LATM Presents - Essential Math Models that Support LSSM Instruction: Area Models – Middle/Jr. High

Students today are being asked to demonstrate certain key skills in mathematics:

- Demonstrate understanding of the math concept, not just the procedure
- Apply their understanding to real world examples
- Use accurate procedures and skills to answer questions
- Demonstrate mathematical reasoning by explaining, justifying, or critiquing with precision

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By the end of the session, participants should:

- Understand the value of new models for helping students develop number sense
- Analyze the progressions of the area model
- Recognize the importance of mathematical discourse and rigor in instruction.

Agenda:

- What is Number Sense?
- Area Model
- Math Discourse and Rigor

What is Number Sense?

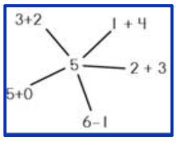
Catherine Kuhns' Definition:

An understanding of numbers so complete that a child knows that 6 is the same as:

- half of 12
- 3 doubled
- 1/3 of 18
- 2 sets of 3
- 3 sets of 2
- 1 more than 5
- 1 less than 7
- add 10 to 6, you'll get 16.

Witzel, Riccomini, and Herlong (2013) describe number sense as follows:

Number sense is an emerging construct that refers to a child's fluidity and flexibility with numbers, the sense of what numbers mean and an ability to perform mental mathematics and to look at the world and make comparisons.



The NMAP (2008) provided the following description of number sense:

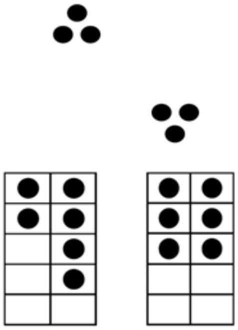
In it's most fundamental form, sense entails an ability to immediately identify the numerical value associated with small quantities;…

This more highly developed form of number sense should extend to numbers written in fraction, decimal, and exponential forms.

- Poor number sense interferes with learning algorithms and number facts and prevents use of strategies to verify if solutions to problems are reasonable.

Area Model

PreK & K

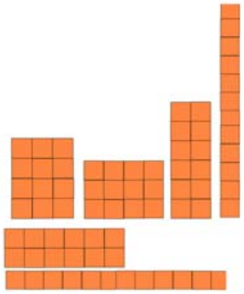


- Fill your ten frame with counters. How many counters are there in all? How many rows do we have? How many are in each row?
- Here are 6 colored tiles. Make a rectangle with your tiles. Some will make a 2x3, some a 3x2, some a 1x6, etc. (K.NBT.A.1)

Area Model

Grades 1-2

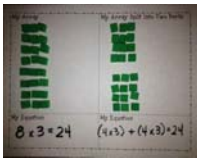
- Continue work with ten frames.
- Here are 12 colored tiles. Make a rectangle with your tiles. Label the row and columns. How many are in each row? If it's a 2x6, can we add 6 and 6 to find our total? If it's a 3x4, can we add 4 and 4 and 4 to get our total? (1.G.A.1, 2.OA.C.4)

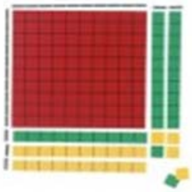


Area Model

Grade 3

- Continue work with colored tiles and arrays.
- Introduce linear pieces. Build a 12x13 first with the linear pieces, then complete the model with the base ten blocks. (3.OA.B.5, 3.MD.C.7, 4.NBT.B.5)
- Have students draw models to represent the linear and area pieces and relate the modules to the standard algorithm.





Area Model

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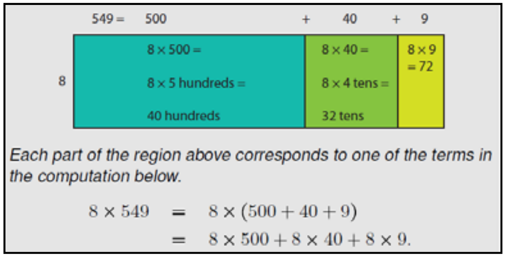
Grade 4

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.



