Preferred Chocolate (IT)

Overview

Students will design and conduct a study to investigate a simple question about which brand of chocolate is preferred. Using results gathered from the study conducted, students will make a recommendation about which chocolate brand should continue to be sold at athletic events for the school.

Standards

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

HSS-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSS-IC.B.3	• HSS-IC.A.1	1. http://www.illustrativemathematics.org/illustrations/122 2. http://www.illustrativemathematics.org/illustrations/1029	• http://www.illustrativemathematics.org/illustrations/186 • http://www.illustrativemathematics.org/illustrations/191 • http://learnzillion.com/lessonsets/475-distinguish-between-surveys-experiments-and-observational-studies-relate-randomization-to-each • http://learnzillion.com/lessonsets/400-recognize-the-purposes-of-and-differences-among-research-methods-including-how-they-relate-to-randomization

Setup/Organization:

- Divide the class into groups of 4-5 students and assign each group the title of Survey, Experiment, or Observational Study. The number of groups will vary but there should be a minimum of three groups so that each type of study is conducted.
- Determine which two brands of chocolate will be used. One possible pairing of brands is Hershey's and Nestlé because of the option to use chocolate baking chips for the group(s) assigned to conduct an experiment. Both brands of baking chips will look the same and limit the opportunity for inaccurate results.

During the Task

- During the planning and discussion phase of the task, the teacher should circulate to not only respond to student questions but to ensure students are considering necessary aspects. Guiding questions will vary by group based on whether they are designing a survey, an experiment, or an observational study.

ALL GROUPS	
Look-Fors (observables)	***Guiding Questions***
Discussions are related to the purpose of the task (to determine which brand of chocolate should continue to be sold).	• What was the original question posed in the task? • Why was this question posed to you? • Do you understand what you are being asked to do? • How can you formulate a research question that captures the problem posed by the task?
Discussions include consideration of design elements (population, brands, location), which will lead to the collection of relevant data.	• What elements are you considering? • Who will make up the population? • Does the task include reference to a required level of confidence? • Is it important to consider where your research will take place? If so, what do you need to consider? • Which elements do you think are essential in answering the research question and providing a meaningful recommendation?
Discussions are centered around data collection.	• What data will be collected? • Is demographic data important? If so, how will you collect this information? • Is the data qualitative or quantitative?
Discussions focus on randomization.	• What is the size of your target population? • Is it necessary to sample the entire population? If no, how will you select your subjects? If yes, what is your rationale? • What sample size is adequate to arrive at a conclusion? • How do you determine an adequate sample size?
Discussions are centered around the feasibility study.	• Is it possible to carry out your study in the required amount of time? • Have you considered the availability of/access to the population? • What tools/resources will you need in order to complete the study? • Do you have access to everything required?
Group members are assigned/take on individual roles.	• What role will each member play in each phase of the process? • Who will lead the efforts? • How will you remain accountable for your tasks?

SURVEY GROUP(S)	
Look-Fors (observables)	***Guiding Questions***
Discussions should include how the population will be surveyed.	• How will you collect the data? • What supplies/tools will be needed in order to collect the data? • Is a paper survey the only option? • What is the most efficient way to capture the needed information?
Discussions are related to how the questions will be asked (regardless of the choice of written or verbal questions).	• How will you ensure the questions are unbiased? • Is it important to consider the order of the questions?
Discussions about the process of conducting the survey should extend beyond the questions included in the survey.	• Who will approach people about completing the survey? • How will you respond when people refuse? • Will this attempt be recorded? • What to partially completed surveys?

EXPERIMENT GROUP(S)	
Look-Fors (observables)	***Guiding Questions***
Discussions should include the logistics of the study.	• How will you conduct the experiment? • What will be needed in order to collect the data?
Discussions focus on whether it is necessary to conduct a blind or double-blind study.	• Does a placebo effect apply to your design? • Will the subjects know what they are sampling? Why or why not? • Will **YOU** know the brand they are sampling?
Discussions about the process of choosing subjects should extend to include the specific data to be collected and the potential for errors or alternate responses.	• Who will approach people about participating in the experiment? • What are the possible responses/outcomes for this trial? • How will you respond when people refuse? • Will this attempt be recorded?

OBSERVATIONAL STUDY GROUP(S)	
Look-Fors (observables)	***Guiding Questions***
Discussions should include details about how students will conduct the experiment relative to the nature of an observational study.	• How will you conduct the observational study? • Will you approach the subjects or will you wait for them to approach you? • What will be needed in order to collect the data?
Based on the research question, students should be discussing the element of choice.	• What role does choice play in your study? • What is the significance of this role?
Discussions about the process of choosing subjects should extend to include the specific data to be collected and the potential for errors or alternate responses.	• How will you respond when people refuse? • Will this attempt be recorded? • How will you record data for a subject who does not follow your instructions—for example, he/she takes both brands of chocolate?

- After the groups have decided on a plan, collect copies of the designs, give feedback with time to make adjustments, and plan to attend the event where each study will take place. The role of the teacher during this time will be that of a supporter and guide if students hit roadblocks that would significantly limit the completion of their study.

After the Task

Have students make connections between what they learned through conducting their own studies and the results of studies they see and/or hear in the news or on the Internet.

Student Instructional Task

Because of new business practices, your school must limit the number of brands to be sold in concession stands during school athletic events. The decision about which brand of chocolate to be sold has been narrowed to two brands: Hershey's and Nestlé. Design and conduct a study that will assist you in making a recommendation of which brand should continue to be sold at athletic events.

Design: All designs must include the following components.

1. Research Question
2. Population
3. Method
    a. Study type
    b. Selection of sample population (sample description, time, place, event)
    c. Data collection
    d. Data analysis

Study Results:

1. Data Summary and Analysis
2. Recommendation

Presentation: Each group will present their design, results, and recommendations to the class. This should include a one-page summary to be distributed to your classmates. As each group presents their results and recommendations, reflect on the questions below.

Class Recommendation: As a group, discuss the following questions and record your responses. This information will be used to formulate a class response.

1. Does the data presented across all groups imply that there may be differences between what people believe they prefer, what people truly prefer, and what people would be likely to buy? If so, explain. Further, how should this be included as a factor when making a recommendation?

2. What role did randomization play in each study?

3. Looking at the data gathered across all of the studies, identify the groups that can be used to make generalizations about the school as a whole. Use data to support and explain these generalizations.

Instructional Task Exemplar Response

Because of new business practices, your school must limit the number of brands to be sold in concession stands during school athletic events. The decision about which brand of chocolate to be sold has been narrowed to two brands: Hershey's or Nestlé. Design and conduct a study that will assist you in making a recommendation of which brand should continue to be sold at athletic events.

SURVEY GROUP EXEMPLAR *(Note: Responses must include all elements as outlined below—because the data will vary by school, the responses will look different but the overall content should be similar. Students should utilize data collection and analysis tools as available.)*

Design:

1. **Research Question**
   Should the school choose Hershey's or Nestlé brand chocolate items to be sold in concession stands at school athletic events?
2. **Population**
   Our survey will be sent out electronically through the East High School communication program to all 1,000 registered users (parents, teachers/staff members, students) since they are the people who are likely to attend school athletic events.
3. **Method**
   a. **Study type** *Survey*
   b. **Selection of sample population** (sample description, time, place, event)
   The survey will be sent to all 1,000 registered users—current East High School parents, teachers/staff members, and students. Responses will be captured through an electronic survey, and to be confident in our recommendation, our response rate needs to be 50% (500 responses). If the response rate is not approaching 50% after two email reminders, we will attend a variety of athletic events to obtain more responses. We will ask all people who enter if they received and responded to the email regarding their preference of Hershey's or Nestlé chocolate. If they have not responded to the electronic survey, we will ask the survey questions and record their responses. If they refuse to complete the survey, we will thank them for their time and will not record this response.
   c. **Data collection** *(Note: Surveys may vary but should include opportunities to collect all data relevant to the research question and described situation. As the teacher reviews group designs, it may be helpful for him or her to ask, "Will these components collectively provide data that translates to information we can use to make a recommendation?")*

   Preferred Brand of Chocolate Survey

   The following items will be included on the survey to be distributed as described above.

   1. *Gender: ___Male ___Female*
   2. *Select the category that best describes you (select ONE):*
      ____student _____teacher/staff member _____ relative of a current student ____other
   3. *On average, how many athletic events do you attend: ____0-3 ____4-5____6 or more*

4. *When purchasing chocolate from school athletic event concession stands, which brand are you more likely to select? ____Hershey's ____Nestlé _____no preference*

d. **Data analysis** *(Note: At the design step of the task, students will include the chart they will use to analyze the responses. A sample set of data is included for this exemplar. All student responses should align to the items included in the survey.)*

The following charts will be used to compile and analyze the responses.

