Understanding Congruence in Terms of Rigid Motions (IT)

Overview

This task allows students to explore triangle congruence in terms of rigid motions.

Standards

HSG.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

HSG.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG.CO.A.5	• 8.G.A.2 • 8.G.A.3	1. http://www.illustrativemathematics.org/illustrations/31 2. http://www.illustrativemathematics.org/illustrations/1549 3. http://www.illustrativemathematics.org/illustrations/1547	• http://www.illustrativemathematics.org/illustrations/646 • http://www.illustrativemathematics.org/illustrations/1231 • http://www.illustrativemathematics.org/illustrations/1232
HSG.CO.B.7	• 8.G.A.2 • HSG-CO.B.6	1. http://www.illustrativemathematics.org/illustrations/1637	• http://www.illustrativemathematics.org/illustrations/1228 • http://www.illustrativemathematics.org/illustrations/1230

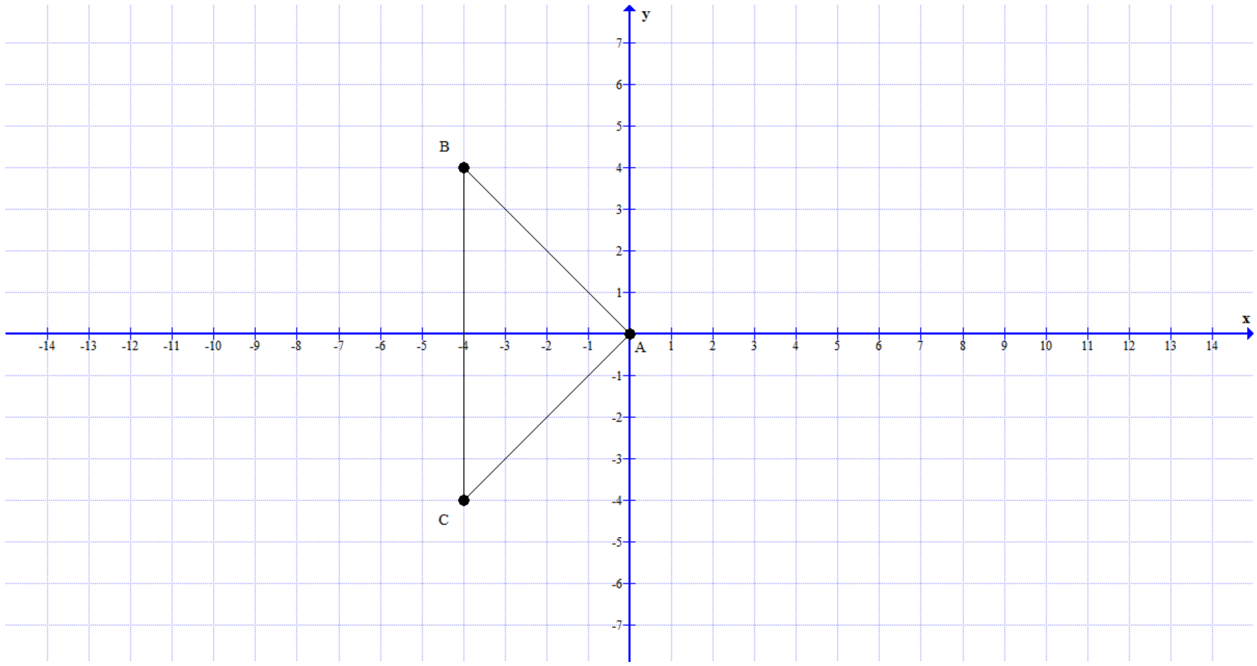
During the Task

While working on this task, students may need help finding the measures of the triangles' angles and sides. Students may choose to use appropriate tools to measure the angles or, as in the Instructional Task Exemplar Response, they may draw an additional segment to form right angles.

After the Task

Have students share the triangles that they created in step 5 with the class. Be sure that they explain how they know that the two triangles are congruent.

Student Instructional Task



1) Reflect Triangle ABC about the y-axis and label the new figure Triangle A'B'C'.

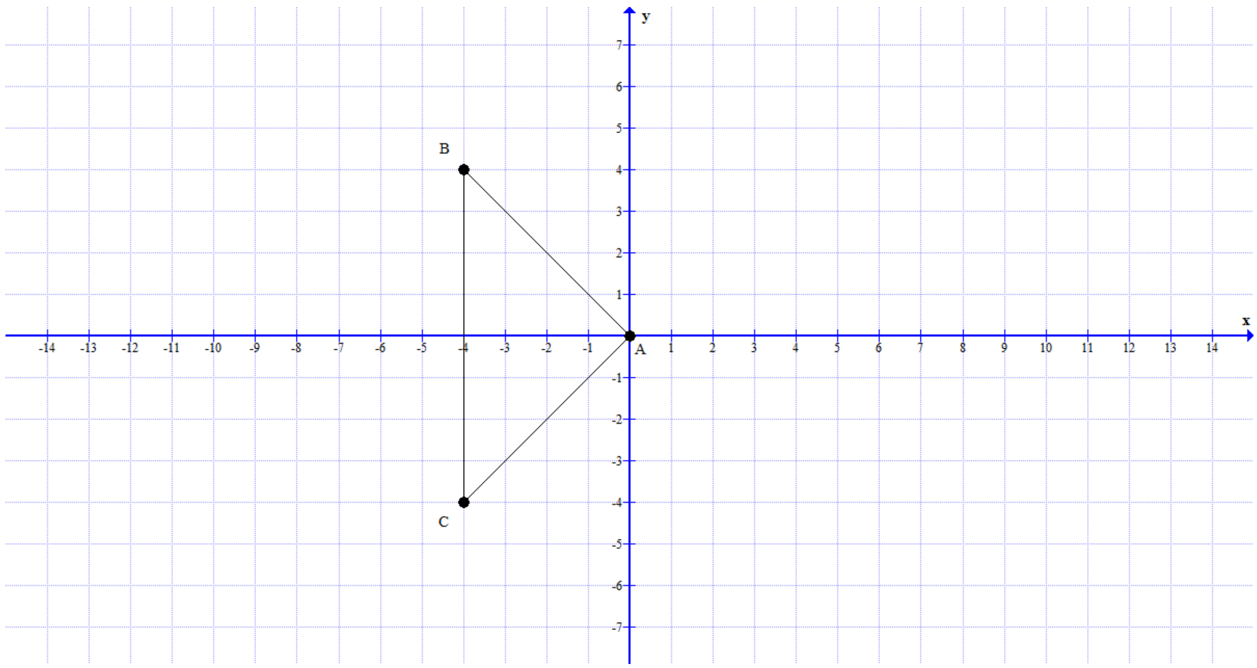
2) Give a sequence of 2 rigid transformations that would also carry Triangle ABC onto Triangle A'B'C'.

