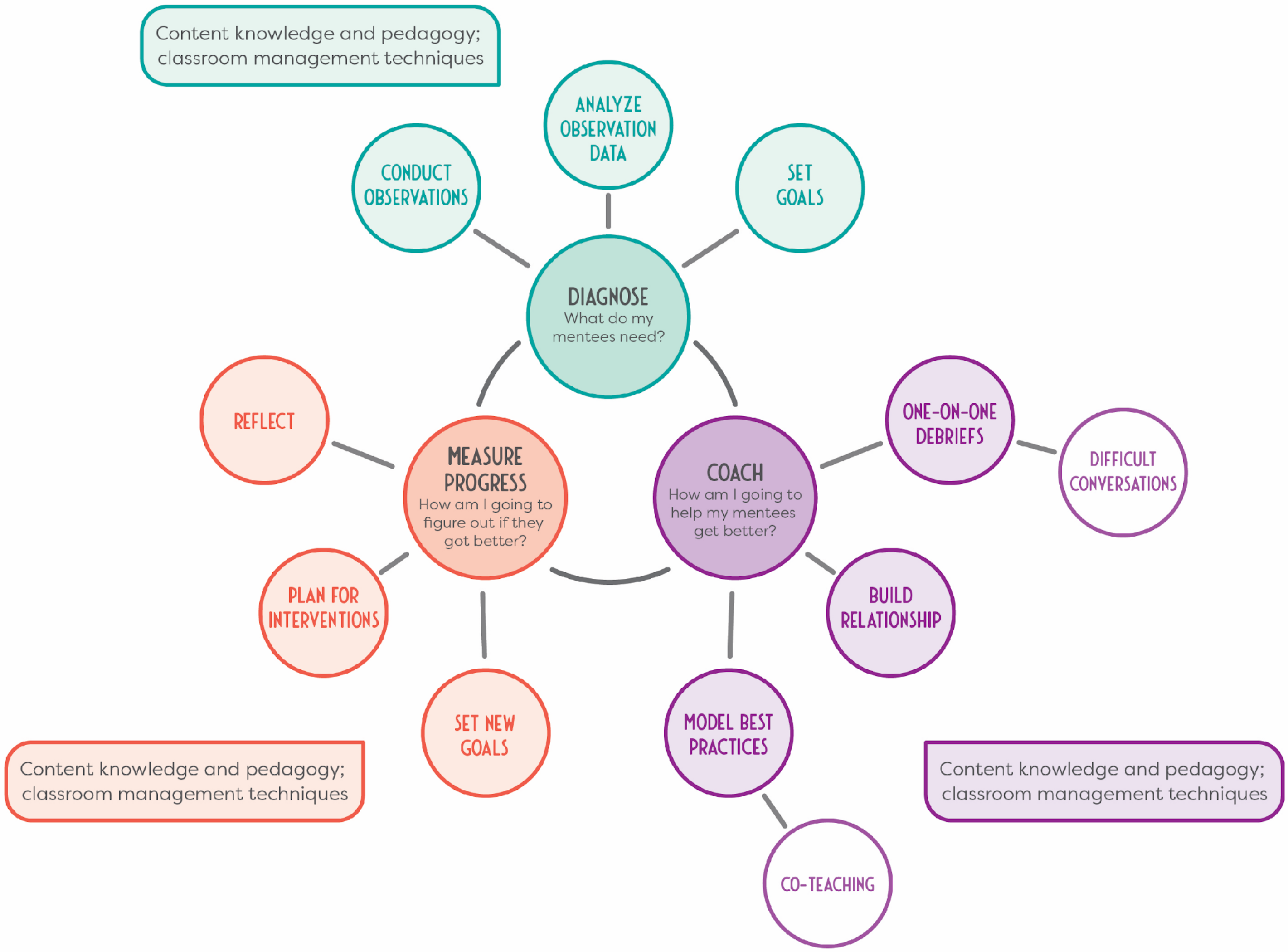
Louisiana Department of Education
Mentor Teacher Training

Module 5:
Mentoring for the Instructional Shifts in Mathematics

Secondary Math Cohort
July, 2019

Facilitated by Learning Forward

The Mentoring Cycle



Mentor Training Course Goals

Mentors will:

- Build strong relationships with mentees.
- Diagnose and prioritize mentee's strengths and areas for growth.
- Design and implement a mentoring support plan.
- Assess and deepen mentor content knowledge and content-specific pedagogy.

Module 5 Outcomes

- Describe the vertical articulation of standards for the big idea: Using multiplicative thinking to reason about ratios and rates.
- Understand how EngageNY resources can be used to support the Louisiana Student Standards for Mathematics content and practice standards and how the practice standards support the key shifts in instruction.
- Write a clear and concise coaching plan that enables you to plan interventions aligned to mentee goals.
- Model best practices through co-teaching.

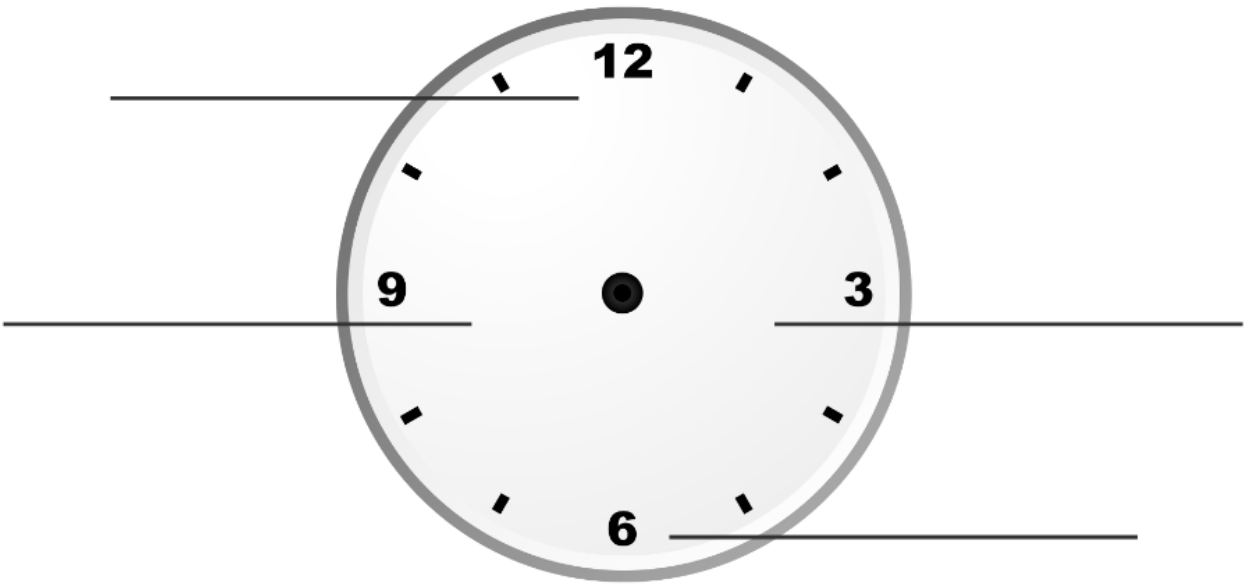
Module 5 Agenda

Morning	**Afternoon**
Welcome and outcomes	Plan for interventions
Investigating aligned tasks	Co-teaching best practices
Exploring vertical alignment in the LSSM	Connection to assessments
Seeing the key shifts in action	Wrap-up

Agreements

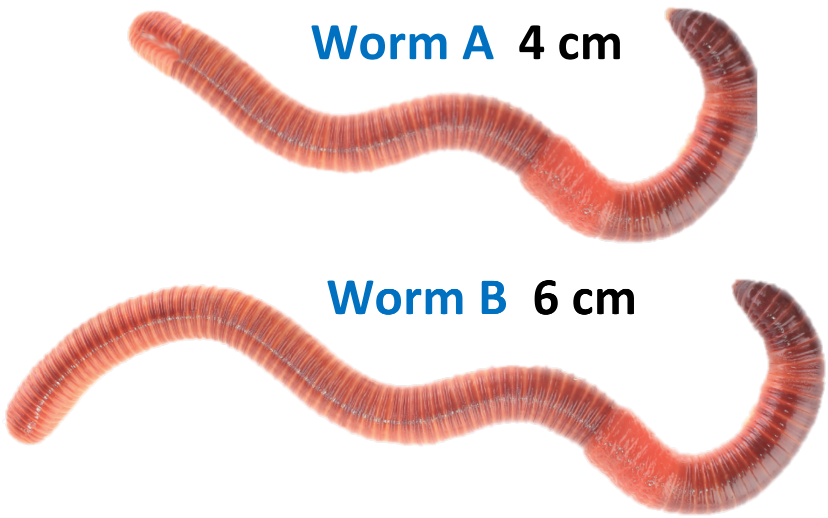
Make the learning meaningful
Engage mentally and physically
Notice opportunities to support the learning of others
Take responsibility for your own learning
Own the outcomes
Respect the learning environment of self and others

Let's Make a Date



Deepening Mathematical Content Knowledge for Effective Instruction

Multiplicative Thinking: Rates and Ratios



How might you compare the lengths of these two worms?