TABLE 1

Sample Population Summary Data		Total	% of Total	# Male	% Male	# Female	% Female
Total Surveyed		510	**100**	245	**48.0**	265	**52.0**
Completion Type	**Completed Online**	248	**48.6**	100	**40.3**	148	**59.7**
	Completed in Person	262	**51.3**	145	**55.3**	117	**44.7**
Respondent Type	**Student**	215	**42.2**	133	**61.9**	82	**38.1**
	Teacher/Staff Member	96	**18.8**	29	**30.2**	67	**69.8**
	Relative	199	**39.0**	105	**52.8**	94	**47.2**
Average Annual Attendance	**0**	45	**8.8**	4	**8.9**	41	**91.1**
	1-3	123	**24.1**	70	**56.9**	53	**43.1**
	4-5	301	**59.0**	202	**67.1**	99	**32.9**
	6 or >	41	**8.0**	33	**98.4**	8	**1.6**

The highest response percentage for each subgroup is noted with a blue shaded cell. The gender with the highest percentage within each category is identified with a yellow shaded cell.

TABLE 2								
Chocolate Preference Data Summary		**Total**	**# Hershey's**	**% Hershey's**	**# Nestlé**	**% Nestlé**	**# No Pref.**	**% No Pref.**
	Total Surveyed	510	300	58.8	183	35.9	27	5.3
Gender	**Male**	245	194	79.2	44	17.9	7	2.9
	Female	265	106	40.0	139	52.5	20	7.5
Completion Type	**Completed Online**	248	104	41.9	129	52.0	15	6.1
	Completed in Person	262	196	74.8	54	20.6	12	4.6
Respondent Type	**Student**	215	166	77.2	35	16.3	14	6.5
	Teacher/Staff Member	96	41	44.8	46	47.9	9	7.3
	Relative	199	93	46.7	102	51.3	4	2.0
Average Annual Attendance	**0**	45	12	26.7	27	60.0	6	13.3
	1-3	123	102	82.9	14	11.4	7	5.7
	4-5	301	171	56.8	122	40.5	8	0
	6 or >	41	15	36.6	20	48.8	6	14.6

The highest preferred percentage in each category is noted with a blue shaded cell.

Study Results: *(Note: The exemplar response below is relative to Tables 1 and 2. Actual student summaries will vary but should accurately reflect the data collected, include an overall summary of the survey respondents, identify key pieces of data related to the research question, and indicate any potential limitations of the study. While most data will be pulled from the summary charts, students may revisit certain aspects of the survey as they work to make a recommendation. The key is that all data presented should remain focused on answering the research question.)*

Population Summary:

1. *Approximately 1,000 people were given an opportunity to complete the survey. Responses were collected from 510 people (51%). The majority of the responses (51.3%) were collected in person during 2 athletic events. Requests for the survey to be completed electronically yielded 248 (48.6%) responses.*
2. *All people who entered the chosen athletic events were asked to complete the survey (if they had not already done so). All potential in-person respondents participated in the survey.*

3. *Females contributed to 52% of responses.*
4. *Based on respondent type, the highest percentages were students (42.2%) and relatives (39%). The category of "other" yielded no data.*
5. *The majority of responses in the area of average annual attendance (67%) came from respondents who attend an average of 4 or more events per year. 8.8% of respondents indicated they attend 0 athletic events per year.*

Data Summary and Analysis:

Because the majority of the response came from in-person respondents, females, students, relatives, and those who attend an average of 4 or more athletic events per year, we chose to focus on the preferences of these subgroups.

Subgroups	% of Total Responses	% Preferring Hershey's	% Preferring Nestlé	% No Preference
Total Population	**100**	**58.8**	**35.9**	**5.3**
Average Attendance: 4 or more	**67.0**	**54.4**	**41.5**	**4.1**
Females	**52.0**	**40.0**	**52.5**	**7.5**
Completed In Person	**51.3**	**74.8**	**20.6**	**4.6**
Students	**42.2**	**77.2**	**16.3**	**6.5**
Relatives	**39.0**	**46.7**	**51.3**	**2.0**

1. *Of the five subgroups selected for further analysis, three groups (average attendance > 4, completed in person, students) prefer Hershey's, and two groups (females and relatives) prefer Nestlé.*
2. *As a group, students have the greatest range in their preference level based on the survey, which indicates 77.2% prefer Hershey's compared to 16.3% preferring Nestlé.*
3. *While females contributed to a large portion of the respondents (52%), the data indicates they are not the group that attends the greatest number of athletic events on average each year. Only 40% (107/265, Table 1) of females indicate they attend 4 or more events on average each year.*
4. *67% (342/510 from Table 1) of respondents indicate they attend 4 or more athletic events each year. When surveyed, these respondents prefer Hershey's over Nestlé chocolate.*

Recommendation: *(Note: Exemplar responses for this portion of the task should include the data used to make a recommendation, limitations of the study, and a final recommendation.)*

Survey Group Recommendation: It is our recommendation that the school choose Hershey's as the chocolate to be sold in athletic event concession stands. The student group was identified with the highest percentage preferred when looking at both brands of chocolate. 77.2% of students surveyed prefer Hershey's compared to 16.3% preferring Nestlé. 67% of the population surveyed indicates they attend 4 or more athletic events per year, and since this group is engaging in activities that give them access to the concession stands more frequently, it is important to look at which chocolate they

prefer. 54.4% prefer Hershey's over Nestlé. Additionally, 58.8% of the 510 survey respondents indicated they prefer Hershey's chocolate. Therefore, the school should select Hershey's chocolate to be sold in athletic event concession stands.

Limitations: While we made efforts to ensure participants completed the survey one time, duplicate responses may have been recorded. If this occurred, we do not feel it would impact our recommendation, considering those responses given face to face were disaggregated from online responses.

EXPERIMENTAL GROUP EXEMPLAR *(Note: Student responses must include all elements as outlined below—because the data will vary by school, the responses will look different but the overall content should be similar. Students should utilize data collection and analysis tools as available.)*

Design:

1. **Research Question**
   Should the school choose Hershey's or Nestlé brand chocolate items to be sold in concession stands at school athletic events?
2. **Population**
   The experimental study will obtain data by sampling the population of people attending athletic events that have concession stands available. Further selection will be enhanced by placing the testing area near the concession stands at each event.
3. **Method**
   a. **Study type** *Double-blind experiment*
   b. **Selection of sample population** (sample description, time, place, event): *The subjects will come from the population of those people attending athletic events that have concession stands available. We will attend a variety of athletic events in order to obtain as many responses as possible. All ages will be considered, and for young participants, adults may help them complete the data collection form. We will not work to eliminate duplicate participants from the study, but if they say they have already completed the taste test, we will not proceed. Our taste test area will be located near the concession stand, and as people approach the concession stand, we will ask them to participate. If they decline, we will thank them for their time and record this as a "declined" attempt.*
   c. **Data collection** *(Note: For an experimental study, the data collection process may vary but should include opportunities to collect all data relevant to the research question and described situation. As the teacher reviews group designs, it may be helpful for him or her to ask, "Will these components collectively provide data that translates to information we can use to make a recommendation?")*

      Subjects will be given two brands of chocolate (Hershey's and Nestlé) and asked to rank their choice with most or least preferred. A response of "no preference" will not be an option. Each subject will be given identical cups from which to taste. Each cup will be filled with one brand of chocolate chips. The subjects will not know which brand is in each cup. Preferences will not be stated verbally in order to minimize the potential for bias of those waiting to participate in the experiment. For double-blinding of the experiment, the chocolate chips will be scooped from two containers labeled only as "X" and "Y." The group member given the role of study manager will be the only one who know which brand is labeled with "X" and which is labeled with "Y." Also,

the order each subject samples the chocolate will be randomized because both cups will be placed in front of the subject and he or she will choose which to sample first.

After each sampling, the subject will complete the following form:

Chocolate Taste Test Experiment
*Check the **one** category that best describes you:*

1. *Gender: _____ Male _____Female*
2. *Age in Years: _____ 0-8 ____ 9-13 ____14-18 ____19-23 ____24-30 ____31-40 ____ >40*
3. *Average Athletic Events Attended per Year: _____1-3 _____4-6 _____7 or more*
4. *Which brand did you prefer? ____Brand X _____Brand Y*

d. **Data Analysis** *(Note: At the design step of the task, students should include the chart they will use to analyze the responses. For a sample set of data (completed tables), see the Survey Group Exemplar. All elements should align to the data being collected.)*

The following tables will be used to compile and analyze the results of the experiment.

