Ice-Cream Cones (IT)

Overview

Students will use their knowledge of surface area and area of circles to find a formula to determine the surface area of a cone. Then students will use that formula and the actual measurements of an ice-cream cone to determine the surface area needed to wrap an ice-cream cone. Students will then be able to use their calculations to estimate how many wrappers can be cut from a sheet of paper with specific dimensions.

Standards

Create equations that describe numbers or relationships.

HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.1	• 7.EE.B.4 • 8.EE.C.7	1. A circle with radius *r* is cut out of a square piece of paper whose side lengths are equal to the diameter of the circle. Write an equation, in terms of *r*, to find the area of the square remaining once the circle is removed. a. $A = 4r^2 - \pi r^2$ *OR* $A = r^2(4 - \pi)$ *OR* $A = (2r)^2 - \pi r^2$ 2. http://www.illustrativemathematics.org/illustrations/1124	• http://www.illustrativemathematics.org/illustrations/643 • http://www.illustrativemathematics.org/illustrations/583 • http://www.illustrativemathematics.org/illustrations/999 • http://learnzillion.com/lessonsets/120-create-equations-and-inequalities-in-one-variable-and-use-them-to-solve-problems

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is lateral area?*** Lateral area is calculated for three-dimensional figures. It is the area of the faces of the figure, not including the base.
- ***What is a net?*** In terms of three-dimensional figures, a net is a two-dimensional representation of the surface of a three-dimensional figure such that if you fold the net, it would create the corresponding three-dimensional figure. The net should include all faces, including the base, of the three-dimensional shape.

During the Task

This task is likely best used with groups of 2-3 students so they can work together to write the equations and work through the problem.

Students will be better positioned to complete this task if they are provided with actual ice-cream cones, or other models of cones, which they can manipulate to create the net. Students will have been introduced to the term *net* in 6th grade. While this may be the first time they have created a net for a cone, they should be able to use their previous understanding of the term *net* to complete the task. If some scaffolding is necessary, use the template of the net given at the end of the task.

Students will have to determine the area of the sector of a circle in order to find the lateral area of a cone. While this is a formula reserved for use in geometry, students should be able to apply their understanding of proportional reasoning in order to create the formula for the area of the sector using the variables *s* and *r* for slant height and radius, respectively. Students may need assistance to see the lateral area as being a portion of a larger circle. Have students trace multiple copies of the sector with edges touching to help them see how they might determine the area of that one part of the circle. Students should be given some time to struggle with this task and persevere through working the problem in order to arrive at the solution.

Some questions that could be asked to prompt student thinking are:

1. What information do you know about the circular lid of the cone that could help you find some information about the lateral area of the cone?
2. How does the circumference of the circular lid compare to the circumference of the larger circle with the center as the vertex of the cone and the radius length equal to the length of the slant height?

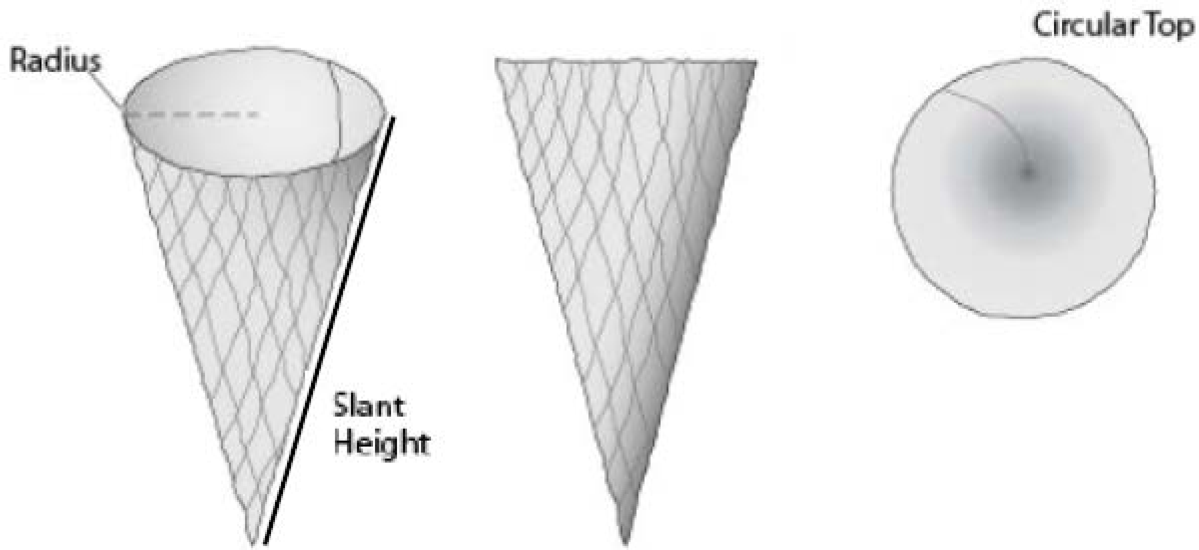
When students are estimating the number of wrappers to be cut in part 5, it may be helpful to give students a copy of the net included in this task or to encourage them to draw a net based on the measurements they found for a cone. It may also be helpful to give students paper they can use to manipulate in order to help make their estimations.

After the Task

Have groups of students explain their estimations for part 5 to the class. Have the class discuss which they think is the best estimation and explain their reasoning. Have students discuss how the answer would change if they had to figure in overlap for the wrapper.

Student Instructional Task

You have been hired by the owner of a local ice-cream parlor to assist in his company's new venture. The company will soon sell its ice-cream cones in the freezer section of area grocery stores. The manufacturing process requires that the ice-cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disk covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones.



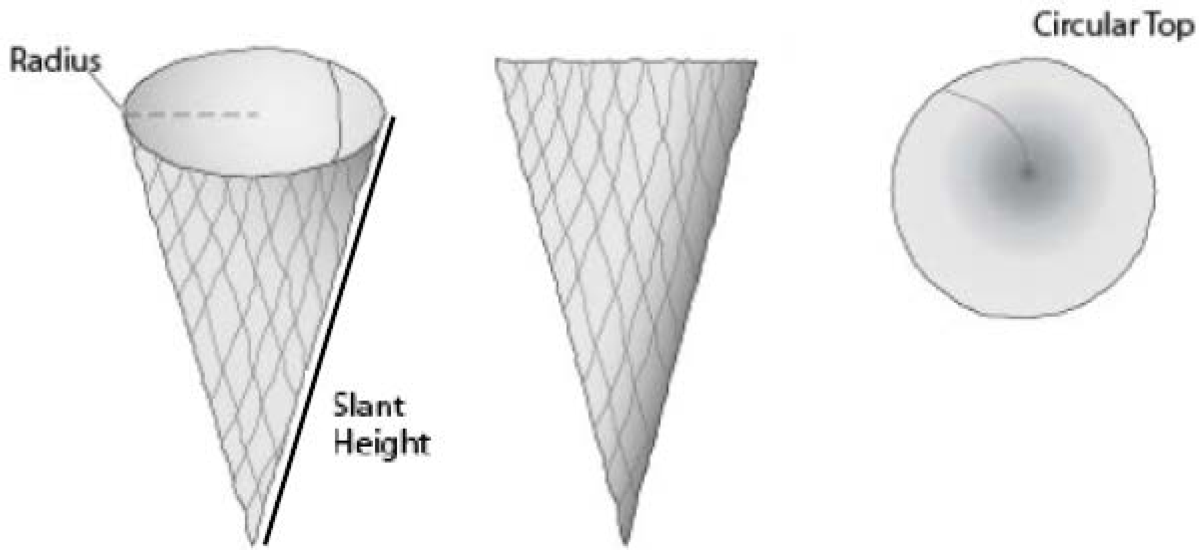
Use a separate sheet of paper for all calculations and explanations.

1. Generate a net of the complete wrapper for the cone and lid, ignoring the overlap required for assembly. The lid should rest on the cone.
2. Use the net of the wrapper without the lid to develop a formula to calculate the lateral area of the cone. The lateral area of a three-dimensional figure is the area of the faces, not including the base. Write or explain the process you used to determine the formula.
3. Provide the ice-cream company owner with a single formula that he can use to find the surface area, *SA*, of a wrapper, including the lid, for any size cone if he knows the radius, *r*, of the base and the slant height, *s*, of the cone.
4. Find the measurements of a real ice-cream cone, either by measuring an actual ice-cream cone, or by researching the dimensions of an ice-cream cone. Use these measurements to calculate the total surface area of the of the ice-cream cone. Show your calculations.
5. The company has large rectangular pieces of paper that measure 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit the cone used in #4 that could be cut from this one piece of paper. Explain your estimate. Would your estimation technique work for wrappers of other sizes or shapes?

Task adapted from http://www.utdanacenter.org/k12mathbenchmarks/tasks/7_coneslaunch.php.

Instructional Task Exemplar Response

You have been hired by the owner of a local ice-cream parlor to assist in his company's new venture. The company will soon sell its ice-cream cones in the freezer section of area grocery stores. The manufacturing process requires that the ice-cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disk covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones.



1. Generate a net of the complete wrapper for the cone and lid, ignoring the overlap required for assembly. The lid should rest on the cone.

   Check that students have an accurate net for the situation. One possible representation is shown below. Note that the placement of the circle may vary.

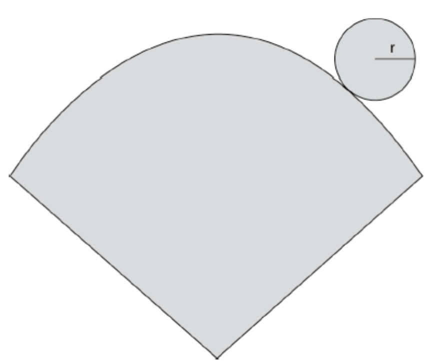
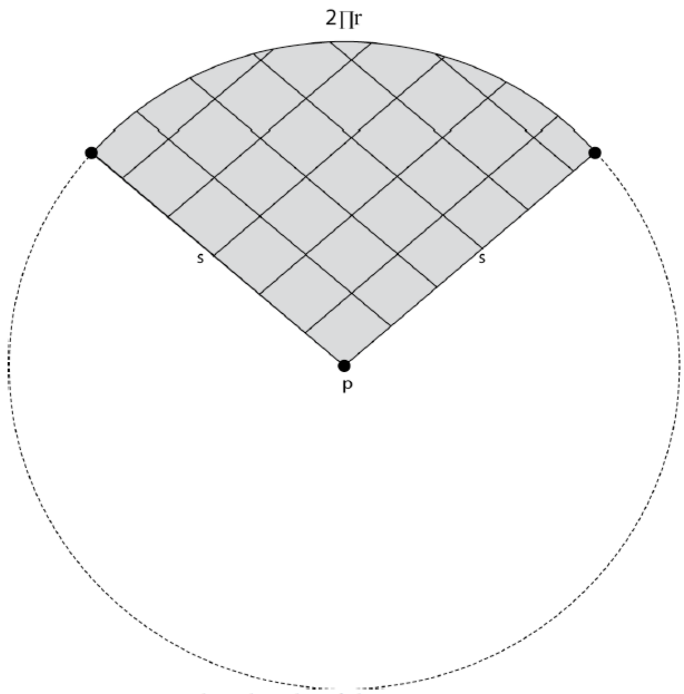


Figure not drawn to scale

2. Use the net of the wrapper without the lid to develop a formula to calculate the lateral area of the cone. The lateral area of a three-dimensional figure is the area of the faces, not including the base. Write or explain the process you used to determine the formula.