Defining the big idea:

Grade 6	Grade 7	Grade 8	Algebra I

Exploring tasks

- What is the math required in each task?
- How does the required math for each task progress?

Grade 6	Grade 7	Grade 8	Algebra I

Reflection

How did looking at the standards verify the order of the tasks?

What evidence of multiplicative thinking did you see in these tasks?

Why is it important for mentors to study standards across grade levels?

Exploring vertical alignment

Why study the vertical alignment of standards?

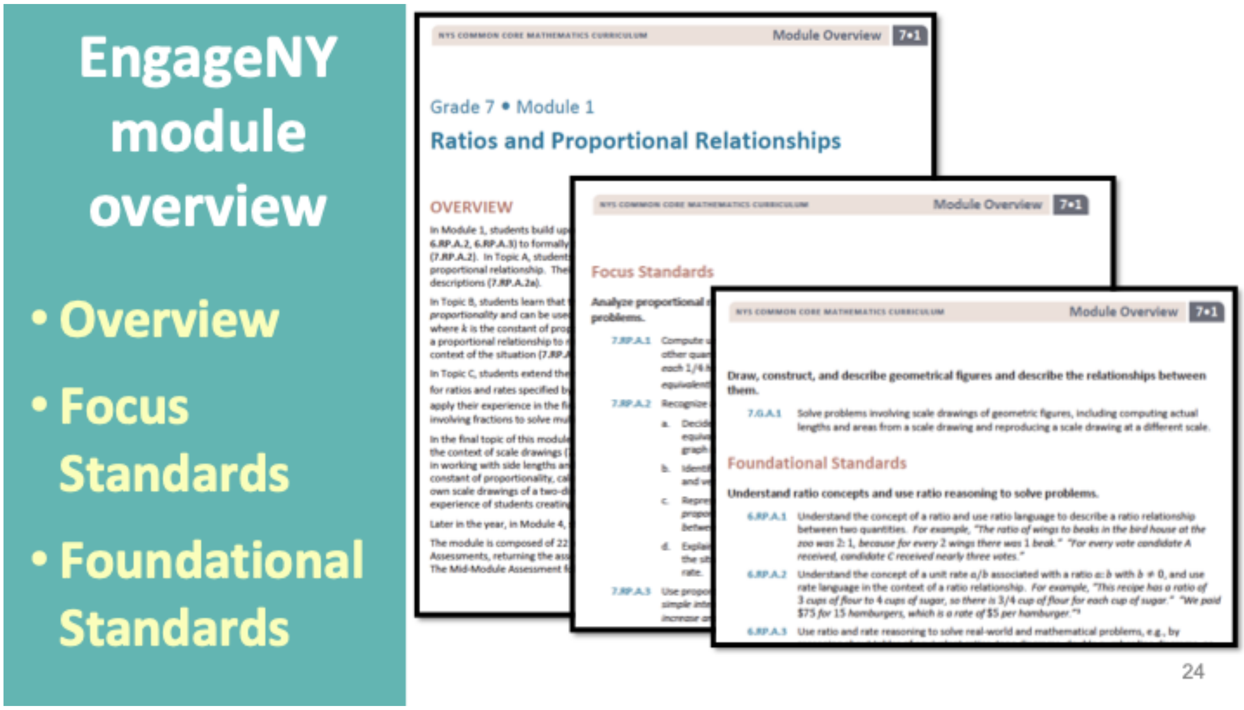
- Helps us understand our grade-level expectations
- Gives us information on student expectations from previous grades so that we can connect to prior learning and remediate, if necessary
- Gives us information on where students will be going with the material
- Gives us an idea of how the math develops and of our role within the larger system
- Provides insights on the type of instruction and experiences we need to provide our students

Mentor Teacher Secondary Math Module 6 17

Notes:

Deepening Mathematical Content Knowledge for Effective Instruction

Engaging in the Math: Multiplicative Thinking: Rates and Ratios



Notes:

New York State Common Core

7 GRADE

Mathematics Curriculum

Module 1 Teacher Materials

Table of Contents[1]

Ratios and Proportional Relationships

[1]Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 7 • Module 1

Ratios and Proportional Relationships

OVERVIEW

In Module 1, students build upon their Grade 6 reasoning about ratios, rates, and unit rates (**6.RP.A.1**, **6.RP.A.2**, **6.RP.A.3**) to formally define proportional relationships and the constant of proportionality (**7.RP.A.2**). In Topic A, students examine situations carefully to determine if they are describing a proportional relationship. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions (**7.RP.A.2a**).

In Topic B, students learn that the unit rate of a collection of equivalent ratios is called the *constant of proportionality* and can be used to represent proportional relationships with equations of the form $y = kx$, where k is the constant of proportionality (**7.RP.A.2b**, **7.RP.A.2c**, **7.EE.B.4a**). Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation (**7.RP.A.2d**).

In Topic C, students extend their reasoning about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers, such as a speed of $\frac{1}{2}$ mile per $\frac{1}{4}$ hour (**7.RP.A.1**). Students apply their experience in the first two topics and their new understanding of unit rates for ratios and rates involving fractions to solve multi-step ratio word problems (**7.RP.A.3**, **7.EE.B.4a**).

In the final topic of this module, students bring the sum of their experience with proportional relationships to the context of scale drawings (**7.RP.A.2b**, **7.G.A.1**). Given a scale drawing, students rely on their background in working with side lengths and areas of polygons (**6.G.A.1**, **6.G.A.3**) as they identify the scale factor as the constant of proportionality, calculate the actual lengths and areas of objects in the drawing, and create their own scale drawings of a two-dimensional view of a room or building. The topic culminates with a two-day experience of students creating a new scale drawing by changing the scale of an existing drawing.

Later in the year, in Module 4, students extend the concepts of this module to percent problems.

The module is composed of 22 lessons; 8 days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Focus Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks $1/2$ mile in each $1/4$ hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$, where r is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4[2] Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

[2]In this module, the equations are derived from ratio problems. 7.EE.B.4a is returned to in Modules 2 and 3.

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was* $2:1$*, because for every* 2 *wings there was* 1 *beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of* 3 *cups of flour to* 4 *cups of sugar, so there is* $3/4$ *cup of flour for each cup of sugar." "We paid* $\$75$ *for* 15 *hamburgers, which is a rate of* $\$5$ *per hamburger."*[3]

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole–number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took* 7 *hours to mow* 4 *lawns, then at that rate, how many lawns could be mowed in* 35 *hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

[3]Expectations for unit rates in this grade are limited to non-complex fractions.

6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of and solve multi-step ratio problems, including cases with pairs of rational number entries; they use representations, such as ratio tables, the coordinate plane, and equations, and relate these representations to each other and to the context of the problem. Students depict the meaning of constant of proportionality in proportional relationships, the importance of $(0,0)$ and $(1, r)$ on graphs, and the implications of how scale factors magnify or shrink actual lengths of figures on a scale drawing.

MP.2 **Reason abstractly and quantitatively.** Students compute unit rates for paired data given in tables to determine if the data represents a proportional relationship. Use of concrete numbers will be analyzed to create and implement equations, including $y = kx$, where k is the constant of proportionality. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, $\text{distance} = \text{rate} \times \text{time}$. In scale drawings, scale factors will be changed to create additional scale drawings of a given picture.

Terminology

New or Recently Introduced Terms

- **Constant of Proportionality** (If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the *constant of proportionality*. For example, if the ratio of y to x is 2 to 3, then the constant of proportionality is $\frac{2}{3}$, and $y = \frac{2}{3}x$.)
- **Miles per Hour** (One *mile per hour* is a proportional relationship between d miles and t hours given by the equation $d = 1 \cdot t$ (both d and t are positive real numbers). Similarly, for any positive real number v, v *miles per hour* is a proportional relationship between d miles and t hours given by $d = v \cdot t$. The unit for the rate, mile per hour (or mile/hour) is often abbreviated as mph.)
- **One-To-One Correspondence Between Two Figures in the Plane (description)** (For two figures in the plane, S and S', a *one-to-one correspondence between the figures* is a pairing between the points in S and the points in S' so that each point P of S is paired with one and only one point P' in S', and likewise, each point Q' in S' is paired with one and only one point Q in S.)

