Lesson Plan:

(This lesson should be adapted, including instructional time, to meet the needs of your students.)

Background Information	
Content/Grade Level	Mathematics/Grade 4
Unit/Cluster:	Use the four operations with whole numbers to solve problems
Essential Questions/Enduring Understandings Addressed in the Lesson	Knowledge of multiplication as repeated addition, and knowledge of multiplication as equal groups and the Commutative Property Ability to solve various types of problems involving multiplication through the use of equations
Standards Addressed in This Lesson	It is critical that the Standards for Mathematical Practice are incorporated in ALL lesson activities throughout the unit as appropriate. It is not the expectation that all eight Mathematical Practice will be evident in every lesson. The Standards for Mathematical Practice make an excellent framework on which to plan your instruction. Look for the infusion of the Mathematical Practices throughout this unit. 4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 – 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. 4.OA.A.3 Solve multi-step problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Lesson Topic	Multiplicative comparisons through the use of word problems
Relevance/Connections	4.NBT.5- Multiplying whole numbers using equations
Student Outcomes	Students will create an equation to represent a word problem using a letter to represent a variable. Explain the multiplicative comparisons represented in equations Justify a solution by explaining their reasoning in creating the equation Incorporate the use of the Properties of Operations (Commutative) in their solution of the problem

Prior Knowledge Needed to Support This Learning	Understanding of the Commutative Property Fluently multiply and divide within 100 Understanding of an array Experience with solving word problems
Method for determining student readiness for the lesson	Give students several multiplication word problems, draw several arrays to represent a multiplication equations.

Learning Experience		
Component	***Details***	***Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?***
Warm Up	Using a whiteboard or large chart, display the multiplication chart for student to view. Ask them to share the similarities and differences they find in the 2 and 4 factor columns with the class. Then have students choose two other factors and brainstorm ways they are alike and different, noting various patterns.	
Motivation	Share the following additive comparison: ***Barb has 12 fish in her fish tank. Rosa has 8 fish in her fish tank. So Barb has 4 more fish than Rosa. OR Rosa has 4 less fish than Barb.*** Then share the following multiplicative comparison: ***Jose has forty dollars in his bank. Robert has eight dollars in his bank. Jose has five times as much money in his bank as Robert has in his.*** Allow time for student to discuss the differences between additive comparisons and multiplicative comparisons. Explain that we will be using multiplicative comparisons to solve problems today.	

Activity 1 UDL Components • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement Key Questions Formative Assessment Summary	UDL Components Expression is present in the activity through drawing a picture to represent their response. Engagement is present in the activity through the use of a task that allows for active participation and exploration. Pose the following problem: *A sunflower seedling is 3 inches tall. Two weeks later, the plant is 4 times taller. How tall is the plant now?* Be prepared to explain why your answer is correct. -Students will write an equation to represent the problem. -Look for two different ways to write the equation (3x4=12, 4x3=12). Draw or model two ways to show these equations. Interpret that 12= 3x4 as a statement that 12 is 3 times as many as 4, and 4 times as many as 3. -Teacher should lead students to understand that identifying the correct model is important to solving a word problem, but the commutative property can be used to help solve the equation. -The purpose of this activity is to bring out the vocabulary that students will need to understand this concept. 12 is 4 times as many as 3. -Give students more opportunities to solve word problems using models and equations. Use CCSS, page 89, Table 2 to see examples of the different types of problems that should be shared. Also see Van de Walle, J.A., Lovin, J.H. (2006) Teaching Student-Centered Mathematics, Grades K-3.Boston, MASS: Pearson Education, Inc. Summary: Students should be able to verbally state that in 5 x 7 = 35, that 35 is 5 times as many as 7, or 35 is 7 times as many as 5. Formative Assessment: A plain pen costs 6 cents. A pen	• **SMP 1:** Make Sense of problems and persevere in solving them As students represent the problems using a variety of models, they need to evaluate their progress and persevere if changes are necessary. • **SMP 2:** Reason abstractly and quantitatively. Students will determine what the numbers in the problem mean and how they relate to each other. • **SMP 4:** Model with mathematics. Students will use equations, arrays, or area models to solve the problem.

	with a school logo costs 2 times as much as a plain pen. How much does a pen with a school logo cost? Students will write an equation to solve the problem, using multiplicative comparison to support their answer.	
Activity 2 UDL Components • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement Key Questions Formative Assessment Summary	UDL Components: Comprehension is present through highlighting patterns, critical features, big ideas, and relationships. _____ Present the following two equations: 5x2=10 and 5x4=20 Students will build a 5 by 2 array and a 5 by 4 array. Ask students "How many times larger is the 5x4 array than the 5x2 array? " It would take two 5x2 arrays to make 20, so the 5 by 4 array is two times larger than a 5 by 2 array. The factor 4 is two times larger than the factor 2, and the product 20 is two times larger than the product 10. Ask students if they feel this would work with other equations. Create other arrays to explore the concept. Example: 3x3=9 3x9=27 (The factor 9 is three times greater than the factor 3, and the product 27 is three times greater than the product 9.) 4x3=12 4x6=24 (The factor 6 is two times the factor 3, and the product 24 is two times the product 12.) Pose similar problems with the product unknown in the 2nd example. 4x2=8 4x4=_____ **Note: After this problem, teachers should point out that in 4 x 4, the factor 4 is 2 times larger than the factor 2 in 4 x 2. The product 16 is two times more than the product 8.	• **SMP 1:** Make Sense of problems and persevere in solving them As students represent the problems using a variety of models, they need to evaluate their progress and persevere if changes are necessary. • **SMP 2:** Reason abstractly and quantitatively. Students will determine what the numbers in the problem mean and how they relate to each other. • **SMP 3:** Construct viable arguments and critique the reasoning of others. Students will justify their solutions and/or challenge those of others in order to come to consensus. • **SMP 4:** Model with mathematics. Students will use equations, arrays, or area models to solve the problem. • **SMP 7:** Look for and make sense of structure to solve the problem. Students will use the structure of the multiplication chart and the relationship of the different factors to justify their thinking.