Teacher notes: To generate a formula for the lateral area of a cone, sketch a circle with the center as the vertex of the cone; the radius has the same measure as the slant height of the cone.

The circumference of the circular top of the cone, where r is the radius of the base of the cone, is 2πr. This is also the length of the arc of the sector.



s = slant height of the ice cream cone

To find the lateral area of the cone, set up a proportion of corresponding ratios:

$$\frac{area\ of\ a\ sector}{area\ of\ circle} = \frac{length\ of\ arc\ of\ sector}{total\ circumference\ of\ circle}$$

$$\frac{A}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

$$\pi s^2 \cdot \frac{A}{\pi s^2} = \frac{2\pi r}{2\pi s} \cdot \pi s^2$$

$$A = \pi r s$$

3. Provide the ice-cream company owner with a single formula that he can use to find the surface area, *SA*, of a wrapper, including the lid, for any size cone if he knows the radius, *r*, of the base and the slant height, *s*, of the cone.

   Lateral area (from #2) + Area of circle lid $SA = \pi rs + \pi r^2$

4. Find the measurements of a real ice-cream cone, either by measuring an actual ice-cream cone, or by researching the dimensions of an ice-cream cone. Use these measurements to calculate the total surface area of the of the ice-cream cone. Show your calculations.

   Teacher notes: Measurements for a standard sugar cone are approximately 11.5 cm for slant height and 2.5 cm for radius. (Answers will vary if a different cone size is used and according to the level of measurement precision.)

   Total surface area: $SA = \pi rs + \pi r^2$

   $SA = \pi(2.5)(11.5) + \pi(2.5)^2$

   $SA = 110.0\ cm^2$

5. The company has large rectangular pieces of paper that measure 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit the cone used in #4 that could be cut from this one piece of paper. Explain your estimate. Would your estimation technique work for wrappers of other sizes or shapes?

   Answers will vary. Students may decide to lay out one copy of the net, then another, to explore a pattern for estimating the number of complete wrappers that could be cut from the large paper. The template on the next page could be used to help students in their estimation.

   Sample response:

   If the cone has a slant height (s) of 11.5 cm and a base radius (r) of 2.5 cm, then the lateral area of the cone is

$$A = \pi * 2.5 * 11.5$$

$$A = 28.75\pi\ cm^2$$

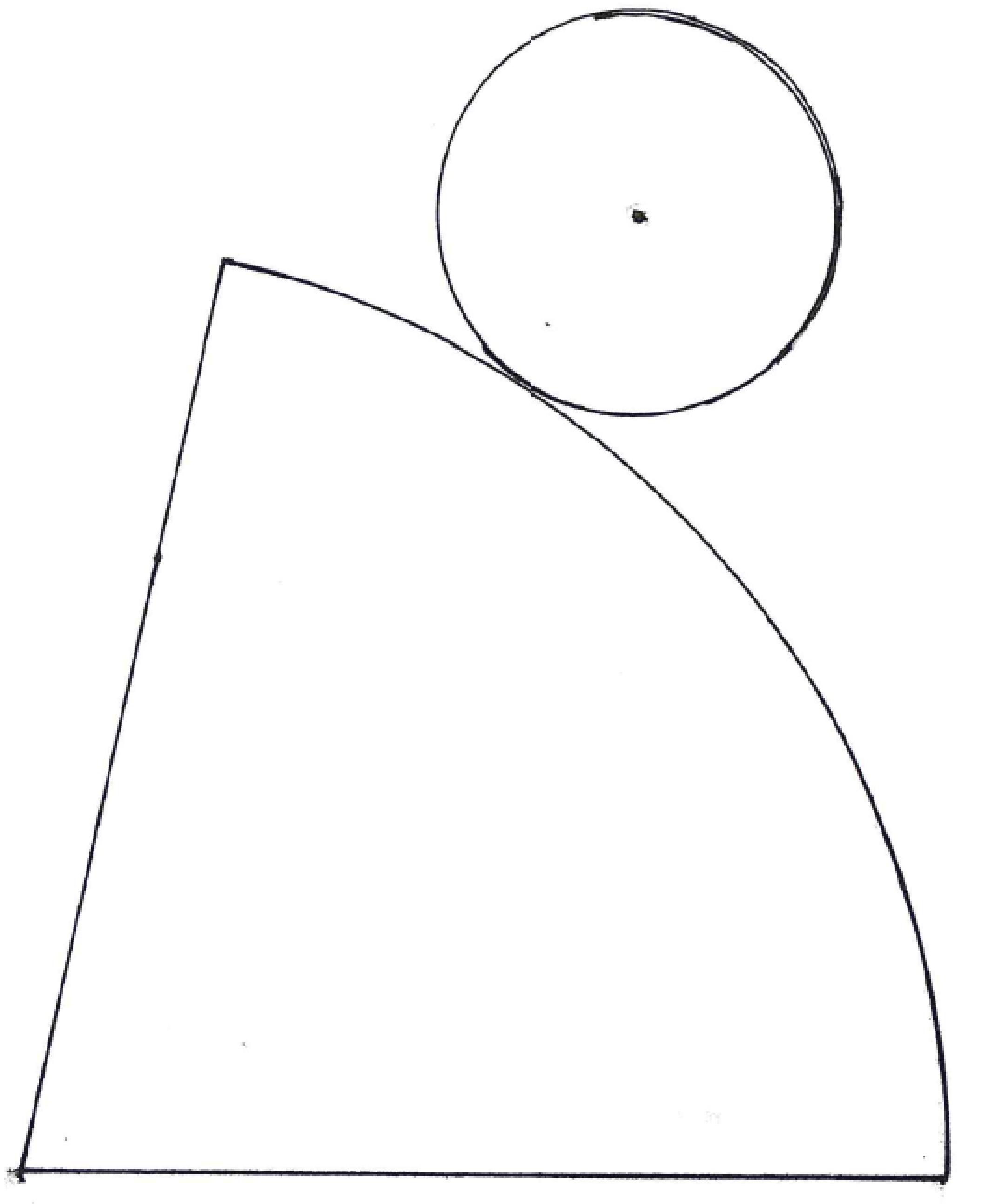
   The area of a circle with a radius of 11.5 cm (the same as the slant height of the cone) is 132.25π cm². Therefore, $\frac{132.25\pi\ cm^2}{28.75\pi\ cm^2} = 4.6$*, which means I can get 4 sectors with the lateral area of the cone from a circle with a radius of 11.5 cm. If I divide the 100 cm x 150 cm paper into squares with side lengths of 23 cm, I can get 24 circles with a radius of 11.5 cm (or diameter of 23 cm). This would be 6 squares by 4 squares (150/23 ≈ 6.5 and 100/23 ≈ 4). If I can get 4 cone pieces (lateral area only) from one circle, I can get 96 cone pieces (lateral area only) from 24 circles (24 x 4).*

   Now I need to determine if I could get at least 96 cone lids (the circular base) from the remaining paper. From the 150 cm length of the paper, there would be 12 cm remaining (150 cm – 6 x 23 cm = 12 cm). So this rectangle would be 12 cm x 100 cm. The circular base has a diameter of 5 cm (2.5 cm x 2). Using the same reasoning as I did with the large circles, I could divide the 12 cm by 100 cm area into squares of 5 cm on each side—there would be two squares by 20 squares (12/5 = 2.4 and 100/5 = 20). This would mean I could cut 2 x 20 small circles from that area, for a total of 40 circles. The remaining area on the 100 cm side would be 8 cm (100 cm – 4 x 23cm = 8 cm) by 138 cm. I would be able to get one 5 cm square from the 8 cm side and 27 squares from the 138 cm side for a total of 27 additional small *circles. Therefore, I would only be able to get 27 circles from this area. That would be a total of 67 circles (29 fewer than I need). From each of the 24 circles created to produce the lateral sides of the cone, there is a sector remaining for each circle from which one small circle (lid) could be cut, giving an additional 24 small circles. The total is now 91 small circles.*

So my estimate is a total of 91 complete cone wrappers from a sheet of paper 100 cm x 150 cm.

Task adapted from [*http://www.utdanacenter.org/k12mathbenchmarks/tasks/7_coneslaunch.php*](http://www.utdanacenter.org/k12mathbenchmarks/tasks/7_coneslaunch.php)*.*

Cone template (slant height = 11.5 cm and radius of circle = 2.5 cm; may not be to scale due to document formatting)



Lemonade Stand (IT)

Overview

Students will use systems of inequalities to decide how many pitchers of regular and strawberry lemonade to make and sell.

Standards

Create equations that describe numbers or relationships.

A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

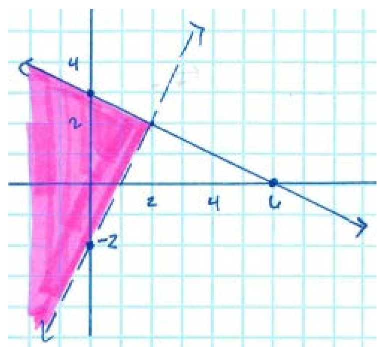
Represent and solve equations and inequalities graphically.

A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-CED.A.3	• HSA-CED.A.1 • HSA-CED.A.2 • HSA-REI.D.10	1. A local bicycle shop makes $75 on each Model A bike and $90 on each Model B bike. The overhead costs for making the bikes are $1,350. Write an inequality to show how many of each bike model must be sold so the company avoids losing money. a. $75a + 90b \geq 1350$ 2. D'Andre earns some money by helping people with their yards. It takes him an average of 2 hours to weed a flowerbed and an average of 1.5 hours to mow and edge a lawn. He can work no more than 6 hours a day. He charges $60 to weed a flowerbed and $75 to mow and edge a lawn. He needs to cover his expenses of $150 to make a profit. Write a system of inequalities that would help determine how many flowerbeds and lawns he could complete on a given day. a. $2f + 1.5m \leq 6$	• http://www.illustrativemathematics.org/illustrations/582 • http://www.illustrativemathematics.org/illustrations/1351 • http://www.illustrativemathematics.org/illustrations/1066 • http://learnzillion.com/lessonsets/667-represent-constraints-by-equations-inequalities-and-systems • http://learnzillion.com/lessonsets/516-represent-constraints-by-linear-equations-inequalities-and-systems-interpret-solutions-as-viable-and-nonviable

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		$60f + 75m \geq 150$ Where *f* = # of flowerbeds and *m* = # of lawns to be mowed and edged 3. http://www.illustrativemathematics.org/illustrations/1351	
HSA-REI.D.12	• HSA-REI.C.5 • HSA-REI.C.6 • HSA-REI.D.11	1. Graph the solution set to the system $\begin{cases} x + 2y \leq 6 \\ y > 2x - 2 \end{cases}$  a. 2. http://www.illustrativemathematics.org/illustrations/644 3. http://www.illustrativemathematics.org/illustrations/1205	• http://www.illustrativemathematics.org/illustrations/1363 • http://www.illustrativemathematics.org/illustrations/1033 • http://www.illustrativemathematics.org/illustrations/618 • http://learnzillion.com/lessonsets/678-graph-the-solutions-to-a-linear-inequality-as-a-halfplane-graph-the-solutions-to-a-system-of-linear-inequalities-as-an-intersection-of-halfplanes • http://learnzillion.com/lessonsets/624-graph-the-solutions-to-a-linear-inequality-as-a-halfplane-and-the-solutions-to-a-system-of-linear-inequalities-as-an-intersection-of-halfplanes

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is a lemonade stand?*** A lemonade stand is a way some young people earn some money. Typically, the person sets up a table in his or her yard or another location that people pass by often. Then the person sells cups of lemonade to those who stop by the stand.

