



THE PROFESSIONAL LEARNING ASSOCIATION

Louisiana Department of Education Mentor Teacher Training

Module 8:
Mathematical Modeling and Productive Classroom Culture

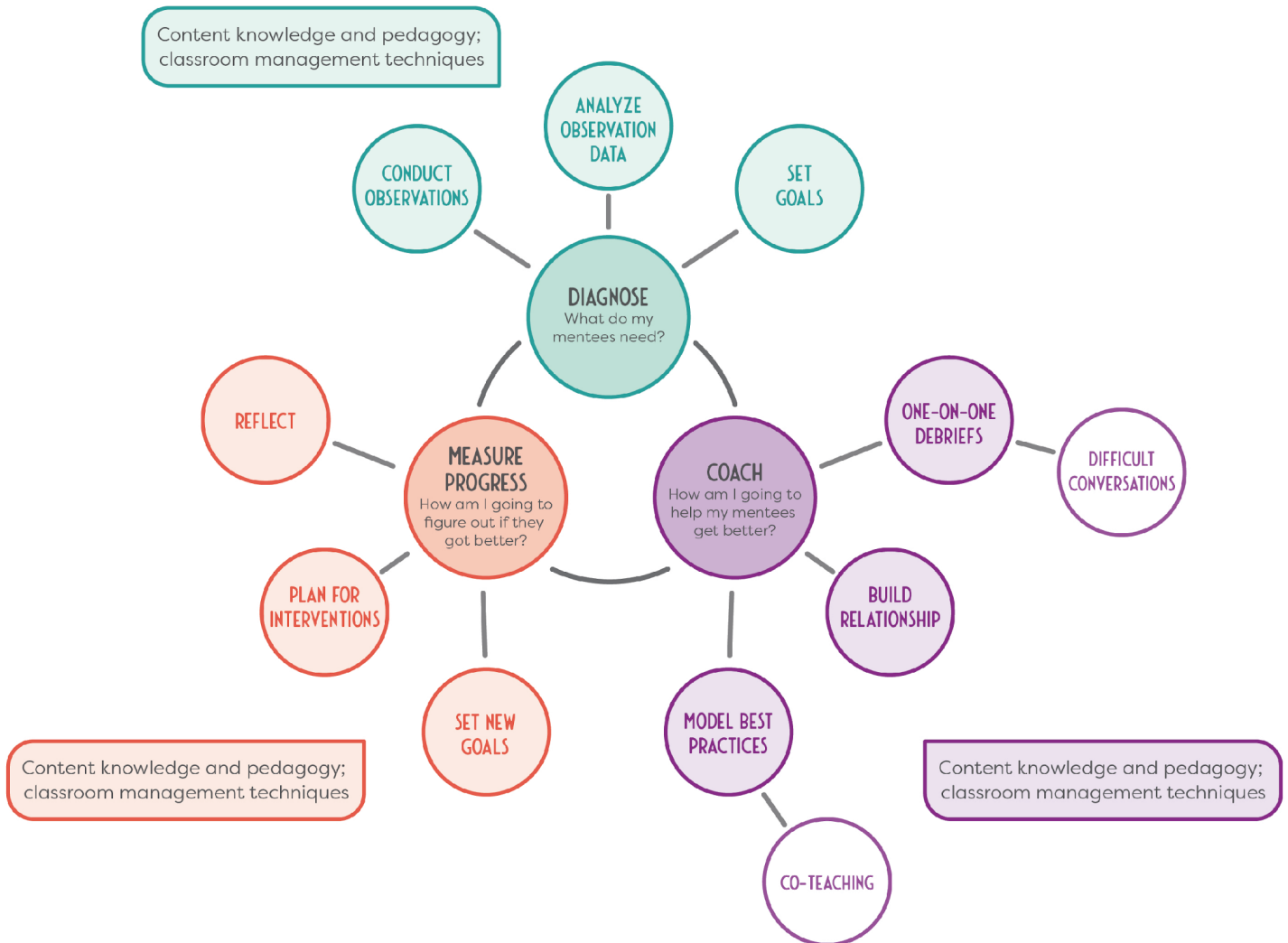
Secondary Math Cohort

January, 2020

Facilitated by Learning Forward



The Mentoring Cycle



Mentor Training Course Goals

Mentors will:

- Build strong relationships with mentees.
- Diagnose and prioritize mentee's strengths and areas for growth.
- Design and implement a mentoring support plan.
- Assess and deepen mentor content knowledge and content-specific pedagogy.

