Connecting Proportional Relationships, Lines, and Linear Equations (IT)

Overview

This instructional task requires students to understand the connections between proportional relationships, lines, and linear equations.

Standards

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

8.EE.B.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standards	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.EE.B.5	• 7.RP.A.2	1. What is the slope between the points (2, 3) and (5, 9)? a. 2 2. http://www.illustrativemathematics.org/illustrations/129 3. http://www.illustrativemathematics.org/illustrations/55 4. http://www.illustrativemathematics.org/illustrations/184	• http://www.illustrativemathematics.org/illustrations/104 • http://www.illustrativemathematics.org/illustrations/1186 • http://www.illustrativemathematics.org/illustrations/1526 • http://learnzillion.com/lessonsets/275-graph-interpret-and-compare-proportional-relationships
8.EE.B.6	• 7.G.A.1 • 7.RP.A.2 • 8.G.A.5	1. How can you tell if two triangles are similar? a. Two triangles are similar if a series of transformations can take one triangle to the other one. 2. http://www.illustrativemathematics.org/illustrations/1537	• http://www.illustrativemathematics.org/illustrations/1082 • http://www.illustrativemathematics.org/illustrations/101 • http://www.illustrativemathematics.org/illustrations/1527 • http://learnzillion.com/lessonsets/274-use-similar-triangles-to-explain-why-the-slope-m-is-the-same-between-two-points-on-a-nonvvertical-line-in-the-coordinate-plane

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

Why would I need to know how much fresh fruit costs by the pound? Fresh fruit and vegetables are often sold by the pound. Sometimes they come in already-weighed bundles with the weight and cost indicated. Sometimes you bag your own and weigh it yourself to estimate how much it will cost.

Why do I need to look at prices for the same item packed differently? Sometimes stores offer the same item in different sizes. Each size will have a different price. Oftentimes it is cheaper by the unit to buy a bigger item. Sometimes, however, it is not, so it is important to figure out the unit price when shopping in order to save money.

During the Task:

Make sure that students are being precise when they graph. They should label the axes and indicate the units. Students may need help plotting points.

Students may struggle with proving that the triangles are similar. If needed, briefly review similarity transformations, congruence transformations, and the angle-angle criterion for similarity.

After the Task:

Have students bring a favorite recipe. Then have them go to the store and figure out how much it would cost to make it buying the smallest available containers of their ingredients and the biggest available containers. Then discuss how much of each ingredient would be left after making your recipe using each. Then have them decide which size container they should buy for each item.

Student Instructional Task

You are shopping for bananas.

Option 1:

Your first option at your local grocery store is purchasing 5 lbs. of bananas for $1.45.

Use this information to complete the following:

1. Fill in the following chart.

Pounds	Cost
1	
2	
3	
4	
5	

2. On the same coordinate plane:
   a. Plot the ordered pairs representing 0 pounds and 1 pound. Connect these points with a line segment.
   b. Plot the ordered pairs representing 2 pounds and 4 pounds. Connect these points with a line segment.
   c. Using similar triangles, explain why the slopes of both line segments are the same.
   d. What is the cost per pound of bananas? How does the cost per pound of bananas relate to the slope of your line segments?

3. Write an equation to represent how much money (y) you would spend for (x) pounds of bananas if 5 lbs. of bananas cost $1.45.

Option 2:

Your second option for purchasing bananas at the grocery store is shown in the graph below.

4. Compare the two options available at your grocery store. Which store offers bananas at the cheaper price? Explain your reasoning.

5. You are going shopping for your favorite fresh fruit or vegetable. Either visit a local store or use the Internet to research the price of the fresh fruit or vegetable. Use your research to complete the following:
    a. Create a table to represent the price of your fruit or vegetable for 0-5 pounds purchased.
    b. Create a graph to represent the price of your fruit or vegetable for 0-5 pounds purchased.
    c. Write an equation based on the graph you created.
    d. What does the slope represent in terms of your fruit or vegetable?
    e. Swap your table with one classmate, your graph with a different classmate, and your equation with a third classmate. Whose fruit or vegetable was cheaper? How do you know? Write a short paragraph to summarize all three sets of information.

Instructional Task Exemplar Response

You are shopping for bananas.

<u>Option 1:</u>

Your first option at your local grocery store is purchasing 5 lbs. of bananas for $1.45.

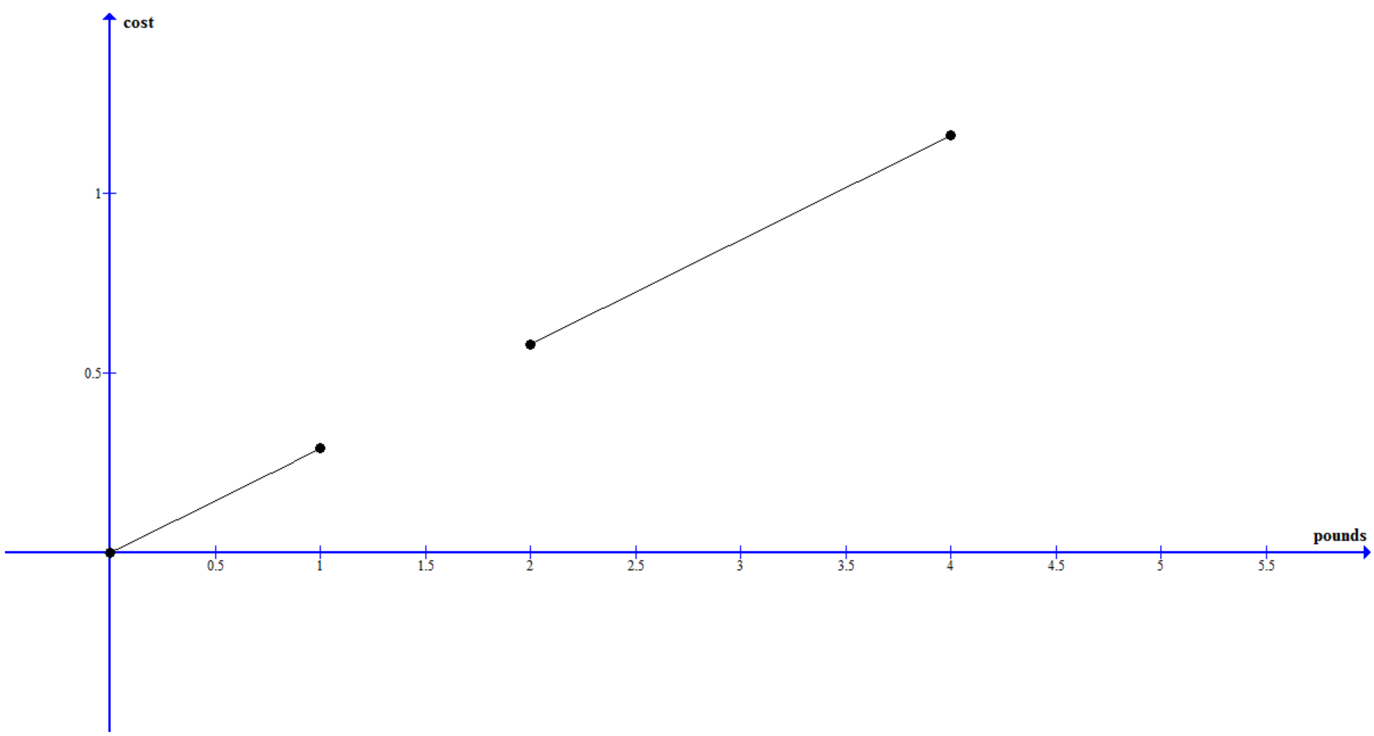
Use this information to complete the following:

1. Fill in the following chart.

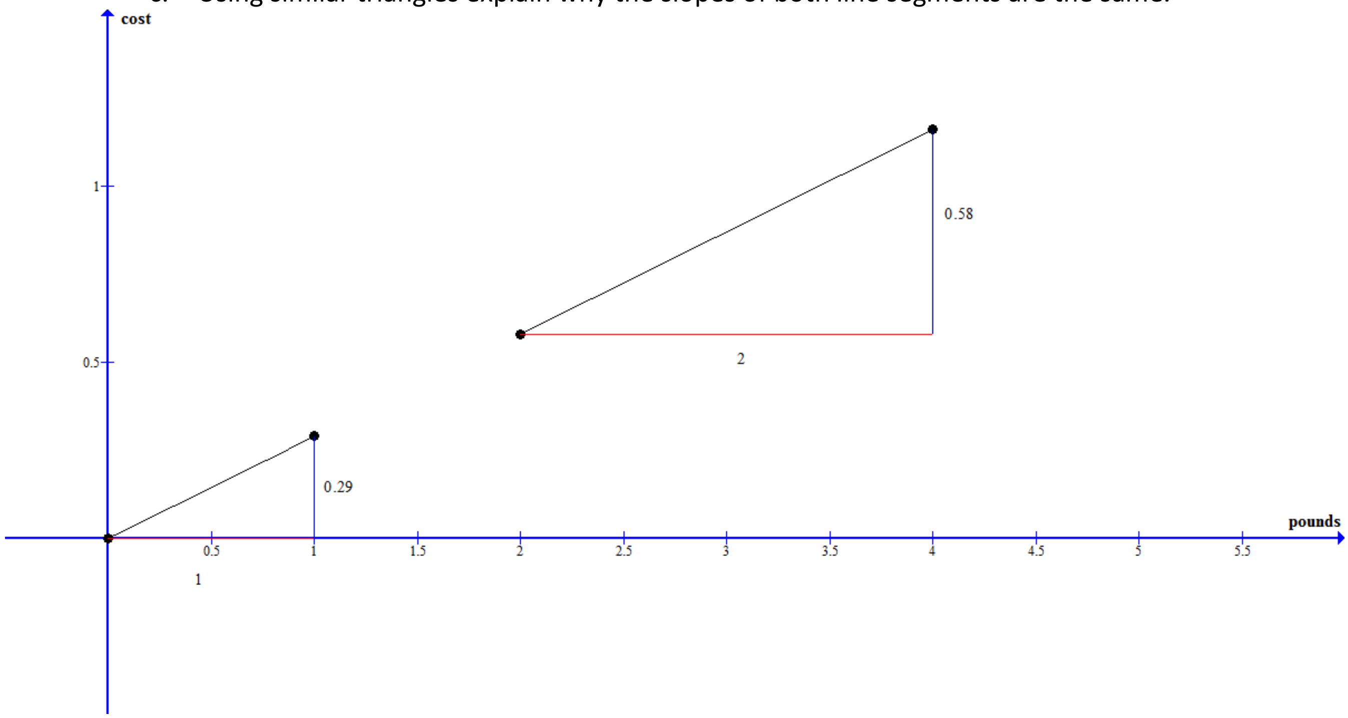
Pounds	Cost
1	*$0.29*
2	*$0.58*
3	*$0.87*
4	*$1.16*
5	*$1.45*

2. On a coordinate plane:
    a. Plot the ordered pairs representing 0 pounds and 1 pound. Connect these points with a line segment.

    b. Plot the ordered pairs representing 2 pounds and 4 pounds. Connect these points with a line segment.



c. Using similar triangles explain why the slopes of both line segments are the same.