3) Find the side lengths and angle measures of both triangles. Show your work, and explain your reasoning.

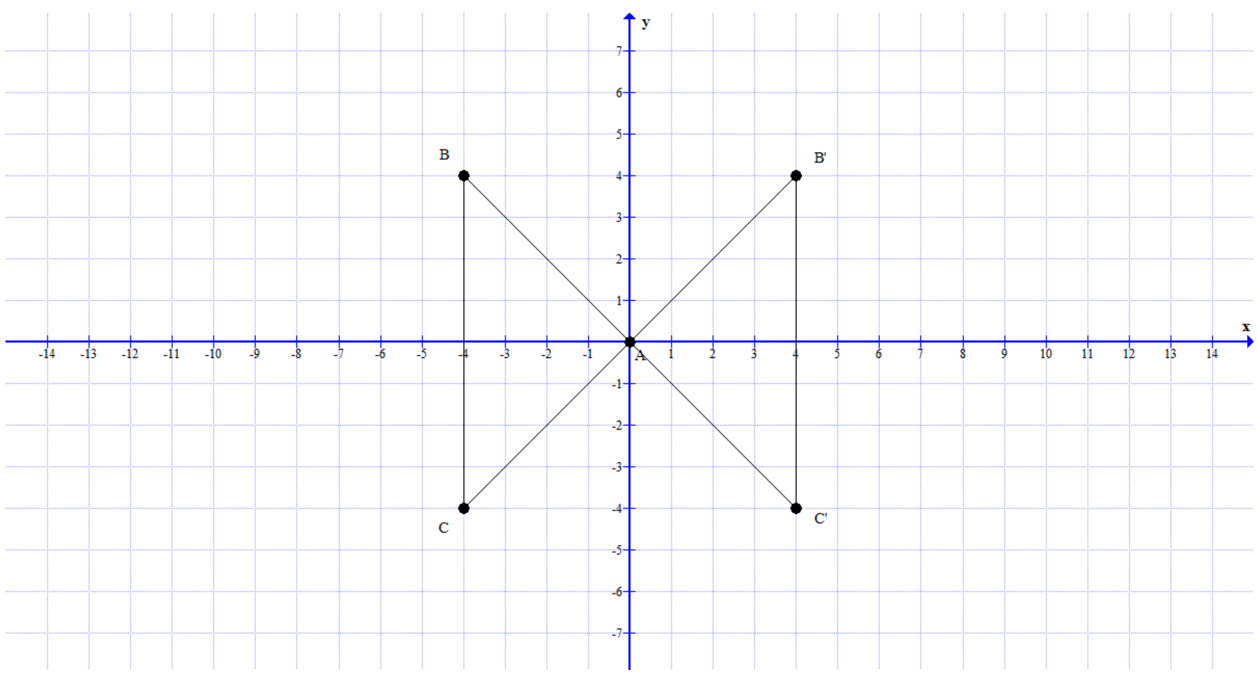
4) What do you know about the two triangles? Explain your reasoning.

5) Draw a new Triangle PQR that is congruent to Triangle ABC, and in a different location than either Triangle ABC or Triangle A'B'C'. Explain how you know Triangle PQR is congruent to Triangle ABC using at least 2 transformations.

Instructional Task Exemplar Response



1) Reflect Triangle ABC about the y-axis and label the new figure Triangle A'B'C'.



2) Give a sequence of 2 rigid transformations that would also carry Triangle ABC onto Triangle A'B'C'.
A rotation of 180° with the origin as the center and a reflection about the x-axis would carry Triangle ABC onto Triangle A'B'C'.

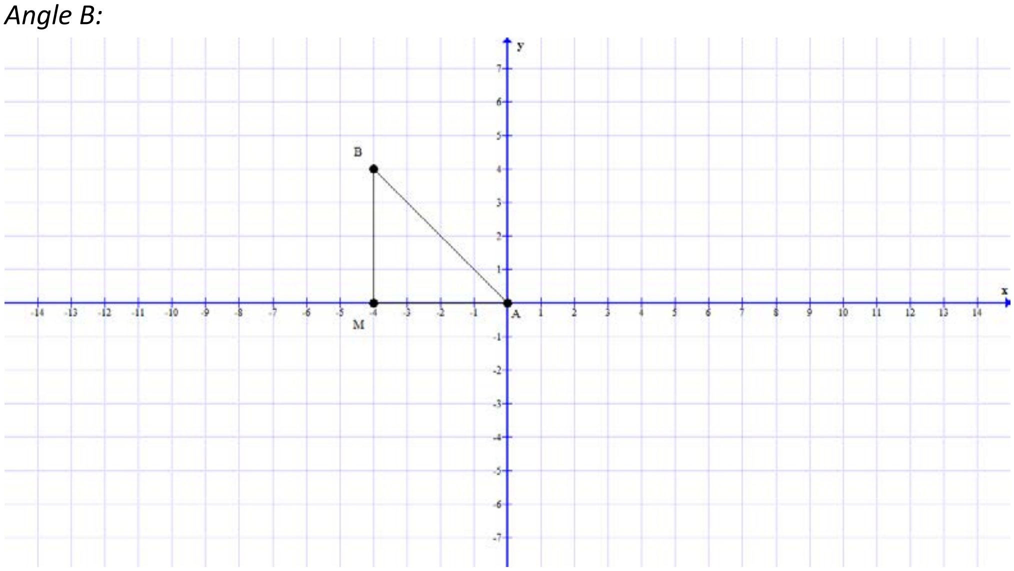
Alternate responses might include:

- *A reflection across the line y = x followed by a rotation of 90° counterclockwise*
- *A reflection across the line y = –x followed by a rotation of 90° clockwise*
- *A translation of 8 units to the right followed by a reflection across the line x = 4*

3) Find the side lengths and angle measures of both triangles. Show your work, and explain your reasoning.

Segment BC: $|4 - -4| = 8$ *units*
Alternate Response: Students might state that since this is a vertical segment, they counted 8 units.

Angle B:



In the above picture, I drew in Point M. This forms an isosceles right triangle. The base angles in an isosceles triangle are congruent, so Angle B measures 45°.