Looking for Evidence of Student Engagement in the Key Shifts

Focus	Evidence
☐ The learning goal(s) of the lesson supports grade level standard(s).	
Coherence	
☐ The lesson intentionally relates new concepts to students' prior skills and knowledge. ☐ Students set the foundation for future learning. ☐ Students access prior learning from major work in the grade in order to support new learning.	
Rigor	
Conceptual Understanding ☐ Students access concepts and ideas from a variety of perspectives. ☐ Students explain mathematical ideas behind a particular concept in a variety of ways. ☐ Students use examples and counterexamples to make and support conjectures applied to one problem to multiple situations. ☐ Students create and use a variety of models to analyze relationships. ☐ Students make use of patterns and structure to compose and decompose numbers, shapes, expressions, and equations. **Procedural Skills and Fluency** ☐ Students select tools (e.g. physical objects, manipulatives, drawings, diagrams, algorithms, or strategies) that are relevant and useful for the task or problem. ☐ Students communicate thinking using appropriate vocabulary, symbols and/or units in precise and accurate ways. ☐ Students look for patterns, generalizations, and shortcuts. ☐ Students are flexible in their use of procedures and skills to solve problems. **Application** ☐ Students decontextualize and contextualize quantities in problem situations. ☐ Students plan and choose a solution pathway when applying their mathematical knowledge to different situations.	

Note: To help educators look for evidence of grade-level-appropriate student engagement in mathematical tasks, these narrative descriptors are adapted from Illustrative Mathematics. (2014, February 12). *Standards for Mathematical Practice: Commentary and Elaborations for K–5 and 6-8*. Tucson, AZ. Available at http://commoncoretools.me/2014/02/12/k-5-elaborations-of-the-practice-standards

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Classwork

Today's Exploratory Challenge is an extension of Lesson 5. You will be working in groups to create a table and graph and to identify whether the two quantities are proportional to each other.

Poster Layout

Use for notes

Problem:	**Table**:
Graph: 	**Proportional or Not? Explanation**:

Gallery Walk

Take notes and answer the following questions:

- Were there any differences found in groups that had the same ratios?
- Did you notice any common mistakes? How might they be fixed?
- Were there any groups that stood out by representing their problem and findings exceptionally clearly?

Poster 1:

Poster 2:

Poster 3:

Poster 4:

Poster 5:

Poster 6:

Poster 7:

Poster 8:

Note about Lesson Summary:

Lesson Summary

The plotted points in a *graph of a proportional relationship* lie on a line that passes through the origin.

Problem Set

Sally's aunt put money in a savings account for her on the day Sally was born. The savings account pays interest for keeping her money in the bank. The ratios below represent the number of years to the amount of money in the savings account.

- After one year, the interest accumulated, and the total in Sally's account was $\$312$.
- After three years, the total was $\$340$. After six years, the total was $\$380$.
- After nine years, the total was $\$430$. After 12 years, the total amount in Sally's savings account was $\$480$.

Using the same four-fold method from class, create a table and a graph, and explain whether the amount of money accumulated and the time elapsed are proportional to each other. Use your table and graph to support your reasoning.

Name ____ Date____________________

Lesson 6: Identifying Proportional and Non-Proportional Relationships in Graphs

Exit Ticket

1. Which graphs in the gallery walk represented proportional relationships, and which did not? List the group number.

   Proportional Relationship   Non-Proportional Relationship

2. What are the characteristics of the graphs that represent proportional relationships?

3. For the graphs representing proportional relationships, what does $(0,0)$ mean in the context of the given situation?

Diamond Reflection
Unpacking the EngageNY Lesson

Observations: How are the mathematics content standards evident in the lesson?

eflections: How does student engagement in the SMPs help develop conceptual understanding, procedural skill and fluency and application skills?

Other thoughts and consideration

Interpretations: What instructional strategies and facilitator moves brought out the content and practice standards?

ecisional: What am I going to do based on my learning?

Module 5 Afternoon Outcomes:

- Write a clear and concise coaching plan that enables you to plan interventions aligned to mentee goals
- Model best practices through co-teaching

Plan for Interventions: 3 Key Components

- Clarify the new learning
- Align the intervention method
- Write a coaching plan

Clarify the new learning

Content	Practice
What does my mentee need to understand?	What do I lean on in my teaching practice in order to do this?
What does the Tier 1 resource recommend?	What does my mentee need to be able to do?
How could my mentee gain this knowledge?	How could my mentee gain this skill?

Sample SMART Goal

I am going to pre-plan interventions using lessons from grade 6, 7, and 8 that are vertically aligned to exploratory Algebra I lesson on graphs of functions and equations from EngageNY so that I understand what students need to learn in order to be successful and can emphasize those skills in the lessons within the unit.	

Types of Co-Teaching

- One teaches, one observes students
- One teaches, one assists
- Station teaching
- Parallel teaching
- Supplemental teaching
- Alternative or differentiated teaching
- **Team Teaching**

Types of Team Teaching

Jigsaw:	Whisper-in:
Teach, pause, discuss:	Share roles:

Mentor Coaching Plan

Mentee SMART goal(s)
I am going to pre-plan interventions using lessons from grade 6, 7, and 8 that are vertically aligned to exploratory Algebra I lesson on graphs of functions and equations from EngageNY so that I understand what students need to learn in order to be successful and can emphasize those skills in the lessons within the unit.

What activities and resources will mentor and mentee engage in to achieve goal(s)?

Specific Activity or Resource	How is it aligned to the goal(s)?	Why will it be effective?	How will you integrate relationship building?	Projected timeline

How will you monitor your mentee's progress toward the identified goals?

Mentor Coaching Plan

Mentee SMART goal(s)

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Mentor Coaching Plan

Mentee SMART goal(s)

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How will you monitor your mentee's progress toward the identified goals?

Reflect: Cumulative Learning

Yesterday I….   **Today I….**   **Now I ….**

Plan for Interventions: Key Takeaway

Coaching plans keep mentor and mentee on track to achieve SMART goals.

Co-Teaching: 3 Key Components

- Co-plan instruction and co-teaching method
- Co-teach the lesson
- Debrief the lesson

Co-Plan Instruction

- Revisit **agreements**
- Confirm the **purpose/goal** of the lesson and **connection to SMART goal**
- Create a **“look-fors” checklist** based on the goal of the lesson or activity
- Select **best model for co-teaching** to achieve student and teacher learning outcome
- **Make thinking visible** as you co-plan what the lesson requires to be successful, including any tweaks you need to make to integrate your chosen co-teaching model

Co-Planning Conversation Transcript (Segment)

Mentor - "Glad we got to meet this morning to talk about how I can best support you in meeting the SMART goal we came up with based on the observation I conducted and our debrief conversation last week. I've been doing some thinking about your goal, and as you saw in the coaching plan, I think one of the ways I can best support you in reaching it is for us to engage in a co-teaching lesson together. You have a strong understanding of the EngageNY lessons components and the skills the students will need to identify proportions and functions, so I think if we were to do some co-teaching as students use those skills to graph functions."

Mentee - "I think I would really like that and I want to be able to provide interventions to students who may need additional scaffolding to be successful with functions. I also know that graphing functions are not my strongest areas."

Mentor - "Sounds great. So per our partnership agreements we set up earlier this year you stated the best time of day to conduct a model, co-teach, or classroom observation is with your third block which starts at 1:00, does that time still work for you?"

Mentee - "Yes, that works perfectly. And I can still conduct debriefs the following day during my planning time at 10:35am - will that work for you?"

Mentor - "Yes, I believe I can ask one of my teammates to watch my class for 20 minutes at that time while we do our debrief of the co-teach. That will work. So, do you have the lesson plan that we are going to use for the co-teach?"

Mentee - "Yes - it's right here. It is from the EngageNY Grade 8, Module 5 Lesson 5."

Mentor - "Okay great - I've been looking through it a little bit as I've prepared for my own class. I also used the coherence maps to identify the aligned grade 6 and 7 standards."

Mentor - "So what are you hoping the outcome of this particular lesson is?"

Mentee - "I am hoping they are able to realize that if a numerical function can be described by an equation, then the graph of the function precisely matches the graph of the equation.."