TABLE A							
Sample Population Summary Data		**Total**	**% of Total**	**# Male**	**% Male**	**# Female**	**% Female**
Total Asked to Participate				**NA**	**NA**	**NA**	**NA**
Declined to Participate				**NA**	**NA**	**NA**	**NA**
Completed Experiment							
Age in Years	**0-8**						
	9-13						
	14-18						
	19-23						
	24-30						
	31-40						
	>40						
Average Annual Attendance	**1-3**						
	4-6						
	7 or >						

TABLE B						
Chocolate Preference Data Summary		**Total**	**# Hershey's**	**% Hershey's**	**# Nestlé**	**% Nestlé**
Total Subjects						
Gender	**Male**					
	Female					
Age in Years	**0-8**					
	9-13					
	14-18					
	19-23					
	24-30					
	31-40					
	>40					
Average Annual Attendance	**1-3**					
	4-6					
	7or >					

Study Results: *(Note: The exemplar response below provides a shell of what the results of an experiment of this type might look like. Actual student summaries will vary but should accurately reflect the data collected, include an overall summary of the subjects, identify key pieces of data related to the research question, and indicate any potential limitations of the study. While most data will be pulled from the summary charts, students may revisit certain aspects of the experiment as they work to make a recommendation. The key is that all data presented should remain focused on answering the research question.)*

Population Summary:

1. *Taste-testing took place at __(#)_events over the course of __(#)_days.*
2. *__(#)__athletic event attendees were asked to participate in the taste test. __(#)__agreed to participate, and __(#)__declined.*
3. *Females contributed to ____% of subjects.*
4. *The majority of the subjects (___%) were _(#)___years of age or older.*
5. *The majority of subjects (_____%) indicated they attend an average of __(#)___or more events per year.*

Data Summary and Analysis:

Because we had an equal number of male and female subjects, the majority of the subjects were in the age category of _(#)__ years of age or older and those who attend _(#)__ or more athletic events per year, we chose to focus on the preferences of these subgroups.

Subgroups (will vary depending on data collected)	**% of Total Responses**	**% Preferring Hershey's**	**% Preferring Nestlé**
Total Population			
Average Attendance: __#__ or more			
Males			
Females			
Age: __#__ or older			

1. *Of the (#) subgroups selected for further analysis, _(#)_ groups (list groups) prefer Hershey's and (#) groups (list groups) prefer Nestlé.*
2. *(Note: Exemplars should highlight a variety of data from the experiment. See Survey Group Exemplar for more examples.)*

Recommendation: *(Note: Exemplar responses for this portion of the task should include the data used to make a recommendation, limitations of the study, and a final recommendation. See Survey Group Exemplar for an example of a recommendation. Potential limitations for this experimental design may be associated with access to a large enough population because of a limited number of athletic events, etc. Students may also point out that while respondents chose one brand over the other based on taste, this may not match their actual purchasing behavior.)*

OBSERVATIONAL STUDY GROUP EXEMPLAR *(Note: Responses must include all elements as outlined below—because the data will vary by school, the responses will look different but the overall content should be similar. Students should utilize data collection and analysis tools as available.)*

Design:

1. **Research Question**
   Should the school choose Hershey's or Nestlé brand chocolate items to be sold in concession stands at school athletic events?
2. **Population**
   The observational study will obtain data by sampling the population of people attending athletic events who purchase chocolate from school athletic events.
3. **Method**
   a. **Study type** *Observational*
   b. **Selection of sample population** (sample description, time, place, event)

The subjects will come from the population of people attending athletic events AND those who purchase chocolate from the school concession stand. We will attend a variety of athletic events in order to maximize the data collected. All people who purchase an item from the concession stand during the event will be considered. When people approach the concession as a group, and one person pays, this will be recorded as one subject. If a person comes to the concession stand more than once, each order will be treated as a new subject.

c. **Data collection** *(Note: For an observational study, the data collection process may vary but should include opportunities to collect all data relevant to the research question and described situation. As the teacher reviews group designs, it may be helpful for him or her to ask, "Will these components collectively provide data that translates to information we can use to make a recommendation?")*

As people approach the concession stand, data will be collected using a chart similar to the one below. We will have a list (including pictures) of Hershey's and Nestlé chocolate items to make brand identification easier.

Subject #	***# of Items Purchased***	***# of Chocolate Items Purchased***	***# of Hershey's Chocolate Items***	***# of Nestlé Chocolate Items***	***# of Other Chocolate Items***
1.					
2.					
3.					
4.					
5.					
6.					
…..					

d. **Data analysis** *(Note: At the design step of the task, students should include the chart they will use to analyze the responses. For a sample set of data (completed tables), see the Survey Group Exemplar. All elements should align to the data being collected.)*

The following table will be used to compile and analyze the results of the observational study.

Observational Study Data Summary	
Total Events Attended	
Total Subjects Observed	
Mean # of Subjects per Event	
Total Number of Items Purchased	
Mean # of Items per Event	
Total # of Chocolate Items Purchased	
Mean # of Chocolate Items per Subject	

Event Summary Data					
Event	**Total Items Purchased**	**# of Hershey's Products**	**% of Hershey's (#H/total items)**	**# of Nestlé Products**	**% of Nestlé (#N/total items)**
Event 1					
Event 2					
Event 3					
Event 4					
Event 5					
Event 6					
Total All					

Events					

Study Results: *(Note: The exemplar response below provides a shell of what the data of an observational study might produce. Actual student summaries will vary but should accurately reflect the data collected, include an overall summary of the subjects, identify key pieces of data related to the research question, and indicate any potential limitations of the study. While most data will be pulled from the summary charts, students may revisit certain aspects of the experiment as they work to make a recommendation. The key is that all data presented should remain focused on answering the research question.)*

Population Summary:

1. *Observations took place at __(#)_events over the course of __(#)_days.*
2. *__(#)__athletic event attendees who purchased items at the concession stand were observed.*
3. *A mean of __(#)_chocolate items per event were purchased compared to a mean of _(#)__total items purchased per event.*
4. *The percentage of Hershey's to total items purchased is ___%.*
5. *The percentage of Nestlé to total items purchased is ___%.*

Data Summary and Analysis:

(Note: Exemplar responses for this section of the observational study should highlight a variety of data from the experiment. See Survey Group Exemplar for more examples.)

Recommendation: *(Note: Exemplar responses for this portion of the task should include the data used to make a recommendation, limitations of the study, and a final recommendation. See Survey Group Exemplar for an example of a recommendation. Potential limitations for this observational design may be associated with the potential for errors on the part of the observers or the concession stand not having an adequate supply of either or both brands of chocolate.)*

ALL GROUPS—EXEMPLAR *(Note: The guidance for the presentation and class recommendation sections is similar regardless of group type.)*

Presentation: *(Note: Each group will present their design, results, and recommendations to the class. This should include a one-page summary to be distributed to classmates. The one-page summary will be a selection of the information from the above sections [data analysis, study results, and recommendation]. As students listen to the results and recommendations, they should be reflecting on the questions under the Class Recommendation section.)*

Class Recommendation: As a group, discuss the following questions and record your responses. This information will be submitted to your teacher, who will review the recommendations and formulate a class response.

1. Does the data presented across all groups imply there may be differences between what people believe they prefer, what people truly prefer, and what people would be likely to buy? If so, explain. Further, how should this be included as a factor when making a recommendation?

(Note: Responses will vary depending on the data collected by all three groups. For example, if all three groups present data summaries that clearly indicate a preference of one chocolate over another, the answer to this question would be: "Based on the data from all three groups, it appears there is no difference between what people believe they prefer, what they truly prefer, and what they would likely buy.")
If there is a difference between the experimental group and the survey or observational study groups, the response should identify the specific data while pointing out the aspect of choice when selecting an item to buy at a concession stand. It should be clear that students understand the link between choice and the two types of studies (survey and observational study).

2. What role did randomization play in each study? *(Note: Responses should note the preplanning and inclusion of participant selection in the method section for each study.)*

   In order to ensure randomization, groups defined the method for identifying their participants prior to the start of the study. The methods presented by each group clearly defined the population and who would participate in the study. They did not change their method during or after the study.

   All users registered for the school communication system were given the same opportunity to reply to the survey, and additional data was collected by giving all people who entered an athletic event a chance to respond. Both approaches satisfy elements of randomization.

   The experimental study was conducted at athletic events, which relates to the research question. Their method was a double-blind taste test in which those administering the samples and the subjects were unaware of which brand was being tasted.

   Groups completing the observational study recorded choices made by the participants attending athletic events when purchasing concessions. The group recorded the choices of all subjects, including those who did not purchase items of either brand of chocolate. No one in the group discussed the purchases with the participants either before or after the purchases were recorded.