	Use this understanding to discover that knowing 12 x 3 = 36 will help them solve the equation 12 x 9 = _____, stating the relationship between the factors 3 and 9, and the product 36 and _____. Students will add 36 three times, or multiply 36 x 3 to get the product 108. Using **SMP 7:** Look for and make sense of structure to solve the problem. Have students explain why their answer is correct. Summary: Through this activity, students should be able to understand that when a factor is increased x number of times, the product also increases the same x number of times. Students will use multiplicative comparisons to help them multiply larger numbers. Formative assessment: 5 x 25 = 125 5 x 50= _____ 7 x 33 = 231 7 x _____ = 693	
Activity 3 UDL Components • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement Key Questions Formative Assessment Summary	UDL Component: Persistence Provide opportunities for collaboration and support Students will use drawings, equations, or multiplicative comparisons to solve word problems using multiplication or division. Word Problems using "times more" and "times less": Use CCSS, page 89, Table 2. Van de Walle, J.A., Lovin, J.H. (2006) Teaching Student-Centered Mathematics, Grades K-3.Boston, MASS: Pearson Education, Inc. Marcus walked 15 miles this week. Larry walked 3 times farther than Marcus. How far did Larry walk? Marcus walked 24 miles this week. Larry walked 4 times less than Marcus. How far did Larry walk? A blue sweatshirt costs $24 and that is 4 times as much as red sweatshirt costs. How much does a red sweatshirt cost?	• **SMP 1:** Make Sense of problems and persevere in solving them As students represent the problems using a variety of models, they need to evaluate their progress and persevere if changes are necessary. • **SMP 4:** Model with mathematics. Students will use equations, arrays, or area models to solve the problem. • **SMP 7:** Look for and make sense of structure to solve the problem. Students will use the structure of the multiplication chart and the relationship of the different factors to justify their thinking.

	A tied bow is 4 inches across. Untied, it is 28 inches. How many times as long is the ribbon untied as it was at first? A new video game costs $12. A used video game costs 3 times less. What is the cost of a used video game? Summary: Discussion of word problem solutions. Formative assessment: Develop similar word problems. Teacher can utilize observation techniques to monitor their understanding of concepts.	
Closure	Allow time for students to create and share puzzle questions that involve the use of multiplicative comparisons. Create a class book of puzzles for their use.	

Supporting Information	
Interventions/Enrichments • Special Education/Struggling Learners • ELL • Gifted and Talented	- Use manipulatives in place of paper and pencil for students with accommodations as stated in their IEP - If possible count in their language. Model visually - Challenge students to create word problems that demonstrate multiplicative comparison
Materials	Graph paper, counting tiles, array models, multiplication charts
Technology	Calculators, document cameras, whiteboards www.mathlanding.org
Resources (must be available to all stakeholders)	*K Counting and Cardinality; K-5 Operations and Algebraic Thinking Progression* document http://ime.math.arizona.edu/progressions/ Van de Walle, J.A., Lovin, J.H. (2006) Teaching Student-Centered Mathematics, Grades K-3.Boston, MASS: Pearson Education, Inc.

Grade 4 Lesson Plan: 4.NF.B.4, Number & Operations – Fractions - Understand a fraction a/b as a multiple of 1/b

(This lesson should be adapted, including instructional time, to meet the needs of your students.)

Background Information	
Content/Grade Level	Mathematics/Grade 4 Domain: 4.OA Number and Operations - Fractions Cluster: Build fractions from unit fraction by applying and extending previous understandings of operations on whole numbers
Unit	**4.NF.B.3-4**: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. ***This lesson addresses only 4.NF.B.4. Other lessons will be included to address the remaining Standards in this Unit.***
Essential Questions/Enduring Understandings Addressed in the Lesson	• What is a **multiple**? • What is a product? • How can I use the denominator as a unit of counting and incorporate this counting as a strategy to make sense of fractions? • When comparing mixed numbers and fractions such as $\frac{3}{2}$ and $1\frac{1}{2}$, how do I explain their relationship? • How will my understanding of different fractions help me understand and communicate information about **equivalent fractions**? • How do benchmark fractions help me compare fractions with different denominators and/or numerators? • Why is it important to compare fractions as representations of equal parts of a whole or of a set? • How will my understanding of whole number computation help me understand computation of fractions and mixed numbers? • Fractions can be used to represent numbers equal to, less than, or greater than 1.
Standards Addressed in This Lesson	**4.NF.B.4:** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. **Teacher Notes:**

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	• The Common Core stresses the importance of moving from concrete fractional models to the representation of fractions using numbers and the number line. Concrete fractional models are an important initial component in developing the conceptual understanding of fractions. However, it is vital that we link these models to fraction numerals and representation on the number line and use them to model addition and subtraction of fractions and mixed numbers. This modeling should also incorporate recording the model in an **equation** so that students can make the connection between the visual model and the numerical representation. • Review the Progressions for Grades 3-5 Number and Operations – Fractions at http://commoncoretools.files.wordpress.com/2011/08/ccss_progression_nf_35_2011_08_12.pdf to see the development of the understanding of fractions as stated by the Common Core Standards Writing Team, which is also the guiding information for the PARCC Assessment development. • When implementing this unit, be sure to incorporate the Enduring Understandings and Essential Questions as a foundation for your instruction. • It is important for students to understand that the denominator names the fraction part that the whole or set is divided into, and therefore is a divisor. The numerator counts or tells how many of the fractional parts are being discussed. • Students should be able to represent fractional parts in various ways. • It is important to make connections between whole number computation and that of fractions and mixed numbers. For example, 5 x 4 can be expressed as 5 groups of 4 or 4 + 4 + 4 + 4 + 4. In the same way, 5 x $\frac{1}{4}$ can be expressed as 5 groups of $\frac{1}{4}$ or $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$. • When **decomposing** fractions, students should use fractions with like denominators and record the **decomposition** in an equation. • It is important to emphasize the use of appropriate fraction vocabulary and talk about fractional parts through modeling with concrete materials. This will lead to the development of fractional number sense needed to successfully compare and compute fractions. • Extending **fraction equivalence** to the general case is necessary to extend arithmetic from whole numbers to fractions and **decimals**. • Standard 4.NF.3 represents an important step in the multi-grade progression for addition and subtraction of fractions. Students extend their prior understanding of addition and subtraction to add and subtract fractions with like denominators by thinking of adding or subtracting so many unit fractions. • Standard 4.NF.4 represents an important step in the multi-grade progression for multiplication and division of fractions. Students extend their developing understanding of multiplication to multiply a fraction by a whole number.
Lesson Topic	Grade 4 Multiplying Fractions

