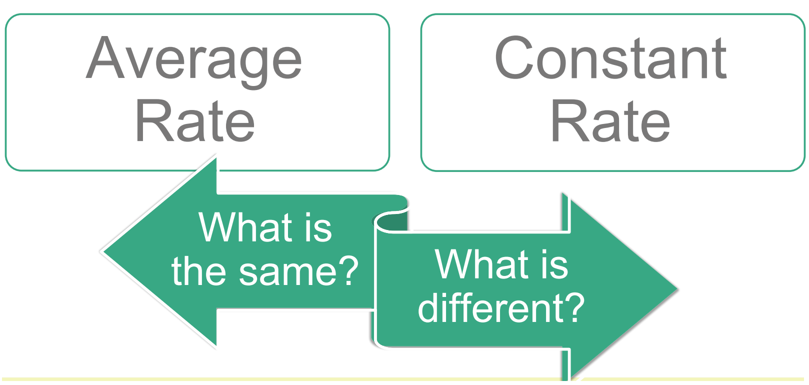
Engaging in the math

Pauline mows a lawn at a constant rate. She mows a 35-square-foot lawn in 2.5 minutes.

How might we express Pauline's average rate for any number of minutes?

Let *y* represent the number of square feet and *t* represent the number minutes.

Talking about the math



Engaging in the math – Practice with a partner

Waters flows at a constant rate out of a faucet. It takes three minutes for 10.5 gallons to flow.

How can you express the number of gallons of water that flows in any number of minutes? Convince us!

- Represent your answer in at least two different ways.
- Write two different ratios for this situation

Exercises

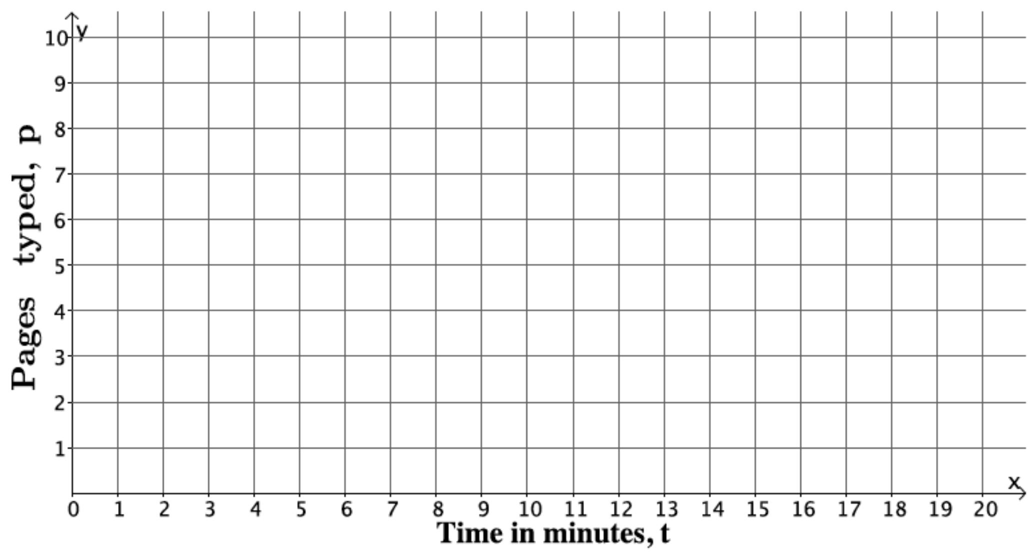
1. Juan types at a constant rate. He can type a full page of text in $3\frac{1}{2}$ minutes. We want to know how many pages, p, Juan can type after t minutes.
   a. Write the linear equation in two variables that represents the number of pages Juan types in any given time interval.

Partner A: Create a TABLE for the situation **Partner B**: Create a GRAPH for the situation

   b. Complete the table below. Use a calculator, and round your answers to the tenths place.

t (time in minutes)	Linear Equation:	p (pages typed)

   c. Graph the data on a coordinate plane.



2. Emily paints at a constant rate. She can paint 32 square feet in 5 minutes. What area, A, in square feet, can she paint in t minutes?

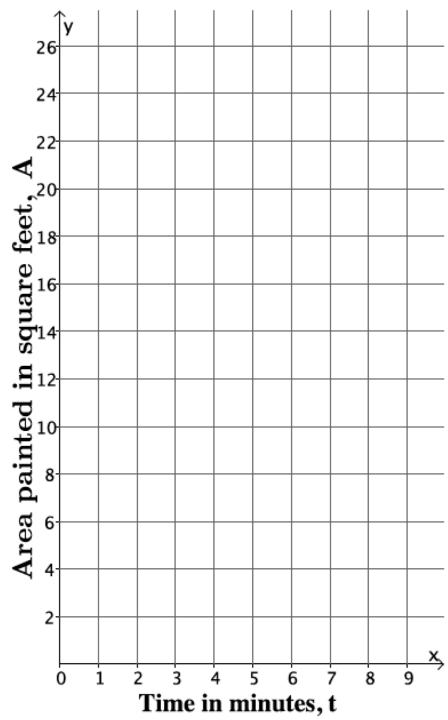
a. Write the linear equation in two variables that represents the number of square feet Emily can paint in any given time interval.

Partner A: Create a GRAPH for the situation **Partner B**: Create a TABLE for the situation

b. Complete the table below. Use a calculator, and round answers to the tenths place.

t **(time in minutes)**	**Linear Equation:**	A **(area painted in square feet)**

c. Graph the data on a coordinate plane.



d. About how many square feet can Emily paint in $2\frac{1}{2}$ minutes? Explain.

Engaging in the math – Closing the lesson

Essential Characteristics	**Application**
Example	**Non-Example**

CONSTANT RATE OF CHANGE

Lesson 11: Constant Rate

Student Outcomes

- Students know the definition of constant rate in varied contexts as expressed using two variables where one is t representing a time interval.
- Students graph points on a coordinate plane related to constant rate problems.

Classwork

Example 1 (6 minutes)

Give students the first question below, and allow them time to work. Ask them to share their solutions with the class, and then proceed with the discussion, table, and graph to finish Example 1.

> **Example 1**
>
> **Pauline mows a lawn at a constant rate. Suppose she mows a 35-square-foot lawn in 2.5 minutes. What area, in square feet, can she mow in 10 minutes? t minutes?**

- What is Pauline's average rate in 2.5 minutes?
  - *Pauline's average rate in 2.5 minutes is* $\frac{35}{2.5}$ *square feet per minute.*
- What is Pauline's average rate in 10 minutes?
  - *Let A represent the square feet of the area mowed in 10 minutes. Pauline's average rate in 10 minutes is* $\frac{A}{10}$ *square feet per mintute.*
- Let C be Pauline's constant rate in square feet per minute; then, $\frac{35}{2.5} = C$, and $\frac{A}{10} = C$. Therefore,

$$\frac{35}{2.5} = \frac{A}{10}$$
$$350 = 2.5A$$
$$\frac{350}{2.5} = \frac{2.5}{2.5}A$$
$$140 = A$$

Pauline mows 140 square feet of lawn in 10 minutes.

- If we let y represent the number of square feet Pauline can mow in t minutes, then Pauline's average rate in t minutes is $\frac{y}{t}$ square feet per minute.
- Write the two-variable equation that represents the area of lawn, y, Pauline can mow in t minutes.

P-05.1

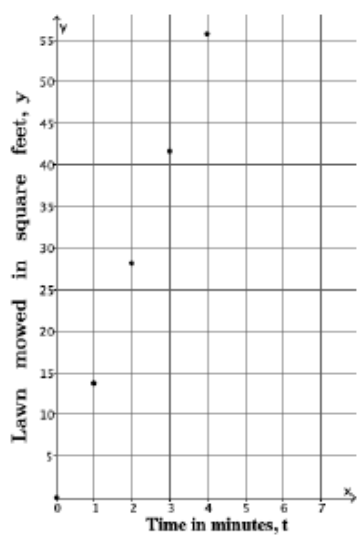
$$\frac{35}{2.5}=\frac{y}{t}$$
$$2.5y=35t$$
$$\frac{2.5}{2.5}y=\frac{35}{2.5}t$$
$$y=\frac{35}{2.5}t$$

MP.7

- What is the meaning of $\frac{35}{2.5}$ in the equation $y=\frac{35}{2.5}t$?
  - *The number $\frac{35}{2.5}$ represents the constant rate at which Pauline can mow a lawn.*
- We can organize the information in a table.

t (time in minutes)	Linear Equation: $y=\frac{35}{2.5}t$	y (area in square feet)
0	$y=\frac{35}{2.5}(0)$	0
1	$y=\frac{35}{2.5}(1)$	$\frac{35}{2.5}=14$
2	$y=\frac{35}{2.5}(2)$	$\frac{70}{2.5}=28$
3	$y=\frac{35}{2.5}(3)$	$\frac{105}{2.5}=42$
4	$y=\frac{35}{2.5}(4)$	$\frac{140}{2.5}=56$

- On a coordinate plane, we will let the x-axis represent time t, in minutes, and the y-axis represent the area of mowed lawn in square feet. Then we have the following graph.