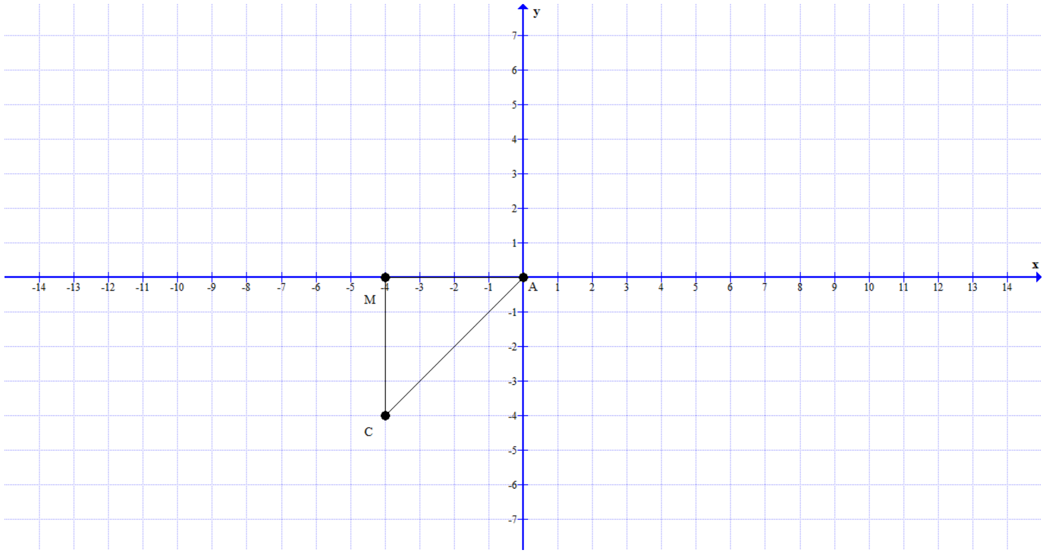
Alternate Response: Students may use a protractor to measure Angle B.

Segment AB:
I can use the Pythagorean Theorem.

$$4^2 + 4^2 = c^2$$
$$16 + 16 = c^2$$
$$32 = c^2$$
$$c = \sqrt{32}$$

Note: Students may find the length of Segment AB using the Distance Formula.

Angle C:



In the above picture, I drew in Point M. This forms an isosceles right triangle. The base angles in an isosceles triangle are congruent, so Angle C measures 45°.

Alternate Response: Students may use a protractor to measure Angle C.

Segment AC:
I can use the Pythagorean Theorem.

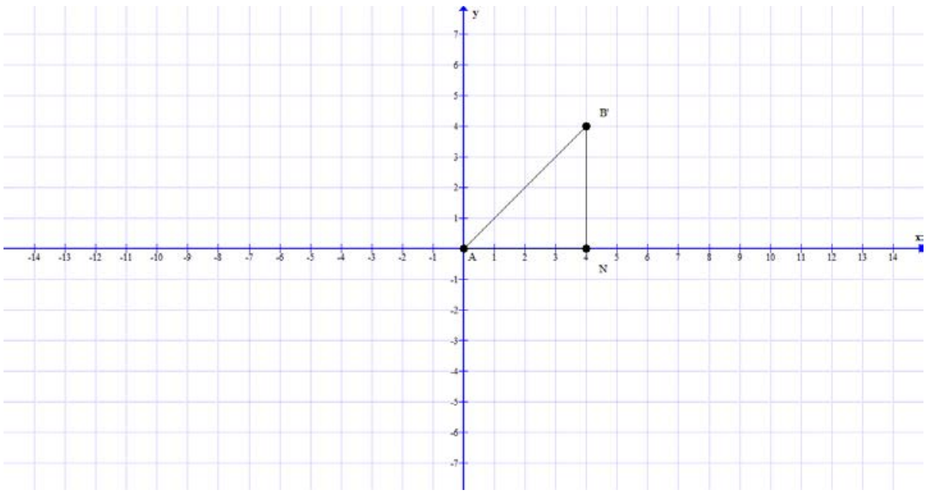
$$4^2 + 4^2 = c^2$$
$$16 + 16 = c^2$$
$$32 = c^2$$
$$c = \sqrt{32}$$

Note: Students may find the length of Segment AC using the Distance Formula.

Angle A: Angle C and Angle B both measure 45°. Therefore, the measure of Angle A is $80° - 45° - 45°\ or\ 90°$.

Segment B'C': $|4 - -4| = 8$
Alternate Response: Students might state that since this is a vertical segment, they counted 8 units.

Angle B′:



In the above picture, I drew in Point N. This forms an isosceles right triangle. The base angles in an isosceles triangle are congruent, so Angle B′ measures 45°.

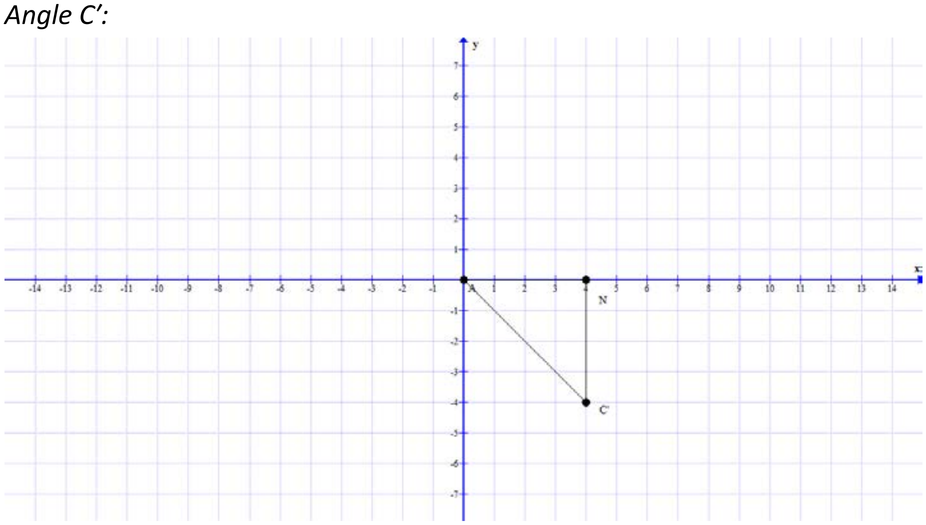
Alternate Response: Students may use a protractor to measure Angle B′.

Segment AB′:
I can use the Pythagorean Theorem.

$$4^2 + 4^2 = c^2$$
$$16 + 16 = c^2$$
$$32 = c^2$$
$$c = \sqrt{32}$$

Note: Students may find the length of Segment AB′ using the Distance Formula.

Angle C′:



In the above picture, I drew in Point N. This forms an isosceles right triangle. The base angles in an isosceles triangle are congruent, so Angle C′ measures 45°.

Alternate Response: Students may use a protractor to measure Angle C'.
Segment A'C':
I can use the Pythagorean Theorem.

$$4^2 + 4^2 = c^2$$
$$16 + 16 = c^2$$
$$32 = c^2$$
$$c = \sqrt{32}$$

Note: Students may find the length of Segment A'C' using the Distance Formula.