Mentor - "Have you thought about which co-teaching model will work best for this particular lesson? I have some ideas, but wanted first to see if you did as well?"

Mentee - "I'm not really sure, I know there are so many different types of co-teaching, but I am not sure what would work best here."

Mentor - "Well, since we may need to provide scaffolding for students, it might work best if we use a combination of station teaching and jigsaw. What do you think?"

Mentee - "I think that sounds great. And please definitely feel free to chime in on anything that is designated as "my part" that you think could be explained in a different way to support understanding."

Mentor - "Same goes for you! So how about we go through the lesson plan together and start discussing who should take the lead on which particular part and where we might incorporate stations as part of the implementation of the co-teach?"

Mentee - "Sounds great…"

Lesson 5: Graphs of Functions and Equations

Student Outcomes

- Students define the graph of a numerical function to be the set of all points (x, y) with x an input of the function and y its matching output.
- Students realize that if a numerical function can be described by an equation, then the graph of the function precisely matches the graph of the equation.

Classwork

Exploratory Challenge/Exercises 1–3 (15 minutes)

Students work independently or in pairs to complete Exercises 1–3.

Exploratory Challenge/Exercises 1–3

1. **The distance that Giselle can run is a function of the amount of time she spends running. Giselle runs 3 miles in 21 minutes. Assume she runs at a constant rate.**

   a. **Write an equation in two variables that represents her distance run, y, as a function of the time, x, she spends running.**

   $$\frac{3}{21} = \frac{y}{x}$$
   $$y = \frac{1}{7}x$$

   b. **Use the equation you wrote in part (a) to determine how many miles Giselle can run in 14 minutes.**

   $$y = \frac{1}{7}(14)$$
   $$y = 2$$

   Giselle can run 2 miles in 14 minutes.

   c. **Use the equation you wrote in part (a) to determine how many miles Giselle can run in 28 minutes.**

   $$y = \frac{1}{7}(28)$$
   $$y = 4$$

   Giselle can run 4 miles in 28 minutes.

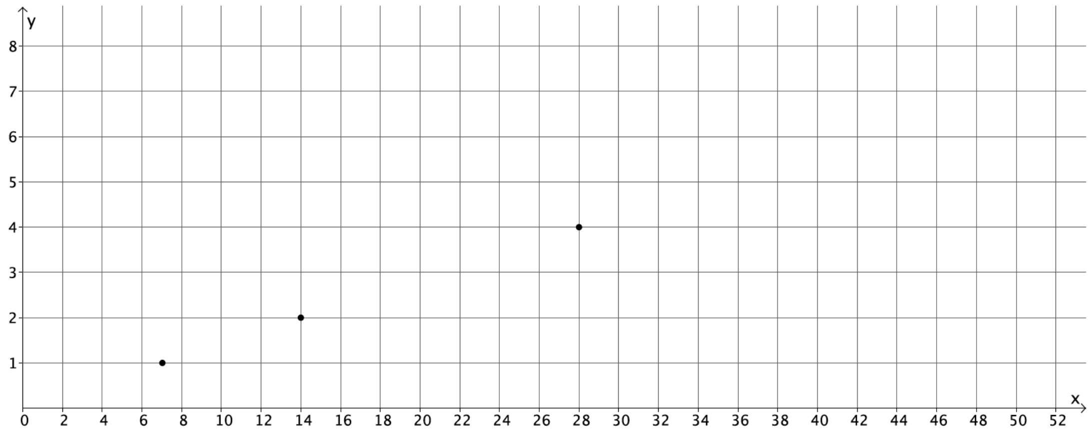
   d. **Use the equation you wrote in part (a) to determine how many miles Giselle can run in 7 minutes.**

   $$y = \frac{1}{7}(7)$$
   $$y = 1$$

   Giselle can run 1 mile in 7 minutes.

e. **For a given input x of the function, a time, the matching output of the function, y, is the distance Giselle ran in that time. Write the inputs and outputs from parts (b)–(d) as ordered pairs, and plot them as points on a coordinate plane.**

$$(14, 2), (28, 4), (7, 1)$$



f. **What do you notice about the points you plotted?**

The points appear to be in a line.

g. **Is the function discrete?**

The function is not discrete because we can find the distance Giselle runs for any given amount of time she spends running.

h. **Use the equation you wrote in part (a) to determine how many miles Giselle can run in 36 minutes. Write your answer as an ordered pair, as you did in part (e), and include the point on the graph. Is the point in a place where you expected it to be? Explain.**

$$y = \frac{1}{7}(36)$$

$$y = \frac{36}{7}$$

$$y = 5\frac{1}{7}$$

$\left(36, 5\frac{1}{7}\right)$ *The point is where I expected it to be because it is in line with the other points.*

i. **Assume you used the rule that describes the function to determine how many miles Giselle can run for any given time and wrote each answer as an ordered pair. Where do you think these points would appear on the graph?**

I think all of the points would fall on a line.

j. **What do you think the graph of all the possible input/output pairs would look like? Explain.**

I know the graph will be a line as we can find all of the points that represent fractional intervals of time too. We also know that Giselle runs at a constant rate, so we would expect that as the time she spends running increases, the distance she can run will increase at the same rate.

k. Connect the points you have graphed to make a line. Select a point on the graph that has integer coordinates. Verify that this point has an output that the function would assign to the input.

Answers will vary. Sample student work:

The point $(42, 6)$ is a point on the graph.

$$y = \frac{1}{7}x$$

$$6 = \frac{1}{7}(42)$$

$$6 = 6$$

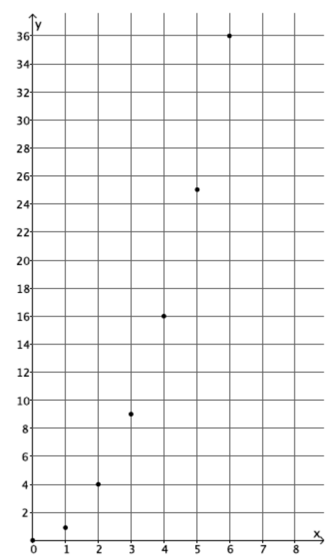
The function assigns the output of 6 to the input of 42.

l. Sketch the graph of the equation $y = \frac{1}{7}x$ using the same coordinate plane in part (e). What do you notice about the graph of all the input/output pairs that describes Giselle's constant rate of running and the graph of the equation $y = \frac{1}{7}x$?

The graphs of the equation and the function coincide completely.

2. Sketch the graph of the equation $y = x^2$ for positive values of x. Organize your work using the table below, and then answer the questions that follow.

x	y
0	0
1	1
2	4
3	9
4	16
5	25
6	36



a. Plot the ordered pairs on the coordinate plane.

b. What shape does the graph of the points appear to take?

It appears to take the shape of a curve.

c. Is this equation a linear equation? Explain.

No, the equation $y = x^2$ is not a linear equation because the exponent of x is greater than 1.

d. Consider the function that assigns to each square of side length s units its area A square units. Write an equation that describes this function.

$$A = s^2$$

e. **What do you think the graph of all the input/output pairs (s, A) of this function will look like? Explain.**

I think the graph of input/output pairs will look like the graph of the equation $y = x^2$. The inputs and outputs would match the solutions to the equation exactly. For the equation, the y value is the square of the x value. For the function, the output is the square of the input.

f. **Use the function you wrote in part (d) to determine the area of a square with side length 2.5 units. Write the input and output as an ordered pair. Does this point appear to belong to the graph of $y = x^2$?**

$$A = (2.5)^2$$
$$A = 6.25$$

The area of the square is 6.25 units squared. $(2.5, 6.25)$ The point looks like it would belong to the graph of $y = x^2$; it looks like it would be on the curve that the shape of the graph is taking.

3. **The number of devices a particular manufacturing company can produce is a function of the number of hours spent making the devices. On average, 4 devices are produced each hour. Assume that devices are produced at a constant rate.**

a. **Write an equation in two variables that describes the number of devices, y, as a function of the time the company spends making the devices, x.**

$$\frac{4}{1} = \frac{y}{x}$$
$$y = 4x$$

b. **Use the equation you wrote in part (a) to determine how many devices are produced in 8 hours.**

$$y = 4(8)$$
$$y = 32$$

The company produces 32 devices in 8 hours.

c. **Use the equation you wrote in part (a) to determine how many devices are produced in 6 hours.**

$$y = 4(6)$$
$$y = 24$$

The company produces 24 devices in 6 hours.