Each part of the region above corresponds to one of the terms in the computation below.

$$\begin{aligned} 8 \times 549 &= 8 \times (500 + 40 + 9) \\ &= 8 \times 500 + 8 \times 40 + 8 \times 9. \end{aligned}$$

Area Model

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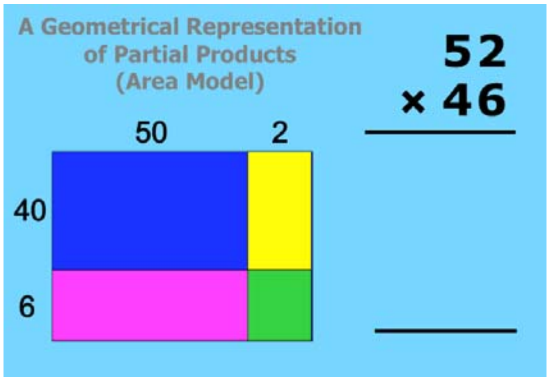
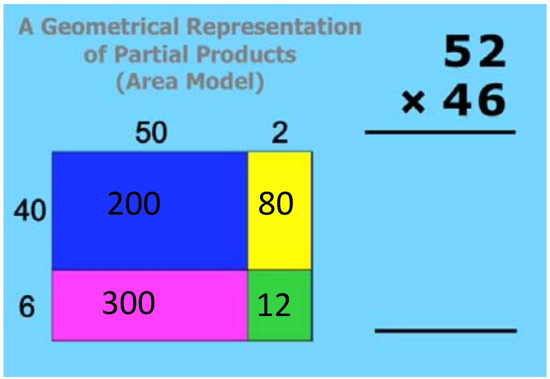

A Geometrical Representation of Partial Products (Area Model)

$$\begin{array}{r} 52 \\ \times\ 46 \\ \hline \\ \hline \end{array}$$

Area Model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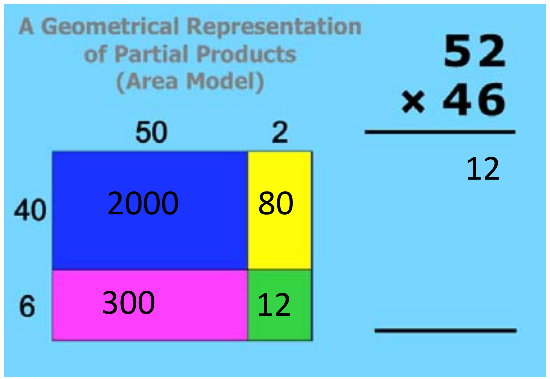
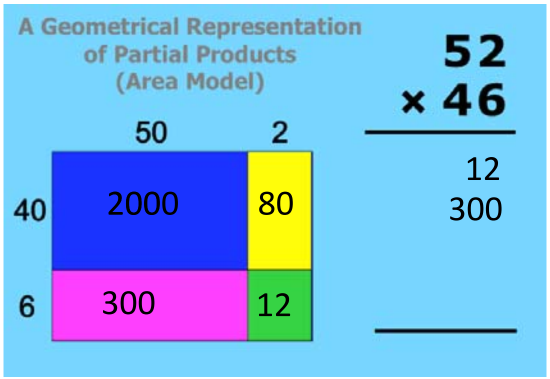

A Geometrical Representation of Partial Products (Area Model)

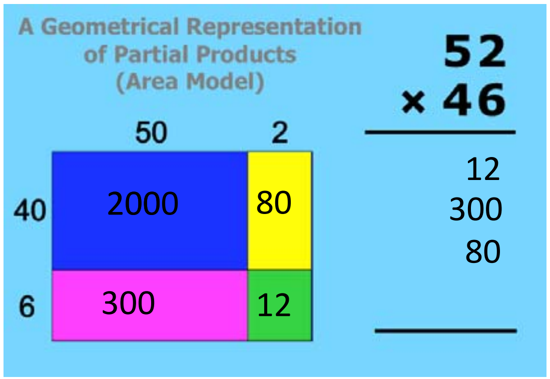
$$\begin{array}{r} 52 \\ \times\ 46 \\ \hline 12 \\ \hline \end{array}$$

Area Model


A Geometrical Representation of Partial Products (Area Model)