During the Task

- Some students may attempt to write and graph a system of equations rather than a system of inequalities. Ask students to discuss how to represent the term *at most* when constructing the models for this task.
- If students forget to shade in the solution for the inequalities, ask them to identify the solution on their graph. Have the students explain what the solution represents. Then ask students if other ordered pairs in the coordinate plane would satisfy the conditions given in the task. Use probing questions and class discussions to guide students to see that the solutions lie in a region bound by the lines and the axes.

- Students have to make some assumptions about how many servings are in the one-gallon pitchers. Students can choose the serving size they wish. They will need to state this assumption in their explanation to determine the amount of money Lilla and Siriana will earn.
- Encourage students to determine if the number of pitchers of lemonade they chose would result in Lilla and Siriana making the most amount of money. This will require them to try other solutions in the system. Have students explain how they determined the number of pitchers that would produce the most earnings.
- For part 2 of the task, students may be assigned a specific scenario or students may choose the scenario. Some of the given options are more difficult and therefore could be reserved for students who have a strong command of the content, while the other scenarios could be used for those students who are still working to master the content.
- During part 2 of the task, if students are struggling to determine how their original recommendation would change, encourage students to use transparencies or sheet protectors as an overlay to create new graphs so they can see the difference in the solution sets.
- For scenario 4, students may choose to use either price: $0.75 or $1.25. Encourage strong groups to investigate both prices and determine which would be the better option.

After the Task

Once students have completed the task, have them share their answers with the class. Have a class discussion about the various procedures students used to complete the task. Also identify the number of pitchers for each type of lemonade that would produce the largest earnings based on the different serving sizes students used. Have students share their work and reasoning based on the given scenarios for part 2. Allow students to critique the reasoning of other students and facilitate a discussion around which recommendations would be best.

For the first scenario, ask the class to determine if there is a better price to charge for the regular lemonade that would allow the friends to make more money after they have paid for supplies. Also, have the class determine if Lilla and Siriana can pay for the supplies and still make a minimum of $300 based on the number of hours each person wants to work. Have students explain their reasoning using graphs, tables, charts, etc. Also, have students determine if they would recommend that Lilla and Siriana should choose the original plan or if they should choose one of the different scenarios.

Student Instructional Task

Lilla and Siriana plan to open a lemonade stand during the summer. The friends have divided the work as described below. The friends agree to sell regular lemonade for $0.75 per serving and strawberry lemonade for $1.25 per serving.

Lilla decides that she will make two flavors of lemonade: regular and strawberry.

- It takes her 10 minutes to make a one-gallon pitcher of regular lemonade.
- It takes her a 15 minutes to make a one-gallon pitcher of strawberry lemonade.
- She plans to spend no more than four hours making lemonade.

Siriana has decided to work outside and sell the lemonade.

- She plans to spend no more than six hours selling lemonade.
- She estimates she will sell one pitcher of lemonade (of either flavor) every third of an hour.

1. How many pitchers of each type of lemonade would you recommend Lilla and Siriana make and sell? Explain your reasoning. Use equations, inequalities, graphs and/or tables to aid your explanation. In your explanation, also include how much money Lilla and Siriana will earn if they sell every serving in every pitcher of lemonade you recommend they make.

2. Choose **one** of the following scenarios to investigate. Determine how your answer might change based on the scenario you choose. Explain your reasoning. Use equations, inequalities, graphs, and/or tables to aid your explanation.
    - Lilla and Siriana have to pay for the supplies to make the lemonade from their earnings. It costs $10 to make a pitcher of regular lemonade and $12 to make a pitcher of strawberry lemonade.
    - Siriana plans to spend no more than 4 hours selling lemonade.
    - Siriana plans to spend no more than 7 hours selling lemonade.
    - The friends sell both types of lemonade for the same price.
    - Lilla and Siriana want their total earnings to be a minimum of $300.

Task adapted with permission from Universal Achievement, LLC.

Instructional Task Exemplar Response

Lilla and Siriana plan to open a lemonade stand during the summer. The friends have divided the work as described below. The friends agree to sell regular lemonade for $0.75 per serving and strawberry lemonade for $1.25 per serving.

Lilla decides that she will make two flavors of lemonade: regular and strawberry.

- It takes her 10 minutes to make a one-gallon pitcher of regular lemonade.
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- She plans to spend no more than four hours making lemonade.

Siriana has decided to work outside and sell the lemonade.

- She plans to spend no more than six hours selling lemonade.
- She estimates she will sell one pitcher of lemonade (of either flavor) every third of an hour.

1. How many pitchers of each type of lemonade would you recommend Lilla and Siriana make and sell? Explain your reasoning. Use equations, inequalities, graphs and/or tables to aid your explanation. In your explanation, also include how much money Lilla and Siriana will earn if they sell every serving in every pitcher of lemonade you recommended they make.

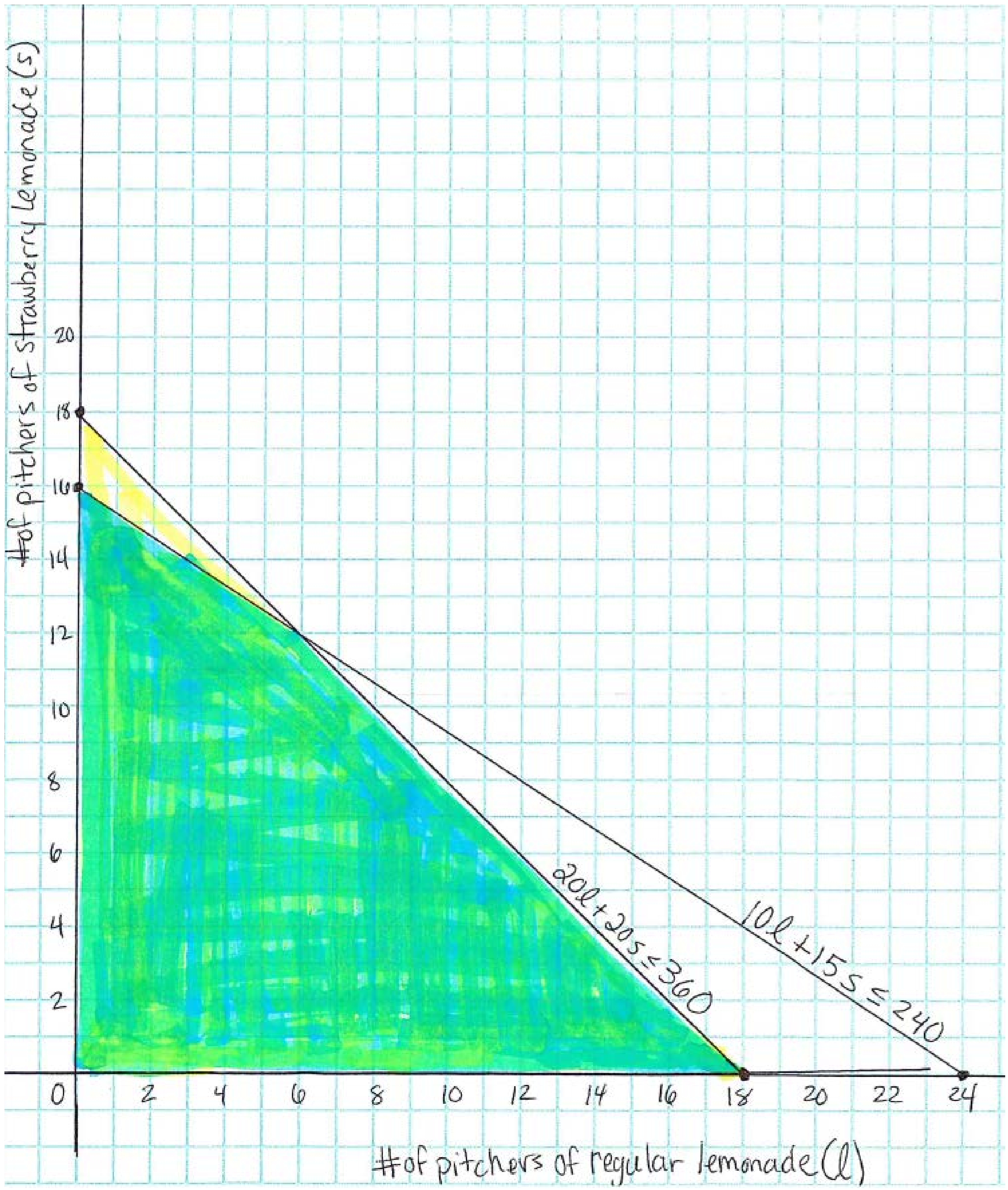
   Sample response: Students have many options to proceed with the work for this task. Below is one way to find an answer.

   Let l = the # of pitchers of regular lemonade to be made and sold; let s = the # of pitchers of strawberry lemonade to be made and sold.

   If Lilla wants to spend no more than four hours making lemonade, then she will spend no more than 4 x 60 minutes or 240 minutes making lemonade. This can be represented by the inequality $10l + 15s \leq 240$, where 10l represents the amount of time (in minutes) Lilla will take to make l pitchers of regular lemonade, and 15s represents the amount of time (in minutes) Lilla will take to make s pitchers of strawberry lemonade.

   If Siriana wants to spend no more than six hours selling lemonade, then she will spend no more than 6 x 60 minutes or 360 minutes selling lemonade. This situation can be represented by the inequality $20l + 20s \leq 360$, where 20l represents the amount of time she would sell l pitchers of regular lemonade and 20s represents the amount of time she would sell s pitchers of strawberry lemonade.

   Below is a graph of this system of inequalities. The green shaded region represents all possibilities for the number of pitchers of each type of lemonade to be made and sold. I used the axes as additional constraints ($l \geq 0$ and $s \geq 0$) because Lilla cannot make a negative number of pitchers of lemonade.



In order to decide the number of pitchers of each type of lemonade to be made and sold, I decided to test different points in the region with the prices they would charge. I think they would want to earn the most money.

