LOUISIANA ASSOCIATION of TEACHERS of MATHEMATICS

LATM Presents - Essential Math Models that Support LSSM Instruction: Utilizing Area Models in High School Mathematics to Deepen Students' Understanding

Students today are being asked to demonstrate certain key skills in mathematics:

- **Demonstrate understanding of the math concept, not just the procedure**
- **Apply their understanding to real world examples**
- **Use accurate procedures and skills to answer questions**
- **Demonstrate mathematical reasoning by explaining, justifying, or critiquing with precision**

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By the end of the session, participants should:

- Understand the value of new models for helping students develop number sense
- Analyze the progressions of the area model

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Area Model

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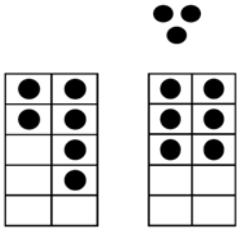
PreK & K

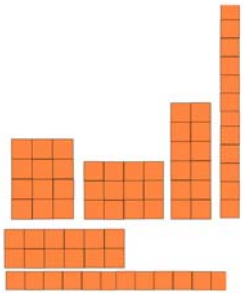
•Fill your ten frame with counters. How many counters are there in all? How many rows do we have? How many are in each row?
•Here are 6 colored tiles. Make a rectangle with your tiles. Some will make a 2x3, some a 3x2, some a 1x6, etc. (K.NBT.A.1)



Area Model

Grades 1-2

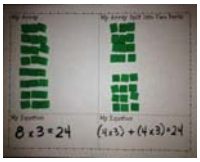
•Continue work with ten frames.
•Here are 12 colored tiles. Make a rectangle with your tiles. Label the row and columns. How many are in each row? If it's a 2x6, can we add 6 and 6 to find our total? If it's a 3x4, can we add 4 and 4 and 4 to get our total? (1.G.A.1, 2.OA.C.4)

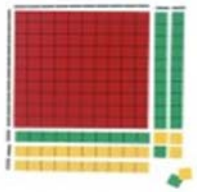


Area Model

Grade 3

•Continue work with colored tiles and arrays.
•Introduce linear pieces. Build a 12x13 first with the linear pieces, then complete the model with the base ten blocks. (3.OA.B.5, 3.MD.C.7, 4.NBT.B.5)
•Have students draw models to represent the linear and area pieces and relate the modules to the standard algorithm.

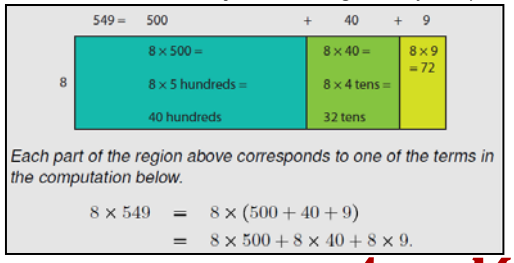




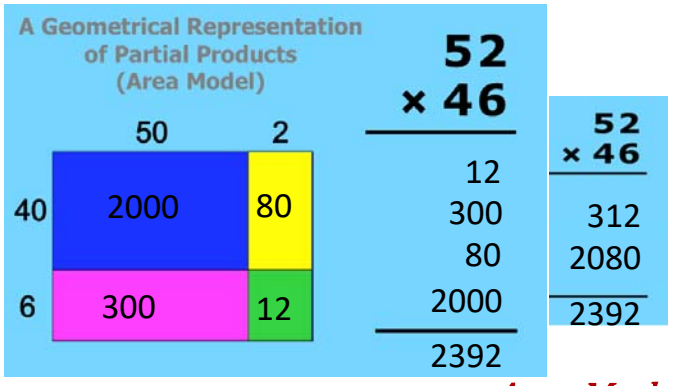
Area Model

Grade 4

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.



Area Model



Area Model

Pablo solved a multiplication problem using two different methods. He made a mistake in either Method W or Method Z.