Module 8 Outcomes

- Examine **modeling with mathematics** and recognize its critical role in the development of students' mathematical thinking.
- Apply knowledge to planning and instruction by determining strategic opportunities to promote rigor in the classroom through student engagement in **modeling with mathematics**.
- Recognize appropriate contexts within EngageNY and other Tier 1 resources for students to **model with mathematics**.
- Promote students' **persistence and effective** effort in the classroom.
- Apply the mentor cycle fluently.

Module 8 Key Takeaways

- Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, understand the situations better, and to improve precision and processes for decision--making.
- Mentors recognize and describe strategic opportunities for students to apply persistence and effective strategies in the EngageNY lessons.
- Mentors can most effectively support mentees through ongoing, repeated mentoring cycles that base goals and success on observable data.

Module 8 Agenda

Morning

Welcome and outcomes
Reflection on Modules 6 and 7
Exploring modeling with mathematics
The role of productive classroom struggle

Afternoon

Scenario Practice
Connection to Assessments
Wrap-up

Agreements

Make the learning meaningful
Engage mentally and physically
Notice opportunities to support the learning of others
Take responsibility for your own learning
Own the outcomes
Respect the learning environment of self and others

Reflection on Learning – Module 6 and 7

Module 6 reflection:

How did you use the Planning Guide tool as part of the mentoring cycle?

Module 7 reflection:

- What step(s) did you take in order to foster a safe and positive environment for student discourse in your classroom?
- How did you apply your understanding of productive math discourse to facilitate your mentee's growth?

Deepening Mathematical Content Knowledge

Exploring Modeling with Mathematics

Exploring modeling with mathematics: Chalk talk

What is modeling with mathematics?

What is NOT modeling with mathematics?

How and why is modeling with mathematics useful?

What structures need to be in place for students to engage in modeling with mathematics?

Notes:

4 Model with mathematics.

Mathematically
proficient
students can
apply the
mathematics

K-12 Louisiana Student Standards for Mathematics: **Standards for Mathematical Practice**

they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

What is Modeling?

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

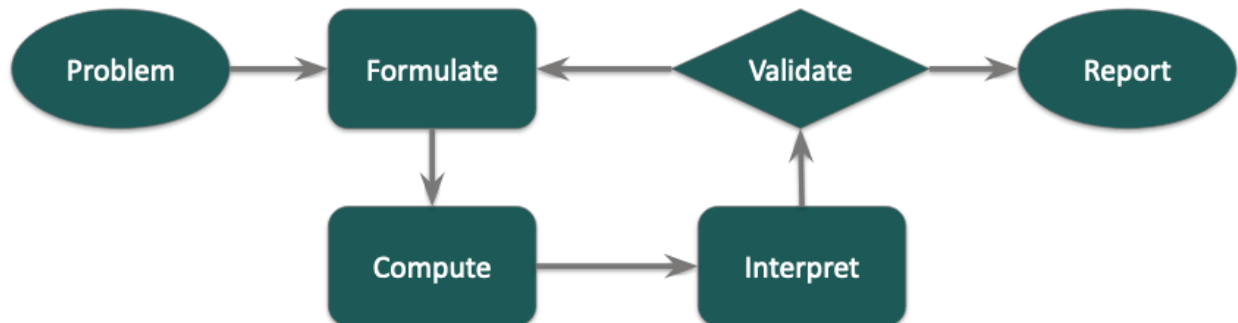
- Estimate how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Plan a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player. Design the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyze the stopping distance for a car.
- Model a savings account balance, bacterial colony growth, or investment growth.
- Engage in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyze the risk in situations such as extreme sports, pandemics, and terrorism.
- Relate population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them.

Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO₂ over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters which are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena)

Mathematical Modeling Summary

Read your grade band document thoroughly. Use the table below for notes.

| LSSM_SMP 4 | HS Modeling Conceptual Category |
|------------|---------------------------------|
| | |