First, I assumed that each serving would be 8 oz. of lemonade—in one gallon of lemonade, there would be sixteen 8 oz. servings. The price for one serving of regular lemonade is $0.75; if I multiply that by 16 servings in *one pitcher, the earnings would be $12 per pitcher. The price for one serving of strawberry lemonade is $1.25; if I multiply that by the 16 servings in one pitcher, the earnings would be $20 per pitcher. Therefore, I can use the expression 12l + 20s to find the total earnings for the number of pitchers sold.*

# of pitchers of regular lemonade sold	***# of pitchers of strawberry lemonade sold***	***Total earnings***
6	*12*	*12(6) + 20(12) = $312*
1	*15*	*12(1) + 20(15) = $312*
3	*14*	*12(3) + 20(14) = $316*
4	*13*	*12(4) + 20(13) = $308*
0	*16*	*12(0) + 20(16) = $320*
9	*9*	*12(9) + 20(9) = $288*
18	*0*	*12(18) + 20(0) = $216*

The most amount of money the girls can earn is $320, but that would mean that they would only serve strawberry lemonade. This may not be a good idea if there are people who don't like strawberry lemonade. So, I'm going to choose the next option, which would be 3 pitchers of regular lemonade and 14 pitchers of strawberry lemonade. Lilla would spend exactly 4 hours making the lemonade (because the point lies on boundary line of the graph of the inequality $10l + 15s \leq 240$ *while Siriana would spend 5 hours and 40 minutes selling the lemonade. The girls would make $316 if they serve 8 oz. at a time.*

2. Choose **one** of the following scenarios to investigate. Determine how your answer might change based on the scenario you choose. Explain your reasoning. Use equations, inequalities, graphs, and/or tables to aid your explanation.
   - Lilla and Siriana have to pay for the supplies to make the lemonade from their earnings. It costs $10 to make a pitcher of regular lemonade and $12 to make a pitcher of strawberry lemonade.
   - Siriana plans to spend no more than 4 hours selling lemonade.
   - Siriana plans to spend no more than 7 hours selling lemonade.
   - The friends sell both types of lemonade for the same price.
   - Lilla and Siriana want their total earnings to be a minimum of $300.
   - Lilla and Siriana want their profit to be a minimum of $300. It costs $10 to make a pitcher of regular lemonade and $12 to make a pitcher of strawberry lemonade.

***Note: Below are sample responses for how the students' answers might change. These responses are based on the recommendation from question 1 above.*

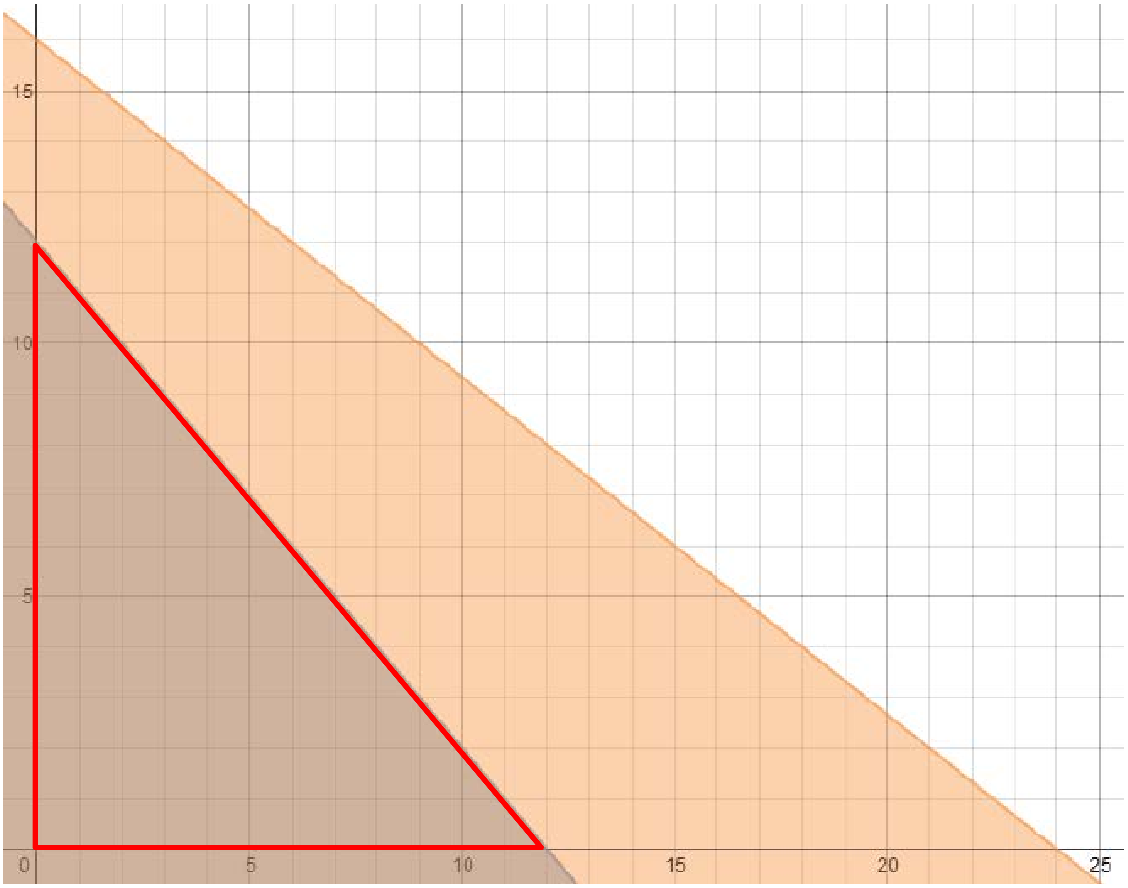
Scenario 1: Lilla and Siriana have to pay for the lemonade with their earnings. It costs $10 to make a pitcher of regular lemonade and $12 to make a pitcher of strawberry lemonade.

The total earnings Lilla and Siriana make can be represented by 12l + 20s, meaning that they will earn $12 per pitcher of regular lemonade and $20 per pitcher of strawberry lemonade. If they have to pay $10 per pitcher of regular lemonade and $12 per pitcher of strawberry lemonade, then they will actually make $2 for every pitcher of regular lemonade (12 – 10) and $8 for every pitcher of strawberry lemonade (20 – 12). Therefore, the amount of money they will have after paying for their supplies can be represented by 2l + 8s. The table below shows their earnings for different combinations of regular and strawberry lemonade.

# of pitchers of regular lemonade sold	***# of pitchers of strawberry lemonade sold***	***Earnings after paying for supplies***
6	*12*	*2(6) + 8(12) = $108*
1	*15*	*2(1) + 8(15) = $122*
3	*14*	*2(3) + 8(14) = $118*
4	*13*	*2(4) + 8(13) = $112*
0	*16*	*2(0) + 8(16) = $128*
9	*9*	*2(9) + 8(9) = $90*
18	*0*	*2(18) + 8(0) = $36*

Based on this table, making 16 pitchers of strawberry lemonade would make the most money again, but I would recommend they make 15 pitchers of strawberry and 1 pitcher of regular in case there are people who do not like strawberry lemonade. If they sell all servings in every pitcher, they would make $122 after paying for supplies.

Scenario 2: Siriana plans to spend no more than 4 hours selling lemonade.



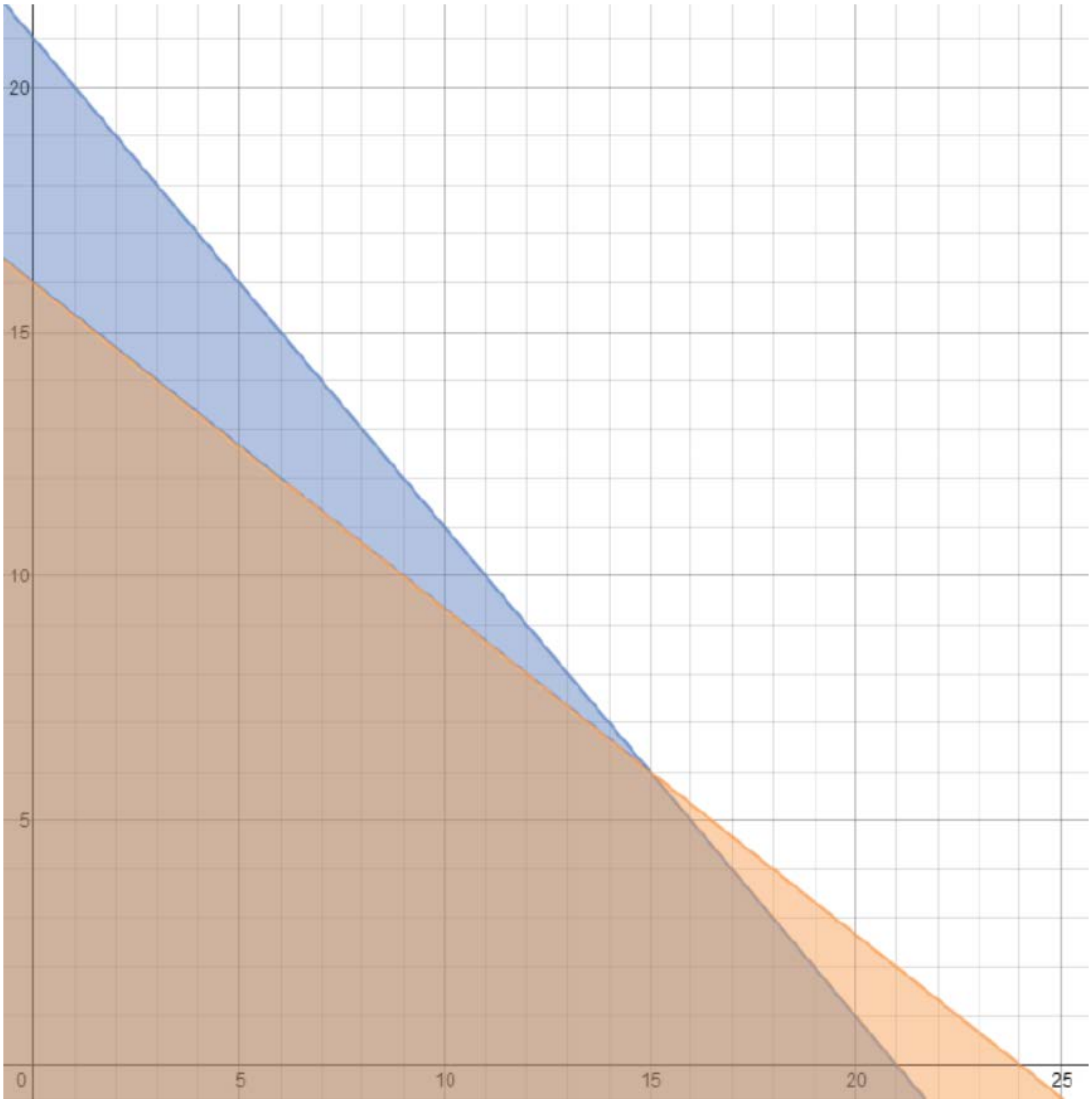
The graph above shows how the solution set changes. There is no intersection, which means that Lilla will always spend less than 4 hours making lemonade. The recommendation must come from the region bounded by the red triangle.

Using the same serving size and cost from my original recommendation, I created the table below to see what combination of pitchers of lemonade would result in the most earnings.

# of pitchers of regular lemonade sold	***# of pitchers of strawberry lemonade sold***	***Total earnings***
0	*12*	*12(0) + 20(12) = $240*
12	*0*	*12(12) + 20(0) = $144*
1	*11*	*12(1) + 20(11) = $232*
1	*10*	*12(1) + 20(10) = $212*
2	*10*	*12(2) + 20(10) = $224*

I know that the maximum number of pitchers to be made is 12 (based on the boundary line for $20l + 20s \leq 420$). I don't think it is a good idea to sell only one type of lemonade, so I recommend making and selling 1 pitcher of regular lemonade and 11 pitchers of strawberry lemonade to earn $232.