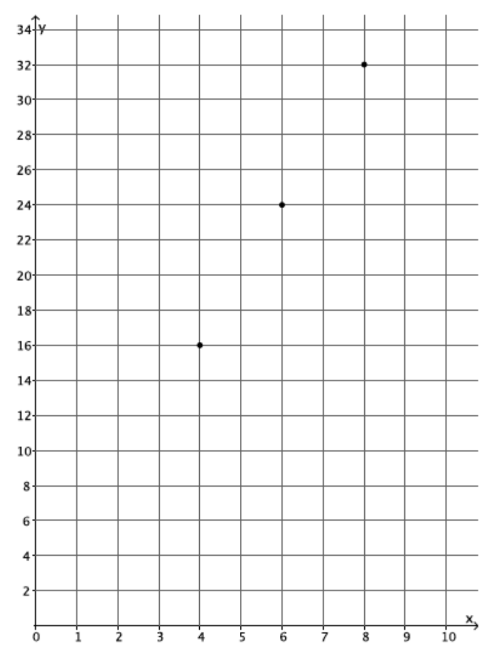
d. **Use the equation you wrote in part (a) to determine how many devices are produced in 4 hours.**

$$y = 4(4)$$
$$y = 16$$

The company produces 16 devices in 4 hours.

e. **The input of the function, x, is time, and the output of the function, y, is the number of devices produced. Write the inputs and outputs from parts (b)–(d) as ordered pairs, and plot them as points on a coordinate plane.**

$(8, 32), (6, 24), (4, 16)$



f. **What shape does the graph of the points appear to take?**

The points appear to be in a line.

g. **Is the function discrete?**

The function is not discrete because we can find the number of devices produced for any given time, including fractions of an hour.

h. **Use the equation you wrote in part (a) to determine how many devices are produced in 1.5 hours. Write your answer as an ordered pair, as you did in part (e), and include the point on the graph. Is the point in a place where you expected it to be? Explain.**

$$y = 4(1.5)$$
$$y = 6$$

$(1.5, 6)$ *The point is where I expected it to be because it is in line with the other points.*

i. **Assume you used the equation that describes the function to determine how many devices are produced for any given time and wrote each answer as an ordered pair. Where do you think these points would appear on the graph?**

I think all of the points would fall on a line.

j. **What do you think the graph of all possible input/output pairs will look like? Explain.**

I think the graph of this function will be a line. Since the rate is continuous, we can find all of the points that represent fractional intervals of time. We also know that devices are produced at a constant rate, so we would expect that as the time spent producing devices increases, the number of devices produced would increase at the same rate.

k. **Connect the points you have graphed to make a line. Select a point on the graph that has integer coordinates. Verify that this point has an output that the function would assign to the input.**

Answers will vary. Sample student work:

The point $(5, 20)$ *is a point on the graph.*

$$y = 4x$$
$$20 = 4(5)$$
$$20 = 20$$

The function assigns the output of 20 *to the input of* 5.

l. **Sketch the graph of the equation $y = 4x$ using the same coordinate plane in part (e). What do you notice about the graph of input/output pairs that describes the company's constant rate of producing devices and the graph of the equation $y = 4x$?**

The graphs of the equation and the function coincide completely.

Discussion (10 minutes)

- What was the equation that described the function in Exercise 1, Giselle's distance run over given time intervals?
  - *The equation was* $y = \frac{1}{7}x$.
- Given an input, how did you determine the output the function would assign?
  - *We used the equation. In place of* x, *we put the input. The number that was computed was the output.*
- So each input and its matching output correspond to a pair of numbers (x, y) that makes the equation $y = \frac{1}{7}x$ a true number sentence?
  - *Yes*

MP.6

Give students a moment to make sense of this, verifying that each pair of input/output values in Exercise 1 is indeed a pair of numbers (x, y) that make $y = \frac{1}{7}x$ a true statement.

- And suppose we have a pair of numbers (x, y) that make $y = \frac{1}{7}x$ a true statement with x positive. If x is an input of the function, the number of minutes Giselle runs, would y be its matching output, the distance she covers?
  - *Yes. We computed the outputs precisely by following the equation* $y = \frac{1}{7}x$. *So* y *will be the matching output to* x.
- So can we conclude that any pair of numbers (x, y) that make the equation $y = \frac{1}{7}x$ a true number statement correspond to an input and its matching output for the function?
  - *Yes*
- And, backward, any pair of numbers (x, y) that represent an input/output pair for the function is a pair of numbers that make the equation $y = \frac{1}{7}x$ a true number statement?
  - *Yes*
- Can we make similar conclusions about Exercise 3, the function that gives the devices built over a given number of hours?

Give students time to verify that the conclusions about Exercise 3 are the same as the conclusions about Exercise 1. Then continue with the discussion.

- The function in Exercise 3 is described by the equation $y = 4x$.
- We have that the ordered pairs (x, y) that make the equation $y = 4x$ a true number sentence precisely match the ordered pairs (x, y) with x an input of the function and y its matching output.
- Recall, in previous work, we defined the *graph of an equation* to be the set of all ordered pairs (x, y) that make the equation a true number sentence. Today we define the *graph of a function* to be the set of all the ordered pairs (x, y) with x an input of the function and y its matching output.
- And our discussion today shows that if a function can be described by an equation, then the graph of the function is precisely the same as the graph of the equation.
- It is sometimes possible to draw the graph of a function even if there is no obvious equation describing the function. (Consider having students plot some points of the function that assigns to each positive whole number its first digit, for example.)

- For Exercise 2, you began by graphing the equation $y = x^2$ for positive values of x. What was the shape of the graph?
  - *It looked curved.*
- The graph had a curve in it because it was not the graph of a linear equation. All linear equations graph as lines. That is what we learned in Module 4. Since this equation was not linear, we should expect it to graph as something other than a line.
- What did you notice about the ordered pairs of the equation $y = x^2$ and the inputs and corresponding outputs for the function $A = s^2$?
  - *The ordered pairs were exactly the same for the equation and the function.*
- What does that mean about the graphs of functions, even those that are not linear?
  - *It means that the graph of a function will be identical to the graph of an equation.*

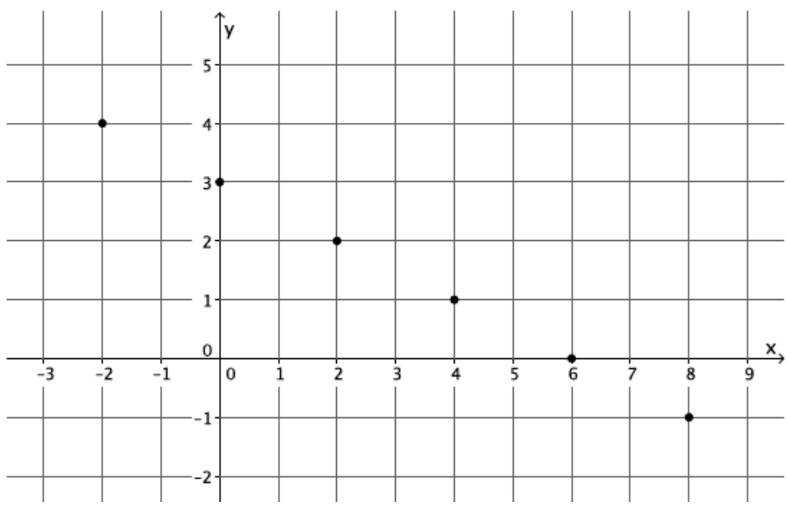
Exploratory Challenge/Exercise 4 (7 minutes)

Students work in pairs to complete Exercise 4.