P-05.2

- Because Pauline mows at a constant rate, we would expect the square feet of mowed lawn to continue to rise as the time, in minutes, increases.

Concept Development (6 minutes)

- In the last lesson, we learned about average speed and constant speed. Constant speed problems are just a special case of a larger variety of problems known as constant rate problems. Some of these problems were topics in Grade 7, such as water pouring out of a faucet into a tub, painting a house, and mowing a lawn.
- First, we define the average rate:

  Suppose V gallons of water flow from a faucet in a given time interval t (minutes). Then, the *average rate* of water flow in the given time interval is $\frac{V}{t}$ in gallons per minute.
- Then, we define the constant rate:

  Suppose the average rate of water flow is the same constant C for *any* given time interval. Then we say that the water is flowing at a *constant rate*, C.
- Similarly, suppose A square feet of lawn are mowed in a given time interval t (minutes). Then, the *average rate* of lawn mowing in the given time interval is $\frac{A}{t}$ square feet per minute. If we assume that the average rate of lawn mowing is the same constant, C, for *any* given time interval, then we say that the lawn is mowed at a constant rate, C.
- Describe the average rate of painting a house.
  - *Suppose A square feet of house are painted in a given time interval t (minutes). Then the average rate of house painting in the given time interval is $\frac{A}{t}$ square feet per minute.*
- Describe the constant rate of painting a house.
  - *If we assume that the average rate of house painting is the same constant, C, over any given time interval, then we say that the wall is painted at a constant rate, C.*
- What is the difference between average rate and constant rate?
  - *Average rate is the rate in which something can be done over a specific time interval. Constant rate assumes that the average rate is the same over any time interval.*
- As you can see, the way we define average rate and constant rate for a given situation is very similar. In each case, a transcription of the given information leads to an expression in two variables.

Example 2 (8 minutes)

> **Example 2**
>
> **Water flows at a constant rate out of a faucet. Suppose the volume of water that comes out in three minutes is 10.5 gallons. How many gallons of water come out of the faucet in t minutes?**

- Write the linear equation that represents the volume of water, V, that comes out in t minutes.

P-05.3

Let C represent the constant rate of water flow.

$\frac{10.5}{3} = C$, *and* $\frac{V}{t} = C$; *then,* $\frac{10.5}{3} = \frac{V}{t}$.

$$\frac{10.5}{3} = \frac{V}{t}$$
$$3V = 10.5t$$
$$\frac{3}{3}V = \frac{10.5}{3}t$$
$$V = \frac{10.5}{3}t$$

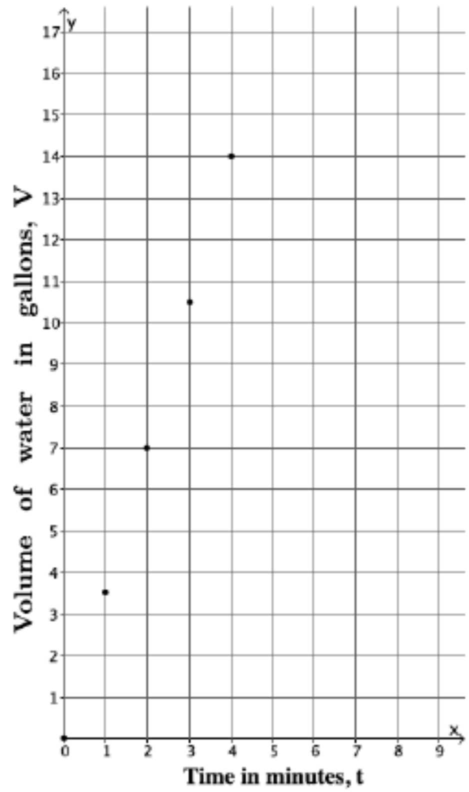
MP.7

- What is the meaning of the number $\frac{10.5}{3}$ in the equation $V = \frac{10.5}{3}t$?
  - *The number $\frac{10.5}{3}$ represents the constant rate at which water flows from a faucet.*
- Using the linear equation $V = \frac{10.5}{3}t$, complete the table.

t (time in minutes)	Linear Equation: $V = \frac{10.5}{3}t$	V (in gallons)
0	$V = \frac{10.5}{3}(0)$	0
1	$V = \frac{10.5}{3}(1)$	$\frac{10.5}{3} = 3.5$
2	$V = \frac{10.5}{3}(2)$	$\frac{21}{3} = 7$
3	$V = \frac{10.5}{3}(3)$	$\frac{31.5}{3} = 10.5$
4	$V = \frac{10.5}{3}(4)$	$\frac{42}{3} = 14$

- On a coordinate plane, we will let the x-axis represent time t in minutes and the y-axis represent the volume of water. Graph the data from the table.

P-05.4



- Using the graph, about how many gallons of water do you think would flow after $1\frac{1}{2}$ minutes? Explain.
  - *After $1\frac{1}{2}$ minutes, between $3\frac{1}{2}$ and 7 gallons of water will flow. Since the water is flowing at a constant rate, we can expect the volume of water to rise between 1 and 2 minutes. The number of gallons that flow after $1\frac{1}{2}$ minutes then would have to be between the number of gallons that flow out after 1 minute and 2 minutes.*
- Using the graph, about how long would it take for 15 gallons of water to flow out of the faucet? Explain.
  - *It would take between 4 and 5 minutes for 15 gallons of water to flow out of the faucet. It takes 4 minutes for 14 gallons to flow; therefore, it must take more than 4 minutes for 15 gallons to come out. It must take less than 5 minutes because $3\frac{1}{2}$ gallons flow out every minute.*
- Graphing proportional relationships like these last two constant rate problems provides us more information than simply solving an equation and calculating one value. The graph provides information that is not so obvious in an equation.

P-05.5

Exercises (15 minutes)

Students complete Exercises 1–3 independently.

Exercises

1. Juan types at a constant rate. He can type a full page of text in $3\frac{1}{2}$ minutes. We want to know how many pages, p, Juan can type after t minutes.

   a. Write the linear equation in two variables that represents the number of pages Juan types in any given time interval.

   Let C represent the constant rate that Juan types in pages per minute. Then,

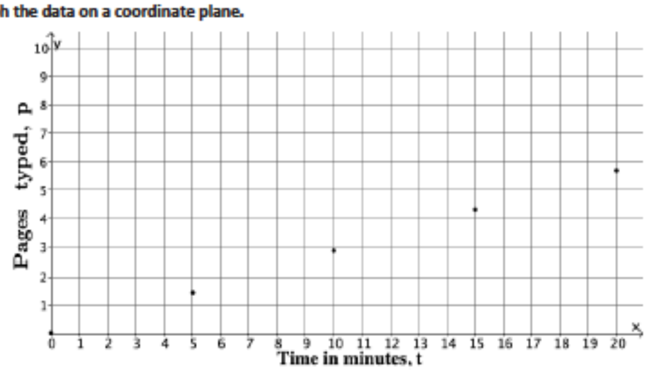
   $\frac{1}{3.5} = C$, *and* $\frac{p}{t} = C$; *therefore,* $\frac{1}{3.5} = \frac{p}{t}$.

$$\frac{1}{3.5} = \frac{p}{t}$$
$$3.5p = t$$
$$\frac{3.5}{3.5}p = \frac{1}{3.5}t$$
$$p = \frac{1}{3.5}t$$

   b. Complete the table below. Use a calculator, and round your answers to the tenths place.

t (time in minutes)	Linear Equation: $p = \frac{1}{3.5}t$	p (pages typed)
0	$p = \frac{1}{3.5}(0)$	0
5	$p = \frac{1}{3.5}(5)$	$\frac{5}{3.5} \approx 1.4$
10	$p = \frac{1}{3.5}(10)$	$\frac{10}{3.5} \approx 2.9$
15	$p = \frac{1}{3.5}(15)$	$\frac{15}{3.5} \approx 4.3$
20	$p = \frac{1}{3.5}(20)$	$\frac{20}{3.5} \approx 5.7$

   c. Graph the data on a coordinate plane.



P-05.6

d. About how long would it take Juan to type a 5-page paper? Explain.

It would take him between 15 and 20 minutes. After 15 minutes, he will have typed 4.3 pages. In 20 minutes, he can type 5.7 pages. Since 5 pages is between 4.3 and 5.7, then it will take him between 15 and 20 minutes.

2. Emily paints at a constant rate. She can paint 32 square feet in 5 minutes. What area, A, in square feet, can she paint in t minutes?

a. Write the linear equation in two variables that represents the number of square feet Emily can paint in any given time interval.