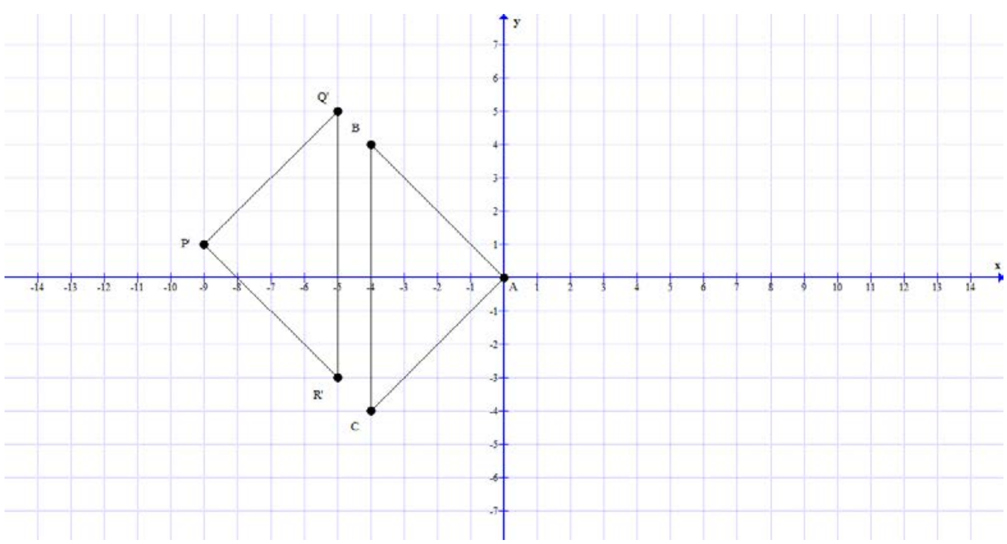
Angle A': Angle C' and Angle B' both measure 45°. Therefore, the measure of Angle A' is $180° - 45° - 45°$ *or* $90°$*.*

4) What do you know about the two triangles? Explain your reasoning.

The two triangles are congruent. From part 3, I can see that the corresponding pairs of sides and angles are congruent, so the two triangles are congruent. Also, since Triangle ABC can be transformed into Triangle A'B'C' through a series of rigid motions, then the two triangles are congruent.

5) Draw a new Triangle P'Q'R' that is congruent to Triangle ABC, and in a different location than either Triangle ABC or Triangle A'B'C'. Explain how you know Triangle PQR is congruent to Triangle ABC using at least 2 transformations.

There are many triangles that would answer this question. Here is a sample:



These triangles are congruent because a series of rigid motions transforms Triangle ABC into Triangle P'Q'R'. Triangle ABC can be reflected across the line, $x = -4.5$*, and translated up 1 unit.*

City Map (IT)

Overview

Students will use their knowledge of geometric terms and their skills with constructions to draw a city map with specific guidelines. After the map has been constructed, students will write proofs and explanations to answer questions about the lines and angles in the map.

Standards

Experiment with transformations in the plane

HSG-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Prove geometric theorems

HSG-CO.C.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent, and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

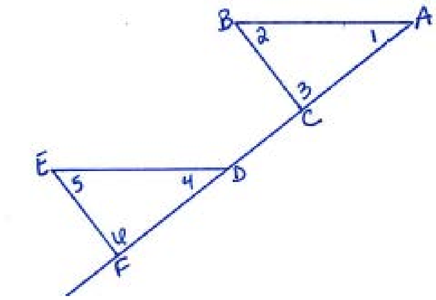
Make geometric constructions

HSG-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-CO.A.1	• 4.MD.C.5 • 4.G.A.1 • 4.G.A.2	1. Define parallel lines. a. Parallel lines are two coplanar lines that do not intersect. **2.** http://www.illustrativemathematics.org/illustrations/1543 3. http://www.illustrativemathematics.org/illustrations/1544	• http://www.illustrativemathematics.org/illustrations/1272 • http://www.illustrativemathematics.org/illustrations/1274 • http://learnzillion.com/lessonsets/808-define-geometric-terms-precisely

HSG-CO.C.9	• 4.MD.C.7 • 7.G.B.5 • 8.G.A.5 • HSG-CO.A.1	1. Given the diagram below, if $\overline{EF} \perp \overline{AF}$, $\overline{BC} \perp \overline{AF}$, and $\angle 5 \cong \angle 2$, prove that $\overline{AB} \parallel \overline{DE}$.  a. By the definition of perpendicular lines, $\angle 6$ and $\angle 3$ are right angles. Since all right angles are congruent, $\angle 6 \cong \angle 3$. By the definition of congruence, $m\angle 5 = m\angle 2$ and $m\angle 6 = m\angle 3$. The sum of the interior angles of a triangle is 180 degrees, so $m\angle 4 + m\angle 5 + m\angle 6 = 180$ and $m\angle 1 + m\angle 2 + m\angle 3 = 180$. By the substitution property of equality $m\angle 4 + m\angle 5 + m\angle 6 = 180$ and $m\angle 1 + m\angle 5 + m\angle 6 = 180$. Then, $m\angle 4 = 180 - (m\angle 5 + m\angle 6)$ and $m\angle 1 = 180 - (m\angle 5 + m\angle 6)$ Therefore, $m\angle 4 = m\angle 1$ and $\angle 4 \cong \angle 1$. $\angle 4$ and $\angle 1$ are formed by two lines and a transversal, which make them corresponding angles. When corresponding angles are congruent, the lines cut by the transversal are parallel. Therefore $\overline{AB} \parallel \overline{DE}$.	• http://www.illustrativemathematics.org/illustrations/1168 • http://www.illustrativemathematics.org/illustrations/59 • http://www.illustrativemathematics.org/illustrations/56 • http://www.illustrativemathematics.org/illustrations/1501 • http://www.illustrativemathematics.org/illustrations/1503
HSG-CO.D.12	• 4.MD.C.6 • 7.G.A.2 • HSG-CO.A.1	**1.** http://www.illustrativemathematics.org/illustrations/966	• http://www.illustrativemathematics.org/illustrations/909

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- **What is a compass?** A compass is used to tell the direction one is traveling. In the case of a map, the compass provides a means of orienting someone reading the map to the directions north, south, east, and west so they can better plan routes for trips.

- **What is a subway?** A subway is a mode of public transportation in some cities. Most subways run on rails like train tracks and are located underground so the subway does not create additional congestion on the highways and surface streets.

During the Task

- The directions in the task indicate students should use a poster board; however, this task can be completed on a regular letter size (8½” x 11”) piece of paper as well. The size of the triangle may need to be adjusted if the task is changed so that students submit the product using a different medium than the poster.
- Students should use a compass, straightedge (ruler), and a protractor for the constructions described in this task.
- Students may attempt to draw parallel lines using their straightedge only. Ask probing questions to get students to think about the properties of parallel lines that will ensure the lines are parallel (if two lines are intersected by a transversal such that corresponding angles are congruent, then the lines must be parallel). Have students use constructions that would ensure congruent angles to create the parallel lines.
- Encourage students to use precise mathematical language as they write their explanations and proofs.
- Look for different explanations from students and have students share their explanations. Allow students to respectfully critique the reasoning of their classmates.

After the Task

Have students construct other polygons like squares, regular hexagons, regular octagons, etc., by copying angles and segments and creating perpendicular lines.