3. Looking at the data gathered across all of the studies, identify the groups that can be used to make generalizations about the school as a whole. Use data to support and explain these generalizations. *(Note: Responses will vary depending on data presented but should not include groups that do not relate to the research question and/or groups that were not included in the data collection. For example, generalizations about 14- to 15-year-old students could only be made if this was a piece of data collected. Generalization is related to the population of interest, and in this task, that population is people attending athletic events.)*

Growing Radishes (IT)

Overview

Students use given data from an experiment involving different fertilizers and a simulation based on re-randomizing the given data to describe differences between the treatment groups in the experiment. Students also determine whether the observed differences are significant.

Standards

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

HSS-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide whether differences between parameters are significant.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSS-IC.B.5	• HSS-IC.A.2 • HSS-IC.B.3	1. Describe the concept of statistical significance. a. A difference is statistically significant if we are reasonably sure that observed differences are not due to sampling variation. If the observed difference would commonly occur due to natural sampling variation, the difference is not significant. 2. Explain how a simulation can determine statistical significance between the means of two groups. a. To determine whether the difference between two groups is statistically significant, all data from both groups can be combined and re-randomized. Then, the combined data is repeatedly, randomly assigned to two groups, and the differences in means is recalculated. The observed difference in means is then compared to the simulated differences in means. If the observed difference, or a value greater than the observed difference, is commonly found in the simulated means, then the observed difference is not significant.	• http://www.illustrativemathematics.org/illustrations/244 • http://www.illustrativemathematics.org/illustrations/125 • http://www.illustrativemathematics.org/illustrations/122 • http://www.illustrativemathematics.org/illustrations/1029 • http://learnzillion.com/lessonsets/402-use-data-from-a-randomized-experiment-to-compare-treatments-evaluate-reports-based-on-data

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is fertilizer, and what is it used for?*** Fertilizer refers to any of a large number of natural and synthetic materials, including manure and compounds containing nitrogen, phosphorus, and potassium, spread on or worked into soil to increase its capacity to support plant growth. Fertilizer is used to increase plant size, healthiness, and/or growth rate.

During the Task

- Students need to understand the difference between an observed difference in two means and a statistically significant difference between two means. If they are unsure of how to answer #1 and/or how to approach #2, discussing this difference may be useful.
- Students may also struggle to understand why re-randomization of data, which have been combined from two groups, gives insight into the statistical significance of the observed differences. To help students see the reasoning behind re-randomizing combined data, have them list all of the data from two groups together. The question students are trying to answer is: "Is there really any difference in these two groups that have been combined?" If one assumes that there is NOT any difference in the two groups, then combining the two groups together and randomly picking half of the data for one group and the remaining data for the other group should produce differences in the means similar to the observed difference. The students could do a few trials of this to see what averages (and what differences) they get to help them understand the data in the re-randomization table.

After the Task

This task helps to establish the distinction between a difference and a statistically significant difference. Differences between groups or subgroups are reported commonly in newspapers (or online news articles). Using a current article making a claim about a difference between groups may be a good way to follow this task. Students could discuss the reported difference and the statistical significance of the difference.

Student Instructional Task

A local farmer has been growing radishes to sell at a farmers market for years. This year, he decided to test two different types of fertilizers. He planted 30 radish plants in an area with consistent lighting conditions. He randomly assigned each plant to one of the following treatments: (1) no fertilizer [control group], (2) Fertilizer A, and (3) Fertilizer B. This resulted in 10 plants per treatment. After the radishes had been growing for four weeks, he measured the height of all of the plants. The heights are shown in the table below.

Plant Heights after 4 Weeks (cm)

No Fertilizer	Fertilizer A	Fertilizer B
4.9	5.0	4.7
3.7	4.5	4.6
4.1	5.9	4.1
4.0	5.7	5.5
3.5	5.2	3.0
4.7	6.2	5.2
5.1	5.2	5.3
3.3	5.1	4.1
4.7	6.0	4.5
4.0	5.4	4.9

1. Compare the center and spread of the three groups. Assume that the three data distributions are approximately unimodal and symmetric.

All of the data from the control group and from the Fertilizer A group were combined and randomly divided into two groups of 10 samples. The means of these new groups (shown as "Sample Group 1 Mean" and "Sample Group 2 Mean") were calculated, and the differences between the mean heights were calculated. This process was repeated for the control group with Fertilizer B. The re-randomization of the data was performed 25 times for each treatment. The results of this re-randomization of the data are shown in the tables below.

Re-randomization of Data

Control and Fertilizer A Group			
Trial #	**Sample Group 1 Mean**	**Sample Group 2 Mean**	**Difference**
1	4.98	4.64	-0.34
2	5.09	4.53	-0.56
3	4.79	4.83	0.04
4	4.68	4.94	0.26
5	4.82	4.8	-0.02
6	4.62	5	0.38
7	5.06	4.56	-0.5
8	4.83	4.79	-0.04
9	4.95	4.67	-0.28
10	4.68	4.94	0.26
11	4.9	4.72	-0.18
12	4.68	4.94	0.26
13	5.16	4.46	-0.7
14	4.81	4.81	0
15	4.75	4.87	0.12
16	4.43	5.19	0.76
17	4.73	4.89	0.16
18	4.77	4.85	0.08
19	5	4.62	-0.38
20	5.01	4.61	-0.4
21	4.57	5.05	0.48
22	4.91	4.71	-0.2
23	4.87	4.75	-0.12
24	4.9	4.72	-0.18
25	4.74	4.88	0.14

Control and Fertilizer B Group			
Trial #	**Sample Group 1 Mean**	**Sample Group 2 Mean**	**Difference**
1	4.7	4.09	-0.61
2	4.59	4.2	-0.39
3	4.32	4.47	0.15
4	4.37	4.42	0.05
5	4.3	4.49	0.19
6	4.18	4.61	0.43
7	4.63	4.16	-0.47
8	4.37	4.42	0.05
9	4.4	4.39	-0.01
10	4.32	4.47	0.15
11	4.36	4.43	0.07
12	4.23	4.56	0.33
13	4.45	4.34	-0.11
14	4.34	4.45	0.11
15	4.36	4.43	0.07
16	4.14	4.65	0.51
17	4.27	4.52	0.25
18	4.4	4.39	-0.01
19	4.63	4.16	-0.47
20	4.37	4.42	0.05
21	4.22	4.57	0.35
22	4.35	4.44	0.09
23	4.18	4.61	0.43
24	4.44	4.35	-0.09
25	4.29	4.5	0.21

2. Based on these data, is the difference between the control group and the group with Fertilizer A significant? Explain how you arrived at your answer.

3. Based on these data, is the difference between the control group and the group with Fertilizer B significant? Explain how you arrived at your answer.

4. Next year, the farmer plans to grow all of the radishes the same way (either with no fertilizer, Fertilizer A, or Fertilizer B). If the cost of Fertilizer A is three times the cost of Fertilizer B, which of the three options would you recommend the farmer choose to use next year? Explain your reasoning, using statistics to support your answer.

Instructional Task Exemplar Response

A local farmer has been growing radishes to sell at a farmers market for years. This year, he decided to test two different types of fertilizers. He planted 30 radish plants in an area with consistent lighting conditions. He randomly assigned each plant one of the following treatments: (1) no fertilizer [control group], (2) Fertilizer A, and (3) Fertilizer B. This resulted in 10 plants per treatment. After the radishes had been growing for four weeks, he measured the height of all of the plants. The heights are shown in the table below.

Plant Heights after 4 Weeks (cm)

No Fertilizer	Fertilizer A	Fertilizer B
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3.5	5.2	3.0
4.7	6.2	5.2
5.1	5.2	5.3
3.3	5.1	4.1
4.7	6.0	4.5
4.0	5.4	4.9

1. Compare the center and spread of the three groups. Assume that the three data distributions are approximately unimodal and symmetric.

   If the distributions are unimodal and symmetric, then the most appropriate measures of center and spread are mean and standard deviation. The mean height with no fertilizer is 4.20 cm, the mean height with Fertilizer A is 5.42 cm, and the mean height with Fertilizer B is 4.59 cm.

   The standard deviation of the data from the group with no fertilizer is 0.59 cm. The standard deviation of the data with Fertilizer A is 0.50 cm and with Fertilizer B is 0.70 cm.

   Both types of fertilizer seemed to produce taller plants (on average, based on the calculated means) than using no fertilizer at all. Also, compared to the group with no fertilizer, the amount of variation in the plant heights was slightly greater with Fertilizer A, and even greater with Fertilizer B.

   ***Note: The standard deviation of the data for all three groups was calculated using a TI-84 Plus calculator. Other technology may produce slightly different results. Students may also find the standard deviation by hand.*

All of the data from the control group and from the Fertilizer A group were combined and randomly divided into two groups of 10 samples. The means of these new groups (shown as "Sample Group 1 Mean" and "Sample Group 2 Mean") were calculated, and the differences between the mean heights were calculated. This process was repeated for the control group with Fertilizer B. The re-randomization of the data was performed 25 times for each treatment. The results of this re-randomization of the data are shown in the tables below.