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Relevance/Connections	It is critical that the Standards for Mathematical Practice are incorporated in ALL lesson activities throughout the unit as appropriate. It is not the expectation that all eight Mathematical Practices will be evident in every lesson. The Standards for Mathematical Practice make an excellent framework on which to plan your instruction. Look for the infusion of the Mathematical Practices throughout this unit. Connections outside the cluster: **4.NF.B.1:** Explain why a fraction a/b is **equivalent** to a fraction (n x a)/(n x b) by using **visual fraction models**, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate **equivalent fractions**. **4.NF.B.2:** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a **visual fraction model**.
Student Outcomes	• Students will demonstrate an understanding that their knowledge of multiplying whole numbers can be applied to multiplication of fractions. For example, 5 x 4 can be expressed as 5 groups of 4 or 4 + 4 + 4 + 4 + 4. In the same way, 5 x $\frac{1}{4}$ can be expressed as 5 groups of $\frac{1}{4}$ or $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$. ▪ Students will demonstrate an understanding that a fraction such as $\frac{6}{5}$ can be written as 6 x $\frac{1}{5}$ and can be thought of as 6 groups of $\frac{1}{5}$.
Prior Knowledge Needed to Support This Learning	▪ Conceptual understanding of multiplication of whole numbers ▪ Conceptual understanding of fractions including unit fractions ▪ Naming fractional parts using fractional models -This includes knowing that, in a set of pattern blocks, it takes 6 green triangles to form 1 whole hexagon. Therefore, each green triangle is $\frac{1}{6}$ of the whole yellow hexagon.
Method for determining student readiness for the lesson	Student Resource Sheet 1: Pretest Teacher Resource Sheet 1: Scoring Guide

Learning Experience		
Component	***Details***	***Which Standards for Mathematical Practice(s) does this address? How is the***

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Learning Experience		
		Practice used to help students develop proficiency?
Warm Up	Allow time for students to complete Resource Sheet 1: Pretest prior to beginning the warm up. Tapping into prior knowledge: 1) Write the math fact on the board/chart paper: 3 x 6 2) Ask students to show multiple representations of 3 x 6 (such as 3 groups of 6 objects, skip counting, 6 + 6 + 6, an array, on a number line, etc.) 3) Have student share their representations and record the various ways on the board or chart paper for all to see. Label each of the ways students showed: repeated addition, skip counting, array, etc. 4) Tell students they will be referring to these charts later in today's lesson.	• **SMP 5:** Use appropriate tools strategically: Students will use relevant models to show multiple representations. • **SMP 6:** Attend to precision: Students will use precise math vocabulary to communicate their reasoning to others.
Motivation	Connect student knowledge of fractions with real-world examples. 1) In pairs, students will list examples (oral or written) where fractions are present in everyday life, such as using a recipe when cooking.	• **SMP 4:** Model with mathematics: Students will apply the mathematics they know to problems arising in everyday life.
Activity 1	***Activity 1:*** Note: Need Student Resource 2: Representing a Fraction 1) Have students work in pairs. Give each pair of students a baggie/container of green triangles (amount may vary from pair to pair) and one yellow hexagon (to model the whole). 2) Tell students that today they will be working on multiplying fractions by whole numbers. 3) Have students discuss with their partner: If the yellow hexagon represents 1 whole, how many red trapezoids does it take to cover the whole? What does the red trapezoid represent? Blue parallelogram? Green triangle? (This should be a	• **SMP 3:** Construct viable arguments and critique the reasoning of others: Students will have a logical progression to their thinking that will lead to making conjectures about the relationship between multiplying fractions and multiplying whole numbers. • **SMP 6:** Attend to precision: Students will use relevant vocabulary in discussion with others and in their own reasoning.

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Learning Experience		
	quick review for students.) 4) Model counting fractional parts. Show students several red trapezoids and model counting: $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$ and so on. Stress with students that they are counting halves when they count the trapezoids so they should be saying, "one half, two halves, three halves, etc. 5) If students need more practice counting fractional pieces, model with the parallelograms. 6) Distribute Student Resource 2: Representing a Fraction – one per student. (Even though students are working in pairs, each should complete his/her own Resource Sheet.) Have students work together to fill in the Resource Sheet. 7) As students are working, monitor student progress and make note of any students who might need additional support and where they seem to be struggling. See Resource Sheet 2A Representing Fractions – Sample Answer Sheet for examples of appropriate student responses. 8) Ask students to refer to their representations of the earlier multiplication problem with whole numbers. Compare what they recorded on their paper to the posted representations. Do they see any similarities? Ask students to discuss with their partner/table group any similarities or differences they see between whole number multiplication and fraction multiplication. 9) Share as a whole group or chart information on a Smart Document.	• **SMP 7:** Look for and make use of structure: Students will look for a pattern when skip-counting by fractional parts.
Activity 2	Note: Need Student Resource 3: Multiplication of Fractions 1) Ask students how else they could represent $\frac{1}{6}$. (of a region, of a set, on a number line).	• **SMP 1:** Make sense of problems and persevere in solving them: As students represent fractions using a variety of model, they need to evaluate their progress and persevere if changes

