Advice for Instruction | 3. Patterns in proportional relationships | Prepare instruction

Prepare instruction

Goals and objectives Topic at a glance Prerequisite skills Resources Language support

Goals and objectives

After finishing the topic **Patterns in proportional relationships**, students will be able to

- decide whether two quantities are in a proportional relationship by looking at a table or graph;
- identify the constant of proportionality, or unit rate, in tables and graphs;
- represent proportional and non-proportional relationships with equations;
- explain what a point (x,y) on the graph of a proportional relationship means;
- solve word problems leading to equations of the form $px + q = r$.

Topic at a glance

This topic, **Patterns in proportional relationships**, is designed to build on students' prior knowledge involving ratios, rates, and proportional reasoning developed in previous topics. Students will learn how to analyze relationships using tables and graphs, and develop algebraic equations that describe the relationships. Students will also explore various patterns, developing skills to predict future iterations of a pattern by developing equations. The ability to analyze relationships will be valuable in future topics as students continue to explore relationships using multiple representations.

This topic is designed to support ten 45-minute blocks of instruction and formative assessment. You may need more or less, depending on your students' familiarity with proportionality, function analysis, and pattern recognition.

This topic provides multiple opportunities for students to engage in literacy tasks. Throughout the topic are opportunities for extended written responses that support a culminating literacy task in Blocks 8 and 9.

Description	Resources	Suggested assignment
Block 1 introduces the graphing of proportional ratios using a paint mixing scenario that students have encountered in a previous topic. Students will compare the graphs of different paint mixture relationships.	*Overview* Block 1 SAS	Block 1 SAS Q2a-c and 3a-c
Block 2 introduces building a table, graph, and equation for proportional relationships.	*Exploring* "Proportional and non-proportional relationships"	*Guided practice* p1-2 *More practice* p1-2

	p1-4 Block 2 SAS	Block 2 SAS Q6a-c
Block 3 guides students in analyzing the differences between proportional and non-proportional relationships in table and graph form.	*Exploring* "Proportional and non-proportional relationships" p5-9 Block 3 SAS	*Guided practice* p3-4 *More practice* p3-5
Block 4 helps students recognize the constant of proportionality, or unit rate, in tables and graphs of proportional relationships.	*Exploring* "Proportional and non-proportional relationships" p10-12 Block 4 SAS	*Guided practice* p5-6 *More practice* p6 Block 4 SAS Q3a-d, 4a-c, and 5a-c
Block 5 gives students an opportunity to describe a relationship using an expression. The emphasis is on the idea that multiple expressions can be used to represent the same pattern.	*Exploring* "Identifying proportional relationships" p1-4 Block 5 SAS	*Guided practice* p7-8 *More practice* p7-9
Block 6 allows student to practice creating generalized algebraic rules from expressions they have developed to describe a specific iteration of a pattern.	*Exploring* "Identifying proportional relationships" p5-8 Block 6 SAS	*Guided practice* p9-10 *More practice* p10 Block 6 SAS Q5a-c and 6a-b
Block 7 gives students time to practice the skills they have developed in Blocks 5 and 6 through applicable reinforcement activities.	Block 7 SAS *Summary* Literacy Task	Literacy Task Block 7 SAS Q1a-g, 2a-f, 3a-c, and 4a-g
Block 8 allows students to use their experiences with completing a table and creating an algebraic rule for a pattern to explore an extended task. This block introduces the culminating literacy task for this topic.	*Mars Task: Tiling squares* Literacy Task Literacy Task Rubric	*Guided practice* p11-15 See *Deliver instruction* Block 8
Block 9 provides time for students to continue the development and revision of their written essay.	Literacy Task Literacy Task Rubric	See *Deliver instruction* Block 9
Block 10 provides time for a topic level assessment.	*Automatically scored* *Constructed response*	None

Prerequisite skills

To be successful with the material in this topic, students should already be able to:

- Explain what makes a relationship proportional

- Describe a relationship using ratios and rates
- Simplify a ratio
- Identify a unit rate
- Map a coordinate pair on a coordinate grid

Resources

LESSON RESOURCES

- Computer with projection device
- Color tiles (Block 5)
- Colored pencils (Block 5)
- Graph paper (Block 5)

RELATED RESOURCES

- Agile Mind **Glossary**

Language support

All students should become proficient in using the core vocabulary of ***factor***, ***constant***, ***variable***, ***coefficient***, ***scale factor***, ***ratio***, ***proportional reasoning***, ***rate***, ***unit rate***, ***constant of proportionality***, ***coordinate pair***, ***proportional linear relationship***, and ***non-proportional linear relationship***.