Re-randomization of Data

Control and Fertilizer A Group			
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9	4.95	4.67	-0.28
10	4.68	4.94	0.26
11	4.9	4.72	-0.18
12	4.68	4.94	0.26
13	5.16	4.46	-0.7
14	4.81	4.81	0
15	4.75	4.87	0.12
16	4.43	5.19	0.76
17	4.73	4.89	0.16
18	4.77	4.85	0.08
19	5	4.62	-0.38
20	5.01	4.61	-0.4
21	4.57	5.05	0.48
22	4.91	4.71	-0.2
23	4.87	4.75	-0.12
24	4.9	4.72	-0.18
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Control and Fertilizer B Group			
Trial #	**Sample Group 1 Mean**	**Sample Group 2 Mean**	**Difference**
1	4.7	4.09	-0.61
2	4.59	4.2	-0.39
3	4.32	4.47	0.15
4	4.37	4.42	0.05
5	4.3	4.49	0.19
6	4.18	4.61	0.43
7	4.63	4.16	-0.47
8	4.37	4.42	0.05
9	4.4	4.39	-0.01
10	4.32	4.47	0.15
11	4.36	4.43	0.07
12	4.23	4.56	0.33
13	4.45	4.34	-0.11
14	4.34	4.45	0.11
15	4.36	4.43	0.07
16	4.14	4.65	0.51
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19	4.63	4.16	-0.47
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21	4.22	4.57	0.35
22	4.35	4.44	0.09
23	4.18	4.61	0.43
24	4.44	4.35	-0.09
25	4.29	4.5	0.21

2. Based on these data, is the difference between the control group and the group with Fertilizer A significant? Explain how you arrived at your answer.

   The difference in the average heights between the control group and the group with Fertilizer A was 1.22 cm. This difference seems very large relative to the total heights of the plants. Once the data were re-randomized (using only the data from the control group and the group with Fertilizer A), the difference in average heights ranged from -0.7 cm to 0.76 cm. Out of 25 trials of re-randomizing the data, an average height difference of 1.22 cm was never found. The greatest difference in averages was 0.76 cm—well below the observed difference of

1.22 cm. Therefore, it appears that the difference in heights was significant and that using Fertilizer A made a significant difference in the heights of radish plants.

3. Based on these data, is the difference between the control group and the group with Fertilizer B significant? Explain how you arrived at your answer.

   The difference in the average heights between the control group and the group with Fertilizer B was 0.39 cm. This difference is notable. However, once the data were re-randomized (using only the data from the control group and the group with Fertilizer B), the difference in average heights ranged from -0.61 cm to 0.51 cm. A difference of 0.39 cm is well within this range. Looking at only the absolute value of the differences in average heights, a difference of 0.39 cm or greater was found 7 out of 25 times. Therefore, according to the simulation, approximately 28% of the time, a difference of 0.39 cm might be observed due to sampling variation. Therefore, I am not very confident that the difference in observed heights is a direct result of using Fertilizer B as opposed to normal sampling variation. The observed difference is not significant.

4. Next year, the farmer plans to grow all of the radishes the same way (either with no fertilizer, Fertilizer A, or Fertilizer B). If the cost of Fertilizer A is three times the cost of Fertilizer B, which of the three options would you recommend the farmer choose to use next year? Explain your reasoning, using statistics to support your answer.

   Sample response:

   Because I am not very confident in the effectiveness of Fertilizer B, I would not choose to purchase that fertilizer at all. The decision for me would be between no fertilizer and Fertilizer A. Because I found a statistically significant difference when using Fertilizer A, I would use this fertilizer for next year's crop, even at the higher cost.

Interpreting Functions (IT)

Overview

Students will interpret functions in a real-world context.

Standards

Interpret functions that arise in applications in terms of the context.

HSF-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

HSF-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSF-IF.B.4	• HSF-IF.A.1 • HSN-Q.A.1	1. http://www.illustrativemathematics.org/illustrations/637 2. http://www.illustrativemathematics.org/illustrations/639 3. http://www.illustrativemathematics.org/illustrations/649	• http://www.illustrativemathematics.org/illustrations/588 • http://www.illustrativemathematics.org/illustrations/589 • http://www.illustrativemathematics.org/illustrations/598 • http://www.illustrativemathematics.org/illustrations/85 • http://www.illustrativemathematics.org/illustrations/473 • http://learnzillion.com/lessonsets/477-graph-quadratic-functions-and-show-intercepts-maxima-and-minima • http://learnzillion.com/lessonsets/470-graph-linear-functions-and-show-intercepts-maxima-and-minima

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSF-IF.B.6	• HSF-IF.A.2	1. What is the rate of change between the points (3, 20) and (-2, 8)? a. 2.4 2. http://www.illustrativemathematics.org/illustrations/577 3. http://www.illustrativemathematics.org/illustrations/686 4. http://www.illustrativemathematics.org/illustrations/1500	• http://www.illustrativemathematics.org/illustrations/599 • http://www.illustrativemathematics.org/illustrations/634 • http://www.illustrativemathematics.org/illustrations/625

Real-World Preparation:

- ***What sport does the New Orleans Pelicans team play?*** The New Orleans Pelicans are a NBA basketball team.
- ***What is a team's record?*** Teams often track their records. A record is a comparison of wins to losses. The fraction formed by the number of wins and the total number of games played is the winning percentage.

During the Task

Remind students that the equation and the graph model the same data. They can use either representation to answer questions. They should decide which is most appropriate for each question.

After the Task

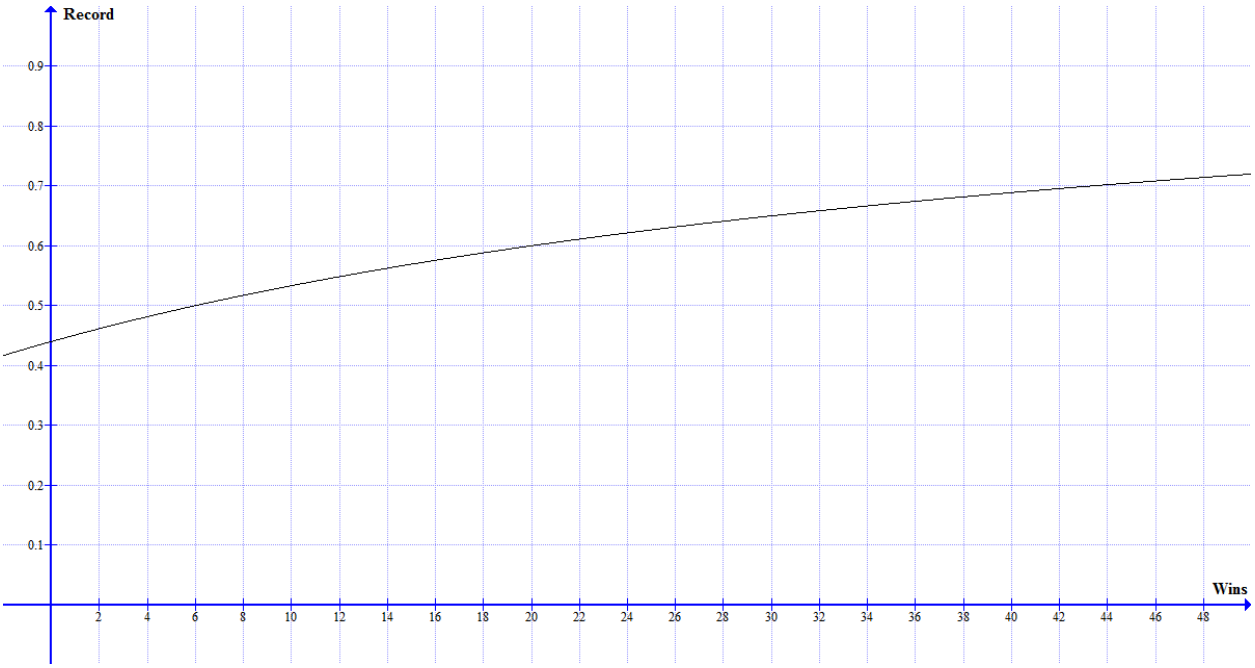
The teacher can have students compare the graph and the team's actual record at various points in the season.

Student Instructional Task

Jeremy is a huge New Orleans Pelicans fan. During the 2013-2014 season, at week 15 the team has 22 wins and 28 losses. Jeremy is worried that they might end the season with a losing record. He decides to see what will happen to the team's record if they win every game for the rest of the season. He writes the equation below. In his equation, *x* represents the number of consecutive wins.

$$W(x) = \frac{22 + x}{50 + x}$$

Jeremy creates the graph below to model his equation.