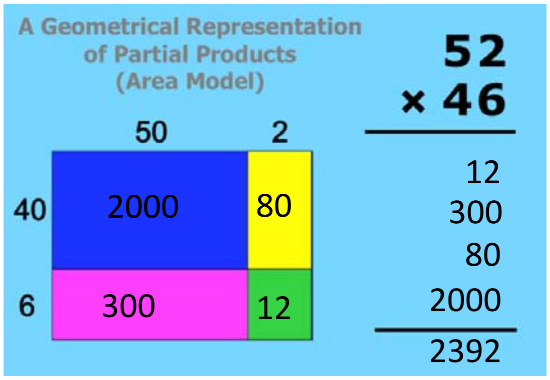
$$\begin{array}{r} 52 \\ \times\ 46 \\ \hline 12 \\ 300 \\ \hline \end{array}$$

Area Model

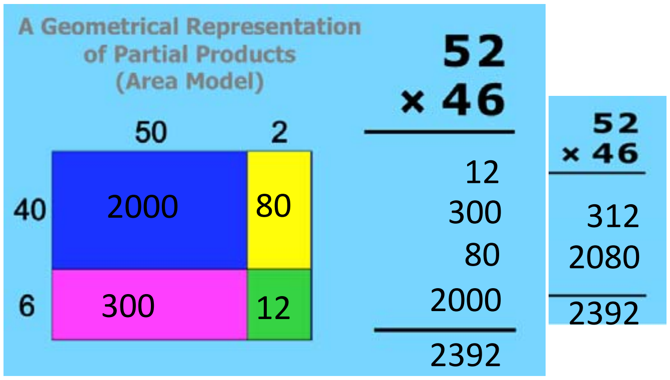

A Geometrical Representation of Partial Products (Area Model)

$$\begin{array}{r} 52 \\ \times\ 46 \\ \hline 12 \\ 300 \\ 80 \\ \hline \end{array}$$

Area Model



Area Model



Handout #1

Area Model

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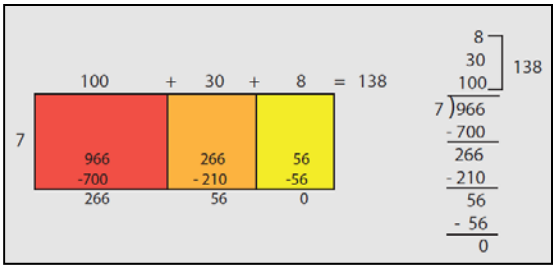
Grade 5



Handout #1

Area Model

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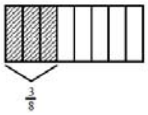
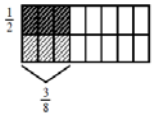
Fractions

1/2 x 3/8

Step 1: Draw a unit rectangle and divide it into 8 pieces vertically. Lightly shade 3 of those pieces. Label it 3/8 .



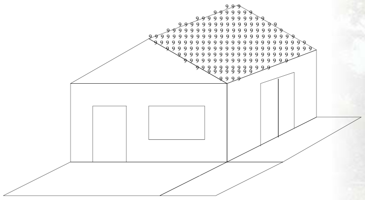
Step 2: Use a horizontal line and divide the unit rectangle in half. Darkly shade 1/2 of 3/8 and label it.



Handout #2

Area Model

Context

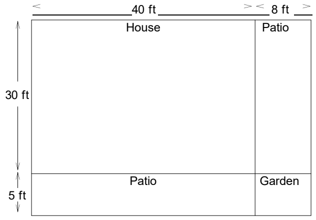


Area Model

Image (CC) NickKarl @Flickr

Building on partial products

Given the dimensions of Peter's house and patios, find the area of the house, each patio, and garden.



Area Model

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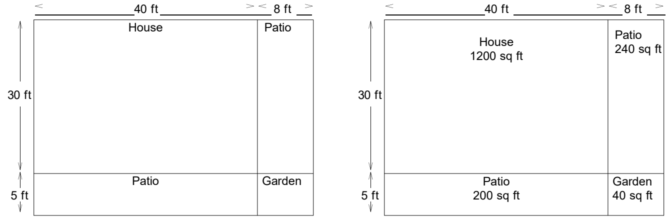


40 ft
8 ft
30 ft
5 ft
House
Patio
Patio
Garden

40 ft
8 ft
30 ft
5 ft
House
1200 sq ft
Patio
240 sq ft
Patio
200 sq ft
Garden
40 sq ft

What is the total area of the house, patios, and garden? Can you find it more than one way?