Exploratory Challenge/Exercise 4

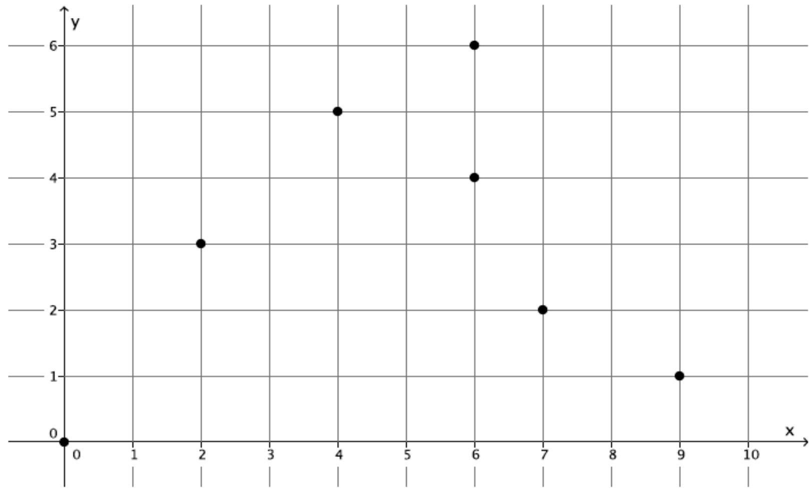
4. **Examine the three graphs below. Which, if any, could represent the graph of a function? Explain why or why not for each graph.**

   Graph 1:



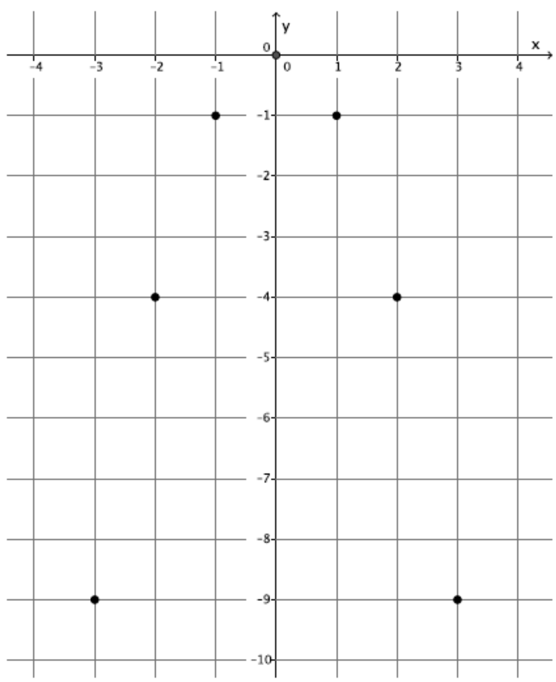
   This is the graph of a function. Each input is a real number x, and we see from the graph that there is an output y to associate with each such input. For example, the ordered pair $(-2, 4)$ on the line associates the output 4 to the input -2.

Graph 2:



This is not the graph of a function. The ordered pairs $(6, 4)$ and $(6, 6)$ show that for the input of 6 there are two different outputs, both 4 and 6. We do not have a function.

Graph 3:



This is the graph of a function. The ordered pairs $(-3, -9)$, $(-2, -4)$, $(-1, -1)$, $(0, 0)$, $(1, -1)$, $(2, -4)$, and $(3, -9)$ represent inputs and their unique outputs.

Discussion (3 minutes)

- The graph of a function is the set of all points (x, y) with x an input for the function and y its matching output. How did you use this definition to determine which graphs, if any, were functions?
  - *By the definition of a function, we need each input to have only one output. On a graph, this means there cannot be two different ordered pairs with the same x value.*
- Assume the following set of ordered pairs is from some graph. Could this be the graph of a function? Explain.

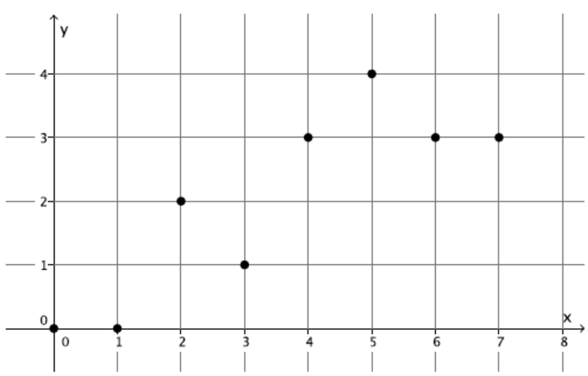
$$(3, 5), (4, 7), (3, 9), (5, -2)$$

  - *No, because the input of 3 has two different outputs. It does not fit the definition of a function.*
- Assume the following set of ordered pairs is from some graph. Could this be the graph of a function? Explain.

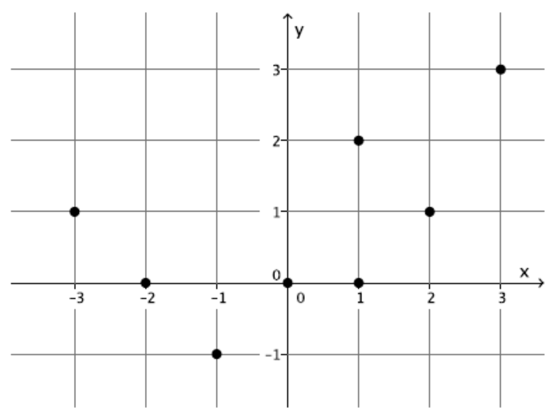
$$(-1, 6), (-3, 8), (5, 10), (7, 6)$$

  - *Yes, it is possible as each input has a unique output. It satisfies the definition of a function so far.*
- Which of the following four graphs are functions? Explain.

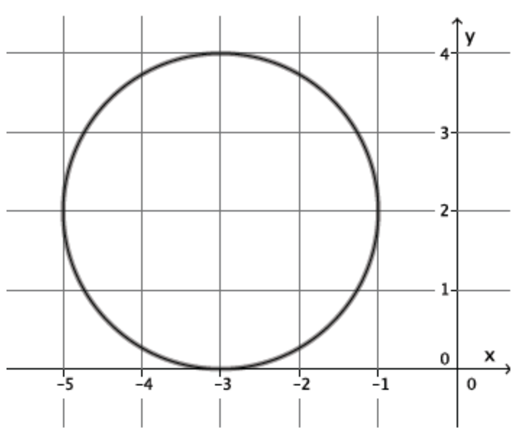
Graph 1:



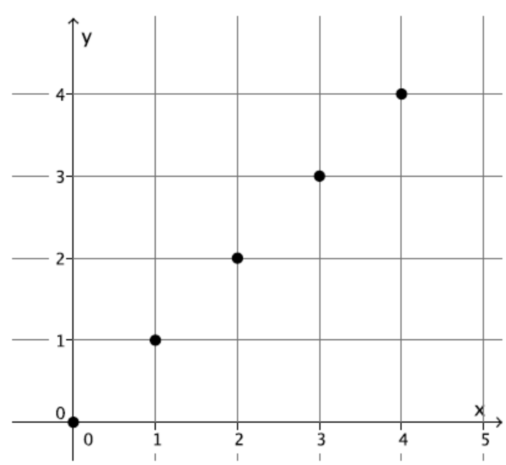
Graph 2:



Graph 3:



Graph 4:



-

- *Graphs 1 and 4 are functions. Graphs 2 and 3 are not. Graphs 1 and 4 show that for each input of x, there is a unique output of y. For Graph 2, the input of $x = 1$ has two different outputs, $y = 0$ and $y = 2$, which means it cannot be a function. For Graph 3, it appears that each value of x between -5 and -1, excluding -5 and -1, has two outputs, one on the lower half of the circle and one on the upper half, which means it does not fit the definition of function.*

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- The graph of a function is defined to be the set of all points (x, y) with x an input for the function and y its matching output.
- If a function can be described by an equation, then the graph of the function matches the graph of the equation (at least at points which correspond to valid inputs of the function).
- We can look at plots of points and determine if they could be the graphs of functions.

Lesson Summary

The graph of a function is defined to be the set of all points (x, y) with x an input for the function and y its matching output.

If a function can be described by an equation, then the graph of the function is the same as the graph of the equation that represents it (at least at points which correspond to valid inputs of the function).

It is not possible for two different points in the plot of the graph of a function to have the same x-coordinate.

Exit Ticket (5 minutes)

Name ______________________________ Date ______________

Lesson 5: Graphs of Functions and Equations

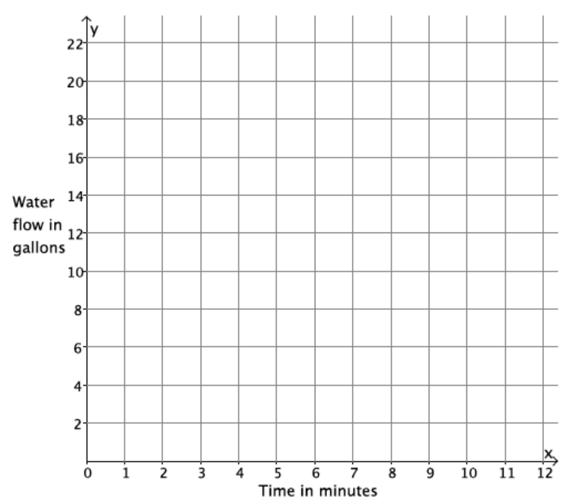
Exit Ticket

Water flows from a hose at a constant rate of 11 gallons every 4 minutes. The total amount of water that flows from the hose is a function of the number of minutes you are observing the hose.

a. Write an equation in two variables that describes the amount of water, y, in gallons, that flows from the hose as a function of the number of minutes, x, you observe it.

b. Use the equation you wrote in part (a) to determine the amount of water that flows from the hose during an
8-minute period, a 4-minute period, and a 2-minute period.

c. An input of the function, x, is time in minutes, and the output of the function, y, is the amount of water that flows out of the hose in gallons. Write the inputs and outputs from part (b) as ordered pairs, and plot them as points on the coordinate plane.