Let C be the constant rate that Emily paints in square feet per minute. Then,

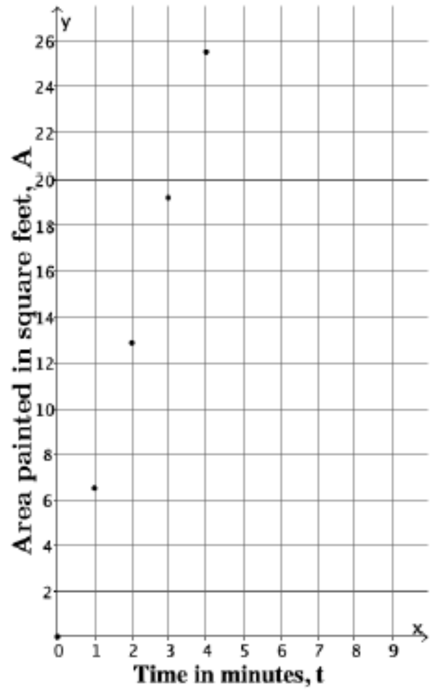
$\frac{32}{5} = C$, and $\frac{A}{t} = C$; therefore, $\frac{32}{5} = \frac{A}{t}$.

$$\frac{32}{5} = \frac{A}{t}$$
$$5A = 32t$$
$$\frac{5}{5}A = \frac{32}{5}t$$
$$A = \frac{32}{5}t$$

b. Complete the table below. Use a calculator, and round answers to the tenths place.

t (time in minutes)	Linear Equation: $A = \frac{32}{5}t$	A (area painted in square feet)
0	$A = \frac{32}{5}(0)$	0
1	$A = \frac{32}{5}(1)$	$\frac{32}{5} = 6.4$
2	$A = \frac{32}{5}(2)$	$\frac{64}{5} = 12.8$
3	$A = \frac{32}{5}(3)$	$\frac{96}{5} = 19.2$
4	$A = \frac{32}{5}(4)$	$\frac{128}{5} = 25.6$

c. Graph the data on a coordinate plane.



d. About how many square feet can Emily paint in $2\frac{1}{2}$ minutes? Explain.

Emily can paint between 12.8 and 19.2 square feet in $2\frac{1}{2}$ minutes. After 2 minutes, she paints 12.8 square feet, and after 3 minutes, she will have painted 19.2 square feet.

3. Joseph walks at a constant speed. He walked to a store that is one-half mile away in 6 minutes. How many miles, m, can he walk in t minutes?

a. Write the linear equation in two variables that represents the number of miles Joseph can walk in any given time interval, t.

Let C be the constant rate that Joseph walks in miles per minute. Then,

$\frac{0.5}{6} = C$, *and* $\frac{m}{t} = C$; *therefore,* $\frac{0.5}{6} = \frac{m}{t}$.

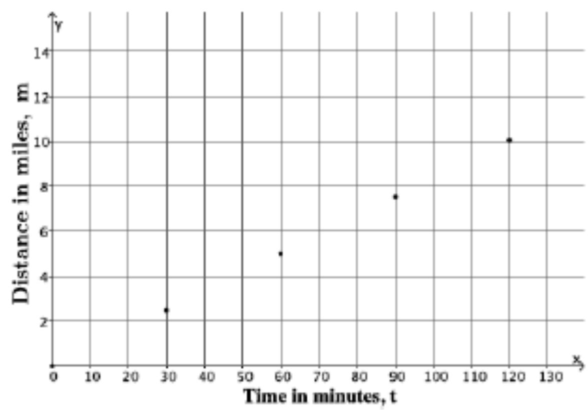
$$\frac{0.5}{6} = \frac{m}{t}$$
$$6m = 0.5t$$
$$\frac{6}{6}m = \frac{0.5}{6}t$$
$$m = \frac{0.5}{6}t$$

P-05.8

b. Complete the table below. Use a calculator, and round answers to the tenths place.

t (time in minutes)	Linear Equation: $m = \frac{0.5}{6}t$	m (distance in miles)
0	$m = \frac{0.5}{6}(0)$	0
30	$m = \frac{0.5}{6}(30)$	$\frac{15}{6} = 2.5$
60	$m = \frac{0.5}{6}(60)$	$\frac{30}{6} = 5$
90	$m = \frac{0.5}{6}(90)$	$\frac{45}{6} = 7.5$
120	$m = \frac{0.5}{6}(120)$	$\frac{60}{6} = 10$

c. Graph the data on a coordinate plane.



d. Joseph's friend lives 4 miles away from him. About how long would it take Joseph to walk to his friend's house? Explain.

It will take Joseph a little less than an hour to walk to his friend's house. Since it takes 30 minutes for him to walk 2.5 miles and 60 minutes to walk 5 miles, and 4 is closer to 5 than 2.5, it will take Joseph less than an hour to walk the 4 miles.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- Constant rate problems appear in a variety of contexts like painting a house, typing, walking, or water flow.
- We can express the constant rate as a two-variable equation representing proportional change.
- We can graph the constant rate situation by completing a table to compute data points.

P-05.9

Lesson Summary

When constant rate is stated for a given problem, then you can express the situation as a two-variable equation. The equation can be used to complete a table of values that can then be graphed on a coordinate plane.

Exit Ticket (5 minutes)

Name ______________________________ Date ______________

Lesson 11: Constant Rate

Exit Ticket

Vicky reads at a constant rate. She can read 5 pages in 9 minutes. We want to know how many pages, p, Vicky can read after t minutes.

a. Write a linear equation in two variables that represents the number of pages Vicky reads in any given time interval.

b. Complete the table below. Use a calculator, and round answers to the tenths place.

t (time in minutes)	Linear Equation:	p (pages read)
0		
20		
40		
60		

c. About how long would it take Vicky to read 25 pages? Explain.

P-05.11

Exit Ticket Sample Solutions

Vicky reads at a constant rate. She can read 5 pages in 9 minutes. We want to know how many pages, p, Vicky can read after t minutes.

a. Write a linear equation in two variables that represents the number of pages Vicky reads in any given time interval.

Let C represent the constant rate that Vicky reads in pages per minute. Then,

$\frac{5}{9} = C$, *and* $\frac{p}{t} = C$; *therefore,* $\frac{5}{9} = \frac{p}{t}$.

$$\frac{5}{9} = \frac{p}{t}$$
$$9p = 5t$$
$$\frac{9}{9}p = \frac{5}{9}t$$
$$p = \frac{5}{9}t$$

b. Complete the table below. Use a calculator, and round answers to the tenths place.

t (time in minutes)	Linear Equation: $p = \frac{5}{9}t$	p (pages read)
0	$p = \frac{5}{9}(0)$	0
20	$p = \frac{5}{9}(20)$	$\frac{100}{9} \approx 11.1$
40	$p = \frac{5}{9}(40)$	$\frac{200}{9} \approx 22.2$
60	$p = \frac{5}{9}(60)$	$\frac{300}{9} \approx 33.3$

c. About how long would it take Vicky to read 25 pages? Explain.

It would take her a little over 40 minutes. After 40 minutes, she can read about 22.2 pages, and after 1 hour, she can read about 33.3 pages. Since 25 pages is between 22.2 and 33.3, it will take her between 40 and 60 minutes to read 25 pages.

Problem Set Sample Solutions

Students practice writing two-variable equations that represent a constant rate.

1. A train travels at a constant rate of 45 miles per hour.

a. What is the distance, d, in miles, that the train travels in t hours?

Let C be the constant rate the train travels. Then, $\frac{45}{1} = C$, *and* $\frac{d}{t} = C$; *therefore,* $\frac{45}{1} = \frac{d}{t}$.

$$\frac{45}{1} = \frac{d}{t}$$
$$d = 45t$$

P-05.12

b. How many miles will it travel in 2.5 hours?

$$d = 45(2.5)$$
$$= 112.5$$

The train will travel 112.5 miles in 2.5 hours.

2. Water is leaking from a faucet at a constant rate of $\frac{1}{3}$ gallons per minute.

   a. What is the amount of water, w, in gallons per minute, that is leaked from the faucet after t minutes?

   Let C be the constant rate the water leaks from the faucet in gallons per minute. Then,

   $\frac{\frac{1}{3}}{1} = C$, *and* $\frac{w}{t} = C$; *therefore,* $\frac{\frac{1}{3}}{1} = \frac{w}{t}$.

$$\frac{\frac{1}{3}}{1} = \frac{w}{t}$$
$$w = \frac{1}{3}t$$

   b. How much water is leaked after an hour?

$$w = \frac{1}{3}t$$
$$= \frac{1}{3}(60)$$
$$= 20$$

   The faucet will leak 20 gallons in one hour.

3. A car can be assembled on an assembly line in 6 hours. Assume that the cars are assembled at a constant rate.

   a. How many cars, y, can be assembled in t hours?

   Let C be the constant rate the cars are assembled in cars per hour. Then,

   $\frac{1}{6} = C$, *and* $\frac{y}{t} = C$; *therefore,* $\frac{1}{6} = \frac{y}{t}$.

$$\frac{1}{6} = \frac{y}{t}$$
$$6y = t$$
$$\frac{6}{6}y = \frac{1}{6}t$$
$$y = \frac{1}{6}t$$

   b. How many cars can be assembled in a week?

   A week is $24 \times 7 = 168$ hours. So, $y = \frac{1}{6}(168) = 28$. Twenty-eight cars can be assembled in a week.

P-05.13

4. A copy machine makes copies at a constant rate. The machine can make 80 copies in $2\frac{1}{2}$ minutes.

a. Write an equation to represent the number of copies, n, that can be made over any time interval in minutes, t.