Scenario 3: Siriana plans to spend no more than 7 hours selling lemonade.



The graph above shows how the solution set changes. The intersection changes to (15, 6), which means that Lilla will spend exactly 4 hours making lemonade and Siriana will spend exactly 7 hours selling lemonade when they have 15 pitchers of regular lemonade and 6 pitchers of strawberry lemonade. The recommendation must come from the region bounded by the red quadrilateral.

Using the same serving size and cost from my original recommendation, I created the table below to see what combination of pitchers of lemonade would result in the most earnings.

# of pitchers of regular lemonade sold	*# of pitchers of strawberry lemonade sold*	*Total earnings*
15	*6*	*12(15) + 20(6) = $300*
6	*12*	*12(6) + 20(12) = $312*
1	*15*	*12(1) + 20(15) = $312*
3	*14*	*12(3) + 20(14) = $316*
4	*13*	*12(4) + 20(13) = $308*
0	*16*	*12(0) + 20(16) = $320*
9	*9*	*12(9) + 20(9) = $288*

I realize that the change in graph simply allows more combinations to be made. However, the total earnings do not change for the different combinations based on Siriana planning to sell for 7 hours instead of 6. Therefore, I will not change my recommendation to make and sell 3 pitchers of regular lemonade and 14 pitchers of strawberry lemonade to earn $316.

Scenario 4: The friends sell both types of lemonade for the same price.

If Lilla and Siriana spend 4 hours and 6 hours, respectively, on their chosen duties, then the solution set will remain the same. For this scenario, the change is in the total earnings. Based on 8 oz. servings, if the friends sell both types of lemonade for $0.75 per serving, they will earn $12 per pitcher. If they sell both types of lemonade for $1.25 per serving, they will earn $20 per pitcher. To make the most money, they should sell both types of lemonade for $1.25.

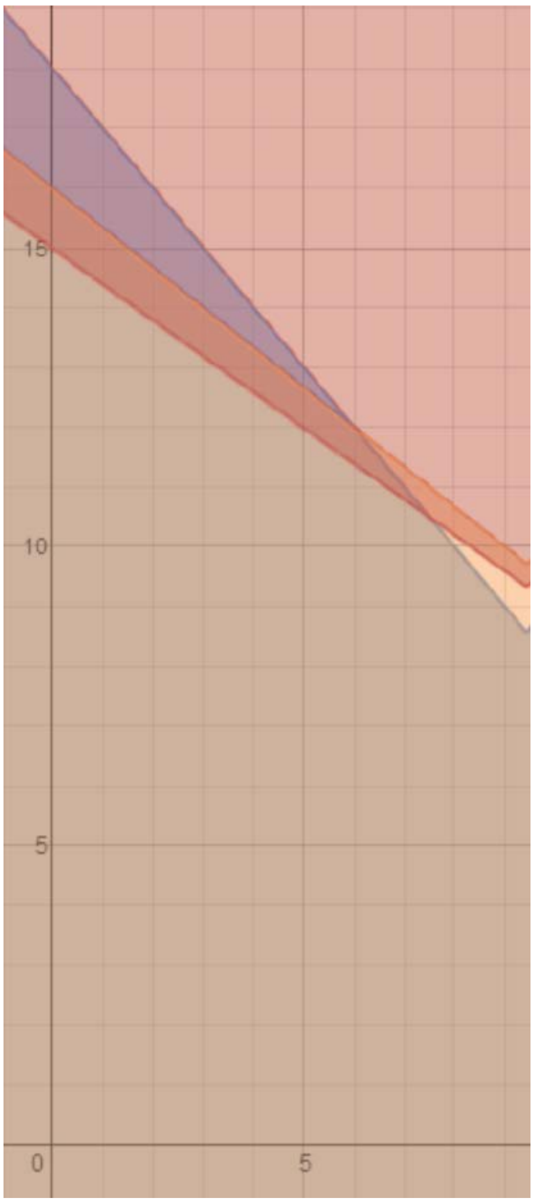
The greatest number of pitchers the friends can make and sell within the time they wish to work is 18. The combination of regular lemonade and strawberry lemonade does not matter here. As long as the friends make and sell a total of 18 pitchers, they will earn $360. Therefore, I recommend that they make and sell 9 pitchers of regular lemonade and 9 pitchers of strawberry lemonade so they might please the most people.

If they decided to sell the lemonade for $0.75, a total of 18 pitchers would still produce the most earnings. However, their earnings will only be $216.

# of pitchers of regular lemonade sold	*# of pitchers of strawberry lemonade sold*	*Earnings at $20 per pitcher*
6	*12*	*20(18) = $360*
1	*15*	*20(16) = $320*

3	*14*	*20(17) = $340*
4	*13*	*20(17) = $340*
9	*9*	*20(18) = $360*
18	*0*	*20(18) = $360*

Scenario 5: Lilla and Siriana want their total earnings to be a minimum of $300.



The graph above shows how the solution set changes with the addition of the constraint $12l + 20s \geq 300$*. There is no intersection of all three boundary lines. The recommendation must come from the region bounded by the red quadrilateral. Because this region is small and the solution must be in whole numbers* *(not allowing for part of a pitcher to be made and sold), the possible solutions are listed in the table below along with the total earnings.*

# of pitchers of regular lemonade sold	***# of pitchers of strawberry lemonade sold***	***Total earnings***
0	*16*	*12(0) + 20(16) = $320*
0	*15*	*12(0) + 20(15) = $300*
1	*15*	*12(1) + 20(15) = $312*
2	*14*	*12(2) + 20(14) = $304*
3	*14*	*12(3) + 20(14) = $316*
4	*13*	*12(4) + 20(13) = $308*
5	*12*	*12(5) + 20(12) = $300*
6	*12*	*12(6) + 20(12) = $312*
7	*11*	*12(7) + 20(11) = $304*

The most money the friends can earn is $320 by selling only strawberry lemonade. However, I believe this would not be a good idea, so my original recommendation to sell 3 pitchers of regular lemonade and 14 pitchers of strawberry lemonade to earn $316 is still my recommendation.

Solving Quadratic Equations (IT)

Overview

Students will reason about the solution methods chosen when solving quadratic equations.

Standards

Understand solving equations as a process of reasoning and explain the reasoning.

HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

HSA-REI.B.4 Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, using the quadratic formula, and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.A.1	• 7.EE.B.4a • 8.EE.C.7	1. Explain the steps to solving a quadratic equation by completing the square. a. First, if *a* is any value other than 1, divide by *a*. Second, subtract the value of the constant term from both sides. Then divide the coefficient of the linear term by 2 and square the result. Add the squared quotient to both sides of the equation. Next, rewrite the perfect square trinomial as a squared binomial. Then take the square root of both sides. Subtract the constant term in this step from both sides. Finally, simplify the resulting solution(s).	• http://www.illustrativemathematics.org/illustrations/392 • http://www.illustrativemathematics.org/illustrations/550 • http://learnzillion.com/lessonsets/495-justify-solutions-to-equations-in-terms-of-equation-properties • http://learnzillion.com/lessonsets/203-solve-and-explain-simple-algebraic-equations

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.B.4	• 8.EE.A.2	1. Solve $2x^2 - 3x = 7$. a. $x = \frac{3 \pm \sqrt{65}}{4}$ 2. http://www.illustrativemathematics.org/illustrations/375	• http://learnzillion.com/lessonsets/24-solve-quadratic-equations • http://learnzillion.com/lessonsets/98-solve-quadratic-equations-by-inspection-taking-square-roots-completing-the-square-the-quadratic-formula-and-factoring • http://learnzillion.com/lessonsets/26-understand-and-choose-methods-to-solve-quadratic-equations

During the Task

- Be sure students are not resorting to the quadratic formula for every solution. Ask students to think about the initial form of the equation before deciding which method they would choose to use. Discuss with students in which situations it might be better to complete the square rather than use the quadratic formula.
- For question 3, some students may struggle to create an equation with a complex solution. Ask students probing questions to remind them that the radicand in the quadratic formula must be negative in order to get a complex solution. Have them identify values for *a*, *b,* and *c* that will force the radicand to be negative.

After the Task

Allow students to discuss the various solution methods chosen for these problems. Provide students with other examples of quadratic equations that represent real-world situations and have students determine which method would be best to solve those equations.

Student Instructional Task

1. Tamara was asked to solve the equation $x^2 + 7x + 8 = -2$. Her work is shown below.

$$x^2 + 7x + 8 = -2$$
$$x^2 + 7x = -10$$
$$x^2 + 7x + \frac{49}{4} = \frac{9}{4}$$
$$\left(x + \frac{7}{2}\right)^2 = \frac{9}{4}$$
$$x + \frac{7}{2} = \pm\frac{3}{2}$$
$$x = -5, -2$$

Show a different method of solving $x^2 + 7x + 8 = -2$. Explain why you chose your method and how the method you chose is different from Tamara's method.

2. The equation $200 = 2.2v + \frac{v^2}{20}$ can be used to approximate the speed of a specific model car (in miles per hour) that took 200 feet to brake.
   a. Based on the given equation, which method of solving a quadratic equation would be most appropriate? Explain your reasoning.

   b. Solve the equation, explaining each step in your solution.

3. Create a quadratic equation that has a complex solution. Use the quadratic formula to verify that the equation you created has a complex solution.

Instructional Task Exemplar Response

1. Tamara was asked to solve the equation $x^2 + 7x + 8 = -2$. Her work is shown below.

$$x^2 + 7x + 8 = -2$$
$$x^2 + 7x = -10$$
$$x^2 + 7x + \frac{49}{4} = \frac{9}{4}$$
$$\left(x + \frac{7}{2}\right)^2 = \frac{9}{4}$$
$$x + \frac{7}{2} = \pm\frac{3}{2}$$
$$x = -5, -2$$

Show a different method of solving $x^2 + 7x + 8 = -2$. Explain why you chose your method and how the method you chose is different from Tamara's method.

Teacher note: Students may choose any method that will produce the correct answer. Check students' work for accuracy.

$$x^2 + 7x + 8 = -2$$
$$x^2 + 7x + 10 = 0$$
$$(x + 5)(x + 2) = 0$$
$$x = -5, -2$$

I chose to use the factoring method because when I subtracted 2 from both sides, the constant term on the left side was 10. I know the factors of 10, 5 and 2, will add to seven, so I could write two factors of $(x + 5)$ *and* $(x + 2)$*. This method is different than Tamara's method of completing the square because I did not have to create a squared binomial or take the square root of both sides of the equation.*

2. The equation $200 = 2.2v + \frac{v^2}{20}$ can be used to approximate the speed of a specific model car (in miles per hour) that took 200 feet to brake.
   a. Based on the given equation, which method of solving a quadratic equation would be most appropriate? Explain your reasoning.

      Sample response:

      The most appropriate method to solve the equation would be to use the quadratic formula because the values for a and b ($\frac{1}{20}$ *and 2.2, respectively) do not make the equation easy to factor or to complete the square.*

   b. Solve the equation, explaining each step in your solution.