Student Instructional Task

Directions for Map Construction:

Note: All streets (lines) constructed should be extended to "run off" the poster board.

1. Begin by sketching a compass to indicate the directions north, south, east, and west. Draw this in the upper left-hand corner of the poster board.
2. Construct an equilateral triangle in the center of your poster board. The sides of the triangle should each measure 4 inches. Use a straightedge to extend the lines, including the sides of each triangle, so the lines "run off" the poster board. Label the vertices A, B, and C. This will give you three streets: $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
3. Construct a street parallel to street $\overline{BC}$. Name this street $\overline{AD}$.
4. Construct a street perpendicular to street $\overline{AD}$ so that it lies to the east of the triangle but does not pass through any point on $\Delta\, ABC$. Label the intersection of the perpendicular street and street $\overline{AD}$ as point E. Label the intersection of the perpendicular street and street $\overline{BC}$ as point F.
5. Label the intersection of streets $\overline{AB}$ and $\overline{EF}$ as point G.
6. Construct a street perpendicular to street $\overline{BC}$ so that it lies to the east of street $\overline{EF}$. Label the intersection of this perpendicular street and street $\overline{AD}$ as point H. Label the intersection of the perpendicular street and street $\overline{BC}$ as point J.
7. Label the intersection of streets $\overline{AB}$ and $\overline{HJ}$ as point K.

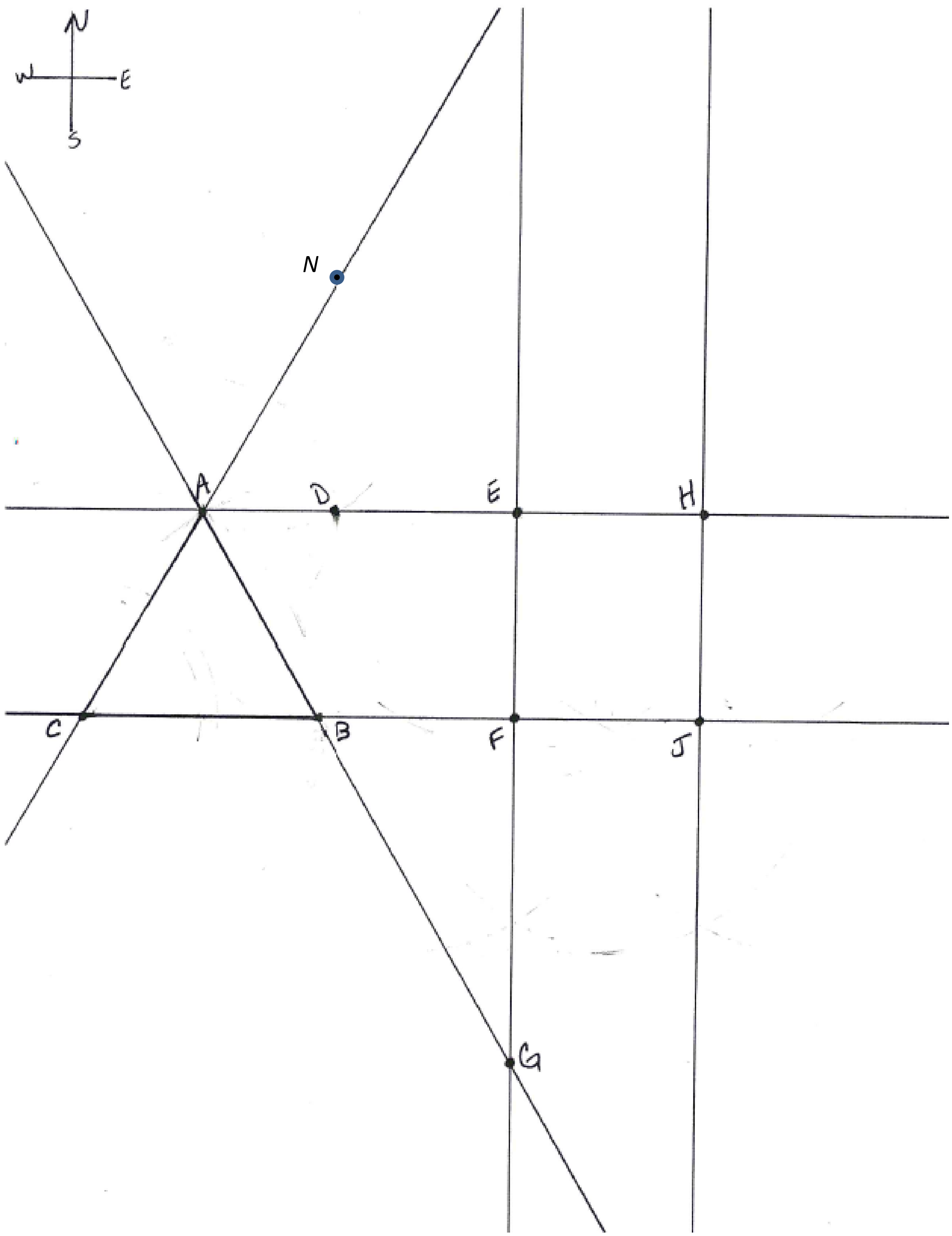
After completing the constructions, answer the questions on the following page.

On a separate sheet of paper, answer the following questions based on the map you created. Use complete sentences in your explanations and justifications. For proofs, you may choose the type of proof you create. Be sure to include logical reasoning to avoid leaving relevant information out of the proof.

1. State the measures of the following angles. Explain how you know the measures of the angles without using a protractor. Verify the angle measures.
    a. measure of Angle BAC
    b. measure of Angle DAB
    c. measure of Angle ABF
    d. measure of Angle BGF
2. What does it mean for two lines to be parallel? Prove that streets $\overline{AD}$ and $\overline{BC}$ are parallel.
3. Are there other pairs of streets that are parallel? Explain your reasoning using the properties of angles and parallel lines.
4. Imagine the section of the city depicted by your map has a subway that runs entirely underground and directly beneath street $\overline{EF}$.
    a. Is the subway parallel to $\overline{EF}$? Explain.
    b. Is the subway parallel to another street? Explain.
    c. Since the subway runs underneath the streets on this map, it will never intersect with any of the streets shown on the map. Does this mean the subway is parallel to some or all of these streets? Explain.

Task adapted from: ***http://www.radford.edu/rumath-smpdc/Resources/src/Walstrum_CivilEng.pdf***

Instructional Task Exemplar Response



Teacher Note: This picture is missing the intersection labeled K. Also, this is not drawn to the scale identified in the task. The construction marks were also removed. Point N was placed to aid in explanations in part two of the task.

Answer the following questions based on the map you created. Use complete sentences in your explanations and justifications. For proofs, you may choose the type of proof you create. Be sure to include logical reasoning to avoid leaving relevant information out of the proof.