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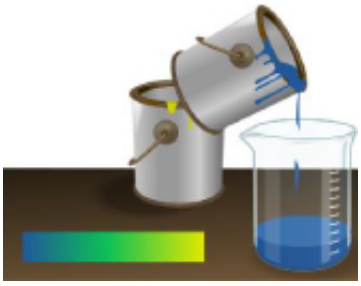
Deliver instruction (Block 1)

Agile Mind materials Opening the lesson Framing questions Lesson activities Suggested assignment

Agile Mind materials

Patterns in proportional relationships:

- *Overview*
- Block 1 Student Activity Sheet: Teacher, Student



Opening the lesson

Ask students to consider how they've used proportional reasoning in the past to solve problems. Use the framing questions to encourage a discussion about their prior experiences.

Framing questions

- What is an example of a ratio?
- How have you used proportional reasoning in previous topics?
- What is an example of a proportion?
- What are two things you might compare with a ratio?

Lesson activities

Overview

Page 1

- This animation provides examples of situations that call for proportional reasoning. Click on each image to view the example. Each example is taken from a scenario in which students previously explored ratios, rates, and proportional reasoning.
- The photograph image reminds students that ratios and proportional reasoning can be used to determine the dimensions of the smaller picture. Ask students to finish the problem to find the missing dimension. Ask what scale factor was used.
- The gas can image provides a visual for calculating the mileage for a vehicle using a unit rate.
- The map image shows how to use the scale of a map to calculate distance. Have students find the distance between two hypothetical cities that are 12 inches apart.
- The statue animation shows how to use ratios in similar triangles to find the height of a distant object. Have students find the height of the statue using proportional reasoning.

Page 2

- This page provides some introduction to the big ideas in this topic, letting students know that they will be working with algebraic rules. The image shows algebraic rules students have seen previously. Discuss each algebraic rule or formula with students, asking what the variables in each one represent. Provide students with some example problems or sketches for the various formulas, or have students create their own scenarios and sketches.

Page 3

- Explore the paint mixing animation. Students may remember this situation from an earlier topic. Play panels 1 and 2, encouraging students to discuss what they remember about the use of ratios in regard to paint mixes. Have students list 5 other combinations of blue and yellow paint that will make the same shade of green. **[SAS, question 1a]**
- Panel 3 directs students to graph the points they listed on the table after providing two examples. Ask students to describe any patterns they see in the graph. **[SAS, question 1b]** Also, draw students' attention to the point (1, 2). What is the significance of this point?
- **Classroom strategy.** Students will determine equivalent ratios and plot the associated points on a graph. Review this process, if necessary. Encourage students to use a pencil to plot the points. If the graph a student creates is not linear, check to see that the initial ratios are equivalent and that the student is accurately plotting points on the graph.
- Panel 4 introduces the vocabulary of ***proportional relationship***.
- **Classroom strategy.** In previous topics, students used proportional reasoning to generate equivalent ratios and to solve problems. This topic deepens students' understanding of proportional reasoning by engaging them in learning about the unique features of proportional relationships—as one variable changes, another variable changes by a constant factor.
- As appropriate, make connections back to the scenarios on page 1. For example, there is a relationship among the side lengths of photos that are not distorted. There is a relationship between the scale on a map and actual distances between cities. There is a relationship between the miles travelled and the gallons of gas used.

Page 4

- This page provides a preview for the discussion of proportional and non-proportional relationships. The graph shown is an example of two different proportional relationships. Tables, verbal descriptions, and graphs will be used to represent these relationships.