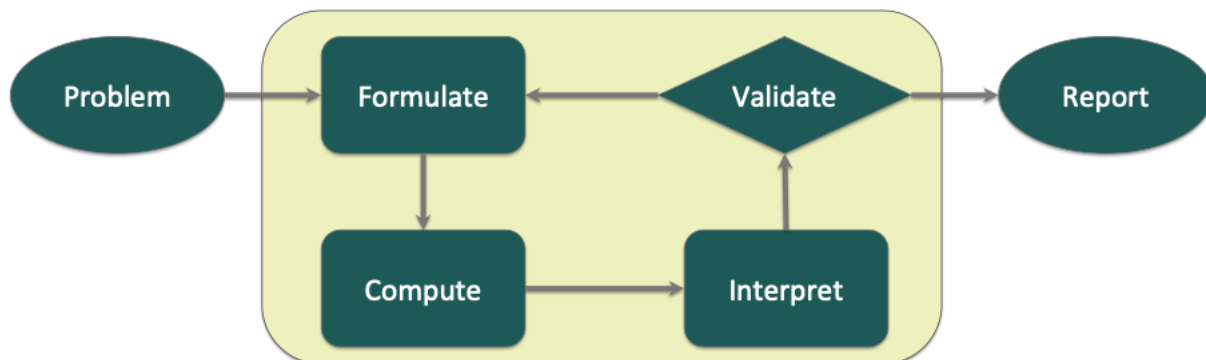
Exploring modeling with mathematics: LSSM

The modeling cycle involves:

- 1) Identifying essential variables in a situation
- 2) Formulating models and creating a representation that describes the relationship between the variables
- 3) Performing operations using those models and drawing conclusions
- 4) Interpreting the results of those operations in context
- 5) Validating the conclusions of those results and improving the model if necessary
- 6) Reporting on the conclusions and the reasoning behind them

Choices, assumptions and approximations are present throughout this cycle.

Exploring modeling with mathematics: LSSM



Exploring Modeling with Mathematics

Exploring Modeling: Notes

Stairs and Super Stairs: Act One
Source: <http://www.101qs.com/2714-super-stairs>

How many steps will he run on the super stairs?

How long will it take him to run them?"

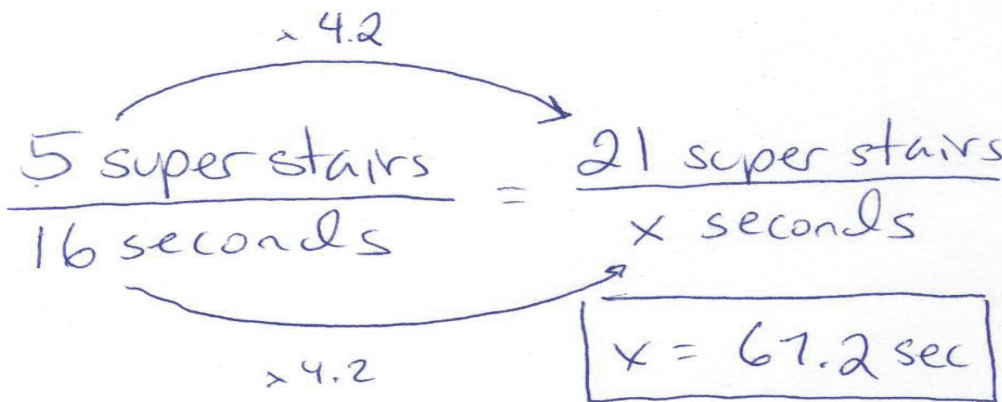
Exploring Modeling with Mathematics: The Sequel

Option 1:

If someone wanted to run 1,000,000 steps, what kind of super staircase would that require?

Option 2:

A student found the answer to the original question a different way.



The student's work shows a proportion:

$$\frac{5 \text{ super stairs}}{16 \text{ seconds}} = \frac{21 \text{ super stairs}}{x \text{ seconds}}$$
 Two curved arrows labeled $\times 4.2$ indicate the student multiplied both the numerator and denominator of the first fraction by 4.2 to get the second fraction. Below the second fraction, the student has boxed the answer:

$$x = 67.2 \text{ sec}$$

Is she right? Why or why not?

Exploring Modeling with Mathematics: Evidence

With a partner from your grade-band who did the same task as you did, respond to the 3 questions below. Find specific evidence in the problem (if present) of using modeling, determine weaknesses in the cycle and determine appropriate adjustments to ensure students have opportunities to engage in the modeling cycle.

Engage NY – Ratio and Rate Problem set (from Module 5)

| |
|--|
| <p>What evidence of modeling is in the problem?</p> |
| <p>What parts of modeling are weak/missing?</p> |
| <p>How can I adjust?</p> |

We saw **evidence** of modeling with mathematics in our task when.... but felt as if we could enhance it by...

How does intentional integration of modeling with mathematics foster rigor in the classroom?

Key Takeaway

Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, understand the situations better, and to improve precision and processes for decision-making.