Exit Ticket Sample Solutions

Water flows from a hose at a constant rate of 11 gallons every 4 minutes. The total amount of water that flows from the hose is a function of the number of minutes you are observing the hose.

a. **Write an equation in two variables that describes the amount of water, y, in gallons, that flows from the hose as a function of the number of minutes, x, you observe it.**

$$\frac{11}{4} = \frac{y}{x}$$
$$y = \frac{11}{4}x$$

b. **Use the equation you wrote in part (a) to determine the amount of water that flows from the hose during an 8-minute period, a 4-minute period, and a 2-minute period.**

$$y = \frac{11}{4}(8)$$
$$y = 22$$

In 8 minutes, 22 gallons of water flow out of the hose.

$$y = \frac{11}{4}(4)$$
$$y = 11$$

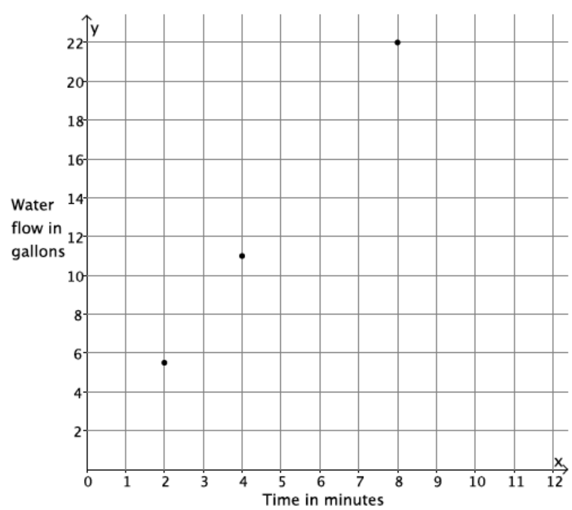
In 4 minutes, 11 gallons of water flow out of the hose.

$$y = \frac{11}{4}(2)$$
$$y = 5.5$$

In 2 minutes, 5.5 gallons of water flow out of the hose.

c. **An input of the function, x, is time in minutes, and the output of the function, y, is the amount of water that flows out of the hose in gallons. Write the inputs and outputs from part (b) as ordered pairs, and plot them as points on the coordinate plane.**

$(8, 22)$, $(4, 11)$, $(2, 5.5)$



Problem Set Sample Solutions

1. The distance that Scott walks is a function of the time he spends walking. Scott can walk $\frac{1}{2}$ mile every 8 minutes. Assume he walks at a constant rate.

   a. Predict the shape of the graph of the function. Explain.

   The graph of the function will likely be a line because a linear equation can describe Scott's motion, and I know that the graph of the function will be the same as the graph of the equation.

   b. Write an equation to represent the distance that Scott can walk in miles, y, in x minutes.

   $$\frac{0.5}{8} = \frac{y}{x}$$
   $$y = \frac{0.5}{8}x$$
   $$y = \frac{1}{16}x$$

   c. Use the equation you wrote in part (b) to determine how many miles Scott can walk in 24 minutes.

   $$y = \frac{1}{16}(24)$$
   $$y = 1.5$$

   Scott can walk 1.5 miles in 24 minutes.

   d. Use the equation you wrote in part (b) to determine how many miles Scott can walk in 12 minutes.

   $$y = \frac{1}{16}(12)$$
   $$y = \frac{3}{4}$$

   Scott can walk 0.75 miles in 12 minutes.

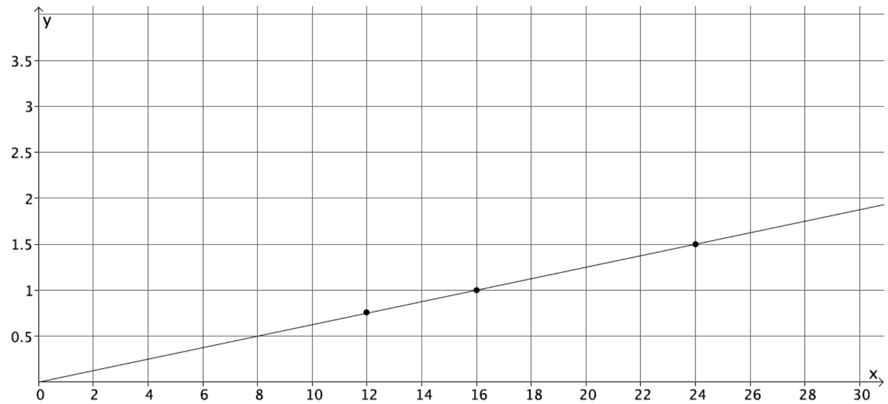
   e. Use the equation you wrote in part (b) to determine how many miles Scott can walk in 16 minutes.

   $$y = \frac{1}{16}(16)$$
   $$y = 1$$

   Scott can walk 1 mile in 16 minutes.

f. **Write your inputs and corresponding outputs as ordered pairs, and then plot them on a coordinate plane.**

$(24, 1.5)$, $(12, 0.75)$, $(16, 1)$



g. **What shape does the graph of the points appear to take? Does it match your prediction?**

The points appear to be in a line. Yes, as I predicted, the graph of the function is a line.

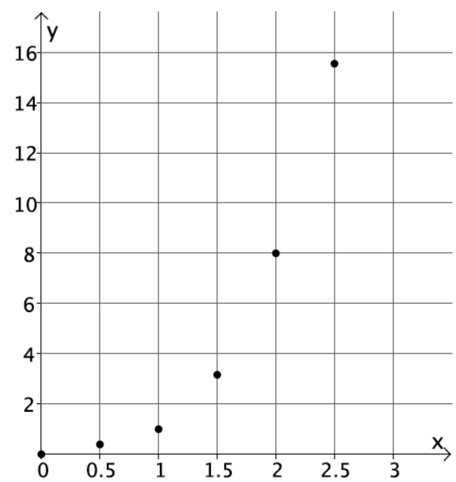
h. **Connect the points to make a line. What is the equation of the line?**

It is the equation that described the function: $y = \frac{1}{16}x$.

2. Graph the equation $y = x^3$ for positive values of x. Organize your work using the table below, and then answer the questions that follow.

x	y
0	0
0.5	0.125
1	1
1.5	3.375
2	8
2.5	15.625

a. Plot the ordered pairs on the coordinate plane.



b. What shape does the graph of the points appear to take?

It appears to take the shape of a curve.

c. Is this the graph of a linear function? Explain.

No, this is not the graph of a linear function. The equation $y = x^3$ is not a linear equation.

d. Consider the function that assigns to each positive real number s the volume V of a cube with side length s units. An equation that describes this function is $V = s^3$. What do you think the graph of this function will look like? Explain.

I think the graph of this function will look like the graph of the equation $y = x^3$. The inputs and outputs would match the solutions to the equation exactly. For the equation, the y-value is the cube of the x-value. For the function, the output is the cube of the input.

e. Use the function in part (d) to determine the volume of a cube with side length of 3 units. Write the input and output as an ordered pair. Does this point appear to belong to the graph of $y = x^3$?

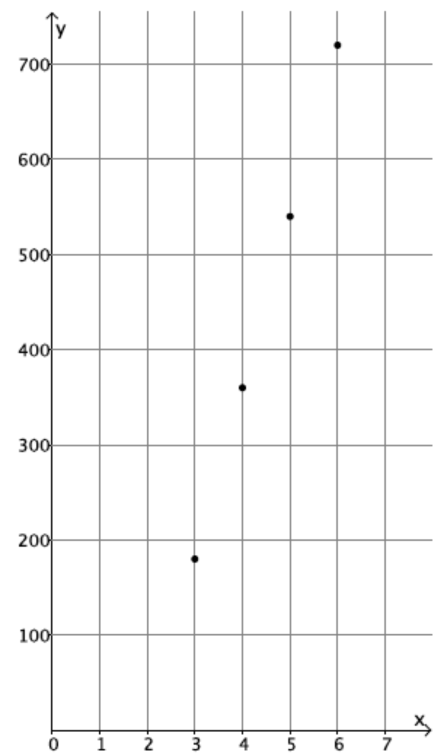
$$V = (3)^3$$
$$V = 27$$

$(3, 27)$ *The point looks like it would belong to the graph of $y = x^3$; it looks like it would be on the curve that the shape of the graph is taking.*

3. Sketch the graph of the equation $y = 180(x - 2)$ for whole numbers. Organize your work using the table below, and then answer the questions that follow.

x	y
3	180
4	360
5	540
6	720

a. Plot the ordered pairs on the coordinate plane.



b. What shape does the graph of the points appear to take?