Let C be the constant rate that copies can be made in copies per minute. Then,

$\frac{80}{2\frac{1}{2}} = C$, and $\frac{n}{t} = C$; therefore, $\frac{80}{2\frac{1}{2}} = \frac{n}{t}$.

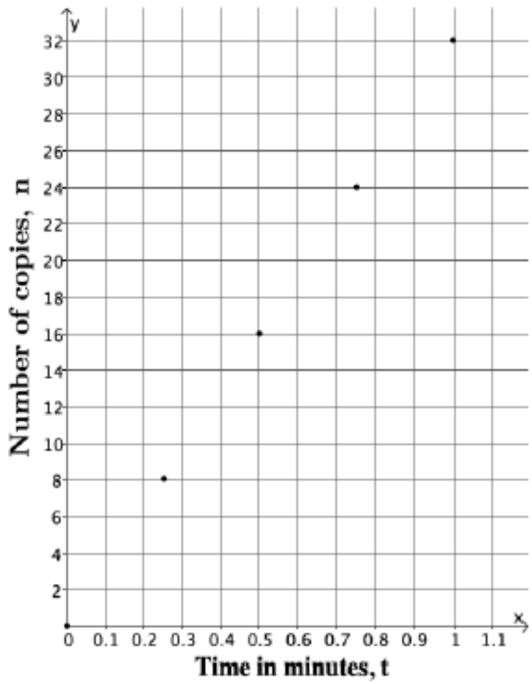
$$\frac{80}{2\frac{1}{2}} = \frac{n}{t}$$
$$2\frac{1}{2}n = 80t$$
$$\frac{5}{2}n = 80t$$
$$\frac{2}{5} \cdot \frac{5}{2}n = \frac{2}{5} \cdot 80t$$
$$n = 32t$$

b. Complete the table below.

t (time in minutes)	Linear Equation: $n = 32t$	n (number of copies)
0	$n = 32(0)$	0
0.25	$n = 32(0.25)$	8
0.5	$n = 32(0.5)$	16
0.75	$n = 32(0.75)$	24
1	$n = 32(1)$	32

P-05.14

c. Graph the data on a coordinate plane.



d. The copy machine runs for 20 seconds and then jams. About how many copies were made before the jam occurred? Explain.

Since 20 seconds is approximately 0.3 of a minute, then the number of copies made will be between 8 and 16 because 0.3 is between 0.25 and 0.5.

5. Connor runs at a constant rate. It takes him 34 minutes to run 4 miles.

a. Write the linear equation in two variables that represents the number of miles Connor can run in any given time interval in minutes, t.

Let C be the constant rate that Connor runs in miles per minute, and let m represent the number of miles he ran in t minutes. Then,

$\frac{4}{34} = C$, and $\frac{m}{t} = C$; therefore, $\frac{4}{34} = \frac{m}{t}$.

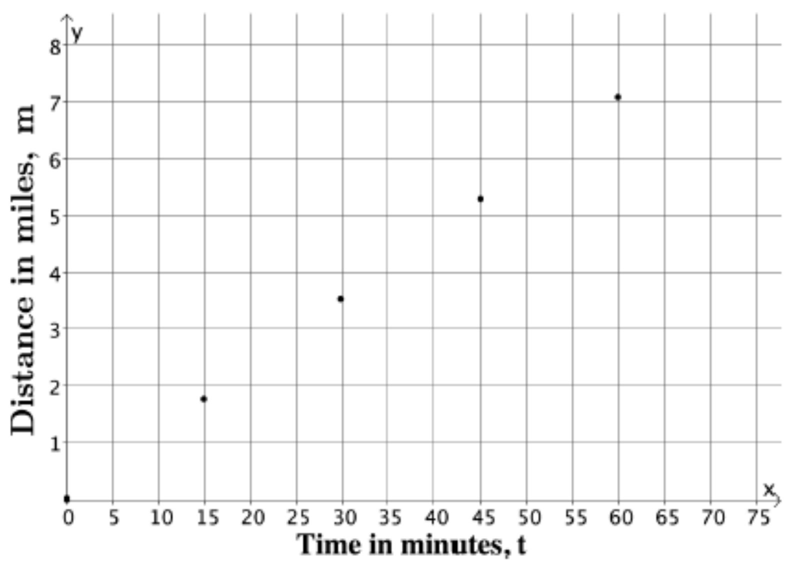
$$\frac{4}{34} = \frac{m}{t}$$
$$34m = 4t$$
$$\frac{34}{34}m = \frac{4}{34}t$$
$$m = \frac{4}{34}t$$
$$m = \frac{2}{17}t$$

P-05.15

b. Complete the table below. Use a calculator, and round answers to the tenths place.

t (time in minutes)	Linear Equation: $m = \frac{2}{17}t$	m (distance in miles)
0	$m = \frac{2}{17}(0)$	0
15	$m = \frac{2}{17}(15)$	$\frac{30}{17} \approx 1.8$
30	$m = \frac{2}{17}(30)$	$\frac{60}{17} \approx 3.5$
45	$m = \frac{2}{17}(45)$	$\frac{90}{17} \approx 5.3$
60	$m = \frac{2}{17}(60)$	$\frac{120}{17} \approx 7.1$

c. Graph the data on a coordinate plane.



d. Connor ran for 40 minutes before tripping and spraining his ankle. About how many miles did he run before he had to stop? Explain.

Since Connor ran for 40 minutes, he ran more than 3.5 miles but less than 5.3 miles. Since 40 is between 30 and 45, then we can use those reference points to make an estimate of how many miles he ran in 40 minutes, probably about 5 miles.

Diamond Reflection
Unpacking the EngageNY Lesson

Observations: How are the mathematics content standards evident in the lesson?

Reflections: How does student engagement in the SMPs help develop conceptual understanding, procedural skill and fluency and application skills?

Other thoughts and considerations

Interpretations: What instructional strategies and facilitator moves brought out the content and practice standards?

Decisional: What am I going to do based on my learning?

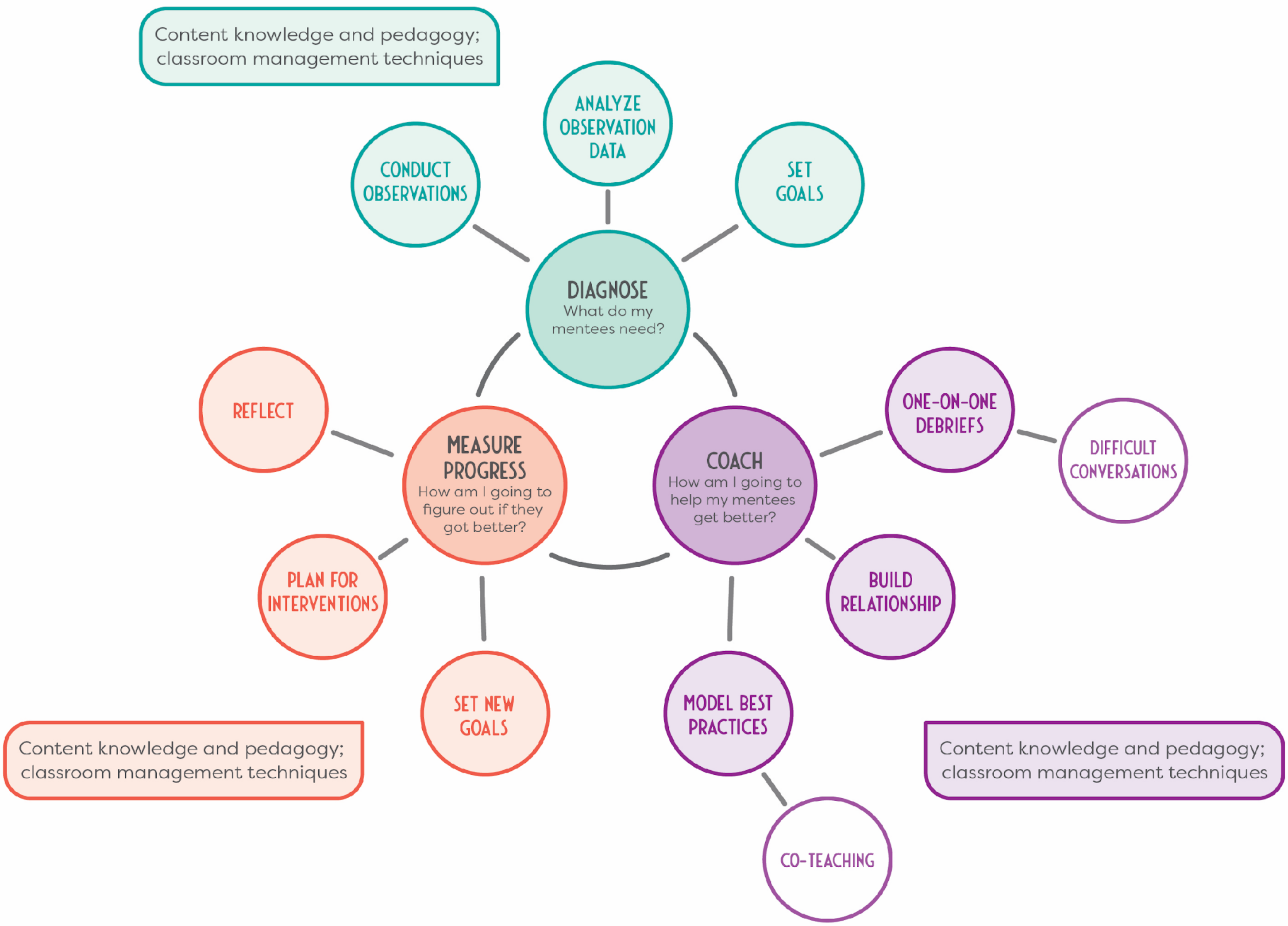
Key Takeaway:
Using a Tier 1 Curriculum ensures that all key shifts in mathematics are being implemented.