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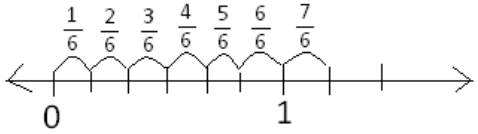
Learning Experience		
	2) Ask students to draw a number line in their Math Journals and represent $\frac{1}{6}$ on the number line. If needed, model drawing a number line that begins with 0 and ends with 1 and ask students how they would determine where $\frac{1}{6}$ would be. Check to make sure students have drawn reasonable representations of $\frac{1}{6}$ on a number line. Ask if anyone used a benchmark fraction to help them decide where $\frac{1}{6}$ should be. 3) Ask students to represent the skip counting from Activity 1 with jumps on their number line. For example, if students had 7 green triangles, or $\frac{7}{6}$, their 4) number line might look like this:  4) Ask students to consider the following multiplication problem: $4 \times \frac{2}{3}$ and record different ways to represent it. 5) Using *Think, Pair, Share*, ask the following questions. a. What does it mean? 4 groups of $\frac{2}{3}$ b. How would it be represented with skip counting? $\frac{2}{3}$ $\frac{4}{3}$ $\frac{6}{3}$ $\frac{8}{3}$ c. Repeated addition? $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$	are necessary. • **SMP 5:** Use appropriate tools strategically: Students will represent fractions using various models such as the number line. • **SMP 6:** Attend to precision: As students are representing fractions on a number line, they need to label the number line consistently and appropriately.

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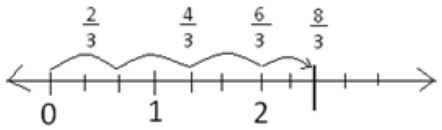
Learning Experience		
	d. On a number line?  5. Distribute Resource Sheet 3: Representing Multiplication of Fractions and ask them to record various ways to represent the following problem: $5 \times \frac{1}{4}$ Allow time for students to share their representations.	
Closure	1) Class discussion: Ask students to share how multiplying a fraction by a whole number is like multiplying a whole number by a whole number. 2) Have students complete Student Resource 4: The Same? 3) Have each student complete the Exit slip – Student Resource 5: Recipe for Brownies.	• **SMP 1:** Make sense of problems and persevere in solving them: Students will explain to themselves the meaning of the process and will monitor and evaluate their understandings. • **SMP 4:** Model with mathematics: Students will apply the mathematics they know to problems arising in everyday life. • **SMP 7:** Look for and make use of structure: Students will look for similar structure between multiplying whole numbers and multiplying fractions.

Supporting Information	
Interventions/Enrichments	**Differentiation: suggestions for meeting the needs of all students:**

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• Special Education/Struggling Learners • ELL • Gifted and Talented	**Struggling Learners:** ▪ Some students, especially ELL students, might need a sheet of vocabulary words with definitions they can refer to throughout the lesson. ▪ Make available: fraction models such as fraction bars so students who need to, can model their problem before representing it on a number line. ▪ Provide cm graph paper during Activity 2 for students having difficulty drawing the number line and making the increments reasonably equal.
Materials	▪ Green triangles from Pattern Blocks – 1 baggie/container of triangles per pair of students. (Each baggie should contain at least 7 triangles and the baggies should have a variety of amounts.) ▪ Pattern blocks – 1 yellow hexagon, 5+ red trapezoids, 8+ blue parallelograms, 10+ green triangles (best if they are overhead pattern blocks, on a Smart Board, etc.) ▪ Chart paper and markers, if not recording warm-up on the board ▪ Math Journals ▪ Resource Sheet 1: Pretest – 1 per student - **Administer prior to the lesson** ▪ Resource Sheet 2: Representing a Fraction – 1 per student ▪ Resource Sheet 2A: Representing a Fraction – Sample Answer Sheet for Teacher ▪ Resource Sheet 3: Representing Multiplication of Fractions – 1 per student ▪ Resource Sheet 4: The Same? – 1 per student ▪ Resource Sheet 5: Exit slip – Recipe for Brownies - 1 per student ▪ Fraction models such as fraction bars available for students who wish to use them
Technology	
Resources (must be available to all stakeholders)	

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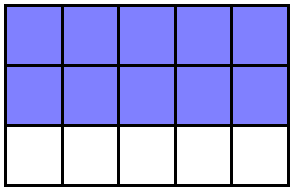
Student Resource Sheet 1 **Name**_____

Pretest

1) Draw a picture that represents 5 x 3

2) Solve this problem in two different ways: 4 x 6

3) What part of the rectangle is shaded? Give your answer as a fraction.



4) Draw a model to show $\frac{2}{5}$

5) Solve: 5 x $\frac{2}{5}$

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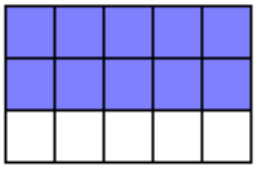
Teacher Resource Sheet 1
Pretest **SCORING GUIDE**

What does the student know? Does the student demonstrate any possible misconceptions?

1) Draw a picture that represents 5 x 3
   Is the student able to represent 5 x 3?
   - **Does the student's representation show 5 groups of 3?**
   - **If the student represents 3 groups of 5, ask the student to explain his/her representation to determine if s/he understands that multiplication means "equal groups of".**

2) Solve: 4 x 6
   Prove that your answer is correct in two different ways.
   - **Is the student able to prove the problem in two different ways?**
   - **Possible ways to prove his/her answer: skip counting, repeated addition, drawing a rectangular array or a dot array, drawing a picture.**

3) What part of the rectangle is shaded? Give your answer as a fraction.



   - **Correct answer: $\frac{10}{15}$ or $\frac{2}{3}$**
   - **If the student is unable to correctly name the fraction, s/he might need more work naming and understanding fractions before this lesson.**

4) Draw a rectangle and shade $\frac{3}{8}$ of the rectangle.
   - **Does the student attempt to divide the rectangle into 8 sections?**
   - **If so, are the sections reasonably equal? If the sections are not equal, does the student understand that the denominator indicates equal parts of the whole?**
   - **If the student does not divide the rectangle into 8 sections, is s/he estimating the answer? Does s/he shade a little less than half of the rectangle?**

5) Draw a picture to solve: 5 x $\frac{3}{4}$
   - **Does the student show 5 groups of $\frac{3}{4}$?**
   - **If so, does the student understand the answer is $\frac{15}{4}$ and not $\frac{15}{20}$?**
   - **Does the student rewrite the answer as a mixed number? ($3\frac{3}{4}$)**
   - **Note: If the student is able to solve the problem because s/he has learned the process of multiplying 5 x 3 and dividing by 4, but is unable to represent the problem by drawing a picture, the student probably has a minimal understanding of multiplying fractions and needs more work to build conceptual understanding.**

If the student is able to answer most, if not all, of these 5 problems and demonstrates a solid conceptual understanding of multiplying a whole number by a fraction, s/he probably needs enrichment – see suggestions provided.