The first triangle is translated to the right 2 units and up 0.58 units. Then the triangle is dilated by a factor of 2. Since we can use a translation and a dilation to make one triangle from the other, we know that they are similar triangles. To find the slope between two points, we find the ratio of the lengths of the legs in a right triangle. The lengths of corresponding sides in similar triangles are also proportional. Therefore, the ratio of the lengths of the legs of the two triangles must be the same, and the two triangles would have the same slope.

We can see this algebraically. Slope is equal to $\frac{y_2-y_1}{x_2-x_1}$*.For the first line segment that would be* $\frac{0.29}{1}$*=0.29, and for the second line segment it would be*$\frac{0.58}{2} = 0.29$*. These line segments have the same slope.*

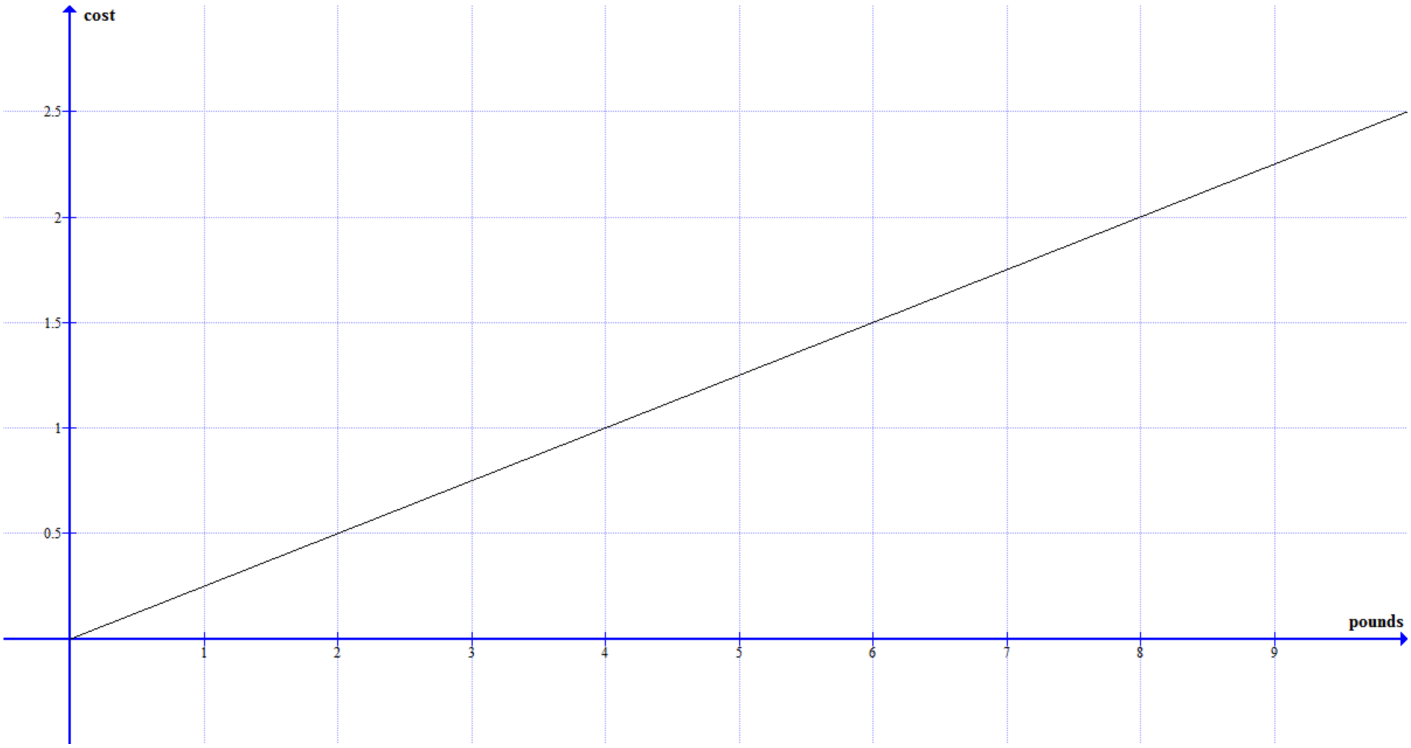
d. What is the cost per pound of bananas? How does the cost per pound of bananas relate to the slope of your line segments?

The bananas cost $0.29 per pound. The cost per pound is the same as the slope of the two line segments.

3. Write an equation to represent how much money (*y*) you would spend for (*x*) pounds of bananas if 5 lbs. of bananas cost $1.45.
y=0.29x

Option 2:

Your second option for purchasing bananas at the grocery store is shown in the graph below.



4. Compare the two options available at your grocery store. Which store offers bananas at the cheaper price? Explain your reasoning.

The slope of the graph for Option 2 is 0.25, so the bananas in Option 2 cost $0.25 a pound. The bananas in Option 1 cost $0.29 a pound. As a result, the bananas in Option 2 are cheaper.

5. You are going shopping for your favorite fresh fruit or vegetable. Either visit a local store or use the Internet to research the price of the fresh fruit or vegetable. Use your research to complete the following:
   a. Create a table to represent the price of your fruit or vegetable for 0-5 pounds purchased.
   b. Create a graph to represent the price of your fruit or vegetable for 0-5 pounds purchased.
   c. Write an equation based on the graph you created.
   d. What does the slope represent in terms of your fruit or vegetable?
   e. Swap your table with one classmate, your graph with a different classmate, and your equation with a third classmate. Whose fruit or vegetable was cheaper? How do you know? Write a short paragraph to summarize all three sets of information.

The answers for this section will vary according to the items chosen and their prices. The results should look much like the results from the previous exercises.

T-Shirt Fundraiser (IT)

Overview

Students will determine the best place to order T-shirts for an upcoming fundraiser and help a new business determine prices for the T-shirts they will sell.

Standards

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.EE.C.8c	• 6.EE.B.5 • 7.EE.B.4a • 8.EE.B.6	1. Write an equation to represent 8 times a number of dollars plus 7 is equal to the total number of dollars. a. 8x + 7 = y 2. Solve the equation 5x + 9 = y when x is equal to 9. a. 54 3. Solve the system $x + y = 27$ $x - y = 9$ a. (18, 9) 4. http://www.illustrativemathematics.org/illustrations/934 5. http://www.illustrativemathematics.org/illustrations/1362	• http://www.illustrativemathematics.org/illustrations/673 • http://www.illustrativemathematics.org/illustrations/1537 • http://learnzillion.com/lessonsets/777-analyze-and-solve-pairs-of-simultaneous-linear-equations-solve-systems-in-two-equations-algebraically • http://learnzillion.com/lessonsets/776-solve-pairs-of-simultaneous-linear-equations-understand-why-solutions-correspond-to-points-of-intersection • http://learnzillion.com/lessonsets/50-graphing-to-solve-systems-of-equations

Real-world preparation: The following questions will prepare students for some of the real-world components of this task:

What is a fundraiser? A fundraiser is an activity held to raise money for an organization or cause.

Why might an organization have a fundraiser? An organization may hold a fundraiser to pay for a special activity.

Why would a company not want to have the highest priced item? A company wants to have prices that are competitive in order for customers to choose them over their competition. If their price is too low, they will make not make a large profit but may be chosen often. If their price is too high, they will not be chosen often and will lose business.

During the Task:

- Look for students struggling with graphing the two situations.
- Students may get stuck determining how to find the price for the new business to sell their T-shirts. Students may need help seeing the area between the two lines as the area the new business wants to fall between. However, some students may do just as well looking at the difference between the two sets of data points, if that is how they chose to represent the data.
- They may also struggle with finding a one-time setup fee, then a per-shirt price. A suggestion is selecting a one-time setup between $0 and $30. Find the difference of this fee and the $100 that 10 shirts should cost. Divide the difference by 10. This will give the per-shirt price.

After the Task:

This can be related to clubs that students are a part of at school or athletic teams. There are times when they need to raise money and need to shop for the best deal in order to make the most profit.

Student Instructional Task

The Math Club has chosen to make T-shirts with the school's logo and sell them for a fundraiser. It is your responsibility to help decide where to purchase the shirts from. There are two local competitors. Use the information from the ads to answer the questions.

1. When is it better to order from *Printin' T's*? When is it better to order from *Bright Stitches*? Explain your choices using a graph, algebra, and/or a table.

2. Choose a company from the two local competitors. Using the company you have chosen, how much must the Math Club sell each shirt for in order to make a profit? Explain your reasons for choosing the company and the price.

3. A third company is going to sell T-shirts in your area. This company doesn't want to ever be the cheapest or most expensive company. They want their T-shirts to be priced between their competitors.
How could the poster for *Your Way T's* be completed to meet these criteria? Explain your reasoning.

Instructional Task Exemplar Response

The Math Club has chosen to make T-shirts with the school's logo and sell them for a fundraiser. It is your responsibility to help decide where to purchase the shirts from. There are two local competitors. Use the information from the ads to answer the questions.

1. When is it better to order from *Printin' T's*? When is it better to order from *Bright Stitches*? Explain your choices using a graph, algebra, and/or a table.

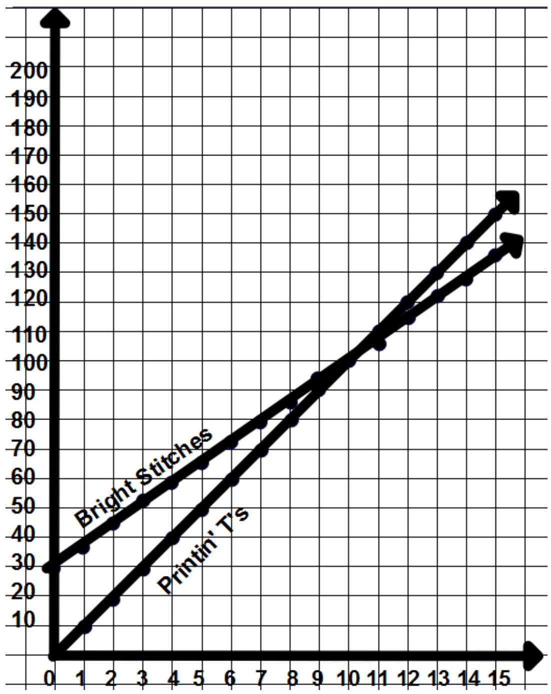
An equation for Printin' T's would be: 10t = c where t is the number of T-shirts and c is the total cost.

An equation for Bright Stitches would be: 30 + 7t = c where t is the number of T-shirts and c is the total cost.

Data points created using the equations:

10t = c		*30 + 7t = c*	
(0, 0)	*(10, 100)*	*(0, 30)*	*(10, 100)*
(1, 10)	*(11, 110)*	*(1, 37)*	*(11, 107)*
(2, 20)	*(12, 120)*	*(2, 44)*	*(12, 114)*
(3, 30)		*(3, 51)*	
(4, 40)		*(4, 58)*	
(5, 50)		*(5, 65)*	
(6, 60)		*(6, 72)*	
(7, 70)		*(7, 79)*	
(8, 80)		*(8, 86)*	
(9, 90)		*(9, 93)*	

When the two equations are graphed, the better value at each point is visible. The better value will be the company represented by the lower line at a given x value.