Key Takeaway:
Having a strong understanding of the instructional shifts in math increases the mentor's ability to coach their mentee's math instruction.

Module 4 Afternoon Outcomes

- Plan for interventions to meet the specific needs of a mentee based on observation data.
- Model best practices to support mentee learning.

The Mentor Cycle



Plan for Interventions: 3 Key Components

- Clarify the new learning
- Align the intervention method
- Write a coaching plan

Clarify the new learning

Content	Practice
What does my mentee need to understand?	What do I lean on in my teaching practice in order to do this?
What does the Tier 1 resource recommend?	What does my mentee need to be able to do?
How could my mentee gain this knowledge?	How could my mentee gain this skill?

Sample SMART Goal 1

Students will, with 80% accuracy during this unit, use multiple methods, including data in a table and on a graph, to show whether the quantities represented in relevant real-world scenarios are proportional.	**What does the mentee need to learn?**

Sample SMART Goal 2

Eighty-eight percent (86%) of students will articulate that scale factor corresponds to the unit rate and constant of proportionality, as measured during this unit, written responses to problems in the context of scale drawings.	**What does the mentee need to learn?**

Summarize: Model vs. Co-Teach - When do we use each method?

Which intervention?

Scenario	SMART Goal	Intervention
Your mentee wants to improve her ability to use a graph to show whether the quantities represented in relevant real-world scenarios are proportional. This is her goal because in a recent observation you noted that she was consistently models proportionality using data tables.	*Students will, with 80% accuracy during this unit, use multiple methods, including data in a table and on a graph, to show whether the quantities represented in relevant real-world scenarios are proportional.*	
Your mentee is trying to increase the number of students who can explain in writing the connection between scale factor, unit rate and the constant of proportionality. When she/he shared samples of student work from yesterday's lesson, she/he was able to point to evidence of students' misuse of the language of mathematics in their written explanations. She/He asked for direction on addressing the gaps.	*Eighty-eight percent (86%) of students will articulate that scale factor corresponds to the unit rate and constant of proportionality, as measured during this unit, written responses to problems in the context of scale drawings.*	

Align the intervention: Overcoming Barriers

Location:	**Time:**
Lesson "bite size":	**Group size:**

Mentor Coaching Plan

Mentee SMART goal(s) *Students will, with 80% accuracy during this unit, use multiple methods, including data in a table and on a graph, to show whether the quantities represented in relevant real-world scenarios are proportional.*

What activities and resources will mentor and mentee engage in to achieve goal(s)?

Specific Activity or Resource	How is it aligned to the goal(s)?	Why will it be effective?	How will you integrate relationship building?	Projected timeline

How will you monitor your mentee's progress toward the identified goals?

Mentor Coaching Plan

Mentee SMART goal(s)
Eighty-eight percent (86%) of students will articulate that scale factor corresponds to the unit rate and constant of proportionality, as measured during this unit, written responses to problems in the context of scale drawings.

What activities and resources will mentor and mentee engage in to achieve goal(s)?

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Mentor Coaching Plan

Mentee SMART goal(s)

What activities and resources will mentor and mentee engage in to achieve goal(s)?

Specific Activity or Resource	How is it aligned to the goal(s)?	Why will it be effective?	How will you integrate relationship building?	Projected timeline

How will you monitor your mentee's progress toward the identified goals?

Plan for Interventions: One Sentence Summary

Key Takeaway: Coaching plans keep mentor and mentee on track to achieve SMART goals.

Model Best Practices: 3 Key Components

- Co-plan instruction
- Model for demonstration
- Debrief

Co-Plan Instruction

- Revisit agreements
- Confirm the purpose/goal and connection to SMART goal
- Confirm that you're modeling
- Make thinking visible as you co-plan the lesson or activity
- Create a "look-fors" checklist based on the goal of the model lesson or activity

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Student Outcomes

- Students examine situations carefully to decide whether two quantities are proportional to each other by graphing on a coordinate plane and observing whether all the points would fall on a line that passes through the origin.
- Students study examples of relationships that are not proportional as well as those that are.

Classwork

Today's Exploratory Challenge is an extension of Lesson 5. You will be working in groups to create a table and graph and to identify whether the two quantities are proportional to each other.

Preparation (5 minutes)

Place students in groups of four. Hand out markers, poster paper, graph paper, and envelopes containing 5 ratios each. (Each group will have identical contents.) Lead students through the following directions to prepare for the Exploratory Challenge.

MP.1 & MP.2

Have the recorder fold the poster paper in quarters and label as follows: Quad 1–Table, Quad 2–Problem, Quad 3–Graph, and Quad 4–Proportional or Not? Explanation.

Instruct the reader to take out the contents of the envelope (located at the end of the lesson), and instruct the group to arrange the data in a table and on a graph.

Instruct the reader to read the problem. The recorder should write the problem on the poster paper. Students use multiple methods to show whether the quantities represented in the envelope are proportional to each other.

Exploratory Challenge (20 minutes)

Give students 15 minutes to discuss the problem and record their responses onto the poster paper. For the last 5 minutes, have groups place their posters on the wall and circulate around the room, looking for the groups that have the same ratios. Have groups with the same ratios identify and discuss the differences of their posters.

Gallery Walk (10 minutes)

In groups, have students observe each poster, write any thoughts on sticky notes, and place them on the posters. Sample posters are provided below. Also, have students answer the following questions on their worksheets:

- Were there any differences found in groups that had the same ratios?
- Did you notice any common mistakes? How might they be fixed?
- Which posters were both visually attractive and informative?

Poster Layout

Use for notes

Group 1 and 8

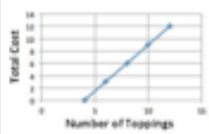
Problem:

A local frozen yogurt shop is known for their monster sundaes. Create a table, and then graph and explain if the quantities are proportional to each other.

Table:

Number of Toppings	*Total Cost of Toppings ($)*
4	0
6	3
8	6
10	9
12	12

Graph:



Explanation:

Although the points appear on a line, the quantities are not proportional to each other because the line does not go through the origin. Each topping does not have the same unit cost.

Group 2 and 7

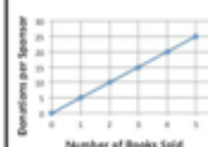
Problem:

The school library receives money for every book sold at the school's book fair. Create a table, and then graph and explain if the quantities are proportional to each other.

Table:

Number of Books Sold	*Donations per Sponsor ($)*
1	5
2	10
3	15
4	20
5	25

Graph:



Explanation:

The quantities are proportional to each other because the points appear on a line that goes through the origin. Each book sold brings in $5.00, no matter how many books are sold.

Group 3 and 6

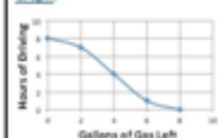
Problem:

Your uncle just bought a hybrid car and wants to take you and your siblings camping. Create a table, and then graph and explain if the quantities are proportional to each other.

Table:

Gallons of Gas Left in Tank	*Hours of Driving*
8	0
6	1
4	4
2	7
0	8

Graph:



Explanation:

The graph is not represented by a line passing through the origin, so the quantities are not proportional to each other. The number of gallons of gas varies depending on how fast or slow the car is driven.

Group 4 and 5

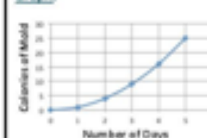
Problem:

For a science project, Eli decided to study colonies of mold. He observed a piece of bread that was molding. Create a table, and then graph and explain if the quantities are proportional to each other.

Table:

Number of Days	*Colonies of Mold*
1	1
2	4
3	9
4	16
5	25

Graph:



Explanation:

Although the graph looks as though it goes through the origin, the quantities are not proportional to each other because the points do not appear on a line. Each day does not produce the same amount of colonies as the other days.

Gallery Walk

Take notes and answer the following questions:

- Were there any differences found in groups that had the same ratios?
- Did you notice any common mistakes? How might they be fixed?
- Were there any groups that stood out by representing their problem and findings exceptionally clearly?