Solution	Explanation
$200 = 2.2v + \frac{v^2}{20}$	
$0 = \frac{v^2}{20} + 2.2v - 200$	*Subtract 200 from both sides to make the equation equal zero.*
$v = \frac{-2.2 \pm \sqrt{2.2^2 - 4\left(\frac{1}{20}\right)(-200)}}{2\left(\frac{1}{20}\right)}$	*Substitute the values of a, b, and c into the quadratic formula.*
$v = \frac{-2.2 \pm \sqrt{44.84}}{\frac{1}{10}}$	*Simplify the radicand.*
$v = 10\left(-2.2 \pm \sqrt{44.84}\right)$	*Simplify the complex fraction by multiplying the numerator and denominator by 10.*
$v = 10\left(-2.2 + \sqrt{44.84}\right) \text{ or } 10\left(-2.2 - \sqrt{44.84}\right)$	*Separate to find both solutions.*
$v \approx 44.96$	*Simplify the first expression because the second will result in a negative value, which does not make sense in this situation.*

The car was traveling at approximately 45 mph if it took 200 feet to brake.

***Note: Students may select other methods to solve this equation. Check their work for accuracy.*

3. Create a quadratic equation that has a complex solution. Use the quadratic formula to verify that the equation you created has a complex solution.

This will have multiple different solutions. Students must use the quadratic formula to show that the equation has a complex solution by having a negative value in the radicand. One possible solution is given below. Check solutions of other students for accuracy.

Sample equation: $3x^2 - 5x = -6$

Sample solution:

$$3x^2 - 5x = -6$$

$$3x^2 + 5x + 6 = 0$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(6)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{25 - 72}}{6}$$

$$x = \frac{5 \pm \sqrt{-47}}{6}$$

This results in a complex solution because the radicand is negative.

***Note: Students in Algebra I are NOT required to write the complex solutions to quadratic equations. They are only required to recognize when complex solutions exist.*

Investment Opportunity (IT)

Overview

Students will plot and use data about revenue from wireless customers and the number of wireless connections. Then the students will use the data and graphs to determine various rates of change and to make a decision about investing in a company for the future.

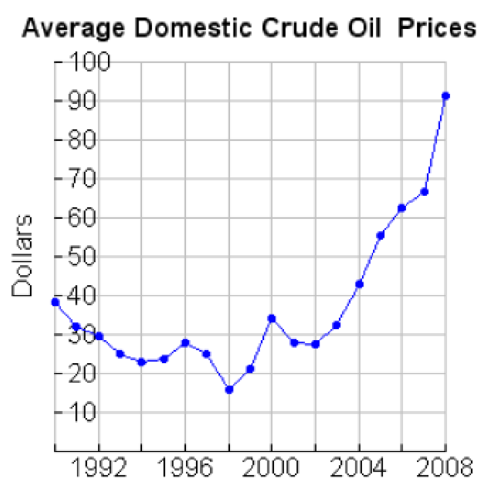
Standards

Interpret functions that arise in applications in terms of the context.

HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them:	Items to Check for Task Readiness	Sample Remediation Items
HSF-IF.B.6	• 8.F.B.4 • HSF-IF.A.2	1. The graph shows the average cost of a barrel of crude oil from 1998 to 2008. Estimate the average rate of change between 1998 and 2004 based on the graph below. Explain the meaning of average rate of change in terms of the information provided.  a. The average rate of change is approximately $4.67 per year. This means the average cost of a barrel of crude oil increased by approximately $4.67 per year from 1998 to 2004. 2. http://www.illustrativemathematics.org/illustrations/577 3. http://www.illustrativemathematics.org/illustrations/1500	• http://www.illustrativemathematics.org/illustrations/1206 • http://www.illustrativemathematics.org/illustrations/383 • http://www.illustrativemathematics.org/illustrations/664

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is revenue?*** Revenue is the amount of money a company makes when providing a service or product for consumers.
- ***What are wireless connections?*** In this task, the term *wireless connections* refers to the number of people with wireless (or cell) service with the company. These connections produce the revenue for the company.

During the Task

- Students may be tempted to find the rate of change by completing the actual math rather than estimating first. Remind them that estimating the rate of change first can help them determine if they have made any major mistakes in the work to find the actual average rate of change.
- For each part of the task, students are finding the average rate of change for two time intervals and then comparing them. The difference is how the information is graphed on the coordinate plane. Students will have to use this information to help them make sense of what the average rate of change means in the context of each part and how that could be useful in determining whether an investment in the company should be made.
- For part 3, students are graphing the revenue with respect to the number of connections. When determining the average rate of change here, students will need to identify the ordered pairs associated with the given time intervals. Also, when calculating the average rate of change, the teacher should monitor groups to be sure that students are not simply dividing the decimal values they obtain from subtracting. Remind them to be precise and pay attention to units for their answers. Guide students to find the average cost per connection rather than cost per million connections.

After the Task

Have groups share their recommendations and their reasoning. Allow students to question each group's recommendations to further their understanding or to critique the reasoning of different groups. This task can be related to finding the average change in the cost of tuition over the past 10-20 years, a factor that may help students decide where to pursue a secondary education.

Student Instructional Task

You have been hired as a financial analyst to help an investor determine whether to invest in various companies. A financial analyst might look at the average rates of change between multiple variables to determine whether the company is in a period of growth.

Below is a table that gives the annual revenue from one company's wireless business and the number of wireless connections that yield this revenue.

Year	**Wireless Revenue (billions of dollars)**	**Wireless Connections (millions)**
2006	38.0	59.1
2007	43.9	65.7
2008	49.3	72.1
2009	60.3	96.5
2010	63.4	102.2
2011	70.2	107.8

1. Graph the data for wireless revenue with respect to time. Be sure to label the axes.
    a. Estimate the rate of change between 2006 and 2008. Estimate the average rate of change between 2008 and 2011. Be sure to include units in the answers.
    b. Support each estimate with calculations.
    c. Describe how the average revenue per year changes between the two time intervals.
2. Graph the data for wireless connections with respect to time. Be sure to label the axes.
    a. Estimate the rate of change between 2006 and 2008. Estimate the rate of change between 2008 and 2011. Be sure to include units in the answers.
    b. Support each estimate with calculations.
    c. Describe how the average connections per year change between the two time intervals.
3. Graph the data for wireless revenue with respect to wireless connections. Be sure to label the axes.
    a. Estimate the rate of change between 2006 and 2008. Estimate the rate of change between 2008 and 2011. Be sure to include units in the answers.
    b. Support each estimate with calculations.

c. Describe how the average revenue per connection changes between the two time intervals.

4. Provide a written recommendation to your employer explaining why you believe they should or should not invest in this company. Be sure to use information from the comparisons above to support your recommendation. Also include any other factors you feel may need to be considered in order to make a final decision.

Task adapted from http://www.whyseemath.com/pdf212/ave_rate_verizon_wksht.pdf. More information found at http://whyseemath.com/wp/surveyofcalc/question/how-do-you-calculate-and-interpret-an-average-rate-of-change/.

Instructional Task Exemplar Response

You have been hired as a financial analyst to help an investor determine whether to invest in various companies. A financial analyst might look at the average rates of change between multiple variables to determine whether the company is in a period of growth.

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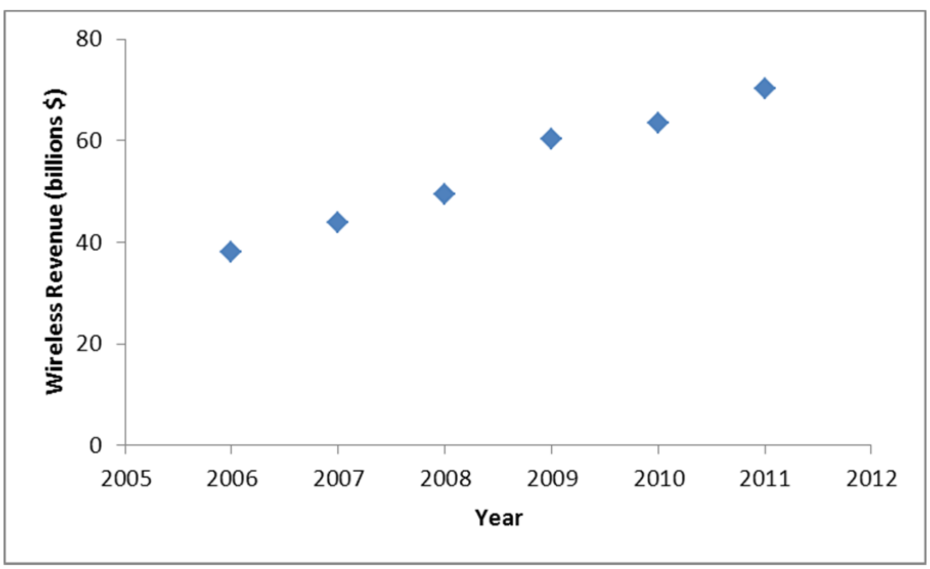
2008	49.3	72.1
2009	60.3	96.5
2010	63.4	102.2
2011	70.2	107.8

1. Graph the data for wireless revenue with respect to time. Be sure to label the axes.



Students may choose to use integers to represent the years (i.e., 0 represents 2006, 1 represents 2007, etc.). If so, they must indicate what the values represent.

a. Estimate the average revenue per year between 2006 and 2008. Estimate the average revenue per year between 2008 and 2011.

Estimated revenue per year for 2006-2008: $\frac{50-40}{2} \approx 5$ *billion dollars per year*

Estimated revenue per year for 2008-2011: $\frac{70-50}{3} \approx 6.67$ *billion dollars per year*

***Students may estimate differently but the values should be close to what is here.*

b. Support each estimate with calculations.

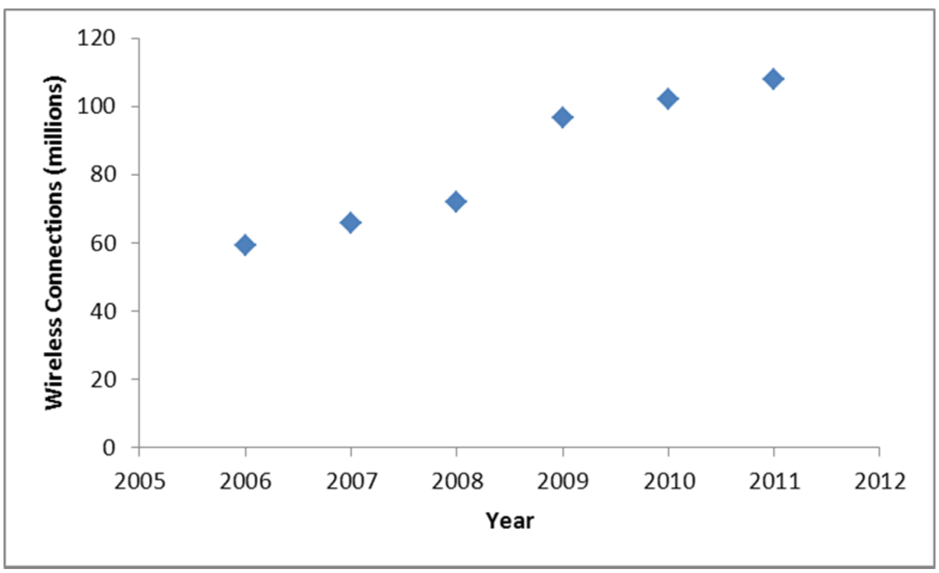
Average revenue per year for 2006-2008: $\frac{49.3-38.0}{2008-2006} = \frac{11.3}{2} \approx 5.65$ *billion dollars per year*

Average revenue per year for 2008-2011: $\frac{70.2-43.9}{2011-2008} = \frac{20.9}{3} \approx 6.97$ *billion dollars per year*

c. Describe how the average revenue per year changes between the two time intervals.