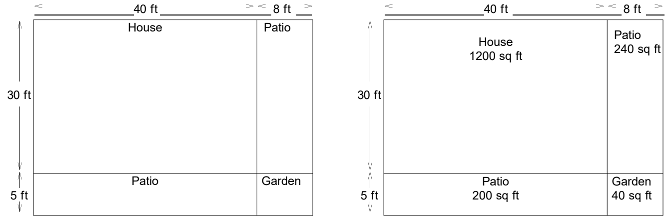
As a sum:

As a product:

Area Model

Building on partial products

Given the dimensions of Peter's house and patios, find the area of the house, each patio, and garden.



40 ft
8 ft
30 ft
5 ft
House
Patio
Patio
Garden

40 ft
8 ft
30 ft
5 ft
House
1200 sq ft
Patio
240 sq ft
Patio
200 sq ft
Garden
40 sq ft

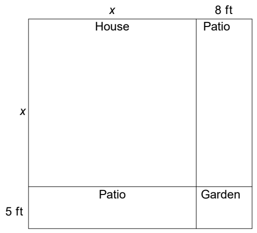
What is the total area of the house, patios, and garden? Can you find it more than one way?

As a sum: $1200 + 240 + 200 + 40 = 1680$ sq ft

As a product: $(40 + 8)(30 + 5) = (48)(35) = 1680$ sq ft

Area Model

Peter's friend Lisa wants to have patios and a garden, too. Peter knows Lisa's house is square, but doesn't know how big, so he just labels the length and width of Lisa's house x.



x
8 ft
x
5 ft
House
Patio
Patio
Garden

How can you write the area of the house?

How can you write the area of each patio?

What is the area of the garden?

How can you write the total area of the house, patios, and garden? Is there more than one way?

As a sum:

As a product:

Area Model

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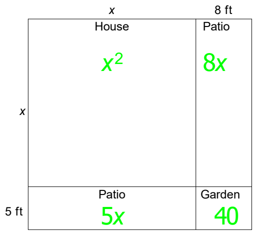


x
8 ft
x
5 ft
House
x^2
Patio
$8x$
Patio
$5x$
Garden
40

How can you write the area of the house?
$x \cdot x = x^2$ sq ft

How can you write the area of each patio?
$8 \cdot x = 8x$ sq ft and $5 \cdot x = 5x$ sq ft

What is the area of the garden?
$5 \cdot 8 = 40$ sq ft

How can you write the total area of the house, patios, and garden? Is there more than one way?

As a sum: $x^2 + 13x + 40$ sq ft

As a product: $(x + 8)(x + 5)$ sq ft

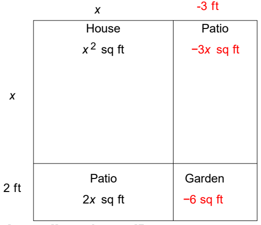
Area Model

Models of to *models for*

Sometimes we **want** to transition away from less-formal contexts and models. Other times we **have** to.

Consider the multiplication of $(x + 2)(x - 3)$:

In context:



x
-3 ft
x
2 ft
House
x^2 sq ft
Patio
$-3x$ sq ft
Patio
$2x$ sq ft
Garden
-6 sq ft

(Not "realistic")

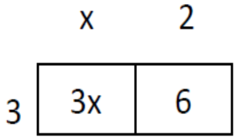
With Algebra Tiles:

(Still useful)

Area Model

Let's use the area model to find this product:

$3(x+2)$.



x
2
3
3x
6

Area Model

Now let's try $2x^2(4x+7)$.

This looks more difficult, but it works exactly the same way.

	$4x$	7
$2x^2$	$8x^3$	$14x^2$

$$2x^2(4x+7)=2x^2\cdot 4x+2x^2\cdot 7$$
$$=8x^3+14x^2$$

Handout #3

Area Model

We can use the same strategy to multiply binomials.

Let's create an area model for $(x+3)(x+2)$.

What dimensions do we need for our area model? Why?

	x	2
x	x^2	2x
3	3x	6

$x\cdot x=x^2$

$x\cdot 2=2x$

$3\cdot x=3x$

$3\cdot 2=6$

$$x^2+2x+3x+6=x^2+5x+6$$

Area Model

Now find the product $(x^2-4)(x+3)$.