Method W

23 × 49
20 × 9 = 180
3 × 9 = 27
20 × 4 = 80
3 × 4 = + 12
299

Method Z

23 × 49

Area Model

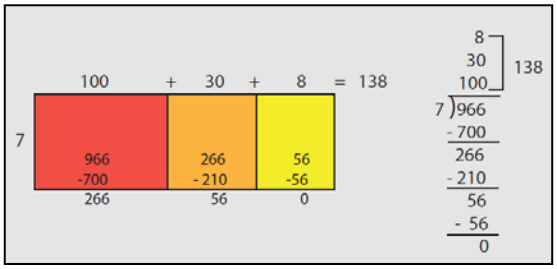
	40	+ 9
20	800	180
+ 3	120	27

Rectangle Sections

1
800
120
180
+ 27
1,127

Identify the method where Pablo made a mistake and explain what he should do to correct it.

Grade 5



Area Model

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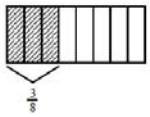
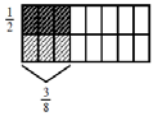
Fractions

1/2 x 3/8

Step 1: Draw a unit rectangle and divide it into 8 pieces vertically. Lightly shade 3 of those pieces. Label it 3/8 .



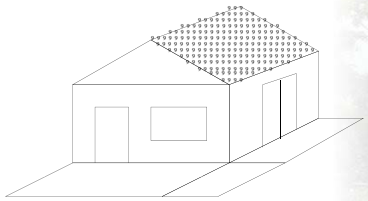
Step 2: Use a horizontal line and divide the unit rectangle in half. Darkly shade 1/2 of 3/8 and label it.

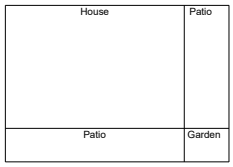


Area Model

Leading the way to excellence in mathematics teaching and learning in Louisiana

Context



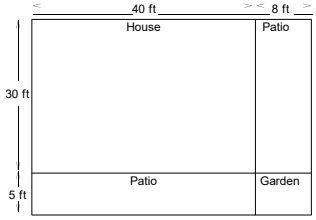


Area Model

Image (CC) NickKarl @Flickr

Building on partial products

Given the dimensions of Peter's house and patios, find the area of the house, each patio, and garden.



Area Model

Building on partial products

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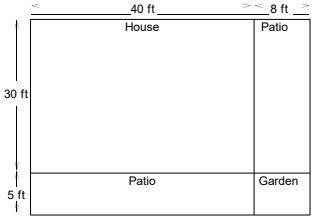
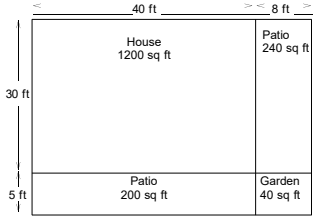




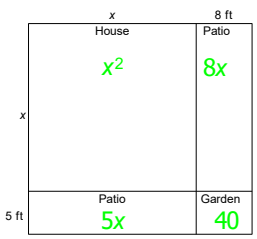
What is the total area of the house, patios, and garden? Can you find it more than one way?

As a sum: 1200 + 240 + 200 + 40 = 1680 sq ft

As a product: (40 + 8)(30 + 5) = (48)(35) = 1680 sq ft

Area Model

Peter's friend Lisa wants to have patios and a garden, too. Peter knows Lisa's house is square, but doesn't know how big, so he just labels the length and width of Lisa's house x.



How can you write the area of the house?
$x \cdot x = x^2$ sq ft

How can you write the area of each patio?
$8 \cdot x = 8x$ sq ft and $5 \cdot x = 5x$ sq ft

What is the area of the garden?
$5 \cdot 8 = 40$ sq ft

How can you write the total area of the house, patios, and garden? Is there more than one way?

As a sum: $x^2 + 13x + 40$ sq ft

As a product: $(x + 8)(x + 5)$ sq ft

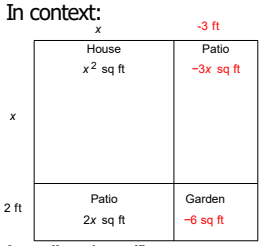
Area Model

Models of to *Models for*

Sometimes we **want** to transition away from less-formal contexts and models. Other times we **have** to.

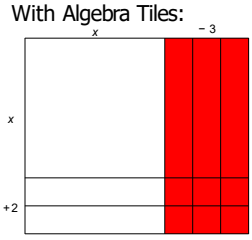
Consider the multiplication of $(x + 2)(x - 3)$:

In context:



(Not "realistic")

With Algebra Tiles:



(Still useful)

Area Model

Let's use the area model to find this product: $3(x+2)$.

	x	2
3	3x	6

Area Model

Now let's try $2x^2(4x+7)$.