The average amount of revenue per year is higher from 2008 to 2011, indicating that the revenue increased faster from 2008 to 2011 than it did between 2006 and 2008.

2. Graph the data for wireless connections with respect to time. Be sure to label the axes.



Students may choose to use integers to represent the years (i.e., 0 represents 2006, 1 represents 2007, etc.). If so, they must indicate what the values represent.

a. Estimate the average connections per year between 2006 and 2008. Estimate the average connections per year between 2008 and 2011.

Estimated connections per year for 2006-2008: $\frac{70-60}{2} \approx 5$ *million connections per year*

Estimated connections per year for 2008-2011: $\frac{110-70}{3} \approx 13.3$ *million connections per year*

***Students may estimate differently but the values should be close to what is here.*

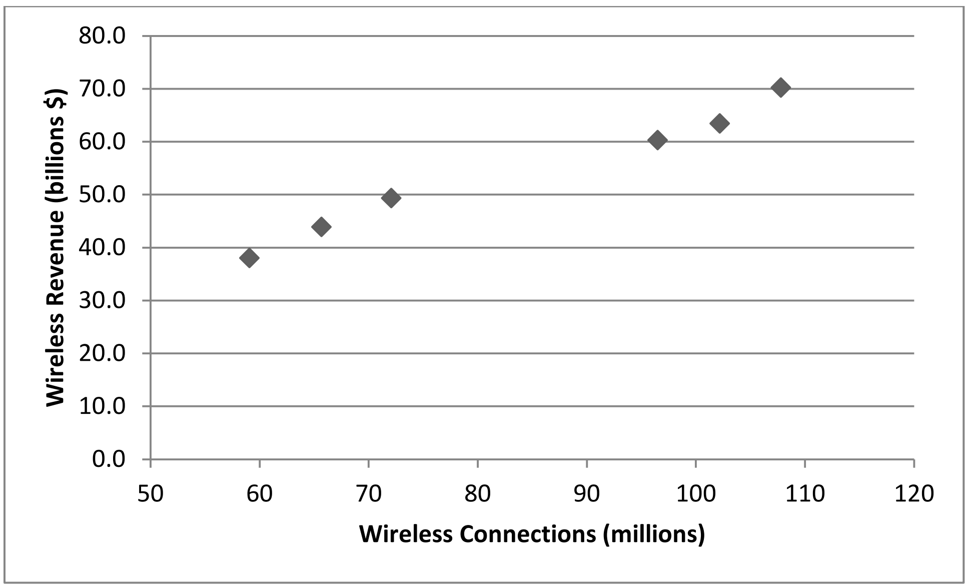
b. Support each estimate with calculations.

Average connections per year for 2006-2008: $\frac{72.1-59.1}{2008-2006}=\frac{13}{2}\approx 6.5$ *million connections per year*

Average connections per year for 2008-2011: $\frac{107.8-72.1}{2011-2008}=\frac{35.7}{3}\approx 11.9$ *million connections per year*

c. Describe how the average connections per year change between the two time intervals.
The average number of connections per year is higher from 2008 to 2011, indicating that each year, more connections are being made.

3. Graph the data for wireless revenue with respect to wireless connections. Be sure to label the axes.



a. Estimate the average revenue per connection between 2006 and 2008. Estimate the average revenue per connection between 2008 and 2011.

Estimated revenue per connection for 2006-2008: $\frac{49-38}{72-60}=\frac{11}{12}\approx 0.917$ *billion dollars per million wireless connections. Since 1 billion divided by 1 million is 1 thousand, this really means $917 per connection from 2006 to 2008.*

Estimated revenue per connection for 2008-2011: $\frac{70-49}{108-73}=\frac{21}{35}\approx 0.6$ *billion dollars per million wireless connections or $600 per connection from 2008 to 2011.*

b. Support each estimate with calculations.

Average revenue per connection for 2006-2008: $\frac{49.3-38.0}{72.1-59.1}=\frac{11.3}{13}\approx 0.869$ *billion dollars per million wireless connections. Since 1 billion divided by 1 million is 1 thousand, this really means $869 per connection from 2006 to 2008.*

Average revenue per connection for 2008-2011: $\frac{70.2-49.3}{107.8-72.1}=\frac{20.9}{35.7}\approx 0.585$ *billion dollars per million wireless connections, or $585 per connection from 2008 to 2011.*

c. Describe how the average revenue per connection changes between the two time intervals.

The comparison of these rates indicates that even though revenue is increasing based on revenue per year, and the number of connections is increasing per year, the amount of money per connection is lower from 2008 to 2011 than it was from 2006 to 2008.

4. Provide a written recommendation to your employer explaining why you believe they should or should not invest in this company. Be sure to use information from the comparisons above to support your recommendation. Also include any other factors you feel may need to be considered in order to make a final decision.

These recommendations will vary. Students should use the information they determined in the first three situations to support their recommendation.

Task adapted from http://www.whyseemath.com/pdf212/ave_rate_verizon_wksht.pdf. More information on responses can be found at http://whyseemath.com/wp/surveyofcalc/question/how-do-you-calculate-and-interpret-an-average-rate-of-change/.

M&M's® Data Analysis (IT)

Overview

Students will analyze given data about bags of M&M's® to determine the relationship between the number of candies in a bag and the net weight of the bag.

Standards

Summarize, represent, and interpret data on two categorical and quantitative variables.

HSS-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

HSS-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

HSS-ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

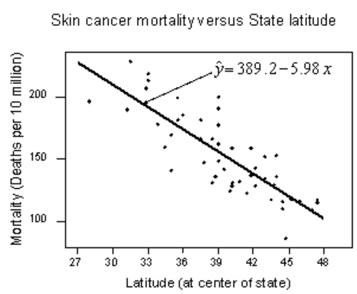
HSS-ID.C.9 Distinguish between correlation and causation.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

<table>
<tr><th>Grade-Level Standard</th><th>The Following Standards Will Prepare Them</th><th>Items to Check for Task Readiness</th><th>Sample Remediation Items</th></tr>
<tr><td>HSS-ID.B.6a</td><td>• 8.SP.A.2
• 8.SP.A.3
• HSS-ID.B.5</td><td>1. Using the data given in the table below, write a linear function to model the relationship between age and glucose level. Use your model to predict the glucose level of a person who is 50 years old.
<table>
<tr><th>Age</th><th>Glucose Level</th></tr>
<tr><td>43</td><td>99</td></tr>
<tr><td>21</td><td>65</td></tr>
<tr><td>25</td><td>79</td></tr>
<tr><td>42</td><td>75</td></tr>
<tr><td>57</td><td>87</td></tr>
<tr><td>59</td><td>81</td></tr>
</table>
a. $y = 0.385x + 65.14$ where *y* is the glucose level and *x* is the age. The predicted glucose level of a person 50 years old is approximately 84.</td><td>• http://www.illustrativemathematics.org/illustrations/1558
• http://www.illustrativemathematics.org/illustrations/1370
• http://www.illustrativemathematics.org/illustrations/123
• http://learnzillion.com/lessonsets/553-represent-and-describe-data-on-two-quantitative-variables-on-a-scatter-plot
• http://learnzillion.com/lessonsets/455-represent-data-on-a-scatter-plot-fit-functions-to-the-data-and-assess-fit</td></tr>
</table>

<table>
<tr><th>Grade-Level Standard</th><th>The Following Standards Will Prepare Them</th><th>Items to Check for Task Readiness</th><th>Sample Remediation Items</th></tr>
<tr><td></td><td></td><td>2. http://www.illustrativemathematics.org/illustrations/1554
3. http://www.illustrativemathematics.org/illustrations/1307</td><td></td></tr>
<tr><td>HSS-ID.C.7</td><td>• 8.SP.A.3
• HSS-ID.B.6c</td><td>1. The following plot shows the relationship between the state latitude and the mortality rate (per 10 million). A function that can model the relationship is given as $y = 389.2 - 5.98x$. What do the slope and y-intercept represent in this situation?



a. The slope of -5.98 means that for every 1 degree the latitude increases, the mortality rate decreases by 5.98 per 10 million. The y-intercept of 389.2 means that at 0 degrees latitude, the mortality rate is 389.2 deaths per 10 million.
2. http://www.illustrativemathematics.org/illustrations/1028</td><td>• http://learnzillion.com/lessonsets/460-interpret-the-slope-and-intercept-of-a-linear-function-in-context
• http://learnzillion.com/lessonsets/457-interpret-the-slope-and-the-intercept-of-a-linear-model-using-data</td></tr>
<tr><td>HSS-ID.C.8</td><td>• HSS-ID.B.6c</td><td>1. Compute the correlation coefficient of the data in the table below. Explain the meaning of the value.
<table><tr><th>Age</th><th>Glucose Level</th></tr><tr><td>43</td><td>99</td></tr><tr><td>21</td><td>65</td></tr><tr><td>25</td><td>79</td></tr><tr><td>42</td><td>75</td></tr><tr><td>57</td><td>87</td></tr><tr><td>59</td><td>81</td></tr></table></td><td>• http://learnzillion.com/lessonsets/584-find-correlation-coefficient-of-a-linear-fit
• http://learnzillion.com/lessonsets/467-compute-and-interpret-the-correlation-coefficient-of-a-linear-fit</td></tr>
</table>

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
		a. The correlation coefficient is approximately 0.530. This suggests that there is a positive linear correlation between age and glucose level. This might suggest that the older a person is, the higher his or her glucose level will be.	
HSS-ID.C.9	• HSS-ID.B.6c	1. Explain the difference between correlation and causation. a. Correlation means that when one variable increases or decreases, so does the other value (even if it is in the opposite direction). Correlation does not imply causation. Causation indicates that one event is the direct result of the other event—causation implies correlation. 2. http://www.illustrativemathematics.org/illustrations/44	• http://learnzillion.com/lessonsets/585-distinguish-between-correlation-and-causation • http://learnzillion.com/lessonsets/468-distinguish-between-correlation-and-causation-and-assess-causation

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is net weight?*** Net weight is the weight of goods in a package and does not include the weight of wrapping material, container, or other packaging. In terms of this task, the net weight would be the weight of the M&M's® in the bag.

During the Task

- Students should be allowed to use technology for all parts of this task. Be sure to review with students the procedure for creating a scatter plot, computing the correlation coefficient, and finding a linear model with the technology available. Graphing calculators or spreadsheet software can be used to complete this task.
- Provide each student with a bag of M&M's® to complete the second part of the task and have students work in groups of 3-4. Students will also need access to a digital scale in order to calculate the net weight of each bag of M&M's®. The science department may be a good source for the digital scales.
- Have students discuss whether the relationship between the number of candies and the weight of the bags is a causal relationship and develop their response to question 8 as a group.