1. What is the y-intercept of this graph? Support your answer using either the equation or graph. What does it represent?

2. Is the function increasing or decreasing? Support your answer using either the equation or graph. Explain your answer in the context of the New Orleans Pelicans' winning percentage.

3. Find the average rate of change from 2 wins to 6 wins and from 14 wins to 18 wins. Compare your answers and explain your answer in the context of the rate of change of the New Orleans Pelicans' winning percentage. Show all of your work.

4. The New Orleans Pelicans will play 82 games during the 2013-2014 season. Is it possible that the team will finish with a winning percentage of 60%? Explain and show your reasoning.

5. Does this function have a horizontal asymptote? Explain your answer in the context of the team's winning percentage.

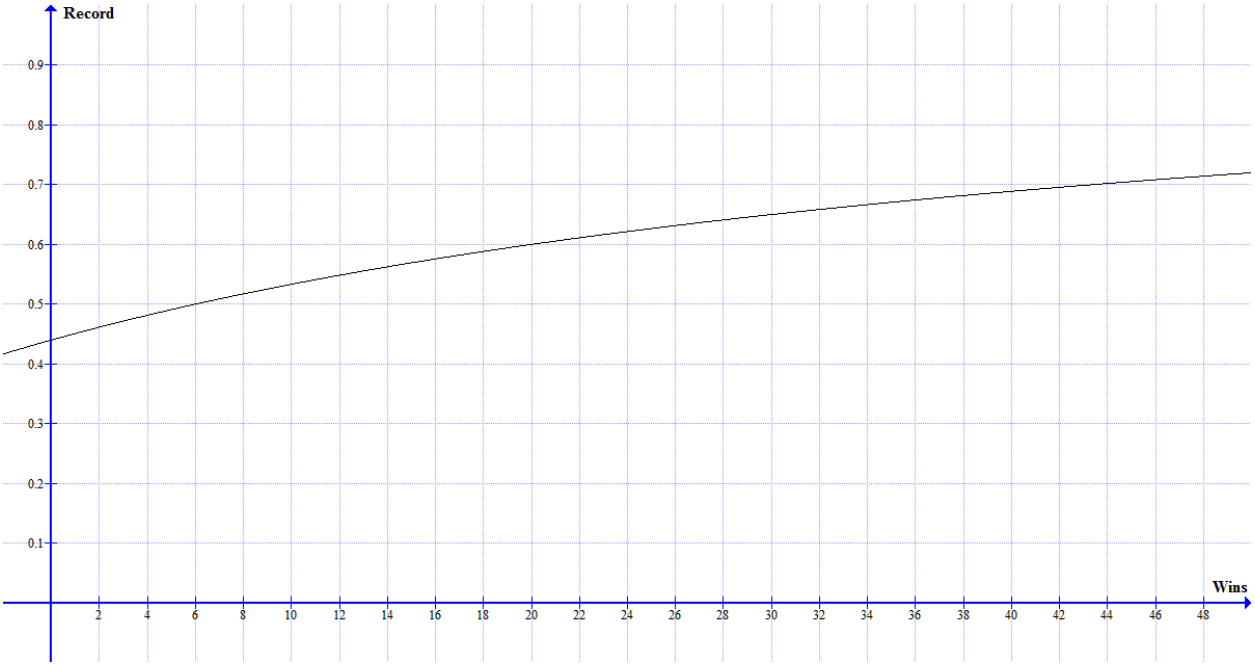
Information found at http://espn.go.com/nba/team/rankings/_/name/no/new-orleans-pelicans.

Instructional Task Exemplar Response

Jeremy is a huge New Orleans Pelicans fan. During the 2013-2014 season, at week 15 the team has 22 wins and 28 losses. Jeremy is worried that they might end the season with a losing record. He decides to see what will happen to the team's record if they win every game for the rest of the season. He writes the equation below. In his equation, *x* represents the number of consecutive wins.

$$W(x) = \frac{22 + x}{50 + x}$$

Jeremy creates the graph below to model his equation.



1. What is the y-intercept of this graph? Support your answer using either the equation or graph. What does it represent?

$$W(x) = \frac{22 + 0}{50 + 0}$$

$$W(x) = \frac{22}{50}$$

The y-intercept is $\frac{22}{50}$ *or 0.44. This y-intercept represents the team's winning percentage at week 15.*

2. Is the function increasing or decreasing? Support your answer using either the equation or graph. Explain your answer in the context of the New Orleans Pelicans' winning percentage.

The function is increasing. The graph is only increasing. This means that the equation is representing the team's winning percentage as they win every game.

3. Find the average rate of change from 2 wins to 6 wins and from 14 wins to 18 wins. Compare your answers and explain your answer in the context of the rate of change of the New Orleans Pelicans' winning percentage. Show all of your work.

From 2 to 6:

$$\frac{0.5 - 0.44}{6 - 0}$$
$$= \frac{0.06}{6}$$
$$= 0.01$$

From 14 to 18:

$$\frac{0.58824 - 0.5625}{18 - 14}$$
$$= \frac{0.02574}{4}$$
$$= 0.006435$$

$$0.01 > 0.006435$$

This means that the New Orleans Pelicans' winning percentage was increasing at a faster rate between wins 2 and 6 than it was between wins 14 and 18.

4. The New Orleans Pelicans will play 82 games during the 2013-2014 season. Is it possible that the team will finish with a winning percentage of 60%? Explain and show your reasoning.

Yes, it is possible that the team finishes with a winning percentage of 60%. I see the point (20, 0.6) on the graph. This means that it would take 20 consecutive wins for the team to have a winning percentage of 60%. They have already played 50 games. If they play an additional 20 games, they will only have played 70 games, less than the total 82 games that they will play during the season.

5. Does this function have a horizontal asymptote? Explain your answer in the context of the team's winning percentage.

This function does have a horizontal asymptote. From the graph, I can see that if I extend the graph, the function will not cross 1. In terms of the team's winning percentage, the team cannot have a record of over 100% wins.

Information found at http://espn.go.com/nba/team/rankings/_/name/no/new-orleans-pelicans

Lifetime Savings (IT)

Overview

Students will derive the formula for the sum of a finite geometric series using a real-world context.

Standards

Write expressions in equivalent forms to solve problems.

HSA-SSE.B.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-SSE.B4	• HSA-SSE.B.3c	1. http://www.illustrativemathematics.org/illustrations/442 2. http://www.illustrativemathematics.org/illustrations/805 3. http://www.illustrativemathematics.org/illustrations/929 4. http://www.illustrativemathematics.org/illustrations/1797	• http://www.illustrativemathematics.org/illustrations/1305

Real-World Preparation:

- ***What does the phrase "compounded annually" mean?*** Interest that is compounded annually is computed at the end of each year and added into the account, thereby increasing the amount of money in the account.
- ***What is an annuity?*** When you contribute the same amount each year to an account, it is called an *annuity*.

During the Task

The task assumes that students have already developed the formula for a geometric series themselves; having students recognize the need for this formula (and look it up, if necessary) allows them to apply their understanding of a geometric series.

When students begin to work on problem 2, be sure that they understand they are only finding the value of the $100 deposited at the end of the second year. Natural inclination might be to find the total value of the account, including the money from the first year. Draw students' attention to the wording in order for them to make sense of the problem.

For problem 5, when students solve the equation, they may round the answer and not check to see if the rounded amount will produce the $150,000. Encourage students to test their answer and try to determine the smallest amount Ms. McCarty would need to save to accumulate a minimum of $150,000 after 70 years.

After the Task

Have students research different savings plans. Using the plans they find, have students determine how much they could accumulate after 70 years if they started saving while in school. Have students determine how much they would have to save each year if they want to reach a particular savings goal. Discuss how developing a savings plan like the one in this task could help students in their future.

Student Instructional Task

For 70 years, Oseola McCarty earned a living washing and ironing other people's clothing in Hattiesburg, Mississippi. Although she did not earn much money, she budgeted her money wisely, lived within her means, and began saving at a very young age. Before she died, she drew worldwide attention by donating $150,000 to the University of Southern Mississippi for a scholarship fund in her name. The fact that Ms. McCarty was able to save so much money and generously gave it away is an inspiration to many others. She was honored with the Presidential Citizens Medal for her generosity. How did she do it?

1. Suppose Ms. McCarty saved $100 and then deposited it at the end of the year in an account that earns 5% interest, compounded annually.

    a. How much will it be worth at the end of the second year? At the end of the third year? At the end of the 70th year? Show your work.

    b. Write an expression that represents the value of an investment of *C* dollars after 70 years. Assume that *C* is deposited at the end of the first year in an account that earns 5% interest, compounded annually.