This looks more difficult, but it works exactly the same way.

	$4x$	7
$2x^2$	$8x^3$	$14x^2$

$$2x^2(4x+7) = 2x^2 \cdot 4x + 2x^2 \cdot 7$$
$$= 8x^3 + 14x^2$$

Handout #1

Area Model

We can use the same strategy to multiply binomials.

Let's create an area model for $(x+3)(x+2)$.

What dimensions do we need for our area model? Why?

	x	2
x	x^2	2x
3	3x	6

$x \cdot x = x^2$

$x \cdot 2 = 2x$

$3 \cdot x = 3x$

$3 \cdot 2 = 6$

$x^2 + 2x + 3x + 6 = x^2 + 5x + 6$

Area Model

Now find the product $(x^2-4)(x+3)$.

What do you notice that is the same as before?

What do you notice that is different?

	x^2	–4
x	x^3	-4x
3	$3x^2$	-12

$$(x^2-4)(x+3) = x^3 + 3x^2 - 4x - 12$$

Handout #2

Area Model

Models of to *models for*

Now consider the multiplication of $(x^2 + 4x + 1)(3x + 4)$

With Algebra Tiles:

Dimensions?

With Box/Table:

	x^2	$+4x$	$+1$
$3x$	$3x^3$	$12x^2$	$3x$
$+4$	$4x^2$	$16x$	4

What role should the FOIL strategy play in this progression?

Area Model

What are the dimensions?

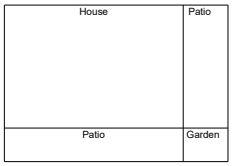
Robin wants patios and a garden next to his house, arranged in a rectangle the same way Peter and Lisa have theirs arranged. Robin has $x^2 + 7x + 10$ sq ft of space. Use Algebra Tiles to model Robin's house, patios, and gardens.

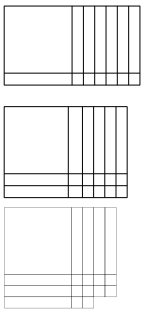
House	Patio
Patio	Garden

Area Model

What are the dimensions?

Robin wants patios and a garden next to his house, arranged in a rectangle the same way Peter and Lisa have theirs arranged. Robin has $x^2 + 7x + 10$ sq ft of space. Use Algebra Tiles to model Robin's house, patios, and gardens.

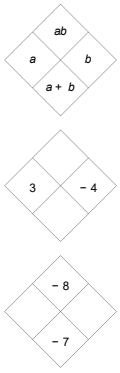


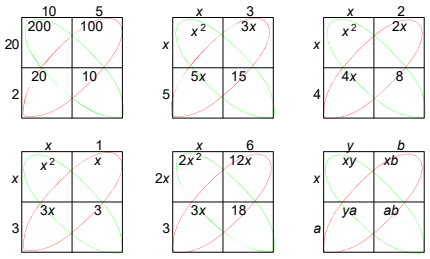


Area Model

Which arrangement makes a rectangle? What are its dimensions?

Models for grouping x :





- What is the pattern here?
- How do you guide this re-invention?

Area Model

Factoring problem string

1. $x^2 + 4x + 3$
2. $x^2 + 6x + 8$
3. $2x^2 + 7x + 3$
4. $8x^2 + 22x + 15$
5. $4x^2 + 16x + 16$
6. $x^2 + 6x + 4$

Problem #4 with box/table:

$8x^2$	
	15

If students are still dependent on Algebra Tiles, where do you expect them to struggle? Why?
What happened with problem #6?

Area Model

Factoring problem string

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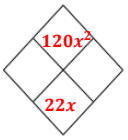

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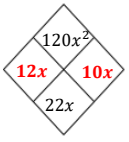

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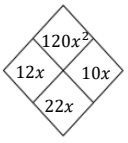
5. $4x^2 + 16x + 16$
6. $x^2 + 6x + 4$

Problem #4 with box/table:

$8x^2$	$10x$
$12x$	15



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What happened with problem #6?