After the Task

It is important to discuss with students the limitations of the linear model they create for question 3. Discuss with students what the intercept means in the context of the task. Ask students to identify what change in the linear model would make the model more accurate.

Have students discuss whether they expected a stronger correlation coefficient. Students should realize that the variance in the weight and the number of candies could be due to the individual weight of the candies, and that this variance might cause the correlation coefficient not to be what one might expect.

Have students use the individual data from the whole class to find the correlation coefficient and a linear model, and then compare it to the data they were given. Have them discuss differences in the function as well as the correlation coefficient.

Student Instructional Task

Part I

The table below gives the color counts and net weight (in grams) for a sample of 30 bags of M&M's®.

Red	Green	Blue	Orange	Yellow	Brown	Weight (g)
15	9	3	3	9	19	49.79
9	17	19	3	3	8	48.98
14	8	6	8	19	4	50.40
15	7	3	8	16	8	49.16
10	3	7	9	22	4	47.61
12	7	6	5	17	11	49.80
6	7	3	6	26	10	50.23
14	11	4	1	14	17	51.68
4	2	10	6	18	18	48.45
9	9	3	9	8	15	46.22
9	11	13	0	7	18	50.43
8	8	6	5	11	20	49.80
12	9	13	2	6	13	46.94
9	7	7	2	18	7	47.98
6	6	6	4	21	13	48.49
4	6	9	4	12	20	48.33
3	5	11	12	11	16	48.72
14	5	6	6	21	6	49.69
5	5	16	12	7	12	48.95
8	9	13	4	15	11	51.71
8	7	7	13	7	18	51.53
9	8	3	8	23	8	50.97
20	2	7	5	13	9	50.01
12	6	1	12	6	19	48.28
8	9	4	6	21	7	48.74
4	6	7	6	14	19	46.72
10	12	11	6	11	7	47.67
5	4	2	9	18	16	47.70
15	11	4	13	7	8	49.40
11	6	7	12	12	13	52.06

Source of data: http://www.math.uah.edu/stat/data/MM.html

1. Create a scatter plot of the data included in the table with the number of candies as the independent variable and the net weight as the dependent variable. Based on the graph you create, describe the relationship between the variables.

2. Find the correlation coefficient of the data. Explain how the correlation coefficient supports your description of the data in the graph.

3. Find a linear function to model the relationship between the variables. Explain what the slope and intercept of the model means in the context of the given data.

4. Based on the linear model, what would be the predicted net weight of a bag of M&M's® that contains 56 candies? Show your work.

5. If the advertised net weight of a bag of M&M's® is 47.9 grams, based on the linear model, how many candies would one expect to be in the bag? Show your work.

Part II

6. Count the number of candies in each bag of M&M's® in your group. Weigh the contents of each bag to the nearest hundredth of a gram. Record the total number of candies and weight for each bag in the table below.

Total Number of Candies	Net Weight

7. Compute the correlation coefficient for the data collected in your group. What does this value tell you about your data?

8. Combine the data from the bags in your group with the original data from part one. Recalculate the correlation coefficient for the new data set (original data plus the group's data). Compare the original correlation coefficient to the new correlation coefficient. Does one correlation coefficient suggest a stronger linear fit than the other? Explain your reasoning.

9. Do you believe a causal relationship exists between the number of candies and the net weight of the bag? Explain your reasoning.

Instructional Task Exemplar Response

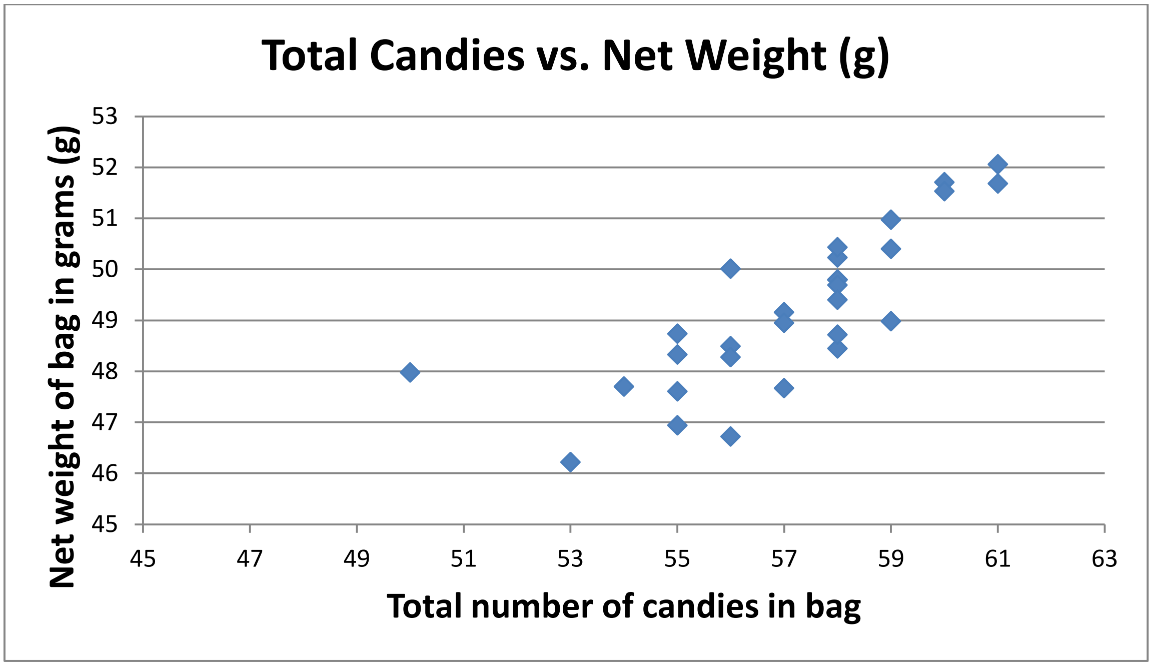
Part I

The table below gives the color counts and net weight (in grams) for a sample of 30 bags of M&M's®.

Red	Green	Blue	Orange	Yellow	Brown	Weight (g)
15	9	3	3	9	19	49.79
9	17	19	3	3	8	48.98
14	8	6	8	19	4	50.40
15	7	3	8	16	8	49.16
10	3	7	9	22	4	47.61
12	7	6	5	17	11	49.80
6	7	3	6	26	10	50.23
14	11	4	1	14	17	51.68
4	2	10	6	18	18	48.45
9	9	3	9	8	15	46.22
9	11	13	0	7	18	50.43
8	8	6	5	11	20	49.80
12	9	13	2	6	13	46.94
9	7	7	2	18	7	47.98
6	6	6	4	21	13	48.49
4	6	9	4	12	20	48.33
3	5	11	12	11	16	48.72
14	5	6	6	21	6	49.69
5	5	16	12	7	12	48.95
8	9	13	4	15	11	51.71
8	7	7	13	7	18	51.53
9	8	3	8	23	8	50.97
20	2	7	5	13	9	50.01
12	6	1	12	6	19	48.28
8	9	4	6	21	7	48.74
4	6	7	6	14	19	46.72
10	12	11	6	11	7	47.67
5	4	2	9	18	16	47.70
15	11	4	13	7	8	49.40
11	6	7	12	12	13	52.06

Source of data: http://www.math.uah.edu/stat/data/MM.html

1. Create a scatter plot of the data included in the table with the number of candies as the independent variable and the net weight as the dependent variable. Based on the graph you created, how would you describe the relationship of the variables?



The graph of the data suggests a positive linear correlation, which would mean that generally, as the number of candies increases, the net weight of the bag of M&M's® increases.

2. Find the correlation coefficient of the data. Explain how the correlation coefficient supports your description of the data in the graph.

***Note: The standard for computing the correlation coefficient specifically indicates the use of technology. The correlation coefficient listed below was found using the TI-84 Plus graphing calculator. Other technology devices may produce slightly different results.*

The correlation coefficient for this data is $r \approx 0.794$. This supports my description that the data has a positive correlation because the value is positive. Correlation coefficients that are close to 1 indicate a strong positive linear relationship, which supports my description as well.

3. Find a linear function to model the relationship between the variables. Explain what the rate of change and constant term of the model represent in the context of the given data.

***Note: The linear model for this sample response was found using a TI-84 graphing calculator. Other technology devices may produce slightly different results. Students may also determine a model without the use of technology, in which case students would need to show their work.*

A linear function to model the relationship could be $y = 0.507x + 20.278$, where x represents the number of candies in the bag and y represents the net weight in grams of the bag of M&M's®. The slope, 0.507, means that for each candy added to the bag, the net weight increases by 0.507 grams. The intercept, 20.278, means that when there are zero candies in the bag, the net weight is 20.278 grams.

4. Based on the linear model, what would be the predicted net weight of a bag of M&Ms® that contains 56 candies? Show your work.

$$y = 0.507(56) + 20.278$$

$$y = 48.67$$

The predicted net weight of a bag of M&M's® that contains 56 candies is 48.67 grams.

5. If the advertised net weight of a bag of *M&M's®* is 47.9 grams, based on the linear model, how many candies would one expect to be in the bag? Show your work.

$$47.9 = 0.507x + 20.278$$

$$47.9 - 20.278 = 0.507x$$

$$27.622 = 0.507x$$

$$\frac{27.622}{0.507} = \frac{0.507x}{0.507}$$

$$54.48 \approx x$$

For the advertised net weight of 47.9 grams, one would expect there to be about 54 candies in the bag.

Part II

6. Count the number of candies in each bag of *M&M's®* in your group. Weigh the contents of each bag to the nearest hundredth of a gram. Record the total number of candies and weight for each bag in the table below.

Total Number of Candies	Net Weight (g)
61	*49.14*
59	*51.62*
52	*48.30*
51	*47.72*

***Note: The data in the table above was randomly generated in order to be able to provide sample responses.*

7. Compute the correlation coefficient for the data collected in your group. What does this value tell you about your data?

***Note: The standard for computing the correlation coefficient specifically indicates the use of technology. The correlation coefficient listed below was found using the TI-84 Plus graphing calculator. Other technology devices may produce slightly different results.*

The correlation coefficient for this data is $r \approx 0.698$. This value indicates there is a strong positive relationship between the number of candies in a bag and the net weight for the group's data.

8. Combine the data from the bags in your group with the original data from part one. Recalculate the correlation coefficient for the new data set (original data plus the group's data). Compare the original correlation coefficient (from question 2) to the new correlation coefficient. Does one correlation coefficient suggest a stronger linear fit than the other? Explain your reasoning.

   The correlation coefficient for the new data set is $r \approx 0.742$. The correlation coefficient for the original data set was $r \approx 0.794$. The original data set has a stronger linear fit because the correlation coefficient for the original data set is closer to 1 than the correlation coefficient for the new data set.

9. Do you believe a causal relationship exists between the number of candies and the net weight of the bag? Explain your reasoning.

   The relationship is causal because the net weight of each bag will only increase if candies are added to the bag (likewise, the net weight would decrease if candies were removed from the bag). The correlation coefficients suggest a strong linear relationship, which supports my belief that this is a causal relationship.