The club should choose Printin' T's when the T-shirt order is less than 10 shirts, because up until that point, it is the better price. The club should choose Bright Stitches when the T-shirt order is more than 10 shirts, because after that point, it is the better price. If the order is exactly 10 shirts, the price will be $100 at both places.

2. Choose a company from the two local competitors. Using the company you have chosen, how much must the Math Club sell each shirt for in order to make a profit? Explain your reasons for choosing the company and the price.
   Sample response: Since the club will most likely sell more than 10 shirts, the club would use Bright Stitches. If 10 shirts were ordered, each shirt would cost $10 to make. After this point, each shirt becomes cheaper. If the shirts are sold for $10, the club would make a profit.

3. A third company is going to sell T-shirts in your area. This company doesn't want to ever be the cheapest or most expensive company. They want their T-shirts to be priced between their competitors.
How could the poster for *Your Way T's* be completed to meet these criteria? Explain your reasoning.

Your Way T's

One-time setup fee: ___
Price per shirt printed: ___

This answer can be different for each student. However, it must meet the criteria of being between the two competitors at all times. There should be an adequate explanation of how the price fits the parameters.

For example: One-time setup fee: $15. Price per shirt printed: $8.50

This would make the shirts equal at $100 and between the other two at all points.

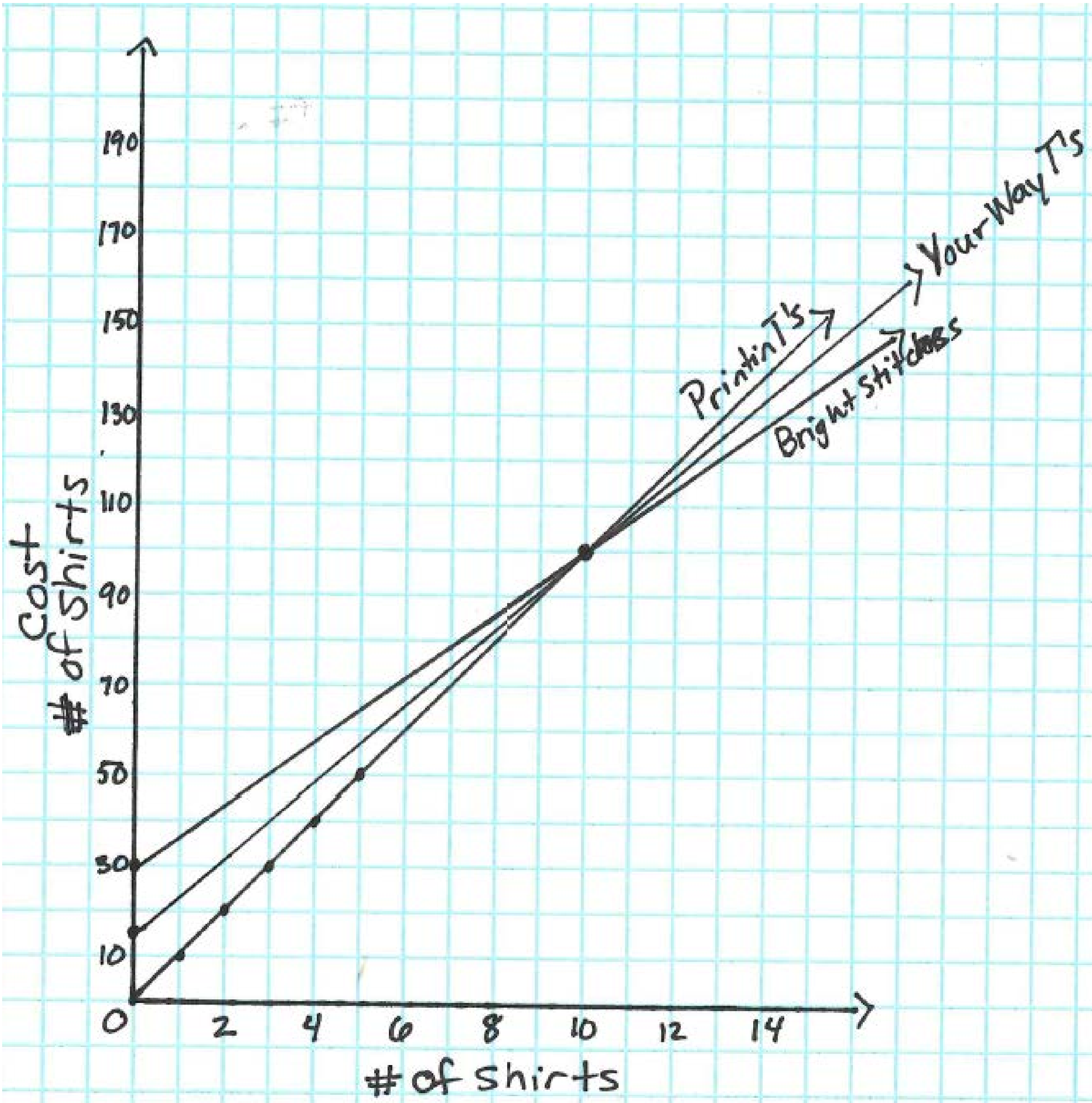
A graph or data points table could be used.

Data points:

$15 + 8.5t = c$

(0, 15)	*(7, 74.50)*	
(1, 23.50)	*(8, 83)*	
(2, 32)	*(9, 91.50)*	
(3, 41.50)	*(10, 100)*	****Notice the point (10, 100). That is a must.*
(4, 49)	*(11, 108.50)*	
(5, 57.50)	*(12, 117)*	
(6, 66)		

Graph for these points proving that it fits the parameters:



Game Design (IT)

Overview

Students will use congruence transformations to create a game on a grid.

Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments, of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.G.A.1	• 7.G.A.2 • 7.G.B.5	1. $\overline{AB}$ is reflected over line *l*. What must be true about the resulting image $\overline{A'B'}$? a. *The resulting image, $\overline{A'B'}$, is the same length as $\overline{AB}$.* 2. $\Delta\, ABC$ is rotated around point *A*. What is true about angle *B'C'A'*? a. *Angle B'C'A' is congruent to angle BCA.*	• http://learnzillion.com/lessonsets/473-verify-properties-of-rotations-reflections-and-translations
8.G.A.2	• 8.G.A.1	1. Two triangles are drawn on a coordinate plane. How can you tell if they are congruent? a. The two triangles are congruent if one triangle can be obtained from the other through a sequence of rotations, reflections, and/or translations. 2. http://www.illustrativemathematics.org/illustrations/646	• http://learnzillion.com/lessonsets/528-understand-congruency-in-twodimensional-figures • http://learnzillion.com/lessonsets/466-assess-congruence-using-rotations-reflections-and-translations

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
		3. http://www.illustrativemathematics.org/illustrations/1228 4. http://www.illustrativemathematics.org/illustrations/1231	
8.G.A.3	• 6.G.A.3 • 8.G.A.1	1. Graph rectangle *RSTU* with vertices $R(2,3), S(3,1), T(-1,-1)$, and $U(-2,1)$. Rotate rectangle *RSTU* 90° clockwise around the origin. What are the coordinates of *R'S'T'U'*? a. *R'(3, -2), S'(1, -3), T'(-1, 1), and U'(1, 2)* 2. http://www.illustrativemathematics.org/illustrations/1243	• http://www.illustrativemathematics.org/illustrations/1188 • http://learnzillion.com/lessonsets/534-describe-the-effect-of-dilations-translations-rotations-and-reflections-on-twodimensional-figures-using-coordinates • http://learnzillion.com/lessonsets/476-describe-the-effects-of-dilations-translations-rotations-and-reflections-using-coordinates

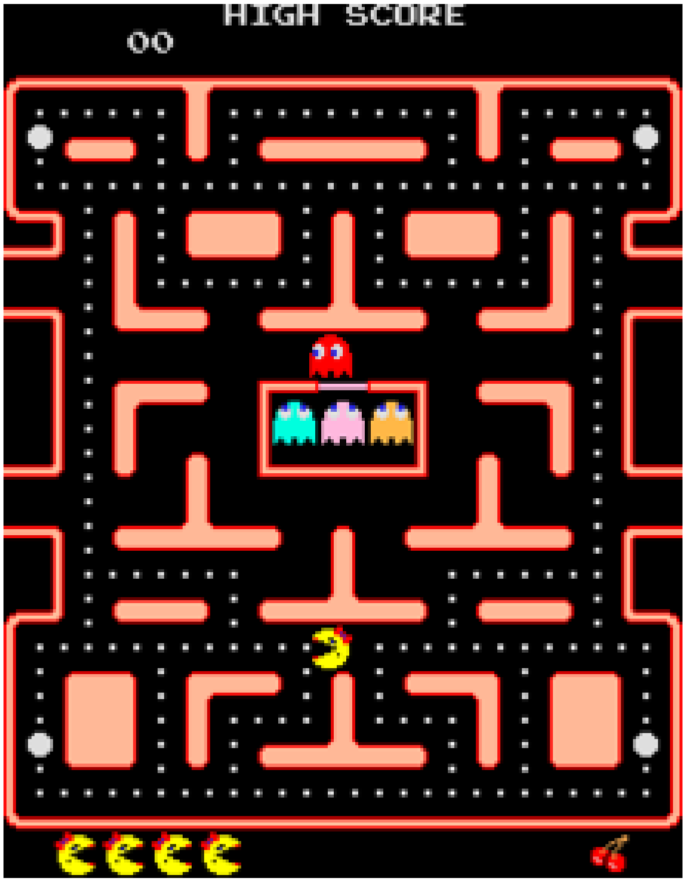
During the Task:

- Students will find it easier to use and describe reflections and translations. They may need some assistance in using and describing rotations to create the game.
- To help students determine whether they have completed all of the requirements, teachers may wish to create a checklist for students to follow.
- Watch for students who may change the shapes, lengths, or measures of angles as they create the game. Discuss with students the meaning of congruence transformations and how they apply to this task.

After the Task:

Have students identify transformations in other areas like classrooms, the gym, cafeteria, school yard, etc. Teachers can also have students find examples of transformations at home. When identifying examples of transformations, students would need to explain how they know the example represents the transformation.