Area Model

Factoring problem string

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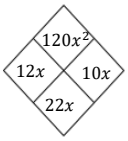
3. $2x^2 + 7x + 3$
4. $8x^2 + 22x + 15$
5. $4x^2 + 16x + 16$
6. $x^2 + 6x + 4$

Problem #4 with box/table:

	$4x$	$+5$
$2x$	$8x^2$	$10x$
$+3$	$12x$	15



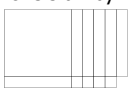
(2x+3)(4x+5)

If students are still dependent on Algebra Tiles, where do you expect them to struggle? Why?
What happened with problem #6?

Area Model

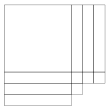
What if we can't make a rectangle?

If you're trying to solve $x^2 + 6x + 4 = 0$, which of these equivalencies offers a way forward?



$x^2 + 6x + 4 \Longleftrightarrow (x + 5)(x + 1) - 1$

$x^2 + 6x + 4 \Longleftrightarrow (x + 4)(x + 2) - 4$



$x^2 + 6x + 4 \Longleftrightarrow (x + 3)^2 - 5$

Area Model

Completing the square of $x^2 + 6x + 4$

x	x
x	x^2	?
	?	

Area Model

Completing the square of $x^2 + 6x + 4$

x	x
x	x^2	?
	?	

x	x	$+3$
x	x^2	$3x$
$+3$	$3x$	

Area Model

Completing the square of $x^2 + 6x + 4$

x	x
x	x^2	?
	?	

x	x	$+3$
x	x^2	$3x$
$+3$	$3x$	

x	x	$+3$
x	x^2	$3x$
$+3$	$3x$	9

-5

$(x + 3)^2 - 5 = 0$

Area Model

Completing the square of $x^2 + 6x + 4$

x	x
x	x^2	?
	?	

x	x	$+3$
x	x^2	$3x$
$+3$	$3x$	

x	x	$+3$
x	x^2	$3x$
$+3$	$3x$	9

-5

$$(x + 3)^2 - 5 = 0$$
$$(x + 3)^2 = 5$$
$$\sqrt{(x + 3)^2} = \pm\sqrt{5}$$
$$x + 3 = \pm\sqrt{5}$$
$$x = -3 \pm \sqrt{5}$$

What role should the quadratic formula play in this progression?

Area Model

Completing the square problem string

1. $x^2 + 4x + 1$
2. $x^2 - 2x + 5$
3. $x^2 + 3x + 4$
4. $2x^2 + 8x + 2$
5. $2x^2 + 5x - 4$

Problem #3 with box/table:

x^2	

If students are still dependent on Algebra Tiles, where do you expect them to struggle? Why?

Area Model

Completing the square problem string

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2. $x^2 - 2x + 5$
3. $x^2 + 3x + 4$
4. $2x^2 + 8x + 2$
5. $2x^2 + 5x - 4$

Problem #3 with box/table:

x^2	3/2 x
3/2 x	

If students are still dependent on Algebra Tiles, where do you expect them to struggle? Why?

Area Model

Completing the square problem string

1. $x^2 + 4x + 1$
2. $x^2 - 2x + 5$
3. $x^2 + 3x + 4$
4. $2x^2 + 8x + 2$
5. $2x^2 + 5x - 4$

Problem #3 with box/table:

x^2	3/2 x
3/2 x	**9/4**

If students are still dependent on Algebra Tiles, where do you expect them to struggle? Why?

Area Model

Completing the square problem string

1. $x^2 + 4x + 1$
2. $x^2 - 2x + 5$
3. $x^2 + 3x + 4$
4. $2x^2 + 8x + 2$
5. $2x^2 + 5x - 4$

Problem #3 with box/table:

x^2	3/2 x
3/2 x	9/4

+7/4

If students are still dependent on Algebra Tiles, do you expect them to struggle? Why?

Area Model

Completing the square problem string

1. $x^2 + 4x + 1$
2. $x^2 - 2x + 5$
3. $x^2 + 3x + 4$
4. $2x^2 + 8x + 2$
5. $2x^2 + 5x - 4$

Problem #3 with box/table:

x^2	3/2 x
3/2 x	9/4

+7/4

$$(x + \frac{3}{2})^2 + \frac{7}{4}$$

If students are still dependent on Algebra Tiles, do you expect them to struggle? Why?