What do you notice that is the same as before?

What do you notice that is different?

	x^2	-4
x	x^3	-4x
3	$3x^2$	-12

$$(x^2-4)(x+3)=x^3+3x^2-4x-12$$

Handout #4

Area Model

Models of to models for

Now consider the multiplication of $(x^2 + 4x + 1)(3x + 4)$

With Algebra Tiles:

Dimensions?

With Box/Table:

	x^2	$+4x$	$+1$
$3x$	$3x^3$	$12x^2$	$3x$
$+4$	$4x^2$	$16x$	4

What role should the FOIL strategy play in this progression?

Area Model

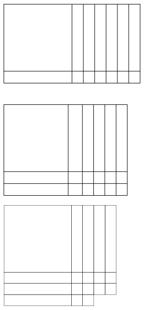
What are the dimensions?

Robin wants patios and a garden next to his house, arranged in a rectangle the same way Peter and Lisa have theirs arranged. Robin has $x^2 + 7x + 10$ sq ft of space. Use Algebra Tiles to model Robin's house, patios, and gardens.

Area Model

What are the dimensions?

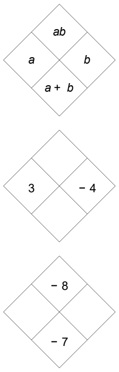
Robin wants patios and a garden next to his house, arranged in a rectangle the same way Peter and Lisa have theirs arranged. Robin has $x^2 + 7x + 10$ sq ft of space. Use Algebra Tiles to model Robin's house, patios, and gardens.

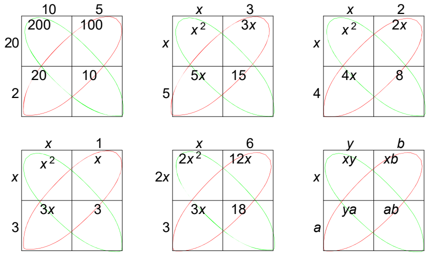


Area Model

Which arrangement makes a rectangle? What are its dimensions?

Models for grouping x :





- What is the pattern here?
- How do you guide this re-invention?

Area Model

Factoring problem string

① $x^2 + 4x + 3$
② $x^2 + 6x + 8$
③ $2x^2 + 7x + 3$
④ $8x^2 + 22x + 15$
⑤ $4x^2 + 16x + 16$
⑥ $x^2 + 6x + 4$

Problem #4 with box/table:

$8x^2$	
	15

If students are still dependent on Algebra Tiles, where do you expect them to struggle? Why?

What happened with problem #6?

Area Model

Factoring problem string

① $x^2 + 4x + 3$
② $x^2 + 6x + 8$
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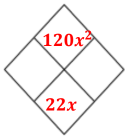

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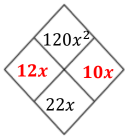

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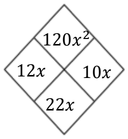
⑥ $x^2 + 6x + 4$

Problem #4 with box/table:

$8x^2$	**$10x$**
$12x$	15



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Area Model

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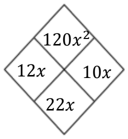
⑥ $x^2 + 6x + 4$

Problem #4 with box/table:

$4x$	**$+5$**
$8x^2$	$10x$
$12x$	15



If students are still dependent on Algebra Tiles, where do you expect them to struggle? Why?

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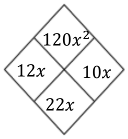
Area Model

Factoring problem string

1. $x^2 + 4x + 3$
2. $x^2 + 6x + 8$
3. $2x^2 + 7x + 3$
4. $8x^2 + 22x + 15$
5. $4x^2 + 16x + 16$
6. $x^2 + 6x + 4$

Problem #4 with box/table:

	$4x$	$+5$
$2x$	$8x^2$	$10x$
$+3$	$12x$	15



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Area Model

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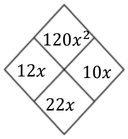
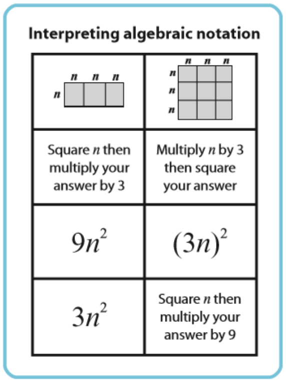


(2x+3)(4x+5)

If students are still dependent on Algebra Tiles, where do you expect them to struggle? Why?
What happened with problem #6?