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Student Resource Sheet 2A Representing a Fraction
Name _____

If the yellow hexagon is equal to 1 whole, what fraction is represented by the green triangle? _____	
How many green triangles are in your bag? _____	***What fraction do your green triangles represent?*** _____
Complete the sentence to describe your set of green triangles: _____ groups of _____	***Write an Equation using multiplication to represent your set:***
Skip counting:	***Write an equation using repeated addition:***

✂

Student Resource Sheet 2A Representing a Fraction
Name _____

If the yellow hexagon is equal to 1 whole, what fraction is represented by the green triangle? _____	
How many green triangles are in your bag? _____	***What fraction do your green triangles represent?*** _____
Complete the sentence to describe your set of green triangles: _____ groups of _____	***Write an Equation using multiplication to represent your set:***

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Skip counting:	***Write an equation using repeated addition:***

✂

Student Resource Sheet 2A **Representing a Fraction – *Sample Answer Sheet***
Name _____

If the yellow hexagon is equal to 1 whole, what fraction is represented by the green triangle? ____$\frac{1}{6}$____	
How many green triangles are in your bag? __*9*___ ***(amount could vary for different pairs of students)***	***What fraction do your green triangles represent?*** __$\frac{9}{6}$___
Complete the sentence to describe your set of green triangles: ___9___ groups of __$\frac{1}{6}$____	***Write an Equation using multiplication to represent your set:*** $9 \times \frac{1}{6} = \frac{9}{6}$
Skip counting: $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}, \frac{7}{6}, \frac{8}{6}, \frac{9}{6}$	***Write an equation using repeated addition:*** $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{9}{6}$

✂

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Student Resource 3 **Representing Multiplication of Fractions**
Name _____

Directions: Complete each of the representations below for the problem: 5 x $\frac{1}{4}$

Equation:	***Complete the sentence:***
_____ X _____ = _____	_____ groups of _____
Skip counting:	***Repeated addition:***
On a number line:	

✂

Student Resource 3 **Representing Multiplication of Fractions**
Name _____

Directions: Complete each of the representations below for the problem: 5 x $\frac{1}{4}$

Equation:	***Complete the sentence:***
_____ X _____ = _____	_____ groups of _____

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Skip counting:	***Repeated addition:***
On a number line:	

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Explain one way that multiplication of fractions is the same as multiplication of whole numbers.

Reflecting on your learning. . . .
Answer 1 of the following questions:

- What is one new thing you learned today?
- What is one thing you found challenging or interesting in today's lesson?
- What is something you need more practice with?

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Student Resource Sheet 5 **Exit Ticket: Recipe for Brownies**

You are using a recipe to make brownies. One tray of brownies calls for $\frac{1}{4}$ cup of sugar. You need to make five trays of brownies for your class. How many cups of sugar will you need to make the brownies? Create a model to solve this problem. Write an equation that represents this problem.

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Grade 4: Unit 4.NF.5-7, Understanding decimal notation for fractions, and compare decimal fractions.

Model Lesson Plan: *Model lesson plans are in the initial stages of creation and will be inserted as available.*

Background Information	
Content/Grade Level	Mathematics/Grade 4 Domain – 4.NF.6 Number and Operations-Fractions (limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100) Cluster: Understand decimal notation for fractions, and compare decimal fractions. • The Common Core stresses the importance of moving from concrete fractional models to the representation of fractions using numbers and the number line. Concrete fractional models are an important initial component in developing the conceptual understanding of fractions. However, it is vital that we link these models to fraction numerals and representation on the number line. This movement from visual models to fractional numerals should be a gradual process as the student gains understanding of the meaning of fractions.
Unit	Use decimal notation for fractions with denominators 10 or 100. ***This is intended to be an introductory lesson for the Standard 4.NF.6. The activities focus on strategies that could be employed to use decimal notation for fractions with denominators 10 and 100.The lesson does not address 4.NF.5 or 4.NF.7. Those topics will be covered in future lessons. The amount of time that should be spent on each activity is dependent upon the needs of the students.***
Essential Questions/Enduring Understandings Addressed in the Lesson	• What is a decimal? • Why is it important to understand the relationship between fractions and decimals? • Why do we use different forms of numbers to represent equivalent values? • How will my understanding of whole numbers and fractions help me understand and use decimals when solving problems? • Numbers can be represented in a variety of forms. • Decimals are an integral part of our daily life and an important tool in solving problems. • Decimals are an important part of our number system. • Decimals provide an easy method for computing or comparing fractions. • Decimals are used to represent money, measurements, etc. • The base-ten place-value system extends infinitely in two directions: to tiny values as well as to large