1. State the measures of the following angles. Explain how you know the measures of the angles without using a protractor. Verify the angle measures.

    a. measure of Angle BAC
    The measure of Angle BAC is 60 degrees. Angle BAC is an angle in the equilateral triangle. All angles in an equilateral triangle are 60 degrees.

    b. measure of Angle DAB
    The measure of Angle DAB is also 60 degrees. Angle DAN is a copy of Angle ACB so it has the same measure as Angle ACB, which is 60 degrees. Angle DAN, Angle CAB, and Angle DAB are adjacent, and their non-common rays form a line, which means the sum of the angle measures is 180 degrees. Since the sum of Angle CAB and Angle DAN is 120 degrees, the measure of Angle DAB is also 60 degrees.

    c. measure of Angle ABF
    The measure of Angle ABF is 120 degrees. Angle ABF and Angle ABC are adjacent, and their non-common rays form a line, so the sum of the measures of the two angles is 180 degrees. Angle ABC is an angle in the equilateral triangle; therefore, its measure is 60 degrees. 180 degrees – 60 degrees = 120 degrees.

    d. measure of Angle BGF
    The measure of Angle BGF is 30 degrees. Angle FBG is 60 degrees because Angle ABC and Angle FBG are vertical angles and vertical angles are congruent, which means they have the same measure. Angle BFG is 90 degrees because street $\overline{EF}$ is perpendicular to street $\overline{BC}$ at point F, which means the angles formed at the intersection (labeled point F) are right angles. Right angles are 90-degree angles. Together, Angle FBG, Angle BFG, and Angle BGF are three angles in right triangle BFG. The sum of the measures of the three angles of a triangle is 180 degrees. 180 degrees – (60 degrees + 90 degrees) = 30 degrees.

2. What does it mean for two lines to be parallel? Prove that streets $\overline{AD}$ and $\overline{BC}$ are parallel.

    If two lines are parallel, they lie in the same plane and they do not intersect. I constructed $\overline{AD}$ by copying Angle ACB to construct Angle NAD. Copying an angle creates two congruent angles so Angle ACB is congruent to Angle NAD. Street $\overline{AC}$ is a transversal of streets $\overline{AD}$ and $\overline{BC}$, which makes Angle ACB and Angle NAD corresponding angles by the definition of corresponding angles. If two lines are cut by a transversal so that two corresponding angles are congruent, then the lines are parallel. Therefore streets $\overline{AD}$ and $\overline{BC}$ *are parallel.*

3. Are there any other pairs of streets that are parallel? Explain your reasoning using the properties of angles and parallel lines.

    Yes, streets $\overline{EF}$ and $\overline{HJ}$ are parallel. Street $\overline{EF}$ was constructed to be perpendicular to street $\overline{AD}$. All four angles formed at the intersection labeled E are right angles, and all right angles are congruent. Because $\overline{EF}$ is a transversal intersecting streets $\overline{AD}$ and $\overline{BC}$*, and they are parallel, street $\overline{EF}$ must be perpendicular to street $\overline{BC}$. Street $\overline{HJ}$ was constructed to be perpendicular to street $\overline{BC}$. By the same reasoning used earlier, street $\overline{HJ}$ must be perpendicular to street $\overline{AD}$. If two lines are perpendicular to the same line then the lines are parallel. Therefore streets $\overline{EF}$ and $\overline{HJ}$ are parallel.*

4. Imagine the section of the city depicted by your map has a subway that runs entirely underground and directly beneath street $\overline{EF}$.

   a. Is the subway parallel to $\overline{EF}$? Explain.

      Yes, the subway is parallel to $\overline{EF}$. Even though the subway is not on the street level, a plane can be formed between the street and the subway. According to the map, both street $\overline{EF}$ and the subway would be running north and south. Since they will never intersect, the street and the subway are parallel.

   b. Is the subway parallel to another street? Explain.

      Yes, the subway is also parallel to $\overline{HJ}$. Even though the subway is not on the street level, a plane can be formed between the street and the subway. According to the map, both street $\overline{HJ}$ and the subway would be running north and south. Since they will never intersect, the street and the subway are parallel.

   c. Since the subway runs underneath the streets on this map, it will never intersect with any of the streets shown on the map. Does this mean the subway is parallel to some or all of these streets? Explain.

      The subway is only parallel to those streets that run north and south. Streets that run any other direction (on the map they would intersect streets $\overline{EF}$ and $\overline{HJ}$) would not be in any same plane as the subway. Two lines that do not intersect and are not in the same plane are considered skew lines, not parallel. Therefore, the subway would be considered skew to all streets that do not run north and south.

Parallelogram Congruence (IT)

Overview

This task allows students to explore congruence and similarity using parallelograms.

Standards

Prove geometric theorems.

HSG-CO.C.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and, conversely, rectangles are parallelograms with congruent diagonals*.

Prove theorems involving similarity.

HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-CO.C.11	• 5.G.B.3 • HSG-CO.B.8 • HSG-CO.C.9	**1.** http://www.illustrativemathematics.org/illustrations/1321 **2.** http://www.illustrativemathematics.org/illustrations/1511 3. http://www.illustrativemathematics.org/illustrations/35	• http://www.illustrativemathematics.org/illustrations/109 • http://www.illustrativemathematics.org/illustrations/339 • http://www.illustrativemathematics.org/illustrations/110 • http://www.illustrativemathematics.org/illustrations/967 • http://learnzillion.com/lessonsets/810-prove-theorems-concerning-triangles-and-parallelograms
HSG-SRT.B.5	• HSG-SRT.A.3 • HSG-CO.B.8	1. http://www.illustrativemathematics.org/illustrations/651	• http://www.illustrativemathematics.org/illustrations/1422 • http://www.illustrativemathematics.org/illustrations/340 • http://learnzillion.com/lessonsets/668-solve-problems-using-congruence-and-similarity-criteria-for-triangles

During the Task

This task addresses similarity and congruence for a specific class of quadrilaterals, namely parallelograms. This task is ideal for hands-on work. For example, only one triangular shape is possible when using three toothpicks, namely an equilateral triangle. For quadrilaterals, on the other hand, four toothpicks can be put together to make any number of rhombuses with that side length.

After the Task

Have students make and verify conjectures about how much information is needed to determine if two quadrilaterals are congruent. For example, for squares one side is enough; for rectangles two adjacent sides are sufficient. Ask students “What information would be needed to show that any two arbitrary quadrilaterals are congruent?”

Student Instructional Task

Rhianna has learned the SSS and SAS congruence tests for triangles, and she wonders if these tests might work for parallelograms.

a. Suppose *ABCD* and *EFGH* are two parallelograms in which all corresponding sides are congruent; that is $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FG}$, $\overline{CD} \cong \overline{GH}$, $\overline{DA} \cong \overline{HE}$. Is it always true that *ABCD* is congruent to *EFGH*? Explain and show your reasoning.

b. Suppose *ABCD* and *EFGH* are two parallelograms with a pair of congruent corresponding sides, $\overline{AB} \cong \overline{EF}$ and $\overline{BC} \cong \overline{FG}$. Suppose also that the included angles $\angle ABC \cong \angle EFG$ are congruent. Are *ABCD* and *EFGH* congruent? Explain and show your reasoning.