Student Instructional Task



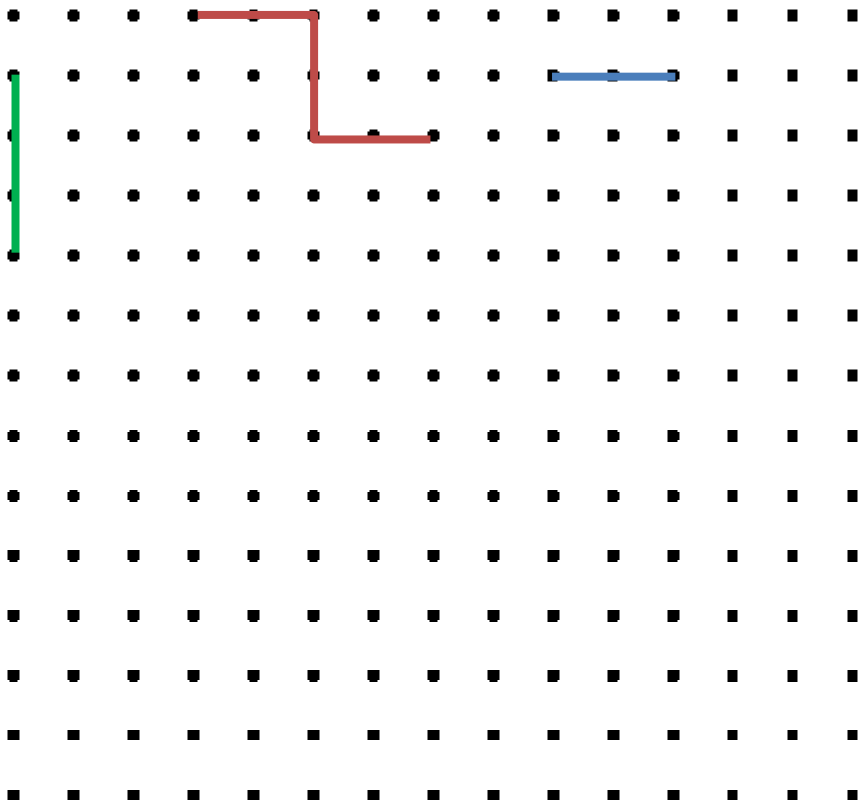
Source for picture: http://en.wikipedia.org/wiki/Ms._Pac-Man

Above is the game screen for Ms. Pac-Man. Reflections, translations, and rotations were used to place the "walls" between which Ms. Pan-Man can travel. Take a minute to locate reflections, translations, and rotations of the "walls" in the screen above.

Design a game of your own below. Create your own game and game characters. Be sure your game meets the following requirements:

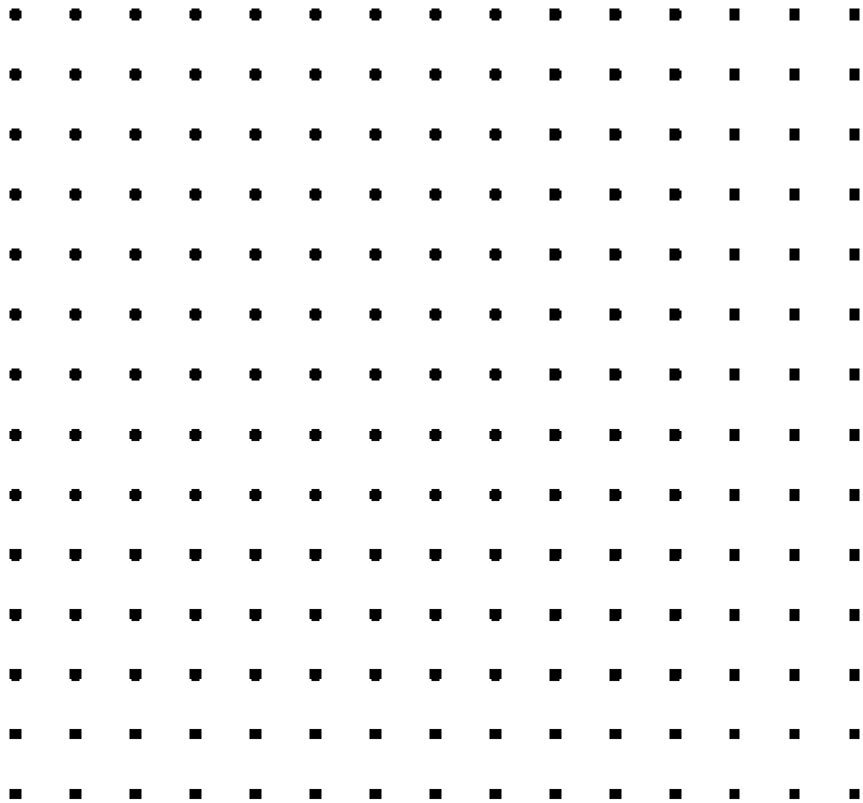
- Identify the goal of your game. The goal may be for your game character to eat all of the dots, as in Ms. Pac-Man, or you may have a different goal altogether.
- Use at least two of the three figures below. You may also use other figures you create. Draw any figures you create below.
- Use at least three different figures in the game.
- Each different figure used must be a different color. When each figure is transformed, the new image must be the same color as the original figure.
- The interior walls of your game must be the figures you have created or chosen and the images of those figures that have been reflected, translated, and rotated.
- There must be at least one use of each type of transformation.
- There must be at least two series of transformations. (You will likely have many more.)
- Exterior walls and other objects may be drawn, if needed, to complete your game.

Use the grid below to draw any additional figures you plan to use in your game that will be transformed using reflections, rotations, and/or translations. Be sure to use a different color for each different figure.



Create your game using the grid on the next page.

Use the blank grid below to create your game.



Complete parts A through H on the following page.

After you have created your game, complete the following about your game:

A. Identify a reflection you used in the game. Explain how you know this is a reflection.

B. Identify a translation you used in the game. Explain how you know this is a translation.

C. Identify a rotation you used in the game. Explain how you know this is a rotation.

D. Identify and list the steps of two transformations that involved a series of reflections, translations, and/or rotations.

Next, exchange games with your partner. Do not show them your answers to parts A through D above. Using your partner's game, complete the following. After you identify the transformations, discuss your findings with your partner.

E. Identify a reflection used in your partner's game. Explain how you know this is a reflection.

F. Identify a translation used in your partner's game. Explain how you know this is a translation.

G. Identify a rotation used in your partner's game. Explain how you know this is a rotation.

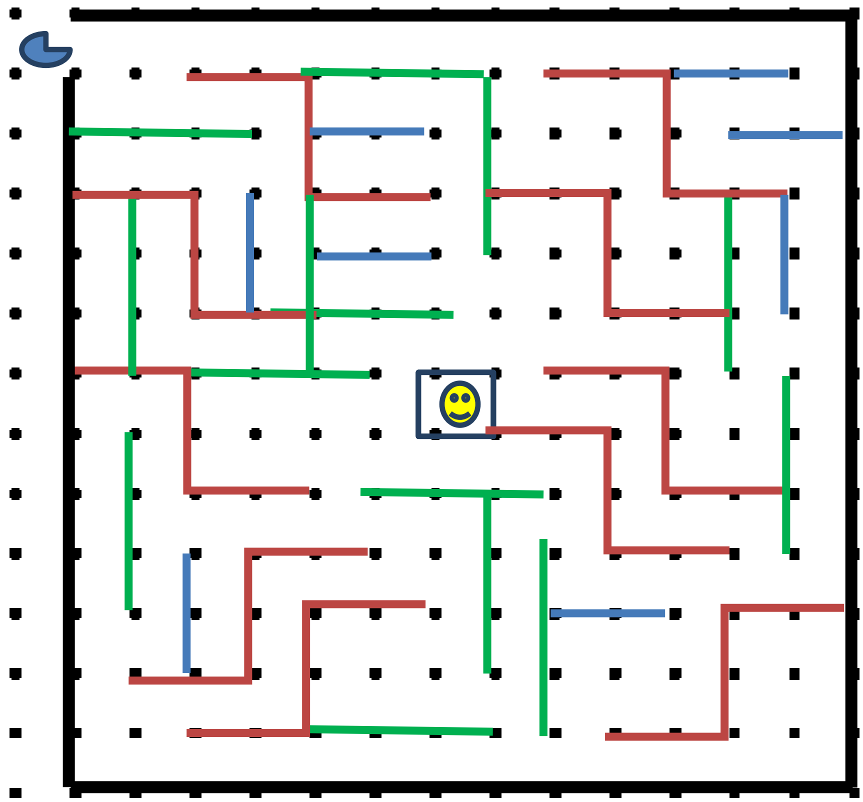
H. Identify and list the steps of one transformation that involves a series of reflections, translations, and/or rotations.

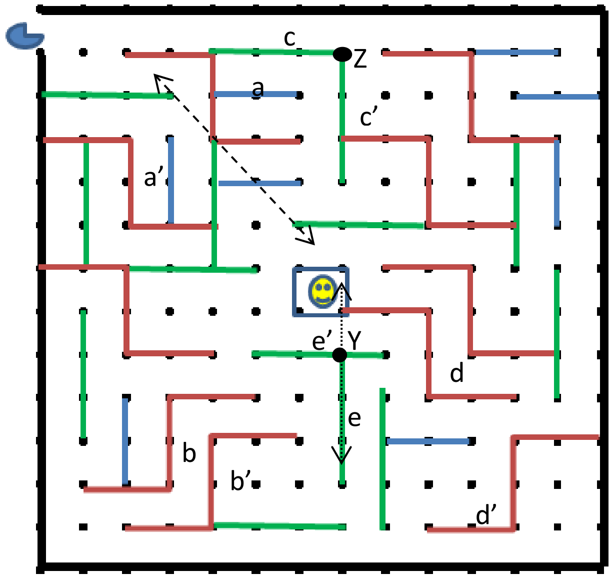
Now, play your partner's game. ☺

Instructional Task Exemplar Response

This is a sample response. This is an open-ended task, and students will create very different games that meet the criteria.

Pac-Man Finds Smiley





After you have created your game, complete the following about your game:

A. Identify a reflection you used in the game. Explain how you know this is a reflection.

$$a \rightarrow a' \textit{reflected over the dashed line.}$$

Check student's explanation.

B. Identify a translation you used in the game. Explain how you know this is a translation.

$$b \rightarrow b' \textit{Translated down one, right one.}$$

Check student's explanation.

C. Identify a rotation you used in the game. Explain how you know this is a rotation.

$$c \rightarrow c' \textit{Rotated } 90^{\circ} \textit{ counterclockwise about point } Z.$$

Check student's explanation.

D. Identify and list the steps of two transformations that involved a series of reflections, translations, and/or rotations.

$$d \rightarrow d' \textit{Reflected over dotted line, then translated down } 3 \textit{ and right } 6.$$

$$e \rightarrow e' \textit{Rotated } 90^{\circ} \textit{ clockwise about point } Y\textit{, then translated right one.}$$

Next, exchange games with your partner. Do not show them your answers to parts A through D above. Using your partner's game, complete the following. After you identify the transformations, discuss your findings with your partner. *Solutions for this section will vary based on student observations.*

Tank Volume (IT)

Overview

This task provides students an opportunity to use their skill and fluency working with volume formulas for cones, cylinders, and spheres in a real-world context. Some of the questions in this task have specific, correct answers (#1, 2, 3, and 5), while other questions (#4 and 6) require students to make assumptions and design tanks to meet given specifications. The dimensions and volumes involve numbers that may be overwhelming to students at first, but as they persevere through the problems they should be able to get answers.