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	values. Between any two place-values, the ten-to-one ratio remains the same. • Any operation you can do with whole numbers, you can do with decimals and fractions as an extension of the whole number system. • The decimal point is a convention that has been developed to indicate the unit's position. The position to the left of the decimal point is the unit that is being counted as singles or ones. • Decimal numbers are another way of writing fractions. It is important to understand the relationship between the two systems. • Decimals are an extension of whole-numbers on the place value chart.
Standards Addressed in This Lesson	4.NF.6 Fractions--Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as* $\frac{62}{100}$*; describe a length as 0.62 meters; locate 0.62 on a number line diagram*. It is critical that the Standards for Mathematical Practice are incorporated in ALL lesson activities throughout the unit as appropriate. It is not the expectation that all eight Mathematical Practices will be evident in every lesson. The Standards for Mathematical Practice make an excellent framework on which to plan your instruction. Look for the infusion of the Mathematical Practices throughout this unit.
Lesson Topic	Use decimal notation for fractions with denominators of 10 or 100.
Relevance/Connections	It is expected that 4.NF.1 – 4.NF.4 will have already been taught when beginning this unit. If not, modify the lessons as needed for your students. The activities below can be used as introductory lessons to decimals over a period of several days or a week. Since PARCC has identified this Cluster as a Major Focus Cluster, emphasis should be placed on developing lessons and activities to support the Standard 4.NF.6. The activities in this lesson only address part of this Standard. Work with meters and the number line are being developed. See Lesson Seeds for ideas.
Student Outcomes	• Students will demonstrate an understanding of whole number place value concepts. • Students will understand how fractions and decimals are related. • Students will write fractions with denominators of 10 and 100 as decimals. • Students continue to develop flexibility in their thinking about fractions and decimals. They can easily move between the two systems. • Students can apply whole number base-ten concepts to decimals, including the ten-to-one ratio that exists between any two place values. • Students understand the decimal point separates whole numbers and decimal numbers.

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Prior Knowledge Needed to Support This Learning	• Understanding that whole number base ten concepts, including the ten-to-one ratio that exists between any two place values. • Knowing the value of coins (dimes and pennies in particular) • Recognizing that a fraction is part of a whole. • Represent fractional models numerically. • Recognizing and representing fractions with denominators of 10 and 100.
Method for determining student readiness for the lesson	Use the motivation included in this lesson as a pre-assessment.
Materials	• Blank paper • 100 pennies per student (Could be real, plastic, or paper) • 10 dimes for every two students (plastic coins or real coins) • Math Journals • Cups with both dimes and pennies (one cup per pair of students) for Activity 2 • Piece=Part=Portion, Fractions=Decimals=Percents (Scott Gifford) • Resource Sheet 1: Representing Tenths & Hundredths • Base ten blocks • Centimeter Grid paper or 10 x 10 grids • Overhead projector or document camera • Cups with unit cubes from a base ten blocks kit (one per pair of students) for Activity 3 • Resource Sheet 2: Scooping for Decimals • Centimeter Grid paper or 10 x 10 grids http://wps.ablongman.com/ab_vandewalle_math_6/0,12312,3547876-,00.html • Scissors for each student • Resource Sheet 3: Decimals Exit Slip • If an interactive white board with virtual manipulatives is available, this should be provided as well

Learning Experience

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Learning Experience		
Component	***Details***	***Which practice(s) does this address? How is this practice used to help students develop proficiency?***
Warm Up		
Motivation	• Ask students (working in pairs) to count out 32 pennies. • Review how many pennies are in a dollar. Ask: *If there are 100 pennies in a dollar and you have 32 pennies, what fraction of a dollar do you have? Write the fraction on your paper.* Note: Students who have difficulty identifying the fraction might need to represent 32 out of 100 using base ten blocks or Digi-Blocks and record their representation on a 10x10 grid. • Share answers. Ask: Is there another way to write 32 pennies? (32 cents 32¢ .32 0.32 thirty two cents)	
Activity 1 UDL Components • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement	UDL Components • **Representation** is present in the activity through supporting the decoding of mathematical symbols and notation. • **Expression** is present in the activity through the use of concrete materials. • **Engagement** is present in the activity through the use of a task that allows for active participation and exploration and through providing learners with as much discretion and autonomy as possible by providing choices in the tools used. • Write on the board: $\frac{32}{100}$ ____ 0.32 Ask students to discuss which symbol (<, >, =) should go in the blank. Students may use manipulatives to support their answer.	**Standards for Mathematical Practice(s):** • **SMP3** is evident in this activity when students work to generate lists about decimals and then discuss them. Regular student interaction and discussion in the mathematics classroom help make this an opportunity for students to construct arguments for their own reasoning, as well as critique the reasoning of others. • **SMP 7** is applied in this activity when students look for and make use of structure with decimal place value. Students may look for a pattern that allows them to develop a generalization about place value. • **SMP 8** is incorporated in this activity as students being to notice that the base ten

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Learning Experience		
	• Distribute blank paper. • Introduce the term ***decimal.*** Ask students to work individually and list everything they know about decimals. Give students about a minute and then have them switch papers with their partner. Now allow another minute as students read what their partner wrote. They may add anything else they know to their partners list. Give students one more minute to find commonalities between their two lists. Circle/highlight ideas they both had listed before switching. They should be prepared to share with the class. • Class sharing: Ask students to share what they wrote on their lists about decimals. During the sharing, students may add new ideas they hear to their list. • Discuss the decimal point and its purpose (symbol that separates decimals and whole numbers, designates part of a whole). o Ask the students questions, such as, "How are fractions and decimals similar?" (Two different ways to write partial quantities.) • This would be a good time to discuss decimal place value. For example, you could write the following numbers on the board: 18.5 185. 1.85 • Even though the digits are the same, the placement of the decimal point changes the amount. Ask students if they can explain how the amounts change. In order to do this, model for students how decimals are read (left to right, like when we read a book; the decimal is read as "and". The number 18.5 is read as, "18 and 5 tenths.)"	place value system extends infinitely in two directions: to tiny values as well as to large values. Between any two place values, the ten-to-one ratio remains the same. Students may begin to see the regularity.