Task adapted from ***http://www.illustrativemathematics.com/illustrations/1517***

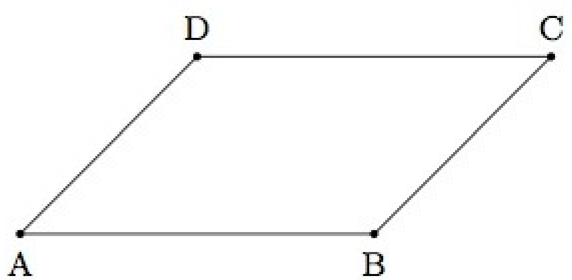
Instructional Task Exemplar Response

Rhianna has learned the SSS and SAS congruence tests for triangles, and she wonders if these tests might work for parallelograms.

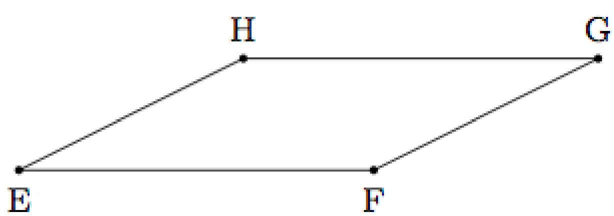
a. Suppose *ABCD* and *EFGH* are two parallelograms in which all corresponding sides are congruent; that is $\overline{AB} \cong \overline{EF}, \overline{BC} \cong \overline{FG},\ \overline{CD} \cong \overline{GH}, \overline{DA} \cong \overline{HE}$. Is it always true that *ABCD* is congruent to *EFGH*? Explain and show your reasoning.

Sample response:

I assumed that the statement "ABCD is always congruent to EFGH if all pairs of corresponding sides are congruent" was true. I attempted to find a counterexample to prove this assumption false. I began by drawing or building a parallelogram. The opposite sides of a parallelogram are congruent, so I will need two pairs of congruent segments:



Leave AD fixed, but push it to the right as if it were hinged to AB at point A. This movement does not change the side lengths, but it does change the angle measures. Therefore, I get a new parallelogram as shown below:

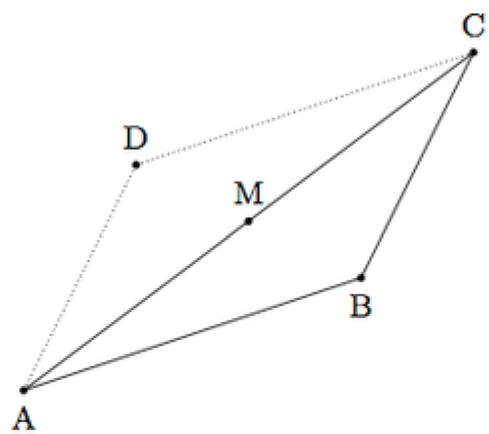


Parallelograms ABCD and EFGH have four pairs of corresponding congruent sides, but the parallelograms are not congruent since they have different angle measures. So it is not always true that ABCD will be congruent to EFGH because I have at least one counterexample to the assumption that they are always congruent.

b. Suppose *ABCD* and *EFGH* are two parallelograms with a pair of congruent corresponding sides, $\overline{AB} \cong \overline{EF}$ and $\overline{BC} \cong \overline{FG}$. Suppose also that the included angles are congruent. Are *ABCD* and *EFGH* congruent? Explain and show your reasoning.

Sample Response:

I know from the SAS triangle congruence test that △ABC is congruent to △EFG. In order to see what happens with the parallelograms ABCD and EFGH, I focus first on ABCD. Note that the vertex D is obtained by rotating vertex B 180 degrees about the midpoint M of $\overline{AC}$*. This is pictured below with the image of B labeled D:*



In other words, the parallelogram ABCD is obtained by connecting to △ABC a second triangle, △CDA, which is congruent to △ABC. The same is true of parallelogram EFGH (which is obtained by connecting △GHE to △EFG), and since △ABC is congruent to △EFG (which implies △CDA is congruent to △GHE), I can conclude that parallelogram ABCD is congruent to parallelogram EFGH.

Task adapted from http://www.illustrativemathematics.com/illustrations/1517

Modeling with Three-Dimensional Figures (IT)

Overview

Students will make decisions about the purchase of an air conditioner based on concepts of density based on volume.

Standards

Apply geometric concepts in modeling situations.

HSG-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

Explain volume formulas and use them to solve problems.

HSG-GMD-A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Prior to the Task

Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-MG.A.2	• 7.G.B.6 • 8.G.C.9 • HSG-GMD.A.1 • HSG-GMD.A.3	1. The volume of a room is 3,300 cubic feet. If 2.5 BTUs per cubic foot are required to cool this room, how many total BTUs are needed? a. 8,250 BTUs **2.** http://www.illustrativemathematics.org/illustrations/1439	• http://www.illustrativemathematics.org/illustrations/266 • http://www.illustrativemathematics.org/illustrations/521 • http://www.illustrativemathematics.org/illustrations/1567 • http://www.illustrativemathematics.org/illustrations/1565 • https://www.khanacademy.org/math/geometry/basic-geometry/volume_tutorial/e/surface-and-volume-density-word-problems
HSG-GMD.A.3	• 8.G.C.9 • HSG-GMD.A.1	1. What is the volume of a rectangular closet with width 5 feet, length 6 feet, and height 8 feet? a. 240 cubic feet **2.** http://www.illustrativemathematics.org/illustrations/527 3. http://www.illustrativemathematics.org/illustrations/514	• http://learnzillion.com/lessonsets/535-use-volume-formulas-for-cylinders-pyramids-cones-and-spheres-to-solve-problems

Real-World Preparation: The following questions will prepare students for some of the real-world components of this task:

- ***What is an air conditioner?*** An air conditioner is a machine used to lower the temperature in a room or building.
- ***What is the difference between a window unit and a central air conditioner?*** A window unit is usually installed in the window of a room and usually cools just one room. A central air conditioner is usually installed outside of a house and usually cools the entire house or building.

During the Task

Students may struggle with finding the prices of air conditioners. You may want to provide a list of possible websites to students. For example, the reference for the Instructional Task Exemplar Response is ***http://www.homedepot.com***.

After the Task

This task shows students how volume is useful in the real world. They are able to see how purchasing decisions can rely on volume.

As an extension activity, students could measure the classroom, calculate the BTUs needed, and decide on an air conditioner.

Student Instructional Task

Sara is in a rock band. Her band practices several nights a week in a building at the back of her parents' property. The building is 22 feet by 24 feet and has 8-foot ceilings.