Standards

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Work with radicals and integer exponents.

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.NS.A.2	• 7.NS.A.2d	1. Approximate the value of each expression: (sample answers are provided; other approximations are dependent upon the rational number used for the irrational value) a. $\pi + \pi$ i. Approximately 6.28 b. 10π i. Approximately 31.4 c. $\frac{4}{3}\pi$ i. Approximately 4 d. $\frac{1}{3}\pi(4)^2$ i. Approximately 16	• http://www.illustrativemathematics.org/illustrations/604 • http://www.illustrativemathematics.org/illustrations/593 • http://learnzillion.com/lessonsets/41-understand-rational-and-irrational-numbers

<table>
<tr><th>Grade Level Standard</th><th>The Following Standards Will Prepare Them:</th><th>Items to Check for Task Readiness:</th><th>Sample Remediation Items :</th></tr>
<tr><td></td><td></td><td>2. http://www.illustrativemathematics.org/illustrations/335
3. http://www.illustrativemathematics.org/illustrations/334
4. http://www.illustrativemathematics.org/illustrations/1538</td><td></td></tr>
<tr><td>8.EE.A.2</td><td>• 6.EE.B.5
• 7.NS.A.3</td><td>1. Solve for x. For irrational answers provide both an exact and approximate answer.
a. $x^2 = 16$
i. $x = 4$
b. $x^2 = 200$
i. $x = \sqrt{200} \approx 14.14$
c. $x^2 = \frac{25}{16}$
i. $x = \frac{5}{4}$
2. Solve for x. For irrational answers provide both an exact and approximate answer.
a. $x^3 = 27$
i. $x = 3$
b. $x^3 = 40$
i. $x = \sqrt[3]{40} \approx 3.42$
c. $x^3 = \frac{64}{125}$
i. $x = \frac{4}{5}$</td><td>• http://www.illustrativemathematics.org/illustrations/673
• http://www.illustrativemathematics.org/illustrations/298
• http://learnzillion.com/lessonsets/351-understand-and-evaluate-square-roots-and-cube-roots
• http://learnzillion.com/lessonsets/45-understand-perfect-cubes-and-cube-roots
• http://learnzillion.com/lessonsets/44-understand-perfect-squares-and-square-roots</td></tr>
<tr><td>8.G.C.9</td><td>• 8.EE.A.2</td><td>1. Write the formula that could be used to calculate the volume of a:
a. Cylinder
i. $V = \pi r^2 h$
b. Cone
i. $V = \frac{1}{3}\pi r^2 h$
c. Sphere
i. $V = \frac{4}{3}\pi r^2$
2. http://www.illustrativemathematics.org/illustrations/520
3. http://www.illustrativemathematics.org/illustrations/517
4. http://www.illustrativemathematics.org/illustrations/521</td><td>• http://learnzillion.com/lessonsets/704-find-volumes-of-cones-cylinders-and-spheres
• http://learnzillion.com/lessonsets/286-know-and-use-the-formulas-for-volumes-of-cones-cylinders-and-spheres</td></tr>
</table>

During the Task:

- Several of the questions in this task are multistep. For example, in #1 students must (a) find the volume of Tank 2 and (b) use this volume along with the provided volume of Tank 1 to determine how many times the volume of Tank 2 is the volume of Tank 1. On any of these questions, students may need help getting started. Questions like, “What is the problem asking you to find?”, “What information do you have?”, “What additional information do you need?”, etc., may help students get started.

- Some students may be intimidated by the arithmetic involved in this task. If students get stuck working with the arithmetic, remind students that they can reduce fractions before multiplying, encourage students to take the arithmetic one step at a time, or provide a calculator to weaker students.

After the Task:

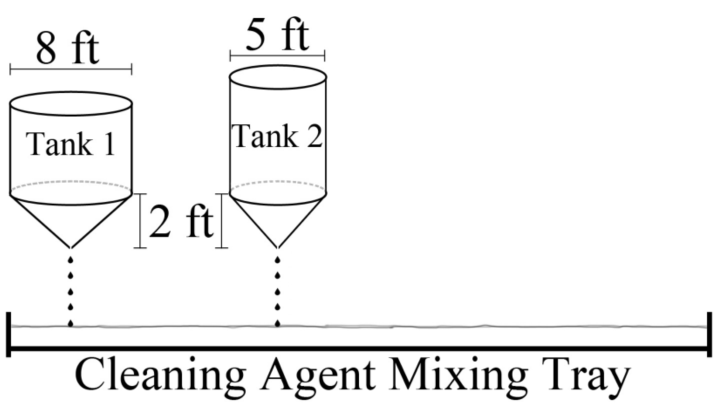
Ultimately, this is a design task. Students need to use what they know about the volume of cylinders, cones, and spheres to perform calculations and then to design tanks to meet specifications. This could be applied to any situation where students need to calculate or approximate volumes of known figures. For example, students may use some form of this in cooking to determine appropriate containers for drinks, soups, gravy, etc., when preparing food for a group of people.

Student Instructional Task

In response to an increase in demand for glass cleaner, the CEO of Shining Glass made the decision to open a new factory to manufacture more glass cleaner. You have been asked to help make the process more efficient. Four of the ingredients are cleaning agents, and these are mixed together before being added to the remaining ingredients.

Your first job is to put together the section of the factory where the four cleaning agents are mixed. When you started working on the project, two of the four tanks of cleaning agents had already been installed. Before deciding what to do with the other two tanks, you need to gather information about Tank 1 and Tank 2. Both tanks are comprised of a cylinder section and a cone section. Both cone sections have a height of 2 ft. The diameter of Tank 1 is 8 ft. and the diameter of Tank 2 is 5 ft. On the side of Tank 1 you found a label stating the volume of the cylinder section only: $V = 96\pi$ ft.^{3}. On the side of Tank 2 you found a label stating the volume of the entire tank: $V = \frac{325}{6}\pi$ ft.^{3}. (When approximating answers use $\pi \approx 3$).

(not necessarily drawn to scale)



1. How many times the total volume of Tank 2 is the total volume of Tank 1? Justify your answer.

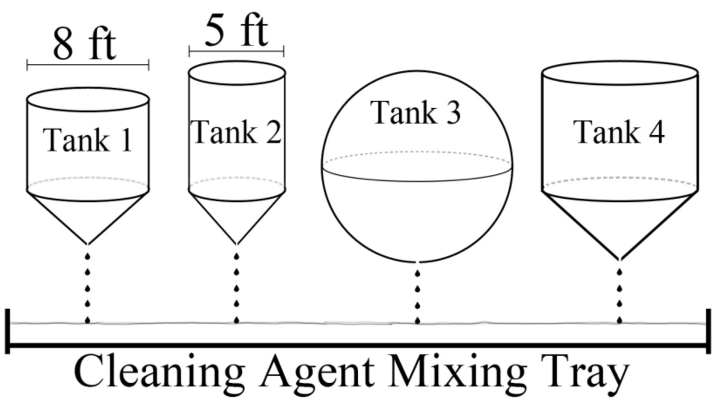
2. The volumes of Tank 1 and Tank 2 were chosen such that using a full tank of each will produce the correct ratio of the two liquids. Both Tank 1 and Tank 2 are set to constantly add liquid to the mixture. If Tank 1 is set to add 16 ft^3 of liquid to the mixture every hour, at what rate should Tank 2 add liquid in order to keep the correct ratio of the two liquids? Show how you found your answer.

The volume of Tank 2 is $\frac{325}{6}\pi$ ft.^{3} and the combined volume of Tank 1 and Tank 2 is $\frac{965}{6}\pi$ ft.^{3} . You now need to add two new tanks for the other two cleaning agents. The third liquid will be dispensed from a spherical tank, and the fourth liquid will be dispensed from a tank with the same basic shape as Tanks 1 and 2. Mixing full tanks of Tanks 1, 2, 3, and 4 will produce the correct ratio of the four liquids.

Tank 3: The cleaning agent mixture will contain more liquid from Tank 3 than from Tanks 1 and 2 combined. To maintain the correct ratio of liquids, the liquid used from Tank 3 should represent $\frac{2000}{965}$ times as much liquid as from Tanks 1 and 2 combined.

Tank 4: The volume of the <u>cone</u> section of Tank 4 is $\frac{10}{13}$ of the entire volume of Tank 2. The <u>cylinder</u> section of Tank 4 needs to hold six times as much liquid as the cone section of Tank 4.

(not necessarily drawn to scale)



3. What would the diameter of Tank 3 need to be? Show all work. *Include the exact diameter and an approximation of the diameter.*

4. What dimensions for Tank 4 would produce the required volume? Justify your answer.

5. Together Tanks 1 and 2 add $\frac{193}{8}\ ft^3$ of liquid to the mixing tray each hour. At what rate should Tank 3 and Tank 4 add liquid in order to keep the correct ratio of liquids? Justify your answer.

6. Two large cylinder-shaped holding tanks (Tank 5 and Tank 6) are used to store the cleaning agent mixture before it is added to the rest of the ingredients. These tanks need to be designed to hold approximately2500 ft^3 of liquid. Tank 6 will be turned sideways on top of Tank 5, so the height of Tank 6 should equal the diameter of Tank 5. Design these two tanks to meet the required specifications. *Show your work.*

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Instructional Task Exemplar Response

In response to an increase in demand for glass cleaner, the CEO of Shining Glass made the decision to open a new factory to manufacture more glass cleaner. You have been asked to help make the process more efficient. Four of the ingredients are cleaning agents, and these are mixed together before being added to the remaining ingredients.

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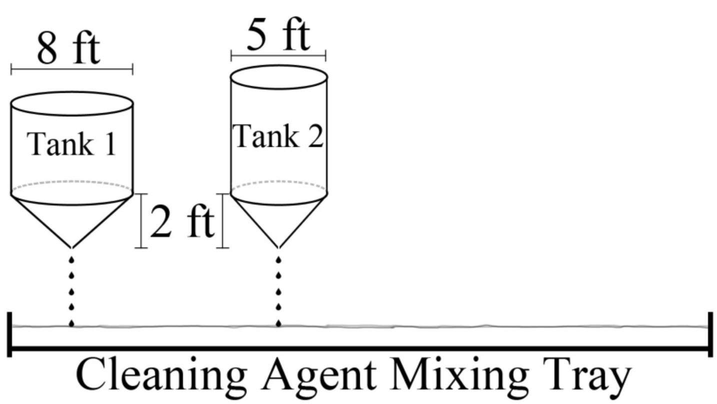
(not necessarily drawn to scale)



1. How many times the total volume of Tank 2 is the total volume of Tank 1? Justify your answer.

Volume of Tank 2: $\frac{325}{6}\pi \text{ ft}^3 \approx \frac{325}{6}(3) \text{ ft}^3 \approx \frac{325}{2} \text{ ft}^3$

Tank 1 (diameter = 8 ft, radius = 4 ft)

Cylinder section: $V = 96\pi \text{ ft}^3$

Cone section: $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4 \text{ ft})^2(2 \text{ ft})$

$V = \frac{1}{3}\pi(16 \text{ ft}^2)(2 \text{ ft})$

$V = \frac{32}{3}\pi \text{ ft}^3$

Total Volume $= 96\pi \text{ ft}^3 + \frac{32}{3}\pi \text{ ft}^3$

$= \frac{288}{3}\pi \text{ ft}^3 + \frac{32}{3}\pi \text{ ft}^3$

$= \frac{320}{3}\pi \text{ ft}^3$

$\approx \frac{320}{3}(3) \text{ ft}^3 \approx 320 \text{ ft}^3$

Tank 1 has a volume of $\frac{320}{3}\pi \text{ ft}^3$ (approximately 320 ft^3) and Tank 2 has a volume of $\frac{325}{6}\pi \text{ ft}^3$ (approximately 162.5 ft^3).