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Learning Experience		
	• Ask students: If these numbers were amounts of money, which would you want, and why?" Allow time for students to • Collect students' lists about decimals and use them to create a class chart for students to refer back to throughout the unit. If you have a dedicated math center in your classroom, you may wish to hang the chart in this area. • Extension Activity: Read Scott Gifford's Piece=Part=Portion, Fractions=Decimals=Percents to the class and discuss.	
Activity 2 UDL Components • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement Key Questions Formative Assessment	UDL Components • **Representation** is present in the activity through using literature to varying the way mathematical ideas are presented. • **Expression** is present in the activity through the use of concrete materials. • **Engagement** is present in the activity through the use of cooperative learning groups. • Have students organize their 32 pennies from the previous activity for easy counting. (Let students decide how they will organize them.) • Students share their different ways of organizing. (Students might organize them in groups of 2's, 5's, 10's, etc.) • Ask students which is the most efficient? (Organizing by 10's and 1's is more efficient than by 2's or 5's. Some students might have organized by 25 and 7 more, but this doesn't allow for skip counting which is a more efficient way to count). • Tell students you operate the Place Value Bank but	**Mathematical Practice(s):** • SMP 1 is evident as students use thinking and reasoning to make sense of problems and persevere in solving them. The problem-solving process includes assessing the reasonableness of answers. • SMP 2 is evident in this activity as students complete the extension activity and practice writing numbers as either decimals and/or fractions by using visual models to represent the problem. • SMP 6 is emphasized in this activity as students accurately represent dimes and pennies in various ways.

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Learning Experience		
	you only have dimes and pennies. Ask students if they were to trade coins with you so they have the least number of coins possible, what coins would they trade? Students should be able to explain how they know this. (30 pennies for 3 dimes) • Ask students how to write 3 dimes and 2 pennies as a fraction ($\frac{32}{100}$). Now ask students if we could represent 3 dimes and 2 pennies in a different way. Give students think time and allow for them to discuss their ideas in pairs before calling on students to share. (Another way to represent 3 dimes and 2 pennies is 32 pennies, which also links to writing the number as a decimal 0.32 or .32). Students may not immediately make the connection between decimals and fractions. Ask students: o What do you notice about these numbers? What do they represent? o Do the different representations change the amount of coins you have? o Why do you think we would write these numbers as a fraction some of the time, and other time as decimals? • Allow time for discussion. Extension Activity: • Allow students to work in pairs. Distribute Math Journals and a cup of pennies and dimes to each pair. • Ask Partner 1 to scoop out a handful of coins. Partner one arranges the coins in front of both partners and then writes the number as a fraction. Partner two writes the number as a decimal. Partners are encouraged to discuss their answers with one another.	

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Learning Experience		
	• Students should record their representations in their Math Journals. • Leave time for discussion and sharing.	
Activity 3 UDL Components • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement Key Questions Formative Assessment	UDL Components • **Representation** is evident in this activity as numbers are represented as both fractions and decimals using concrete materials as well as numbers and symbols. • **Expression** is present in this activity as students are given the option to use virtual manipulatives (if available). Expression is also present in the extension activity when grid paper is given as an option. • **Engagement** is present in this activity when learners are invited to share their personal responses in their math journals. • Distribute Resource Sheet 1: Representing Tenths & Hundredths and base ten blocks to each student. You may use Resource Sheet 1 as a warm up. Ask students to complete Resource Sheet 1 independently. (Students should have had multiple experiences using base ten blocks and should be able to identify a flat as a 100 block, a long as a ten stick, and a unit as one cube prior to completing this activity). If an interactive white board and virtual manipulatives are available, these should be offered to students as well. • Ask a few students to share their solutions to Resource Sheet 1 using a document camera or overhead projector. An interactive white board could also be used, if applicable. • Distribute Resource Sheet 2: Scooping for Decimals to each student. Also distribute two base ten flats	**Mathematical Practice(s):** • SMP 3 is evident in this activity as students discuss solutions with their partners. They may use the information gleaned from class discussions to explain in their math journals how they know their answers to the extension activity is correct. • SMP 5 is evident in this activity as students represent decimals and fractions using base ten blocks.

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Learning Experience		
	(100 block) and a cup of unit cubes to each pair of students. • Assign the flat the value of "one". • Ask students to work in pairs. Students take turns scooping a handful of unit cubes and placing them on their flat. • After each turn, both partners shade the grid on Resource Sheet 2 to show the cubes placed on the flat. • Partners should discuss and decide how to write a decimal and a fraction on their grid on Resource Sheet 2 after each turn. • Bring the class together after pairs of students have had a chance to complete Resource Sheet 2. Discuss students' work. • Extension Activity: Distribute grid paper and scissors to each student. • Write the following problem on the board for students to solve: *If I use a flat to represent one whole, a long to represent tenths, and a unit to represent hundredths, what decimal numbers can I represent using exactly ten pieces?* • Students should shade in their numbers using the grid paper provided. They may cut out their grids and glue them into their math journal and explain how they know their answers are correct. (Asking students to create their own grids gives them an opportunity to count ten rows of ten to ensure they have created a grid of exactly 100 units. For students who have difficulty, you can cut out grids in advance	

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Learning Experience		
	and hand these out). • A whole class or small group discussion should follow this activity. • Note: Base ten blocks are used to solve this problem. Students could also find the biggest or smallest possible number using ten pieces. • Distribute math journals. Allow students to share their personal responses to this activity. You may choose to ask them to write a one-minute response, such as "what is the most important thing you learned in math class today?", or "Tell me one thing that was unclear about today's math lesson."	
Closure	Discussion: • How are decimals like fractions? • How are decimals like whole numbers? • Where have you seen decimals used before? (at home, on a grocery store receipt, at a restaurant, etc.) • Refer back to the chart created in Activity 1 from students' lists about what they know about decimals. Address any misconceptions and add any new ideas. Keep this as you continue to develop lessons for this Standard. • Distribute Resource Sheet 3: Decimals Exit Slip and base ten blocks.	**Mathematical Practice(s):** • Construct viable arguments and critique the reasoning of others • Look for and make use of structure

Supporting Information	
Interventions/Enrichments • Students with Disabilities/Struggling Learners	• Embedded within the lesson.

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Technology	• http://themathgames.com/our-games/decimal-games/place-value/decimal-place-value-math-game (Decimal game) • http://www.mathsisfun.com/decimals-menu.html (Introduction to decimals) http://www.sheppardsoftware.com/mathgames/decimals/DecimalModels10.htm (decimal matching game) • http://wps.ablongman.com/ab_vandewalle_math_6/0,12312,3547876-,00.html (Blackline masters)
Resources	See Unit Resources link.