Poster 1:

Poster 2:

Poster 3:

Poster 4:

Poster 5:

Poster 6:

Poster 7:

Poster 8:

Note about Lesson Summary:

Closing (5 minutes)

- Why make posters with others? Why not do this exercise in your student books?
  - *We can discuss with others and learn from their thought processes. When we share information with others, our knowledge is tested and questioned.*
- What does it mean for a display to be both visually appealing and informative?
  - *For a display to be both visually appealing and informative, the reader should be able to find data and results fairly quickly and somewhat enjoyably.*
- Suppose we invited people from another school, state, or country to walk through our gallery. What would they be able to learn about ratio and proportion from our posters?
  - *Hopefully, after looking through the series of posters, people can learn and easily determine for themselves if graphs represent proportional and non-proportional relationships.*

> Lesson Summary
>
> **The plotted points in a *graph of a proportional relationship* lie on a line that passes through the origin.**

Exit Ticket (5 minutes)

Name ____ Date________________

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Exit Ticket

1. Which graphs in the gallery walk represented proportional relationships, and which did not? List the group number.

   Proportional Relationship   Non-Proportional Relationship

2. What are the characteristics of the graphs that represent proportional relationships?

3. For the graphs representing proportional relationships, what does $(0, 0)$ mean in the context of the given situation?

Exit Ticket Sample Solutions

1. Which graphs in the art gallery walk represented proportional relationships, and which did not? List the group number.

Proportional Relationship	Non-Proportional Relationship	
Group 2	*Group 1*	*Group 5*
Group 7	*Group 3*	*Group 6*
	Group 4	*Group 8*

2. What are the characteristics of the graphs that represent proportional relationships?

   Graphs of groups 2 and 7 appear on a line and go through the origin.

3. For the graphs representing proportional relationships, what does $(0, 0)$ mean in the context of the situation?

   For zero books sold, the library received zero dollars in donations.

Problem Set Sample Solutions

Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios below represent the number of years to the amount of money in the savings account.

- After one year, the interest accumulated, and the total in Sally's account was $312.
- After three years, the total was $340. After six years, the total was $380.
- After nine years, the total was $430. After 12 years, the total amount in Sally's savings account was $480.

Using the same four-fold method from class, create a table and a graph, and explain whether the amount of money accumulated and time elapsed are proportional to each other. Use your table and graph to support your reasoning.

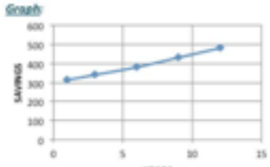
Problem:

Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping the money in the bank. The ratios below represent the number of years to the amount of money in the savings account. Create a table and a graph, and explain whether the quantities are proportional to each other.

Table:

Years	Savings ($)
1	312
3	340
6	380
9	430
12	480

Graph:



Explanation:

The graph is not a graph of a proportional relationship because, although the data appears to be a line, it is not a line that goes through the origin. The amount of interest collected is not the same every year.

Modeling Best Practices: Co-Planning Conversation Transcript

***Mentor** - "Glad we got to meet this morning to talk about how I can best support you in meeting the SMART goal we came up with based on the observation I conducted and our debrief conversation last week. I've been doing some thinking about your goal, and as you saw in the coaching plan, I think one of the ways I can best support you in reaching it is to model a small portion of an EngageNY lesson during which I can work with your students on using graphs to determine proportionality"*

***Mentee** - "Yes, my students are really good at using data tables, but I can't seem to get them to use both tables and graphs."*

***Mentor** - "Great! So I was looking at what you have coming up in your lesson plans and it looks like you are doing the Math 7, Module 1, Lesson 6 tomorrow, is that right?"*

***Mentee** - "Yes."*

***Mentor** - "So I was thinking I could come into your classroom after you've gotten the lesson started. Students could have already opening exercise classwork, and then when it comes to the time to start talking about using graphs to determine proportional relationships, that's when I can take over."*

***Mentee** - "So I would get the lesson started and then you would take over the second part? I think that sounds good. But could you be in there for the part that I am teaching too in case it isn't going well?*

***Mentor** - "Sure, I think I can work that with my schedule. I can come in for 1st period on A-day."*

***Mentor** - "This is the lesson that is coming up tomorrow, "*Identifying Proportional and Non-Proportional Relationships in Graphs" *and I think it aligns well with your SMART goal. Students will have the opportunity to use both data tables and graphs. So as I look through this lesson plan I want to look for opportunities to reinforce the use of graphs and prompt students to use more than one method to support their answers...this may come up in the teaching notes or in the sample responses included in the lesson.*

***Mentor** - "The last thing we need to discuss is what you'll be doing while I am modeling. You should definitely be observing both me and the students, but I want us to come up with a specific look-fors checklist for you to complete while you observe me teaching."*

***Mentee** - "Okay, that sounds good. I know one thing I really want to watch out for is how you get students to make the connection between the data table and the graph."*

Mentor - *"That's a great thing to put on the checklist* ***[fills out a checklist using the template]*** *So you'll want to make note of what wording and/or signals I use to remind students to connect the data tablet to the graph.*

I think another item for the checklist is how I phrase the questions in such a way that signal to students to verify their thinking and use precise mathematical language rather than just answering with phrases like, "they're the same", "they change by the same amount". Let's add that onto our list as well…
[Mentee adds this to their checklist.]

Mentor - *"Well, I look forward to seeing you Thursday at 8:15 for this lesson. When is a good time for you to meet after the lesson on Thursday to debrief?*

Mentee - *"I could do Friday during our lunch time - will that work?"*

Mentor - *"Sounds great - I will see you Thursday!"*

Model Best Practices: "Look-Fors" Checklist

Sample checklist from co-planning conversation transcript

Look-Fors	Observation Notes
• How to get students to use what they know to try out a solution pathway without giving them too many "hints"	
• Types of questioning used that scaffolds student thinking/learning	
• How students visualize the problem	
• The wait time provided for processing & discussing	

TRY IT OUT: Model Best Practices: "Look-For's" Checklist

Look-For's	Observation Notes

Look-For's Checklist

Look-For's	Observation Notes

Lesson 17: The Unit Rate as the Scale Factor

Student Outcomes

- Students recognize that the enlarged or reduced distances in a scale drawing are proportional to the corresponding distances in the original picture.
- Students recognize the scale factor to be the constant of proportionality.
- Given a picture or description of geometric figures, students make a scale drawing with a given scale factor.

Classwork

Scaffolding:
- Give the measurements of the original image lengths for the table prior to beginning Example 1.
- Challenge students by giving problems that use different units of measurement and have them compare the scale factors.

Example 1 (7 minutes): Jake's Icon

After reading the prompt with the class, discuss the following questions:

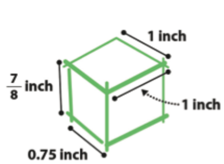
- What type of scale drawing is the sticker?
  - *It is an enlargement or a magnification of the original sketch.*
- What is the importance of proportionality for Jake?
  - *If the image is not proportional, it looks less professional. The image on the sticker will be distorted.*
- How could we go about checking for proportionality of these two images? (Have students record steps in their student materials.)
  - *Measure corresponding lengths and check to see if they all have the same constant of proportionality.*

As a class, label points correspondingly on the original sketch, and then on the sticker sketch. Use inches to measure the distance between the points and record on a table.

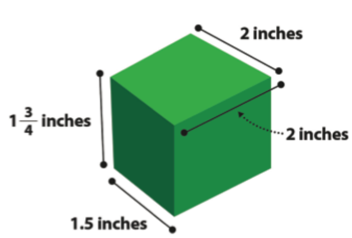
Example 1: Jake's Icon

Jake created a simple game on his computer and shared it with his friends to play. They were instantly hooked, and the popularity of his game spread so quickly that Jake wanted to create a distinctive icon so that players could easily identify his game. He drew a simple sketch. From the sketch, he created stickers to promote his game, but Jake wasn't quite sure if the stickers were proportional to his original sketch.

Original Sketch:



Sticker:



Original	Sticker
1 in.	2 in.
$\frac{3}{4}$ in.	$1\frac{1}{2}$ in.
1 in.	2 in.
$\frac{7}{8}$ in.	$1\frac{3}{4}$ in.

Steps to check for proportionality for scale drawing and original object or picture:

1. ***Record the lengths of the scale drawing on the table.***

Key Idea:

The *scale factor* can be calculated from the ratio of any length in the scale drawing to its corresponding length in the actual picture. The scale factor corresponds to the unit rate and the constant of proportionality.

Scaling by factors *greater than* 1 enlarges the segment, and scaling by factors *less than* 1 reduces the segment.

- What relationship do you see between the measurements?
  - *The corresponding lengths are proportional.*
- Is the sticker proportional to the original sketch?
  - *Yes, the sticker lengths are twice as long as the lengths in the original sketch.*
- How do you know?
  - *The unit rate,* 2*, is the same for the corresponding measurements.*
- What is this called?
  - *Constant of proportionality*

Introduce the term *scale factor* and review the key idea box with students.