Further questions

- What are some other algebraic rules that you have seen or used in previous topics or courses?

Suggested assignment

- Block 1 Student Activity Sheet, questions 2a-c and 3a-c

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Deliver instruction (Block 3)

Agile Mind materials Opening the lesson Framing questions Lesson activities Suggested assignment

Agile Mind materials

Patterns in proportional relationships:

- *Exploring* "Proportional and non-proportional relationships"
- Block 3 Student Activity Sheet: Teacher, Student

Opening the lesson

Review the concepts students have learned about proportional relationships, including the constant of proportionality.

Framing questions

How can you tell if a relationship is proportional? Knowing what you do about proportional relationships, how might you be able to tell if a relationship is not proportional?

Lesson activities

Exploring "Proportional and non-proportional relationships"

Page 5

- This page introduces the first of two scenarios in which students will compare proportional and non-proportional relationships. Play the animation or ask a student to come to the class computer and slide the pizzas into the oven. Ask students to record the results as each pizza slides into the oven. **[SAS, question 1]** At the end of Panel 1, ask students to share their ideas for an algebraic rule to represent the relationship between the number of pizzas and the cost. Play panels 2-4 so students can verify their thinking. Then play panel 5 and engage students in a brief discussion about how the table, graph and algebraic rule are related. Ask,
  - *How is the price of each pizza represented in each representation?*
  - *Is this relationship proportional? How do you know?*
  - *If the relationship is proportional, what is the constant of proportionality?*
- Have students complete the reinforcement questions. **[SAS, question 2]**
- **Classroom strategy** Encourage students to reason abstractly and quantitatively with question 2 on their Student Activity Sheets. Encourage them to write equations to represent the situations in question 2a and 2b. Some students may not recall how to solve an “undoing” situation with an equation. Encourage them to reason about

the situation in 2b using inverse operations. Formal work with solving equations symbolically will occur in a later topic. Discuss the case in 2c where 0 pizzas are purchased.

Page 6

- The pizza scenario changes so there is a delivery charge added, creating a non-proportional situation. Before playing panel 1, have students think about the tables of values, graph, and algebraic rule and offer suggestions for how the representations will be different for this situation. Then show the rest of the animation, having students record the representations. **[SAS, question 3]**
- Have students answer the extension questions. **[SAS, question 4]** Discuss the case where 0 pizzas are purchased. Does the answer found using the algebraic rule make sense? Why or why not?
- **Classroom strategy** Students may construct a variety of arguments for the zero pizzas situation. This is an opportunity to foreshadow the work they will do in 8th grade with domain and range. There are often situations that are modeled by an algebraic rule, but that only make sense for certain values. In the pizza situation with the delivery charge, only whole number values greater than or equal to 1 make sense for the scenario. Students may also be interested in the practical upper limits of this situation. This presents a great opportunity to reinforce the mathematical practice of constructing viable arguments and critiquing the reasoning of others.

Page 7

- Students are guided through a comparison of the proportional and non-proportional tables and graphs. Discuss each comparison before continuing to the next panel. **[SAS, question 5]**
- **Classroom strategy.** Some students may have difficulty understanding why the non-proportional graph does not start at the origin. Clarify this point before moving on.

Page 8

- Have students work with a partner to answer questions that require further analysis of proportional and non-proportional relationships described by the pizza scenarios. Discuss students' answers as a class. **[SAS, questions 6-7]**

Page 9

- A summary of the differences between proportional and non-proportional relationships is provided, along with a discussion about inputs and outputs as they relate to the variables in these relationships.
- **Classroom strategy.** If students are having difficulty with the idea of inputs and outputs, compare the process column of the table to a machine in which a rule is applied to each input in order to produce the output, using the same process for every row in the table.

Further questions

- How can you tell if a scenario describes a proportional or non-proportional relationship?
- Describe one scenario that has a proportional relationship and one that has a non-proportional relationship.

Suggested assignment

- *Guided practice*, pages 3-4
- *More practice*, pages 3-5