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Resource Sheet 1

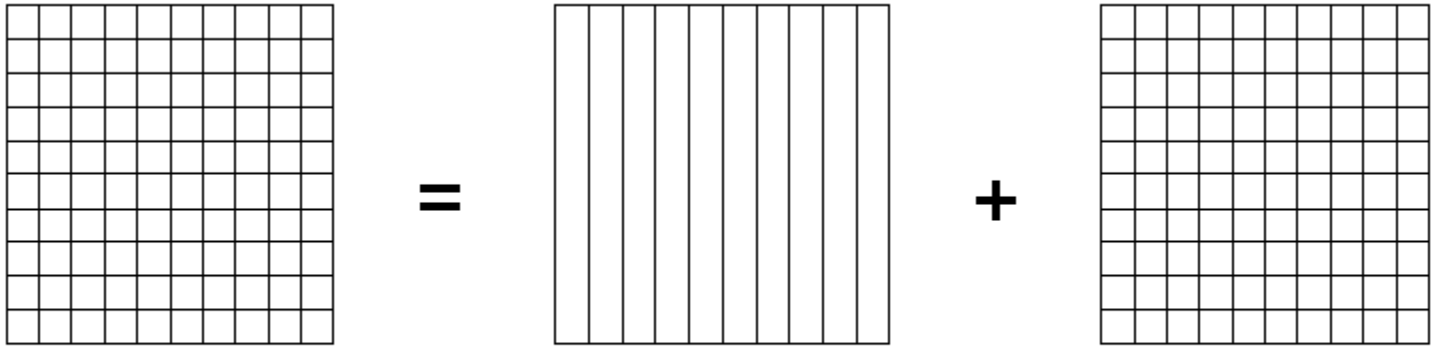
Representing Tenths & Hundredths

Name: _____

Directions:

1. On the grids below:

- Represent and label $\frac{32}{100}$.
- Represent and label the two fractional parts that have a sum of $\frac{32}{100}$ using tenths and hundredths



_____ _____ _____

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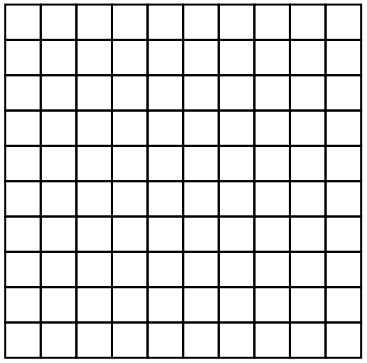
Resource Sheet 2 (1 of 2)

Scooping for Decimals

Name: _____

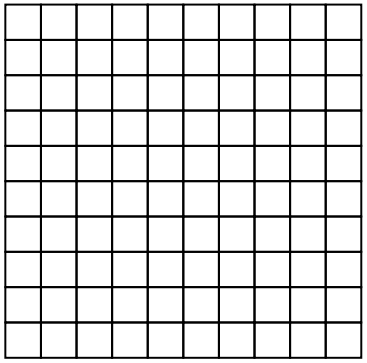
Directions:

- Shade the grid to show the cubes you placed on the flat.
- Write a decimal and fraction below the grid.



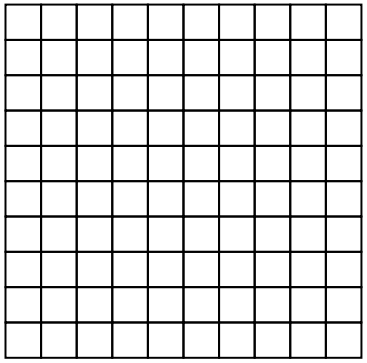
Fraction: _____

Decimal: _____



Fraction: _____

Decimal: _____



Fraction: _____

Decimal: _____

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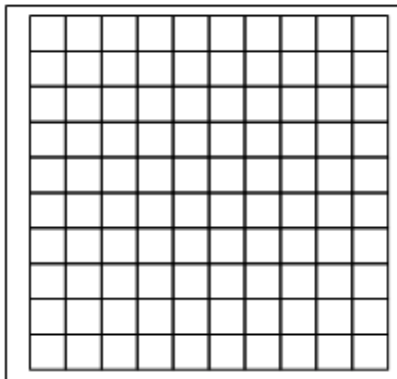
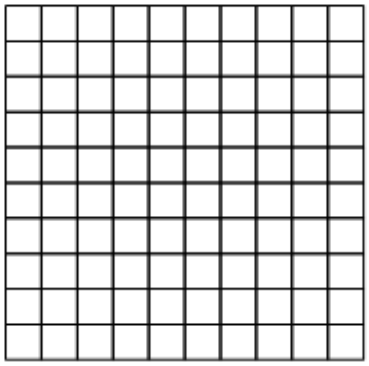
Resource Sheet 2 (2 of 2)

Scooping for Decimals



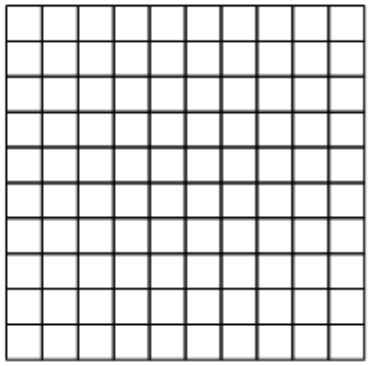
Fraction: _____

Decimal: _____



Fraction: _____

Decimal: _____



Fraction: _____

Decimal: _____

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Resource Sheet 3

Decimal Exit Ticket

Name: _____

Kelsey was asked to use a flat to represent one whole, a long to represent tenths, and a unit to represent hundredths. Below are some decimal numbers Kelsey represented using exactly five pieces. Place an X next to each decimal number that you believe was made with *only* 5 pieces.

_____ **A. 2.12**

_____ **B. 0.50**

_____ **C. 1.43**

_____ **D. 4.01**

_____ **E. 0.83**

If Kelsey could only use tenths and hundredths to represent 2.12, how many pieces would she need to use to do so? Explain how you know this.

_____

_____

_____

_____

_____

_____

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