Persistence and effective effort

Persistence is continuation of effort and striving in the face of difficulty, opposition, or failure: it is a key characteristic of successful people across professional and academic disciplines. Persistence is evidenced by willingness to continue to try in the face of challenge. For students, this persistence can be a driving force to help them achieve their academic and personal goals.



But where does persistence in the face of adversity come from, and how can educators and parents promote it? Persistence, or **academic resilience**—academic achievement despite a challenging or threatening circumstance in the educational process—is often described as an outcome of high motivation and positive self-beliefs. For example, students with a strong belief in their own capabilities are shown to persist longer through academic difficulty.

Adults play a significant role in helping students develop persistence and apply **effective effort**—proven strategies paired with attitudes and beliefs that enable students to see the greatest benefit. They can reinforce a view of intelligence as malleable, or changeable, provide frequent and specific feedback to students on their academic progress, and encourage students to reflect on their own experiences with overcoming challenges and succeeding.



Educators can foster in students specific skills that are associated with effective effort, such as time management, organizational strategies, and goal setting. When students have a plan for how to deal with challenging academic situations that includes metacognitive strategies, or strategies for examining their own thinking, they are better equipped to continue to try when they struggle.

And classroom practice and routines influence students' effort. Students are more likely to persist in productive ways when teachers:

- Allow time for reflection and discussion.
- Include opportunities for error analysis on assignments and tests.
- Use questioning to uncover confusion as opposed to giving an answer right away.
- Assign tasks that have clear outcomes and rubrics so students can self-assess.
- Praise effort and processes as opposed to outcomes. Encourage students to use multiple problem-solving strategies. Foster a community of learners in their classrooms.
- Push students to honestly evaluate their struggles and their attributions for success and failure.

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Used with permission of Agile Mind, Inc., the article "Persistence and effective effort," which appears in An Educator's Course in Academic Youth Development, by the Charles A. Dana Center and Agile Mind, Inc., available to users of Dana Center/Agile Mind programs in mathematics and science.



National School Reform Faculty

Harmony
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Text Rendering Experience

Developed in the field by educators affiliated with NSRF.

Purpose

To collaboratively construct meaning, clarify, and expand our thinking about a text or

Roles

A facilitator to guide the process.

A scribe to track the phrases and words that are

Set Up

Take a few moments to review the document and mark the sentence, the phrase, and the word that you think is particularly important for our work.

Steps

1. First Round: Each person shares a *sentence* from the document that he/she thinks/feels is particularly significant.
2. Second Round: Each person shares a *phrase* that he/she thinks/feels is particularly significant. The scribe records each phrase.
3. Third Round: Each person shares the *word* that he/she thinks/feels is particularly significant. The scribe records each word.
4. The group discusses what they heard and what it says about the document.
5. The group shares the words that emerged and any new insights about the
6. The group debriefs the text rendering

Citation: This resource, used for the Louisiana Department of Education LSSM professional learning experiences, is from the National School Reform Faculty. Harmony Education Center. Available at https://www.nsrharmony.org/system/files/protocols/text_rendering_0.pdf

Protocols are most powerful and effective when used within an ongoing professional learning community such as a Critical Friends Group® and facilitated by a skilled coach. To learn more about professional learning communities and seminars for new or experienced coaches, please visit the National School Reform Faculty website at www.nsrharmony.org.

Thinking about Thinking: Self-Reflection Tool

Use these prompts to help you reflect on the problem-solving experience.

| |
|---|
| 1. When I first read the problem, I thought.... |
| 2. I started this problem by... |
| 3. My initial strategy got me to this point in the problem... |
| 4. I made a plan when... |
| 5. When I got stuck I... |
| 6. I knew I was stuck because... |
| 7. The approach I took with this problem (would or would not) work on other problems because... |
| 8. If I were given a similar task, I would do the following differently... |
| 9. What did I learn from this problem that I could use in solving other problems? |
| 10. What have I learned about myself as a problem solver / learner? |

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The Charles A. Dana Center at The University of Texas at Austin

Revisit the **EngageNY lesson, Math7, M1, TA, L6 Teacher** and identify the places that support students in persistence and effective effort. As you look through the lesson, look for the following:

- Places where students are likely to struggle
- Places where you can model that mistakes and taking risks are how we learn
- Places where you can provide feedback to students that encourage metacognition and other effective effort strategies.
- Places where you can apply the specific actions that might change the overall culture of learning in your classroom.

Key Takeaways

- Mentors recognize and describe strategic opportunities for students to apply persistence and effective effort strategies in the EngageNY lessons.