- Is the new figure larger or smaller than the original?
  - *Larger*
- What is the scale factor for the sticker? How do you know?
  - *The scale factor is two because the scale factor is the same as the constant of proportionality. It is the ratio of a length in the scale drawing to the corresponding length in the actual picture, which is* 2 *to* 1*. The enlargement is represented by a number greater than* 1*.*
- Each of the corresponding lengths is how many times larger?
  - *Two times*
- What can you predict about an image that has a scale factor of 3?
  - *The lengths of the scaled image will be three times as long as the lengths of the original image.*

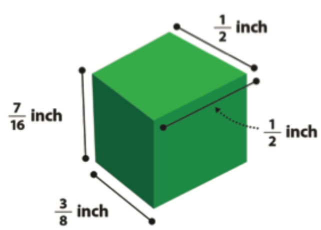
Exercise 1 (5 minutes): App Icon

Give students time to work with partners to record the lengths (in inches) of the app icon that correspond to the lengths in Example 1 on tables.

- What was the relationship between the sticker and the original sketch?
  - *The sticker is larger than the original.*

- What was the constant of proportionality, or scale factor, for this relationship?
  - 2
- What is the relationship between the icon and the original sketch?
  - *The icon is smaller than the original sketch.*
- What was the constant of proportionality, or scale factor, for this relationship?
  - $\frac{1}{2}$
- How do we determine the scale factor?
  - *Measure the lengths of the app icon and the corresponding lengths of the original sketch and record the data. Using the data, determine the constant of proportionality.*
- What does the scale factor indicate?
  - *A scale factor less than* 1 *indicates a reduction from the original picture, and a scale factor greater than* 1 *indicates a magnification or enlargement from the original picture.*

Exercise 1: App Icon



Original	App Icon
1 in.	$\frac{1}{2}$ in.
$\frac{3}{4}$ in.	$\frac{3}{8}$ in.
1 in.	$\frac{1}{2}$ in.
$\frac{7}{8}$ in.	$\frac{7}{16}$ in.

Example 2 (7 minutes)

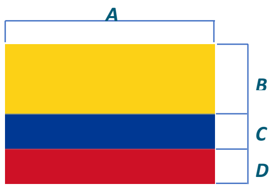
Begin this example by giving the scale factor, 3. Demonstrate how to make a scale drawing using the scale factor. Use a table or an equation to show how you computed your actual lengths. Note that the original image of the flag should be 1 inch by $1\frac{1}{2}$ inches.

- Is this a reduction or an enlargement?
  - *An enlargement*
- How could you determine that it was an enlargement even before seeing the drawing?
  - *A scale factor greater than one represents an enlargement.*
- Can you predict what the lengths of the scale drawing will be?
  - *Yes, they will be* 3 *times as large as the actual picture.*
- What steps were used to create this scale drawing?
  - *Measure lengths of the original drawing and record onto a table. Multiply by* 3 *to compute the scale drawing lengths. Record and draw.*
- How can you double check your work?
  - *Divide the scale lengths by* 3 *to see if they match actual lengths.*

Example 2

Use a scale factor of 3 to create a scale drawing of the picture below.

Picture of the flag of Colombia:



A. $1\frac{1}{2}\text{ in.} \times 3 = 4\frac{1}{2}\text{ in.}$

B. $\frac{1}{2}\text{ in.} \times 3 = 1\frac{1}{2}\text{ in.}$

C. $\frac{1}{4}\text{ in.} \times 3 = \frac{3}{4}\text{ in.}$

D. $\frac{1}{4}\text{ in.} \times 3 = \frac{3}{4}\text{ in.}$

Exercise 2 (6 minutes)

Have students work with partners to create a scale drawing of the original picture of the flag from Example 2 but now applying a scale factor of $\frac{1}{2}$.

- Is this a reduction or an enlargement?
  - *This is a reduction because the scale factor is less than one.*
- What steps were used to create this scale drawing?
  - *Compute the scale drawing lengths by multiplying by* $\frac{1}{2}$ *or dividing by* 2*. Record. Measure the new segments with a ruler and draw.*

Exercise 2

Scale Factor $= \frac{1}{2}$

Sketch and notes:

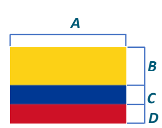
A. $1\frac{1}{2}\text{ in.} \times \frac{1}{2} = \frac{3}{4}\text{ in.}$

B. $\frac{1}{2}\text{ in.} \times \frac{1}{2} = \frac{1}{4}\text{ in.}$

C. $\frac{1}{4}\text{ in.} \times \frac{1}{2} = \frac{1}{8}\text{ in.}$

D. $\frac{1}{4}\text{ in.} \times \frac{1}{2} = \frac{1}{8}\text{ in.}$

Picture of the flag of Colombia:



Example 3 (5 minutes)

After reading the prompt with the class, discuss the following questions:

- What is the shape of the portrait?
  - *Square*
- Will the resulting picture be a reduction or a magnification?
  - *It will be a reduction because the phone picture is smaller than the original portrait. Also, the scale factor is less than one, so this indicates a reduction.*
- One student calculated the length to be 2 inches, while another student's response was $\frac{1}{6}$ of a foot. Which answer is more reasonable?
  - *Although both students are correct,* 2 *inches is more reasonable for the purpose of measuring and drawing.*
- What will the scale drawing look like?
  - *The scale drawing should be a square measuring* 2 *inches by* 2 *inches.*

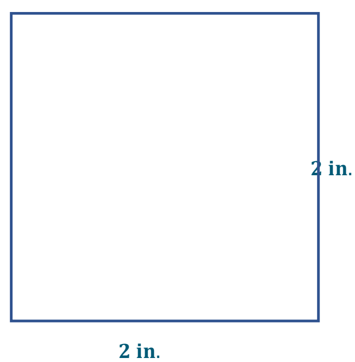
Example 3

Your family recently had a family portrait taken. Your aunt asks you to take a picture of the portrait using your phone and send it to her. If the original portrait is 3 feet by 3 feet, and the scale factor is $\frac{1}{18}$, draw the scale drawing that would be the size of the portrait on your phone.

Sketch and notes:

$3 \times 12 \text{ in.} = 36 \text{ in.}$

$36 \text{ in.} \times \frac{1}{18} = 2 \text{ in.}$



Exercise 3 (5 minutes)

Read the problem aloud, and ask students to solve the problem with another student.

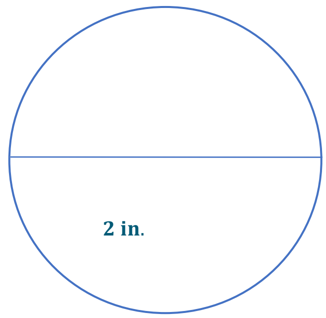
- What is the diameter of the window in the sketch of the model house?
  - 2 *inches*

Exercise 3

John is building his daughter a doll house that is a miniature model of their house. The front of their house has a circular window with a diameter of 5 feet. If the scale factor for the model house is $\frac{1}{30}$, make a sketch of the circular doll house window.

$5 \times 12 \text{ in.} = 60 \text{ in.}$

$60 \text{ in.} \times \frac{1}{30} = 2 \text{ in.}$



Closing (5 minutes)

- How is the constant of proportionality represented in scale drawings?
  - *Scale factor*
- Explain how to calculate scale factor.
  - *Measure the actual picture lengths and the scale drawing lengths. Write the values as a ratio of the length of the scale drawing length to the length of the actual picture.*
- What operation(s) is (are) used to create scale drawings?
  - *After the lengths of the actual picture are measured and recorded, multiply each length by the scale factor to find the corresponding scale drawing lengths. Measure and draw.*

Exit Ticket (5 minutes)

Name ____
Date____________________

Lesson 17: The Unit Rate as the Scale Factor

Exit Ticket

A rectangular pool in your friend's yard is $150 \text{ ft.} \times 400 \text{ ft.}$ Create a scale drawing with a scale factor of $\frac{1}{600}$. Use a table or an equation to show how you computed the scale drawing lengths.

Exit Ticket Sample Solutions

A rectangular pool in your friend's yard is 150 ft. $\times$ 400 ft. Create a scale drawing with a scale factor of $\frac{1}{600}$. Use a table or an equation to show how you computed the scale drawing lengths.

Actual Length	*Scale Length*
150 ft.	150 ft. *multiplied by* $\frac{1}{600} = \frac{1}{4}$ ft., *or* 3 in.
400 ft.	400 ft. *multiplied by* $\frac{1}{600} = \frac{2}{3}$ ft., *or* 8 in.

Problem Set Sample Solutions

1. Giovanni went to Los Angeles, California, for the summer to visit his cousins. He used a map of bus routes to get from the airport to his cousin's house. The distance from the airport to his cousin's house is 56 km. On his map, the distance was 4 cm. What is the scale factor?

   The scale factor is $\frac{1}{1{,}400{,}000}$. *I had to change kilometers to centimeters or centimeters to kilometers or both to meters in order to determine the scale factor.*

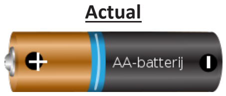
2. Nicole is running for school president. Her best friend designed her campaign poster, which measured 3 feet by
   2 feet. Nicole liked the poster so much, she reproduced the artwork on rectangular buttons that measured 2 inches by $1\frac{1}{3}$ inches. What is the scale factor?

   The scale factor is $\frac{2}{3}$.