Area Model

Dividing polynomials

The last problem in multiplying polynomials was $(x^2 + 4x + 1)(3x + 4)$:

	x^2	$+4x$	$+1$
$3x$	$3x^3$	$12x^2$	$3x$
$+4$	$4x^2$	$16x$	4

Knowing the patterns of like terms, can you fill in what's missing if the product is $6x^3 + 17x^2 + 16x + 6$?

$6x^3$		$4x$
$9x^2$	$12x$	

What are the factors (dimensions) of the second box?

Area Model

Dividing polynomials

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	$3x^2$		
	$6x^3$		$4x$
	$9x^2$	$12x$	

What are the factors (dimensions) of the second box?

Area Model

Dividing polynomials

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$3x$	$3x^3$	$12x^2$	$3x$
$+4$	$4x^2$	$16x$	4

Knowing the patterns of like terms, can you fill in what's missing if the product is $6x^3 + 17x^2 + 16x + 6$?

	$3x^2$		
$2x$	$6x^3$		$4x$
$+3$	$9x^2$	$12x$	

What are the factors (dimensions) of the second box?

Area Model

Dividing polynomials

The last problem in multiplying polynomials was $(x^2 + 4x + 1)(3x + 4)$:

	x^2	$+4x$	$+1$
$3x$	$3x^3$	$12x^2$	$3x$
$+4$	$4x^2$	$16x$	4

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Area Model

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The last problem in multiplying polynomials was $(x^2 + 4x + 1)(3x + 4)$:

	x^2	$+4x$	$+1$
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Knowing the patterns of like terms, can you fill in what's missing if the product is $6x^3 + 17x^2 + 16x + 6$?

	$3x^2$	$+4x$	$+2$
$2x$	$6x^3$	$8x^2$	$4x$
$+3$	$9x^2$	$12x$	6

What are the factors (dimensions) of the second box?

$(3x^2 + 4x + 2)(2x + 3)$

Area Model

Knowing patterns of like terms, can you fill in what's missing if the product is $x^3 + 8x^2 + 19x + 20$?

x	x^3		
$+5$			

How is this different than asking students to divide $\frac{x^3+8x^2+19x+20}{x+5}$?

Handout #3 & #4

Area Model

Knowing patterns of like terms, can you fill in what's missing if the product is $x^3 + 8x^2 + 19x + 20$?

	x^2		
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How is this different than asking students to divide $\frac{x^3+8x^2+19x+20}{x+5}$?

Handout #3 & #4

Area Model

Knowing patterns of like terms, can you fill in what's missing if the product is $x^3 + 8x^2 + 19x + 20$?

	x^2		
x	x^3		
$+5$	$5x^2$		

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Handout #3 & #4

Area Model

Knowing patterns of like terms, can you fill in what's missing if the product is $x^3 + 8x^2 + 19x + 20$?

	x^2		
x	x^3	$3x^2$	
$+5$	$5x^2$		

How is this different than asking students to divide $\frac{x^3+8x^2+19x+20}{x+5}$?

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Area Model

Knowing patterns of like terms, can you fill in what's missing if the product is $x^3 + 8x^2 + 19x + 20$?

	x^2	$+3x$	
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	x^2	$+3x$
x	x^3	$3x^2$
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$+5$	$5x^2$	$15x$	20

How is this different than asking students to divide $\frac{x^3+8x^2+19x+20}{x+5}$?

Handout #3 & #4

Area Model

Dividing polynomials problem string

1. $\frac{x^3+8x^2+23x+24}{x+3}$
2. $\frac{4x^3+11x^2+11x+10}{x+2}$
3. $\frac{3x^4+17x^3+10x^2+x+5}{x+5}$
4. $\frac{x^2+4x+6}{x+5}$

What role should long and synthetic division play in this progression?

Area Model

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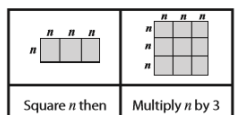
Interpreting algebraic notation



Square n then multiply your answer by 3	Multiply n by 3 then square your answer
$9n^2$	$(3n)^2$
$3n^2$	Square n then multiply your answer by 9

http://maths-no-fear.wikispaces.com/file/view/Malcolm+Swan-Improving+learning+in+mathematics-challenges+and+strategies.pdf

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- National Council of Teachers of Teachers of Mathematics (NCTM). 2014. *Principles to Action: Ensuring Mathematical Success for All.* Reston, VA: NCTM.
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Thank you for your attendance.

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