(Volume of Tank 2)(x) = (Volume of Tank 1)

$\left(\frac{325}{6}\pi \text{ ft}^3\right)x = \frac{320}{3}\pi \text{ ft}^3$

$x = \frac{\frac{320}{3}\pi \text{ ft}^3}{\frac{325}{6}\pi \text{ ft}^3} = \frac{320}{3} \div \frac{325}{6} = \frac{320}{3} \cdot \frac{6}{325} = \frac{128}{65}$

Tank 1 has a volume $\frac{128}{65}$ times the volume of Tank 2.

2. The volumes of Tank 1 and Tank 2 were chosen such that using a full tank of each will produce the correct ratio of the two liquids. Both Tank 1 and Tank 2 are set to constantly add liquid to the mixture. If Tank 1 is set to add 16 $ft.^3$ of liquid to the mixture every hour, at what rate should Tank 2 add liquid in order to keep the correct ratio of the two liquids? Show how you found your answer.

$$\left(\text{Volume of Tank 2}\right)\left(\frac{128}{65}\right) = \left(\text{Volume of Tank 1}\right)$$

$$\left(\text{Rate for Tank 2}\right)\left(\frac{128}{65}\right) = \left(\text{Rate for Tank 1}\right)$$

$$\text{Rate for Tank 2} = \left(16\ \frac{ft^3}{hr}\right)\cdot\left(\frac{65}{128}\right)$$

$$\text{Rate for Tank 2} = \frac{(\cancel{16}^{\,1})(65)}{\cancel{128}_{\,8}}\ \frac{ft^3}{hr}$$

$$\text{Rate for Tank 2} = \frac{65}{8}\ \frac{ft^3}{hr}$$

The volume of Tank 2 is $\frac{325}{6}\pi$ ft.^{3} and the combined volume of Tank 1 and Tank 2 is $\frac{965}{6}\pi$ ft.^{3} . You now need to add two new tanks for the other two cleaning agents. The third liquid will be dispensed from a spherical tank, and the fourth liquid will be dispensed from a tank with the same basic shape as Tanks 1 and 2. Mixing full tanks of Tanks 1, 2, 3, and 4 will produce the correct ratio of the four liquids.

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Tank 4: The volume of the <u>cone</u> section of Tank 4 is $\frac{10}{13}$ of the entire volume of Tank 2. The <u>cylinder</u> section of Tank 4 needs to hold six times as much liquid as the cone section of Tank 4.

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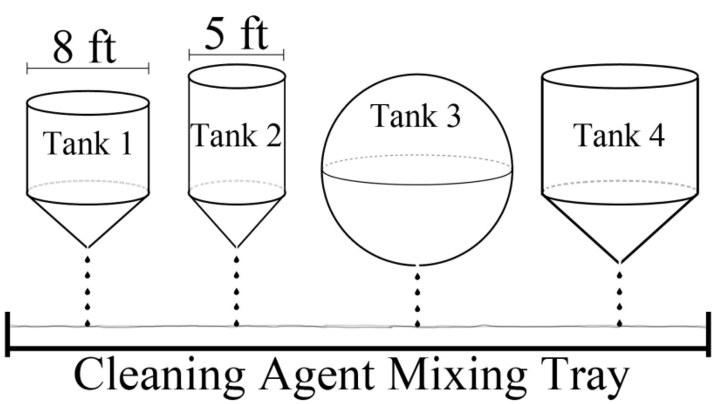


3. What would the diameter of Tank 3 need to be? Show all work. *Include the exact diameter and an approximation of the diameter.*

Tank 3 Volume $= \left(\frac{2000}{965}\right)\left(\frac{965}{6}\pi \text{ ft}^3\right) = \frac{1000}{3}\pi \text{ ft}^3$

For a sphere: $V = \frac{4}{3}\pi r^3$

$\frac{1000}{3}\pi \text{ ft}^3 = \frac{4}{3}\pi r^3$

$\frac{1000\pi}{3} \cdot \frac{3}{4\pi} \text{ ft}^3 = r^3$

$250 \text{ ft}^3 = r^3$

$r = \sqrt[3]{250} \text{ ft} \approx 6.3 \text{ ft}$ → Diameter: $2\sqrt[3]{250} \text{ ft} \approx 12.6 \text{ ft}$

4. What dimensions for Tank 4 would produce the required volume? Justify your answer.

***Note: The work shown here is a sample answer. Students may find other dimensions that work, as long as they can justify their answer.*

Cone section

$$\frac{10}{13}\left(\frac{325}{6}\pi\ ft^3\right) = \frac{125}{3}\pi\ ft^3$$

Cylinder Section

$$6\left(\frac{125}{3}\pi\ ft^3\right) = 250\pi\ ft^3$$

Different combinations of heights + radii could work. I will use a radius of 5 ft.

Cone section: $V = \frac{125}{3}\pi\ ft^3 = \frac{1}{3}\pi(5\ ft)^2 h$

$$h = \frac{\frac{125}{3}\pi\ ft^3}{\frac{25}{3}\pi\ ft^2} = \underline{5\ ft}$$

Cylinder Section: $V = 250\pi\ ft^3 = \pi(5\ ft)^2 h$

$$h = \frac{250\pi\ ft^3}{25\pi\ ft^2} = \underline{10\ ft}$$

The radius could be 5 feet, and then the height of the cone section will be 5 feet and the height of the cylinder section would be 10 ft.

5. Together Tanks 1 and 2 add $\frac{193}{8}\ ft.^3$ of liquid to the mixing tray each hour. At what rate should Tank 3 and Tank 4 add liquid in order to keep the correct ratio of liquids? Justify your answer.

Tank 3

$$\frac{\text{Volume of Tank 3}}{\text{Volume of Tanks 1+2}} = \frac{\text{Rate for Tank 3}}{\text{Rate for Tanks 1+2}} \rightarrow \frac{\frac{1000}{3}\pi\ ft^3}{\frac{965}{6}\pi\ ft^3} = \frac{x}{\frac{193}{8}\ \frac{ft^3}{hr}}$$

$$x = \frac{1000}{3}\cdot\frac{6}{965}\cdot\frac{193}{8}\ \frac{ft^3}{hr} = \frac{(1000)(2)}{(5)(8)}\ \frac{ft^3}{hr} = \frac{(125)(2)}{5}\ \frac{ft^3}{hr} = \boxed{50\ \frac{ft^3}{hr}}$$

Tank 4

$$\text{Volume} = \frac{125}{3}\pi\ ft^3 + 250\pi\ ft^3 = \frac{875}{3}\pi\ ft^3$$

$$\frac{\text{Volume of Tank 4}}{\text{Volume of Tanks 1+2}} = \frac{\text{Rate for Tank 4}}{\text{Rate for Tanks 1+2}} \rightarrow \frac{\frac{875}{3}\pi\ ft^3}{\frac{965}{6}\pi\ ft^3} = \frac{x}{\frac{193}{8}\ \frac{ft^3}{hr}}$$

$$x = \frac{875}{3}\cdot\frac{6}{965}\cdot\frac{193}{8}\ \frac{ft^3}{hr} = \frac{(875)(2)}{(5)(8)}\ \frac{ft^3}{hr} = \boxed{\frac{175}{4}\ \frac{ft^3}{hr}}$$

6. Two large cylinder-shaped holding tanks (Tank 5 and Tank 6) are used to store the cleaning agent mixture before it is added to the rest of the ingredients. These tanks need to be designed to hold approximately2500 $ft.^3$ of liquid. Tank 6 will be turned sideways on top of Tank 5, so the height of Tank 6 should equal the diameter of Tank 5. Design these two tanks to meet the required specifications. *Show your work.*

Tank 5

Assume a radius of 8 feet and a height of 10 feet.

$$V = \pi(8\ ft)^2(10\ ft) = 640\pi\ ft^3$$

This is an approximate volume of $640(3)\ ft^3 = 1920\ ft^3$

Tank 6 needs to hold approximately $(2500 - 1920)\ ft^3$ of liquid and have a height of 16 ft.

$$580\ ft^3 = \pi(r)^2(16\ ft)$$

$$\frac{580\ ft^3}{16\pi\ ft} = r^2$$

$$r^2 \approx \frac{\overset{145}{\cancel{580}}}{(\underset{4}{\cancel{16}})(3)}\ ft^2 = \frac{145}{12}\ ft^2$$

$$r \approx \sqrt{\frac{145}{12}\ ft^2} \approx \sqrt{12.1}\ ft \approx 3.5\ ft$$

These dimensions should result in a combined volume approximately equal to $2500\ ft^3$.

Tank 5	Tank 6
Radius: 8 feet	Radius: 3.5 feet
Height: 10 feet	Height: 16 feet

Pythagorean Theorem Proof (IT)

Overview

Students will demonstrate their ability to work through and explain a proof of the Pythagorean Theorem. This task does not address the converse of the Pythagorean Theorem. The task walks the student step-by-step through a proof of the Pythagorean Theorem. To end the task, students pull all of the steps together to explain the proof using the provided diagram.

Students will cut out figures to complete this proof. The pieces will either need to be pre-cut, or students will need to cut them out. Keeping everything neat and together is easier if the students glue their figures to a separate sheet of paper. Provide students with additional paper and glue sticks as needed.

Standards

Understand and apply the Pythagorean Theorem.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness:	Sample Remediation Items :
8.G.B.6	• 7.G.B.6	1. You find a triangle on your desk. Answer the following questions: a. How can you tell if it is a right triangle using only a protractor? i. Measure the angles. If the triangle has a 90° angle, then the triangle is a right triangle. b. If it is a right triangle, which side will be the longest? i. The side across from the biggest angle (the hypotenuse, because it is across from the right angle.) 2. How can you find the area of a rectangle? a. Multiply the base by the height 3. How can you find the area of a square? a. Square the length of one side OR multiply the base by the height 4. Given the area of a square, how can you find the length of each side? a. Find the square root of the area 5. http://www.illustrativemathematics.org/illustrations/724	• http://learnzillion.com/lessonsets/450-explain-a-proof-of-the-pythagorean-theorem-and-its-converse • http://learnzillion.com/lessonsets/279-prove-and-apply-the-pythagorean-theorem-to-determine-unknown-side-lengths-in-righttriangles

During the Task:

The teacher should monitor the progress of students/groups to make sure:

1. Students are using the correct pieces and arranging them correctly as they go through the proof.
2. Students are correctly labeling and determining lengths in terms of a, b, and/or c.
3. Students are correctly calculating areas and relating areas of Figures 1 and 3.