Analyze Observation Data

| Strengths: What was effective about the lesson in regards to the focus area? In which “look fors” did the observee excel? What specific actions did the observee take that enabled them to be successful in the focus area? What specifically were the students able to do as a result of those actions? | Areas for Growth: What was ineffective about the lesson in regards to the focus area? Which “look fors” is the observee trying and on the verge of doing? Which “look fors” is the observee ready to try next? Where are there areas of missed opportunity? | Prioritize One Area for Growth: In your opinion, which area for growth could have the biggest impact on the observee and their students? What might you recommend the observee change or modify in their focus area based on your observation? What big takeaway do you hope the observee gains as a result of the debrief conversation? |
|---|--|--|
| 1. | 1. | |
| 2. | 2. | |
| 3. | 3. | |

Draft SMART Goals

1. **Discuss:** What about the upcoming instruction would inform your SMART goals?

2. **Discuss:** Where might the priority area of growth and the upcoming instruction align?

3. **Draft 2 SMART goals:**

SMART Goal 1:

SMART Goal 2:

Plan to Engage in One-on-One Debrief

| Suggested Guiding Questions for Discussion | Planning Notes (observer completes prior to debrief) | Debrief Meeting Notes |
|---|--|-----------------------|
| Primary Questions | | |
| Your focus area was _____. How do you think the lesson went with [your focus area]? | | |
| What are you noticing about [your focus area]? | | |
| What is important to you about [action observee took in focus area]? | | |
| Feedback Questions | | |
| You said your instructional goal for your students was [goal]. How well do you feel they accomplished it? | | |
| What is the student work/data showing about their progress to this goal? | | |
| In what ways did the lesson go as you had planned? | | |
| In what ways did things happen that you did not expect? | | |
| What other ways are there to try [action observee took in focus area]? | | |
| Feedback on Prioritized Area of Growth | | |
| You _____ and the impact of that is _____. I suggest you _____. | | |
| Closing Questions | | |
| What can you change before you try this again? | | |
| What can you learn before you try this again? How might you approach learning it? | | |
| Are there ways you think I can be helpful to you with your learning? | | |

Mentor Coaching Plan

Mentee SMART goal(s)

What activities and resources will mentor and mentee engage in to achieve goal(s)?

| Specific Activity or Resource | How is it aligned to the goal(s)? | Why will it be effective? | How will you integrate relationship building? | Projected timeline |
|-------------------------------|-----------------------------------|---------------------------|---|--------------------|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

How will you monitor your mentee's progress toward the identified goals?

Set New Goals Guiding Template

Step One: Examine All Data

| Guiding Questions | Analysis Notes |
|--|----------------|
| What data are we looking at? | |
| What is being measured in each assessment? | |
| How did various populations of students perform? Are all students being positively impacted? | |
| What areas of student performance are demonstrating the goal is or is not being met? | |
| Do patterns exist in the data? | |
| What confirms what we already know? | |
| What surprises us? | |

Step Two: Identify Progress

| Guiding Questions | Analysis Notes |
|---|----------------|
| <p>Has the teacher made progress toward their goal? What evidence exists to support that?</p> | |
| <p>What additional evidence, if any, is necessary to show adequate progress toward the goal?</p> | |
| <p>Does enough evidence exist to support that the teacher has adequately met their goal? Describe the evidence.</p> | |
| <p>Could the teacher benefit from continued work on this goal?</p> | |

Step Three: Determine Next Steps

| Suggested Guiding Questions for Discussion | Planning Notes (mentor completes prior to conversation) | Meeting Notes |
|--|---|---------------|
| Step One: Examine All Data | | |
| Your SMART goal is _____. How do you think it's going in meeting your goal? | | |
| What actions/supports have best supported you in working on this goal? | | |
| I brought some data from our time working together including _____. What evidence here exists to support your work on this goal? | | |
| Step Two: Identify Progress | | |
| How do you feel about the progress you've made toward meeting your SMART goal? | | |
| What, if any, additional work could be done in continuing to address this SMART goal? | | |
| Step Three: Determine Next Steps | | |
| Do we have a sufficient amount of evidence to support that your SMART goal was met | | |

| | | |
|---|--|--|
| | | |
| <p>(If the answer to the above question is no) What next steps should we take to continue working on this goal? I.e. another model or co-teach, observation with feedback, etc.</p> | | |
| <p>(If the answer to the above question is yes) Do you have another focus area in mind that we can set a new SMART goal for?</p> | | |
| <p>(If the answer to the above question is yes) Would you like to participate in a new observation and see what new areas to grow in come through as a result?</p> | | |