It appears to take the shape of a line.

c. Is this graph a graph of a function? How do you know?

It appears to be a function because each input has exactly one output.

d. Is this a linear equation? Explain.

Yes, $y = 180(x - 2)$ is a linear equation. It can be rewritten as $y = 180x - 360$.

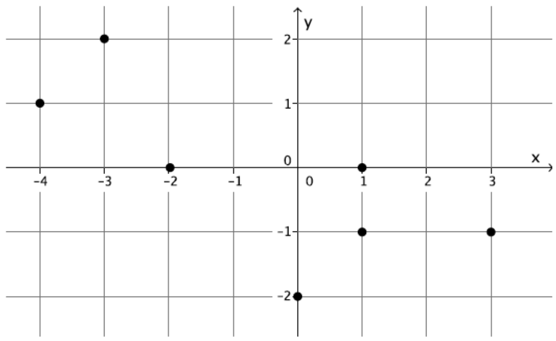
e. The sum S of interior angles, in degrees, of a polygon with n sides is given by $S = 180(n - 2)$. If we take this equation as defining S as a function of n, how do you think the graph of this S will appear? Explain.

I think the graph of this function will look like the graph of the equation $y = 180(x - 2)$. The inputs and outputs would match the solutions to the equation exactly.

f. Is this function discrete? Explain.

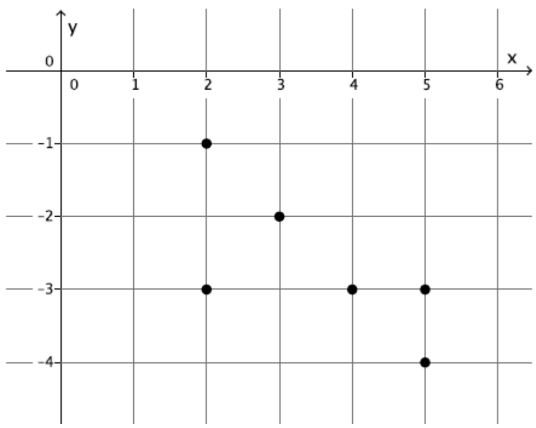
The function $S = 180(n - 2)$ is discrete. The inputs are the number of sides, which are integers. The input, n, must be greater than 2 since three sides is the smallest number of sides for a polygon.

4. Examine the graph below. Could the graph represent the graph of a function? Explain why or why not.



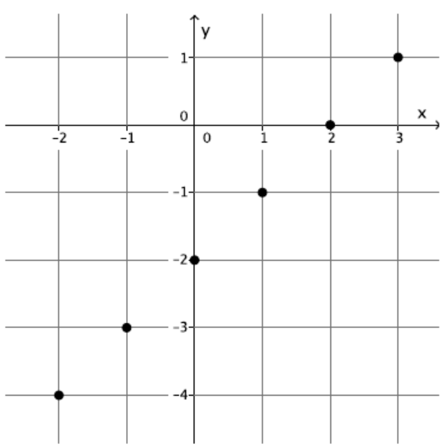
This is not the graph of a function. The ordered pairs $(1, 0)$ and $(1, -1)$ show that for the input of 1 there are two different outputs, both 0 and -1. For that reason, this cannot be the graph of a function because it does not fit the definition of a function.

5. Examine the graph below. Could the graph represent the graph of a function? Explain why or why not.



This is not the graph of a function. The ordered pairs $(2, -1)$ and $(2, -3)$ show that for the input of 2 there are two different outputs, both -1 and -3. Further, the ordered pairs $(5, -3)$ and $(5, -4)$ show that for the input of 5 there are two different outputs, both -3 and -4. For these reasons, this cannot be the graph of a function because it does not fit the definition of a function.

6. Examine the graph below. Could the graph represent the graph of a function? Explain why or why not.



This is the graph of a function. The ordered pairs $(-2, -4)$, $(-1, -3)$, $(0, -2)$, $(1, -1)$, $(2, 0)$, and $(3, 1)$ represent inputs and their unique outputs. B

Look-For Checklist

Look-For's	Observation Notes
• Using a recording sheet with anticipated responses to take notes while students are working	

Look-For Checklist

Look-For's	Observation Notes

Video: Co-Teaching a Lesson

Reflect on Co-Teaching

- What are you most looking forward to when it comes to co-teaching with your mentee?

Debrief Co-Taught Lesson

- Mentor and mentee both reflect using look-fors
- What worked and what can be improved upon
- Review the lesson impact on student learning
- Reflect on co-teaching and how to strengthen in the future

Co-Teaching: Debrief the Lesson

Suggested Guiding Questions for Discussion	Debrief Planning Notes	Debrief Meeting Notes
Primary Questions		
How did this co-teach lesson or activity help you and your students in reaching desired outcomes?		
What was most effective about the co-teaching strategy on impacting student learning and teaching practices?		
What was not effective about the co-teaching strategy on impacting student learning and teaching practices?		
Application Questions		
What will you continue implementing into your teaching practice as a result of this co-teach?		
What would you change/modify if you were teaching this lesson on your own and why?		
Clarifying Questions		
What are, if any, lingering questions you may have regarding how the lesson went or the implementation of the co-teach strategy used?		
Closing Questions		
What is/are the top learnings you are taking away from this co-teaching experience?		
How can I support you as you continue working on this SMART goal?		
How can we improve our agreements and processes for future co-teaching opportunities?		

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Debriefing Conversation Transcript (Segment)

Mentor - "Thanks for taking the time to meet with me. I had a great time co-teaching with you in your classroom and now just want to take some time to debrief about how it went and hopefully some new learning that occurred for you during this process."

Mentee - "Yeah I really enjoyed co-teaching with you as well."

Mentor - "So how do you think the co-teach lesson went overall?"

Mentee - "Well I really enjoy co-teaching. It was nice to have another adult in the room to bounce ideas off of in real time and to have that in the moment support when working on this goal. Overall I was very happy with the lesson. I feel like the students were able to develop deeper knowledge and skills with graphing functions."

Mentor - "That's great! I agree - I feel like the students did a great job working together. What do you think was most effective about us team teaching that directly impacted student learning and your teaching practices?"

Mentee - "I really liked how we had the lesson divided up ahead of time because I knew exactly what areas I needed to focus on. I also liked how I got to see you in action working with the students who needed scaffolding. I gained some new ideas on how to support students who may have skill gaps in future lessons."

Mentor - "That's wonderful! I also thought you did a great job setting the expectations for the students and posing powerful questions. Is there anything looking back, that you would change or modify about how the lesson went?"

Mentee - "Hmmmm, let me think for a minute. Maybe the pacing. I think because there was two of us teaching sometimes we took longer to explain things cause we continued to bounce ideas off of each other and therefore lost track of time a little bit and had to rush at the end of the lesson."

Mentor - "I agree with you on that point. If we do another co-teach together, which I hope we do, this is something we can both work on together. So what are your top take-aways from this co-teaching experience?"

Mentee - "Well, obviously one of the most important skills students need in order to be successful in graphing functions is to understand that the graph of the function is same as the graph of the equation"

Mentor - "So is this something you plan on continuing to focus on during the remainder of this unit?"

Mentee - "Definitely."

Mentor - "How can I support you as you continue working on this goal?"

Mentee - "I think I would like you to come observe me again on a lesson that focuses on graphing linear equations. I'd like to get your feedback on how I do teaching a lesson focused on that on my own. The next couple of lessons I feel good about because they are really focused on constant rate of change."

Mentor - "I can definitely do that, just let me know when that particular lesson is coming up in your scope and sequence and we will see what we can work out in our schedules."

Co-Teaching: Key Takeaway

Mentors use co-teaching to demonstrate growing confidence in mentees and support achievement of their SMART goals.

Make a Commitment to Start the Year Strong!

1. How will you establish a strong relationship with your mentee?

2. How will you engage in beginning of the year mentoring?

Module 4-5 Survey

http://tinyurl.com/y5kyoz9c