Toward the end of the task, students are asked if this proof could be generalized to other types of triangles. The anticipated response is not a formal proof of why this could or could not work. Students can informally answer this question using what they know about types of triangles. For example, the same sets of figures could not be formed with acute or obtuse triangles, as this proof depends on making rectangles and squares using repeated copies of the same triangle. Putting together multiple copies of the same acute or obtuse triangle will form parallelograms, not rectangles. This connection between parallelograms and triangles is first developed in 6^{th} grade as students start to investigate the area formulas of parallelograms and triangles.

After the Task:

If this task is used when students have had limited exposure to the Pythagorean Theorem, students may later be asked to use the Pythagorean Theorem to find missing side lengths. For then they will have established why the Pythagorean Theorem is true.

For students who are having trouble really understanding the proof with the variables a, b, and c, the teacher could ask them to walk through this proof again with known side lengths that are Pythagorean triples like 3, 4, and 5 or 7, 24, and 25.

The teacher should relate the usefulness of the Pythagorean Theorem to any real-world situation involving right triangles, for example, building a truss, constructing a frame (with a diagonal support) as the foundation of a stage or box, calculating the required length of a cable to support a cell tower, etc.

Student Instructional Task

As you work through this lesson, use the Pythagorean Theorem Proof page at the end of this lesson.

1. Cut out all of the shapes (including the Area and Figure boxes) on the Pythagorean Theorem Proof page at the end of the lesson.
2. You should now have eight congruent triangles cut out. These triangles are all right triangles. Draw a right angle on each triangle to indicate which of the three angles is the right angle.
3. Label the hypotenuse of each triangle as length "c," label the longer leg of each triangle "b," and label the shorter leg of each triangle "a."
4. Determine and label the side lengths of the two largest squares in terms of "a," "b," and/or "c."
5. Arrange four triangles and one square in the configuration below, and place the "Figure 1" label under the shapes. Tape or glue these to a separate sheet of paper.

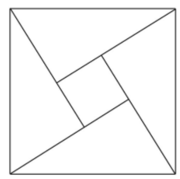


Figure 1

Is Figure 1 a square? *Support your answer with evidence about both the side lengths and angle measures.*

6. Place one of the "Area:" rectangles under Figure 1, and write the area in terms of "a," "b," and/or "c" on the "Area:" rectangle. Explain here how you determined the area of Figure 1.

7. Leave Figure 1 arranged as it is. There should be four more triangles and one more square that are the same sizes as the five shapes used in Figure 1. Arrange these in the configuration below, and place the “Figure 2” label under the shapes. Tape or glue this next to Figure 1.

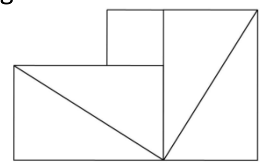
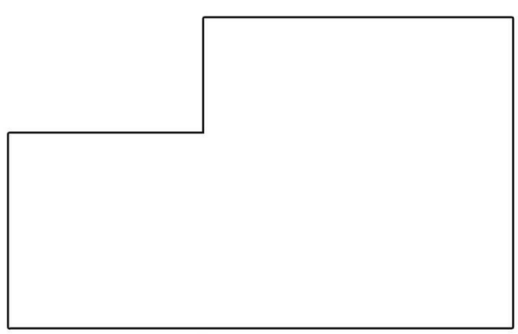


Figure 2

8. Place one of the “Area:” rectangles under Figure 2, and write the area in terms of “a,” “b,” and/or “c” on the “Area:” rectangle. Explain here how you determined the area of Figure 2.

9. Is the total area of Figure 2 larger, smaller, or the same as the total area of Figure 1? Provide evidence to support your answer.

10. The perimeter of Figure 2 should be shaped like the diagram below.



Determine the lengths of all 6 sides of this diagram in terms of “a,” “b,” and/or “c.” Label all of these lengths on the diagram above.

11. Arrange the two remaining squares in the configuration below, and place the “Figure 3” label under the shapes. Tape or glue these next to Figure 2.

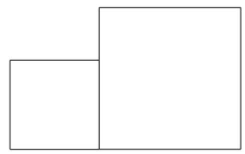


Figure 3

12. Place one of the “Area:” rectangles under Figure 3, and write the area in terms of “a,” “b,” and/or “c” on the “Area:” rectangle. Explain here how you determined the area of Figure 3.

13. Compare the perimeters of Figure 2 and Figure 3.

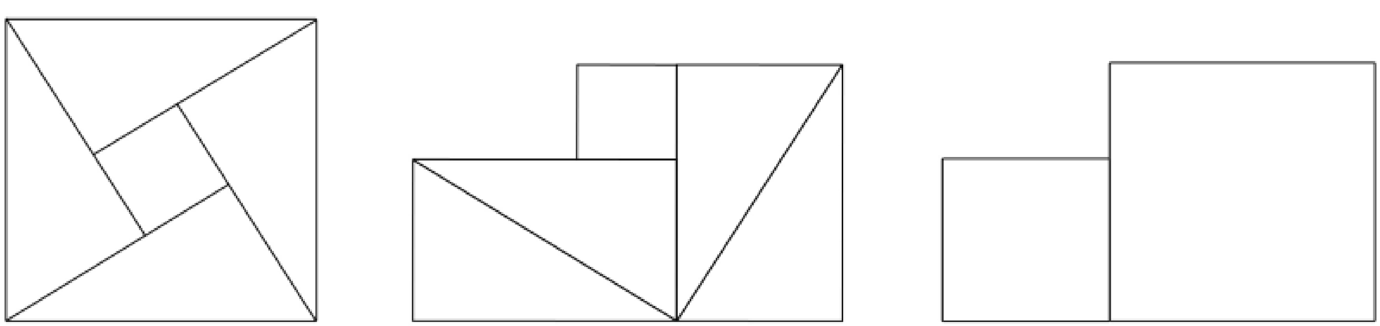
14. Compare the total areas of Figures 1, 2, and 3.

15. Write an equation relating your expression for the area of Figure 1 and your expression for the area of Figure 3. How do these areas and this equation support what you know about the Pythagorean Theorem?

16. Make a conjecture about why this same process could not be repeated with acute or obtuse triangles.

17. Would your equation from Problem #15 be true for acute or obtuse triangles? *Explain.*

18. The basic shapes from this proof are shown below. Use these figures to explain a proof of the Pythagorean Theorem. Label any side lengths needed for the proof, and write explanations below the figures.



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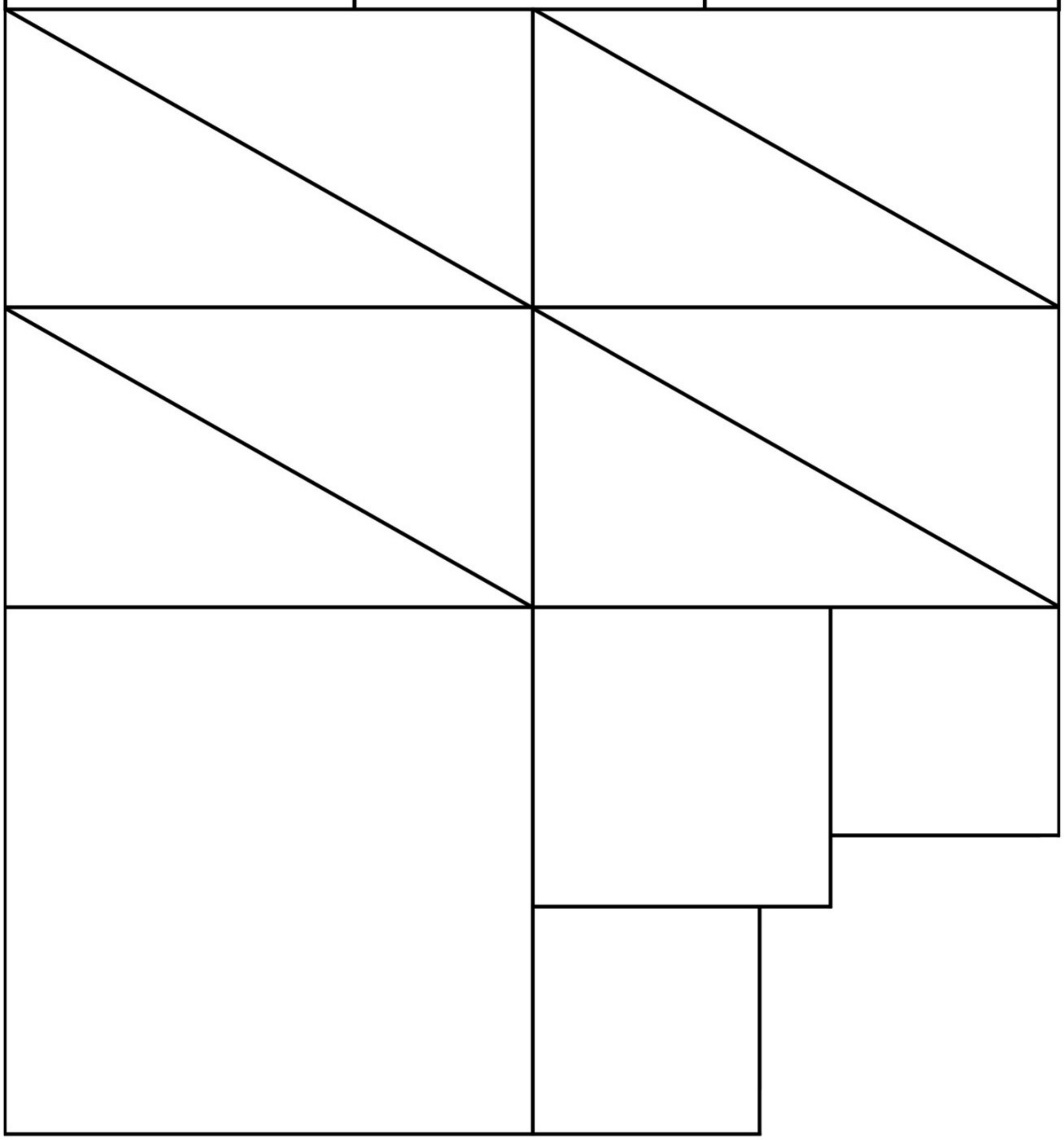
Pythagorean Theorem Proof

(cut out all of the shapes below)

Figure 1	Figure 2	Figure 3
Area:	Area:	Area:



Instructional Task Exemplar Response

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3. Label the hypotenuse of each triangle as length "c," label the longer leg of each triangle "b," and label the shorter leg of each triangle "a."
4. Determine and label the side lengths of the two largest squares in terms of "a," "b," and/or "c."
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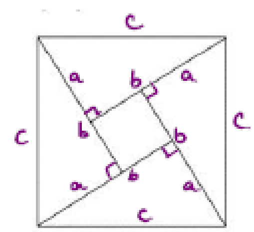
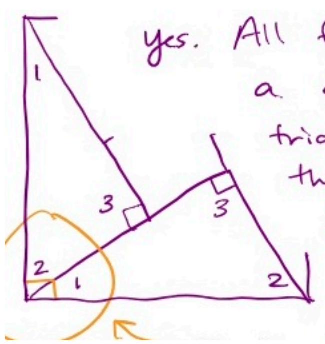


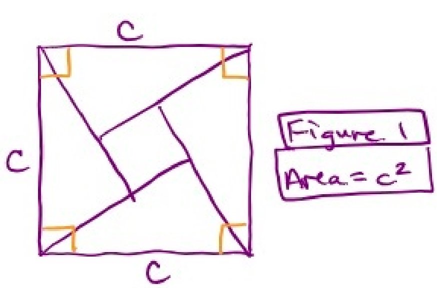
Figure 1

Is Figure 1 a square? *Support your answer with evidence about both the side lengths and angle measures.*



yes. All four sides have a side length of "c." Also, to be a square, the corners must be right angles. Since the triangles are congruent, the angles in each triangle must have the same measure. If ∠3 is a right angle m∠1 + m∠2 must add to 90° (to get 180° total degrees). If m∠1 + m∠2 = 90°, then each corner of the square must be a right angle.