3. Find the scale factor using the given scale drawings and measurements below.

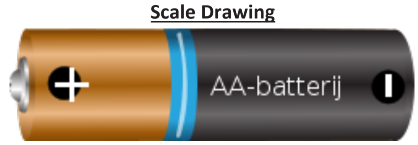
   Scale Factor: $\frac{5}{3}$

Actual



3 cm

Scale Drawing

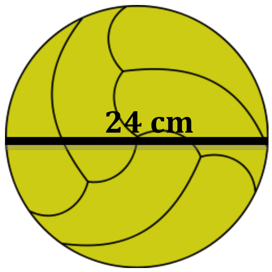


5 cm

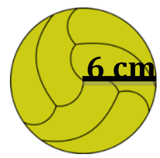
4. Find the scale factor using the given scale drawings and measurements below.

Scale Factor: $\frac{1}{2}$ *****Compare diameter to diameter or radius to radius.***

Actual Picture



Scale Drawing

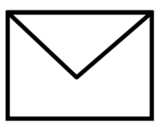


5. Using the given scale factor, create a scale drawing from the actual pictures in centimeters:

a. Scale factor: 3

Small Picture : 1 in.

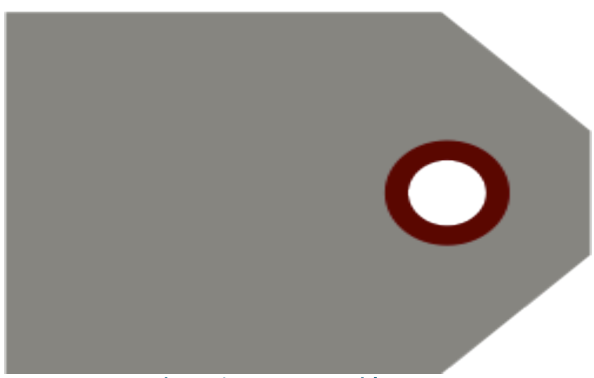
Large Picture: 3 in.



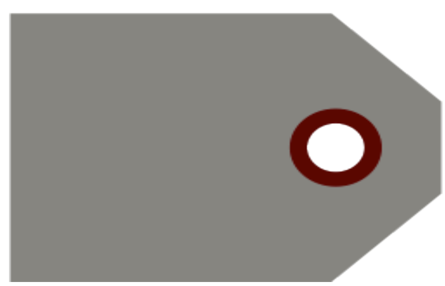
1 in.

3 in.

b. Scale factor: $\frac{3}{4}$



Actual Drawing Measures: 4 in.



Scale Drawing Measures: 3 in.

6. **Hayden likes building radio-controlled sailboats with her father. One of the sails, shaped like a right triangle, has side lengths measuring 6 inches, 8 inches, and 10 inches. To log her activity, Hayden creates and collects drawings of all the boats she and her father built together. Using the scale factor of $\frac{1}{4}$, create a scale drawing of the sail.**

   A triangle with sides 1.5 inches, 2 inches, and 2.5 inches is drawn.

Scaffolding:

Extension: Students can enlarge an image they want to draw or paint by drawing a grid using a ruler over their reference picture and drawing a grid of equal ratio on their work surface. Direct students to focus on one square at a time until the image is complete. Have students compute the scale factor for the drawing.

Model for Demonstration

- Share with students about this growth opportunity
- Make your thinking and decision making visible
- Step in and out of the teacher role vs. mentor role
- Encourage mentee to watch how students respond to the instruction
- The mentee should be actively engaged using the checklist
- Remember you don't have to model the ENTIRE lesson - keep it focused!

Sentence Starters for Stepping In and Out of Modeling

- Did you notice how I just _____________?
- I am about to try _________, so watch how I do that.
- When I did ________________, what did you notice about students reactions?
- I was hoping _______________ would occur, but then I had to adjust by _________.
- That strategy did not seem to work, so now I am going to try ____________ and see if the results are different.

Sample Modeling For Demonstration Transcript

Mentee *- "Good morning class! Remember how I told you all yesterday that I was going to have a friend come by our classroom today and help us work on some things? Well here she is! This is Mrs. _________ - Can you all say hi? Mrs. ______ is such an awesome teacher and she has agreed to help me with a personal goal that I am working on. So today she is going to be your guest teacher and I am going to be watching very closely as she works on our next EngageNY lesson with you all. "*

Mentor *- "That's right! I am so happy to be here and am looking forward to teaching you all today. I also want to let you all in on a little secret - we are all learners in this classroom. All of you, your teacher, and me - today we are all going to be learners. So while I am teaching you today, there might be something really specific I want to point out or tell your teacher so I might pause the lesson a few times and ask you to talk to a shoulder partner, or to think silently for a minute or two so I can go chat with your teacher and point out some things about our lesson today that are working or maybe even not working. Can you all help me with that today? Awesome!"* ***(Mentee goes and sits down ready to observe the mentor teacher in action.)***

Mentor - ***(Skips to the portion of the lesson where groups are creating their posters and I'm talking with a group 2)*** *Okay, now that you had a chance to review the ratios in your envelope, before you create the graph, I want you to think about what you expect to see in the graph. If the two quantities are proportional, what should the graph look like?* ***(A student explains answers the question.)*** *That is right – if the quantities are proportional, the graph should be a straight line and the vertical and horizontal distance between the points will be constant. As your group works together, think about how the display of proportionality in the graph compares to the display of proportionality in the data table. Alright, back to work!*

Mentor - ***(goes up to the mentee as if students are working and you are now discussing a bit of what's happening with the mentee)*** *So one of the things that is really helpful about the EngageNY lesson plans are the sample student responses.* ***(points to an example of this in the lesson plan.)***

Mentee *- That's good - I think that will help me remember to look and prompt for more detail in student responses.*

Debrief Model Teaching

- Mentee reflects on what they observed using the checklist
- Mentee identifies the reasons, processes, and/or strategies that made the teaching successful or not successful
- Mentee makes a plan for applying the new learning into their practice

The purpose of modeling is LEARNING. Amplify learning in the debriefing.

Model Best Practices: Debrief the Lesson

Suggested Guiding Questions for Discussion	Debrief Meeting Notes
Primary Questions	
How did this model lesson or activity help you?	
What did you see that was effective? (Encourage mentee to use their checklist from the observation)	
What did you see that was ineffective? (Encourage mentee to use their checklist from the observation)	
Application Questions	
What will you integrate into your teaching? How will you do that?	
What would you change/modify if you were teaching this lesson and why?	
Clarifying Questions	
What parts of what I was modeling during this lesson or activity still need further clarification?	
Closing Questions	
What is/are the top learnings you are taking away from the model lesson or activity?	
How can I support you as you begin to integrate what you are learning?	

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Closing Questions	
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How can I support you as you begin to integrate what you are learning?	

Sample Debriefing a Model Lesson Transcript

Mentor - *"Thanks for taking the time to meet with me. I had a great time modeling in your classroom and now just want to take some time to debrief about what you observed and hopefully some new learning that occurred for you during this process."*

Mentee - *"Sounds good."*

Mentor - *"So just thinking about the model lesson overall, how did do you think it helped you with regards to your SMART goal?"*

Mentee - *"Well, I enjoyed getting to see someone else teach proportional and non-proportional realtionships in my classroom with my students. A lot of times people have just told me ideas to try or read this blog for new ideas and while that is great, it was much better to see these new ideas live in person with my own students."*

Mentor - *"That's great to hear. It sounds like seeing someone else teach had a powerful impact on you. So tell me, what were some things you observed using your checklist that were effective in the lesson?"*

Mentee - *"Well starting off it was helpful to be part of the planning process. I noticed you went through the EngageNY lesson and made note of where the questions about using graphs to identify proportional relationships were going to come up. This showed me how you already knew when and where in the lesson students may struggle, and you were prepared ahead of time because of making those notes as we went through the lesson plan. During the lesson I noticed when students gave you answers that were incomplete you prompted them to use the language of mathematics and the evidence in their work to provide more detail.*

Mentor - *"I'm so glad you noticed that. Often times students just need quick, simple reminders to "speak the math" and use evidence. Those simple reminders over and over again will eventually get ingrained in their brains to where it becomes more natural for them to go back into the text."*

Mentor - *"What will you integrate into your teaching as a result of what you saw during the model?"*

Mentee - *"I plan to take the time prior to the lesson or activity to deliberately plan for those* *questions and know exactly where they come up in my lesson so I can be better prepared to support my students in being fully ready to use more than one method to identify proportionality. I also have some new tools in my toolbox to help students when they are struggling in finding evidence."*

Mentor - *"That's great - I am so glad to hear that. So how can I continue to support you as you integrate this new learning into your practice?"*

Mentee - *"I think it would be helpful for you to come observe me teaching another lesson that is focused on proportionality and giving me some feedback on how I utilize these new strategies with my students. I would love some help in determining if my new learning is truly having the impact on student learning that I need it to."*

Mentor - *"I can definitely do that. When would you like me to come?..."*

Key Takeaway:
Mentors use model teaching to demonstrate practices they expect to see mentees use to address their SMART goals.