Area Model



Interpreting algebraic notation

Square n then multiply your answer by 3	Multiply n by 3 then square your answer
$9n^2$	$(3n)^2$
$3n^2$	Square n then multiply your answer by 9

http://maths-no-fear.wikispaces.com/file/view/Malcolm+Swan-Improving+learning+in+mathematics-challenges+and+strategies.pdf

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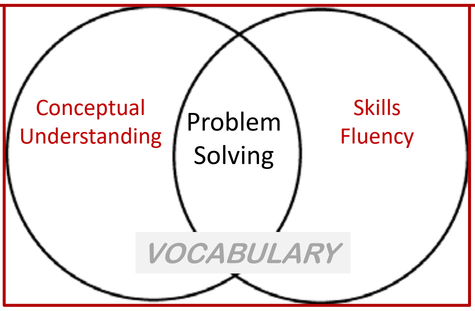
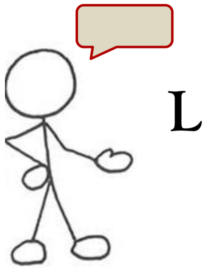
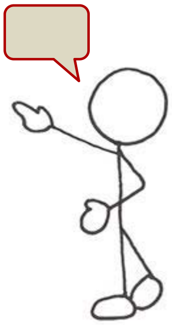
Both conceptual understanding and skill fluency are necessary for students to become effective problem solvers.





Let’s
TALK



Discourse

Begin with questions that:

- Are challenging without being overwhelming
- Require students to think
- Are focused not only on right answers but also on sparking thought
- Invite a discussion or debate about the math
- Challenge students to show their thinking symbolically
- Focus on multiple ways of explaining

Discourse

Examples of questions that can start discourse:

- The answer is 5. What is the question?
- Think of a number made up of tens and ones. Switch the number of tens and the number of ones. What happens to the value of your number? Why?
- How is adding 42+38 like adding 52+28? How is it different?
- You can represent a multiplication using only base ten rods. What numbers might you have multiplied?
- An expression involving the variable x has the value 10 when $x = 4$. What could the expression be?
- You are measuring 10 things using inches. When you make a line plot, there are a few tall lines of x's, then a break, and then a few more tall lines of x's. What might you be measuring and what might the lengths be?

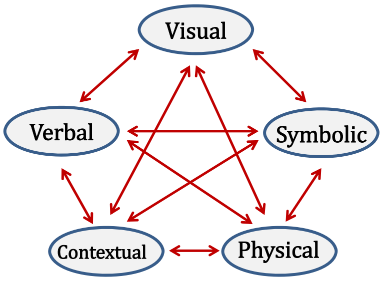
Discourse

Asking the right questions the right way…

- No Hands Up
- All Students Respond
- Planning

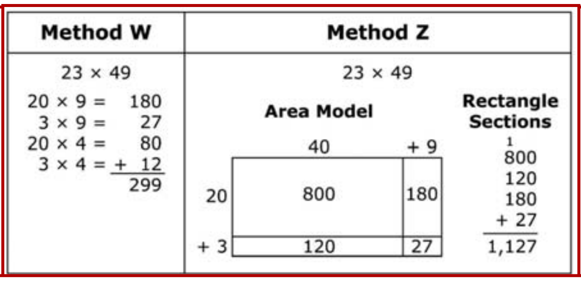
Discourse

According to NCTM's recent publication, Principles to Action, "Effective teaching helps students make connections among visual, symbolic, verbal, contextual, and physical mathematical representations."



Making Connections

Pablo solved a multiplication problem using two different methods. He made a mistake in either Method W or Method Z.



Method W	Method Z
23 × 49 20 × 9 = 180 3 × 9 = 27 20 × 4 = 80 3 × 4 = + 12 299	23 × 49 Area Model 40 + 9 20: 800, 180 + 3: 120, 27 Rectangle Sections 1 800 120 180 + 27 1,127

Identify the method where Pablo made a mistake and explain what he should do to correct it.

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Thank you for your attendance.

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