6. Place one of the "Area:" rectangles under Figure 1, and write the area in terms of "a," "b," and/or "c" on the "Area:" rectangle. Explain here how you determined the area of Figure 1.



Since Figure 1 is a square, and all sites have a length of "c" the area is:

Area = $c \cdot c = c^2$

7. Leave Figure 1 arranged as it is. There should be four more triangles and one more square that are the same sizes as the five shapes used in Figure 1. Arrange these in the configuration below, and place the “Figure 2” label under the shapes. Tape or glue this next to Figure 1.

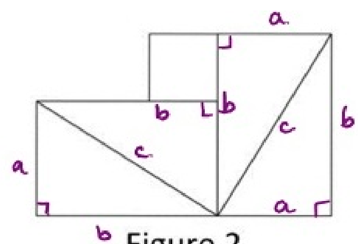
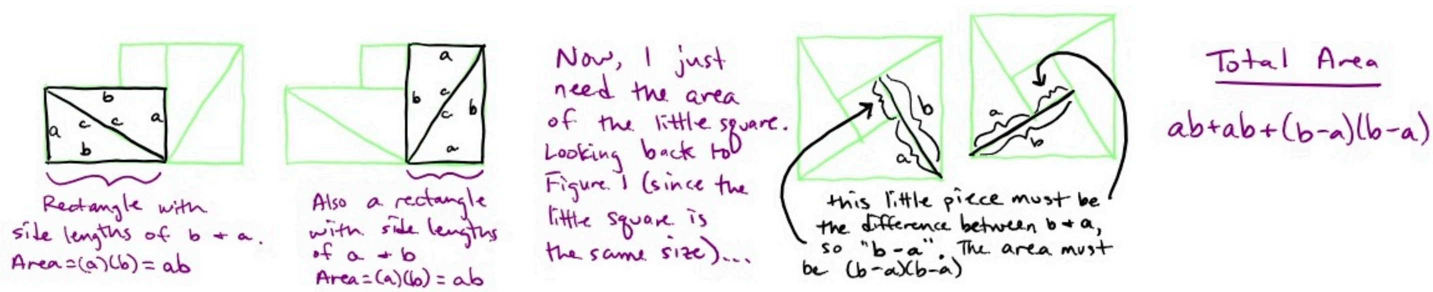


Figure 2

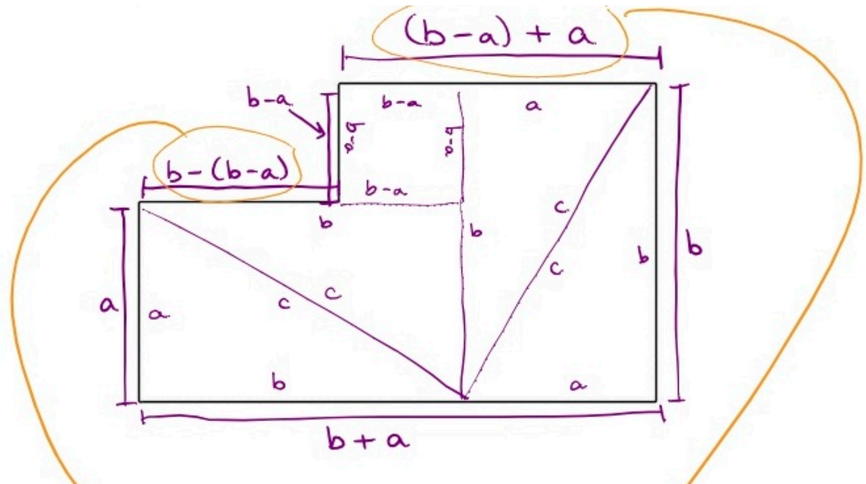
8. Place one of the “Area:” rectangles under Figure 2, and write the area in terms of “a,” “b,” and/or “c” on the “Area:” rectangle. Explain here how you determined the area of Figure 2.



9. Is the total area of Figure 2 larger, smaller, or the same as the total area of Figure 1? Provide evidence to support your answer.

The same. Even though the expressions don't look the same (c^2 vs. $2ab+(b-a)(b-a)$), the areas must be the same because the two figures are made up of the same 5 shapes.

10. The perimeter of Figure 2 should be shaped like the diagram below.

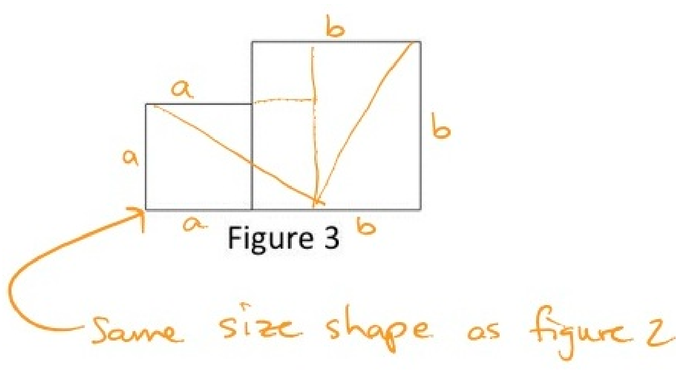


Determine the lengths of all 6 sides of this diagram in terms of “a”, “b”, and/or “c”. Label all of these lengths on the diagram above.

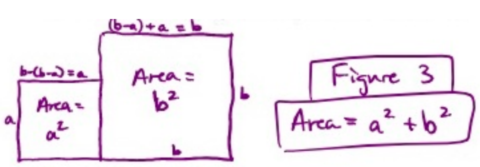
$b-(b-a)=$
$=b-b+a$
$=a$

$(b-a)+a=$
$=b-a+a$
$=b$

11. Arrange the two remaining squares in the configuration below, and place the “Figure 3” label under the shapes. Tape or glue these next to Figure 2.



12. Place one of the “Area:” rectangles under Figure 3, and write the area in terms of “a,” “b,” and/or “c” on the “Area:” rectangle. Explain here how you determined the area of Figure 3.

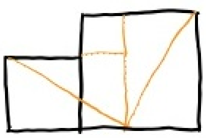


I used the perimeters from #10 to figure out the side lengths of these two squares. Then, I held the squares next to the sides of one triangle to verify that the side lengths were indeed "a" + "b"

13. Compare the perimeters of Figure 2 and Figure 3.

The perimeters are the same. Each side on the outer edge of Figure 1 has the same length as the corresponding side on figure 2.

14. Compare the total areas of Figures 1, 2, and 3.



All three areas must be exactly the same. Figure 3 could be divided like Figure 2 and all of the lengths are what they are supposed to be.

15. Write an equation relating your expression for the area of Figure 1 and your expression for the area of Figure 3. How do these areas and this equation support what you know about the Pythagorean Theorem?

$c^2 = a^2 + b^2$

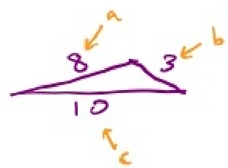
This is the Pythagorean theorem. With any right triangles I should be able to repeat this procedure, so $c^2 = a^2 + b^2$ should be true for any right triangles.

16. Make a conjecture about why this same process could not be repeated with acute or obtuse triangles. ****Note: This is a sample response. Other explanations are acceptable if they are grounded in solid mathematical reasoning.***

With acute or obtuse triangles these configurations could not be made. Right angles could not be formed to create rectangles + squares like we did above.

17. Would your equation from Problem #15 be true for acute or obtuse triangles? *Explain.*
***Note: This is a sample response. Other explanations are acceptable if they are grounded in solid mathematical reasoning.*

Not necessarily. These same figures could not be made, so the equation may not be true. But, looking at a non-right triangle,



$8^2 + 3^2 \stackrel{?}{=} 10^2$

$64 + 9 \stackrel{?}{=} 100$

$73 \neq 100$

Does not work

18. The basic shapes from this proof are shown below. Use these figures to explain a proof of the Pythagorean Theorem. Label any side lengths needed for the proof, and write explanations below the figures.

***Note: This is a sample explanation of this proof. Other explanations should be reviewed for accuracy and conceptual understanding of the Pythagorean Theorem.*

Let all triangles be congruent right triangles with side lengths a, b, + c.

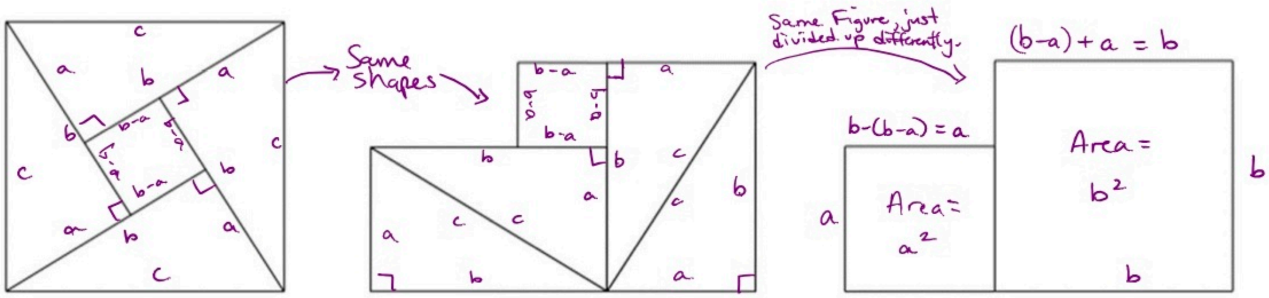


Figure 1 — Area = c^2

Figure 2 — Area = $2ab + (b-a)(b-a)$

Figure 3 — Area = $a^2 + b^2$

All three figures must have the same area, therefore

$$c^2 = 2ab + (b-a)(b-a) = a^2 + b^2$$

and

$$\underline{c^2 = a^2 + b^2}$$