2. Now suppose Ms. McCarty saved another $100 in the **second** year and then deposited it in her account at the end of that year.

    a. How much will this $100 deposit be worth at the end of the third year? At the end of the fourth year? At the end of the 70th year? Show your work.

    b. Write an expression that represents the value of an investment of *C* dollars after 69 years. Assume that *C* is deposited at the end of the second year in an account that earns 5% interest, compounded annually.

3. Suppose Ms. McCarty saved $100 **each and every year** for 70 years. Each time, she deposited it in her account at the end of the year.

   a. How much money would Ms. McCarty have saved without interest? What would be the total amount in the account after 70 years? Show your work.

   b. Write an expression that represents the value of an investment of *C* dollars deposited each year for 70 years. Assume as above that it is always deposited at the end of the year in an account that earns 5% interest, compounded annually.

4. Had Ms. McCarty saved $1,000 each year, how much would she have had after 70 years under the same conditions? Justify your answer.

5. How much would she have to save each year in order to accumulate $150,000 after 70 years? Are you surprised by the answer? Show your work and explain your reasoning.

6. The *future value* (*FV*) of an annuity is the total value of the annuity after a certain number of years. The formula for the future value of an annuity is shown below.

$$FV = C \cdot \frac{(1+r)^t - 1}{r}$$

   Based on the work you did above, what is the meaning of *C* in this context? What is the meaning of *r* in this context? What is the meaning of *t* in this context?

Task adapted from http://www.illustrativemathematics.org/illustrations/1283.

Instructional Task Exemplar Response

For 70 years, Oseola McCarty earned a living washing and ironing other people's clothing in Hattiesburg, Mississippi. Although she did not earn much money, she budgeted her money wisely, lived within her means, and began saving at a very young age. Before she died, she drew worldwide attention by donating $150,000 to the University of Southern Mississippi for a scholarship fund in her name. The fact that Ms. McCarty was able to save so much money and generously gave it away is an inspiration to many others. She was honored with the Presidential Citizens Medal for her generosity. How did she do it?

1. Suppose Ms. McCarty saved $100 and then deposited it at the end of the year in an account that earns 5% interest, compounded annually.

   a. How much will it be worth at the end of the second year? At the end of the third year? At the end of the 70th year? Show your work.

      Sample answer:

      Note that computations are rounded to the nearest cent.

Year	***Value (in dollars) at the end of the year***
1	*100*
2	$100 \cdot (1.05) = 105$
3	$[100 \cdot (1.05)] \cdot 105 = 100 \cdot (1.05)^2 = 110.25$
4	$[100 \cdot (1.05)^2] \cdot 105 = 100 \cdot (1.05)^3 = 115.76$
70	$100 \cdot (1.05)^{69} = 2897.76$

      The investment will be worth $105 at the end of the second year, $110.25 at the end of the third year, and $2,897.76 at the end of the 70th year.

   b. Write an expression that represents the value of an investment of *C* dollars after 70 years. Assume that *C* is deposited at the end of the first year in an account that earns 5% interest, compounded annually.

      Replacing 100 by C, an expression that represents the value of an investment of C dollars after 70 years is $C \cdot (1.05)^{69}$.

2. Now suppose Ms. McCarty saved another $100 in the **second** year and then deposited it in her account at the end of that year.

   a. How much will this $100 deposit be worth at the end of the third year? At the end of the fourth year? At the end of the 70th year? Show your work.

Year	***Value (in dollars) at the end of the year***
2	100
3	$100 \cdot (1.05) = 105$
4	$[100 \cdot (1.05)] \cdot 105 = 100 \cdot (1.05)^2 = 110.25$
5	$[100 \cdot (1.05)^2] \cdot 105 = 100 \cdot (1.05)^3 = 115.76$
70	$100 \cdot (1.05)^{68} = 2759.77$

   $100 deposited at the end of the second year will be worth $105 at the end of the third year, $110.25 at the end of the fourth year, and $2,759.77 at the end of the 70th year.

   b. Write an expression that represents the value of an investment of *C* dollars after 69 years. Assume that *C* is deposited at the end of the second year in an account that earns 5% interest, compounded annually.

   An expression that represents the value of an investment of C dollars deposited at the end of the second year after 69 years is $C \cdot (1.05)^{68}$.

3. Suppose Ms. McCarty saved $100 **each and every year** for 70 years. Each time, she deposited the money in her account at the end of the year.

   a. How much money would Ms. McCarty have saved without interest? What would be the total amount in the account after 70 years? Show your thinking.

   Ms. McCarty saved $100 \cdot 70$ *or $7,000 over the 70 years. The statements below show how to find the ending value of each of these deposits:*

- *The $100 invested at the end of year 1 is worth* $100 \cdot (1.05)^{69}$ *at the end of the 70 years.*
- *The $100 invested at the end of year 2 is worth* $100 \cdot (1.05)^{68}$ *at the end of the 70 years.*
- *The $100 invested at the end of year 3 is worth* $100 \cdot (1.05)^{67}$ *at the end of the 70 years.*
- *...and so on, until...*
- *The $100 invested at the end of year 69 is worth* $100 \cdot (1.05)^{1}$ *at the end of the 70 years.*
- *The $100 invested at the end of year 70 is worth* $100 \cdot (1.05)^{0}$ *at the end of the 70 years.*

Now add these up.

$$100 \cdot (1.05)^{69} + 100 \cdot (1.05)^{68} + 100 \cdot (1.05)^{67} + \ldots + 100 \cdot (1.05)^{1} + 100 \cdot (1.05)^{0}$$

The sum is a finite geometric series, and can be evaluated as

$$100 \cdot \frac{(1.05)^{70} - 1}{1.05 - 1} = 58852.85$$

I conclude that she would have accumulated $58,852.85 altogether by setting aside $100 each year for 70 years.

***Note: the calculation could also be written as*

$$100 \cdot \frac{1 - (1.05)^{70}}{1 - 1.05} = 58852.85$$

b. Write an expression that represents the value of an investment of *C* dollars deposited each year for 70 years. Assume as above that it is always deposited at the end of the year in an account that earns 5% interest, compounded annually.

An expression that represents the value of an investment of C dollars deposited at the end of each year for 70 years is

$$C \cdot \frac{(1.05)^{70} - 1}{.05}$$

***Note: the expression could also be written as*

$$C \cdot \frac{1 - (1.05)^{70}}{1 - 1.05}$$

4. Had Ms. McCarty saved $1,000 each year, how much would she have had after 70 years under the same conditions? Justify your answer.

I can use the last part of the problem above to find this amount:

$$1000 \cdot \frac{(1.05)^{70} - 1}{.05} = 588528.51$$

She would have saved $588,528.51 altogether.

Alternate response:

Ms. McCarty is saving 10 times as much each year, so the total she saves will be 10 times as great. She would have saved $588,528.51 altogether.

5. How much would she have to save each year in order to accumulate $150,000 after 70 years? Are you surprised by the answer? Show your work and explain your reasoning.

Using the expression I wrote in part 3:

$$C \cdot \frac{(1.05)^{70} - 1}{.05} = 150000$$

$$C \cdot 588.53 = 150000$$

$$C = \frac{150000}{588.53}$$

$$C = 254.87$$

Sample student response:

If Ms. McCarty saves $254.87 each year, she would actually be short by $1.74. If she saves $255.88 each year, she would have $150,004.15, $4.15 more than she needs. Therefore, Ms. McCarty would need to save approximately $254.88 each year to accumulate at least $150,000 in 70 years.

This is very surprising. I would have thought that she needed to save a lot more each month to accumulate that much money.

***Note: If student work is correct based on incorrect expressions from earlier problems, the answer should be counted as correct. Also, students may provide various responses about whether the amount of money Ms. McCarty would need to save is surprising—students should be able to justify their reasoning.*

6. The *future value* (*FV*) of an annuity is the total value of the annuity after a certain number of years. The formula for the future value of an annuity is shown below.

$$FV = C \cdot \frac{(1 + r)^t - 1}{r}$$

Based on the work you did above, what is the meaning of *C* in this context? What is the meaning of *r* in this context? What is the meaning of *t* in this context?

Based on the work above, C is the amount of money added to the annuity every year, r is the annual interest rate (expressed as a decimal), and t is the number of years the same amount of money is deposited in the annuity.

Radical Equations (IT)

Overview

Students will apply their knowledge of solving radical equations to identify errors and determine whether given equations are always true, sometimes true, or never true using the set of real numbers as the domain.

Standards

Understand solving equations as a process of reasoning and explain the reasoning.

HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

HSA-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.A.1	• HSA-REI.B.4	1. Explain the steps you would take to solve $\sqrt{4x-3}=7$. a. Assuming there is a value for x that would make this equation true, I would begin by squaring both sides of the equation to get $4x-3=49$. Then I would solve this linear equation by adding 3 to both sides, then dividing by 4. This gives the result of x = 13. Finally, I would check my solution by substituting 13 for x in order to be sure it is not an extraneous solution. $\sqrt{4(13)-3}=\sqrt{49}=7$, so the solution is 13.	• http://www.illustrativemathematics.org/illustrations/618 • http://www.illustrativemathematics.org/illustrations/1690 • http://learnzillion.com/lessonsets/495-justify-solutions-to-equations-in-terms-of-equation-properties • http://learnzillion.com/lessonsets/203-solve-and-explain-simple-algebraic-equations

Grade-Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSA-REI.A.2	• HSA-REI.A.1	1. Solve $\sqrt{x+3}=2$. State any extraneous solutions. a. $x=1$; there are no extraneous solutions. 2. Solve $5-x=\sqrt{2x-10}$. State any extraneous solutions. a. $x=5$; there is one extraneous solution: $x=7$. 3. Solve $\sqrt{x^2+16}=x+4$. State any extraneous solutions. a. $x=0$; there are no extraneous solutions. 4. http://www.illustrativemathematics.org/illustrations/391	• http://learnzillion.com/lessons/1458-solve-radical-algebraic-equations • http://learnzillion.com/lessons/1459-fina-extraneous-solutions-in-algebraic-radical-expressions

During the Task

- Students may struggle to explain their reasoning for each step in problem 1. Ask students to think about why they are choosing certain operations as they go through each step. Encourage them to use precise vocabulary when explaining their reasoning for each step.
- Students who have struggled with multiplication of polynomials may not realize the mistake in problem 2. Have students try to work the same problem on their own before looking at the steps for Ashley's work. Students should always check their own answers for extraneous solutions. If students find a solution that works from the values they found, students can then compare their work with the steps given for Ashley.
- Allow students to work together for problem 3 in determining whether the statements are always true, sometimes true, or never true. Students will spend time trying to determine how to show that their choice is correct. In most cases, solving the equation in the traditional manner will work. However, for some of these items, students will need to use their number sense and reasoning to determine what is happening with the solutions to the given equations. Encourage students to explain to each other why they believe a given equation is always true, sometimes true, or never true, and have students critique each other's arguments.
- At the end of problem 3, it is a good idea to have all of the groups share their responses to see how the class labeled each item. Have a class discussion in which students defend their choices and others critique their reasoning.

After the Task

Lead a discussion with the class to have students make connections between solving radical equations and the work with rational exponents and simplifying radical expressions. After discussing the solutions to problem 3, have students use graphing technology to verify their claims based on the truth of the given statements.

Student Instructional Task

1. Find all real solutions for $4 = \sqrt{3x + 7}$. Show your work and explain your reasoning for each step. Identify any extraneous solutions.

2. Ashley solved the equation $5x - 3 = \sqrt{9 - 15x}$. Her work is shown below.

   $5x - 3 = \sqrt{9 - 15x}$

   Step 1: $(5x - 3)^2 = \left(\sqrt{9 - 15x}\right)^2$

   Step 2: $25x^2 + 9 = 9 - 15x$

   Step 3: $25x^2 + 9 - 9 + 15x = 0$

   Step 4: $25x^2 + 15x = 0$

   Step 5: $(5x)(5x + 3) = 0$

   Step 6: $5x = 0$ or $5x + 3 = 0$

   Step 7: $x = 0$ or $x = -\frac{3}{5}$

   Ashley made an error in her work. Identify Ashley's error and find the correct solution set. Be sure to state any extraneous solutions. Show all of your work.

3. Decide whether each of the statements below is always true, sometimes true, or never true for the set of real numbers. If you decide a statement is sometimes true, state the values of *x* for which the statement is true. Explain why the statement is true for only those values. If you decide the statement is always true or never true, explain your reasoning. Remember, substituting a few values for *x* is not enough to explain your reasoning.

   a. $\sqrt{2x-3} = \sqrt{2x} - \sqrt{3}$

   b. $\sqrt{-x} = -\sqrt{x}$

   c. $\sqrt[3]{-x} = -\sqrt[3]{x}$

   d. $-\sqrt{x^2} = -x$

   e. $\sqrt{8x+3} = -1$

   f. $\sqrt{16x^2} \cdot \sqrt{4} = 8x$

Instructional Task Exemplar Response

1. Find all real solutions for $4 = \sqrt{3x + 7}$. Show your work and explain your reasoning for each step. Identify any extraneous solutions.

$4 = \sqrt{3x + 7}$	*Assume there is a value for x that will make this equation true.*
$4^2 = \left(\sqrt{3x + 7}\right)^2$	*The inverse operation for the square root is to square the expression, so I squared both sides of the equation.*
$16 = 3x + 7$	*Each side of the equation is simplified.*
$16 - 7 = 3x + 7 - 7$	*Subtract 7 from both sides in order to isolate the variable expression, then simplify.*
$9 = 3x$	
$\frac{9}{3} = \frac{3x}{3}$	*Divide both sides of the equation by 3 to isolate x, then simplify.*
$x = 3$	

The solution is $x = 3$. There are no extraneous solutions. I know this because substituting 3 for x in the original equation results in 4 = $\sqrt{16}$, which is a true statement.

2. Ashley solved the equation $5x - 3 = \sqrt{9 - 15x}$. Her work is shown below.

	$5x - 3 = \sqrt{9 - 15x}$
Step 1:	$(5x - 3)^2 = \left(\sqrt{9 - 15x}\right)^2$
Step 2:	$25x^2 + 9 = 9 - 15x$
Step 3:	$25x^2 + 9 - 9 + 15x = 0$
Step 4:	$25x^2 + 15x = 0$
Step 5:	$(5x)(5x + 3) = 0$
Step 6:	$5x = 0$ or $5x + 3 = 0$
Step 7:	$x = 0$ or $x = -\frac{3}{5}$

Ashley made an error in her work. Identify Ashley's error and find the correct solution(s). Be sure to state any extraneous solutions. Show all of your work.

Ashley's error occurred in Step 2. Ashley did not square the binomial correctly. She should have gotten $25x^2 - 30x + 9 = 9 - 15x$ for Step 2. The correct solution is $x = \frac{3}{5}$. There is one extraneous solution, $x = 0$. I know this because substituting 0 for x in the original equation results in the equation – 3 = 3, which is not true.

$25x^2 - 30x + 9 - 9 + 15x = 0$
$25x^2 - 15x = 0$
$(5x)(5x - 3) = 0$
$5x = 0$ or $5x - 3 = 0$
$x = 0$ or $x = \frac{3}{5}$

3. Decide whether each of the statements below is always true, sometimes true, or never true for the set of real numbers. If you decide a statement is sometimes true, state the values of *x* for which the statement is true. Explain why the statement is true for only those values. If you decide the statement is always true or never true, explain your reasoning. Remember, substituting a few values for *x* is not enough to explain your reasoning.

a. $\sqrt{2x-3} = \sqrt{2x} - \sqrt{3}$

Sometimes true—the equation is only true when $x = \frac{3}{2}$. When I solved the equation for x, I only obtained one value for the solution. When I checked by substitution, $\frac{3}{2}$ made the equation true.

$\left(\sqrt{2x-3}\right)^2 = \left(\sqrt{2x} - \sqrt{3}\right)^2$
$2x - 3 = 2x - 2\sqrt{6x} + 3$
$-6 = -2\sqrt{6x}$
$3 = \sqrt{6x}$
$(3)^2 = \left(\sqrt{6x}\right)^2$
$9 = 6x$
$\frac{3}{2} = x$

b. $\sqrt{-x} = -\sqrt{x}$

Sometimes true—the equation is only true when x = 0. If x is positive, then the left side of the equation becomes the square root of a negative number, which does not exist in the real number system. If x is negative, then the right side contains the square root of a negative number, which does not exist in the real number system. Since 0 does not have an opposite, when x = 0, the equation will be true.

c. $\sqrt[3]{-x} = -\sqrt[3]{x}$

Always true—the cube root of a negative number is negative, and the cube root of a positive number is positive.

$\left(\sqrt[3]{-x}\right)^3 = \left(-\sqrt[3]{x}\right)^3$
$-x = -x$

Since the resulting equation is a true statement, this is always true.

d. $\sqrt{8x+3} = -1$

Never true—the left side of the equation asks for the positive square root of the quantity. The positive root will never be equal to a negative number.

e. $\sqrt{16x^2} \cdot \sqrt{4} = 8x$

Sometimes true—this is only true for $x \geq 0$. When the left side is simplified, the result is $8|x|$. This is to ensure that we obtain a positive result for the left side. If x is negative, then the equation would not be true when evaluated.