The central air conditioner in the building broke last week. Sara's parents told her that since she is the only one who uses the building, Sara has to pay to put in a new air conditioner if she wants the building to be cool when her band practices. Sara has to decide if they should purchase a replacement central air conditioning unit or a window unit.

1) A BTU, or British Thermal Unit, is a measurement of heat used to determine the cooling capacity of an air conditioner. One BTU is approximately the amount of heat put off by one kitchen match, so running a 1,000 BTU air conditioner is like extinguishing 1,000 matches in one hour.

   A good estimate for the cooling requirement of a building with 8-foot ceilings is 2.5 BTUs per cubic foot. Based on this estimate, how many BTUs does Sara's air conditioner need to have? Explain your reasoning and show your work. Source: **http://www.ehow.com/info_10044382_recommended-btu-air-conditioner-per-cubic-foot.html**

2) For rooms that regularly contain more than two people, an air conditioner needs an extra 600 BTUs per person. Sara's band has a total of seven people. How many additional BTUs does she need to add to her required BTUs? Show your work.

3) Air conditioners are available with BTUs in increments of 1,000. Based on your calculations, what is the minimum number of BTUs her air conditioner needs to have? Justify your answer.

4) Central air conditioners are often measured using tons in increments of half a ton. One ton of refrigeration can remove 12,000 BTUs of heat in one hour. How many tons would Sara's unit need to be? Justify your answer.

5) Research the prices and operating costs of central air conditioners and window units online. Select a central air conditioner and a window unit for Sara. Create equations for the central unit and window unit you chose that would tell the total cost, C, for operating the unit after x years.

6) Which type of air conditioner, a replacement central air conditioner or a window unit, should Sara buy? Explain your reasoning using the information you gathered about the cost of the air conditioners.

Instructional Task Exemplar Response

Sara is in a rock band. Her band practices several nights a week in a building at the back of her parents' property. The building is 22 feet by 24 feet and has 8-foot ceilings.

The central air conditioner in the building broke last week. Sara's parents told her that since she is the only one who uses the building, Sara has to pay to put in a new air conditioner if she wants the building to be cool when her band practices. Sara has to decide if they should purchase a replacement central air conditioning unit or a window unit.

1) A BTU, or British Thermal Unit, is a measurement of heat used to determine the cooling capacity of an air conditioner. One BTU is approximately the amount of heat put off by one kitchen match, so running a 1,000 BTU air conditioner is like extinguishing 1,000 matches in one hour.

   A good estimate for the cooling requirement of a building with 8-foot ceilings is 2.5 BTUs per cubic foot. Based on this estimate, how many BTUs does Sara's air conditioner need to have? Explain your reasoning and show your work. Source: **http://www.ehow.com/info_10044382_recommended-btu-air-conditioner-per-cubic-foot.html**
   First, find the volume of the room, and then multiply by the number of BTUs per cubic foot.

   Volume = length X width X height

$$V = 22 \times 24 \times 8$$
$$V = 4224\ ft^3$$
$$4224\ ft^3 \times 2.5\ BTUs/ft^3$$
$$= 10{,}560\ BTUs$$

2) For rooms that regularly contain more than two people, an air conditioner needs an extra 600 BTUs per person. Sara's band has a total of seven people. How many additional BTUs does she need to add to her required BTUs? Show your work.

$$7\ people - 2\ people = 5\ people$$
$$= 5 \times 600 = 3000\ BTUs$$

3) Air conditioners are available with BTUs in increments of 1,000. Based on your calculations, what is the minimum number of BTUs her air conditioner needs to have? Justify your answer.

$$10{,}560\ BTUs + 3000\ BTUs$$
$$= 13{,}560\ BTUs$$

   Sara will need an air conditioner for $14{,}000\ BTUs$ *in order to meet the minimum of 13,560 BTUs.*

4) Central air conditioners are often measured using tons in increments of half a ton. One ton of refrigeration can remove 12,000 BTUs of heat in one hour. How many tons would Sara's unit need to be? Justify your answer.

$$14{,}000\ BTUs \div 12{,}000\ BTUs$$
$$\approx 1.167\ tons$$

   Sara will need $1.5\ tons$ *of refrigeration because 1 ton will not be enough.*

5) Research the prices and operating costs of central air conditioners and window units online. Select a central air conditioner and a window unit for Sara. Create equations for the central unit and window unit you chose that would tell the total cost, *C*, for operating the unit after *x* years.

Sample response:
Sara found a replacement central air conditioner for $1,799. It has an estimated yearly operating cost of $122.

Central air conditioner:

$$C = 1799 + 122x, x = \#years\ used\ and\ C = Cost$$

Sara found a window unit for $399. It has an estimated yearly operating cost of $206:

$$C = 399 + 206x, x = \#years\ used\ and\ C = Cost$$

Source for air conditioner prices: [**http://www.homedepot.com**](http://www.homedepot.com)

6) Which type of air conditioner, a replacement central air conditioner or a window unit, should Sara buy? Explain your reasoning using the information you gathered about the cost of the air conditioners.

This answer will vary based on the prices and equations in part 5. Sample response:

I decided to find out how long it would take for the cost to run the different types of air conditioners to be the same.

$$C = 1799 + 122x$$
$$C = 399 + 206x$$
$$1799 + 122x = 399 + 206x$$
$$1400 = 84x$$
$$16.67 = x$$

At 16.67 years the cost of the two units would be the same.

$$C = 399 + 206(16.67)$$
$$C = 399 + 3{,}434.02$$
$$C = 3{,}833.02$$

The cost would be $3,833.02.

Note: If the cost is calculated using $\frac{1400}{84}$ *instead of 16.67, the answer is $3,832.33.*

If Sara is planning on using the practice space less than 16.67 years, she should purchase the window unit. If Sara is planning on using the practice space more than 16.67 years, she should replace the central air conditioner.

Exploring Similar Triangles (IT)

Overview
Students will be given two triangles and use a sequence a transformations to prove that they are similar.

Standards
Understand similarity in terms of similarity transformations.

HSG-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prior to the Task
Standards Preparation: The material in the chart below illustrates the standards and sample tasks that are pre-requisites for student success with this task's standards.

Grade Level Standard	The Following Standards Will Prepare Them	Items to Check for Task Readiness	Sample Remediation Items
HSG-SRT.A.3	• 8.G.A.4 • HSG-SRT.A.2	1. Explain using words how to prove that a two-dimensional figure is similar to another. a. In order to prove that a two-dimensional figure is similar to another two-dimensional figure, you must be able to draw the second figure from the first using a sequence of rotations, reflections, translations, and/or dilations.	• http://www.illustrativemathematics.org/illustrations/603 • http://learnzillion.com/lessonsets/775-establish-the-aa-criterion-for-triangle-similarity • http://learnzillion.com/lessonsets/540-use-the-properties-of-similarity-transformations-to-establish-the-aa-criterion

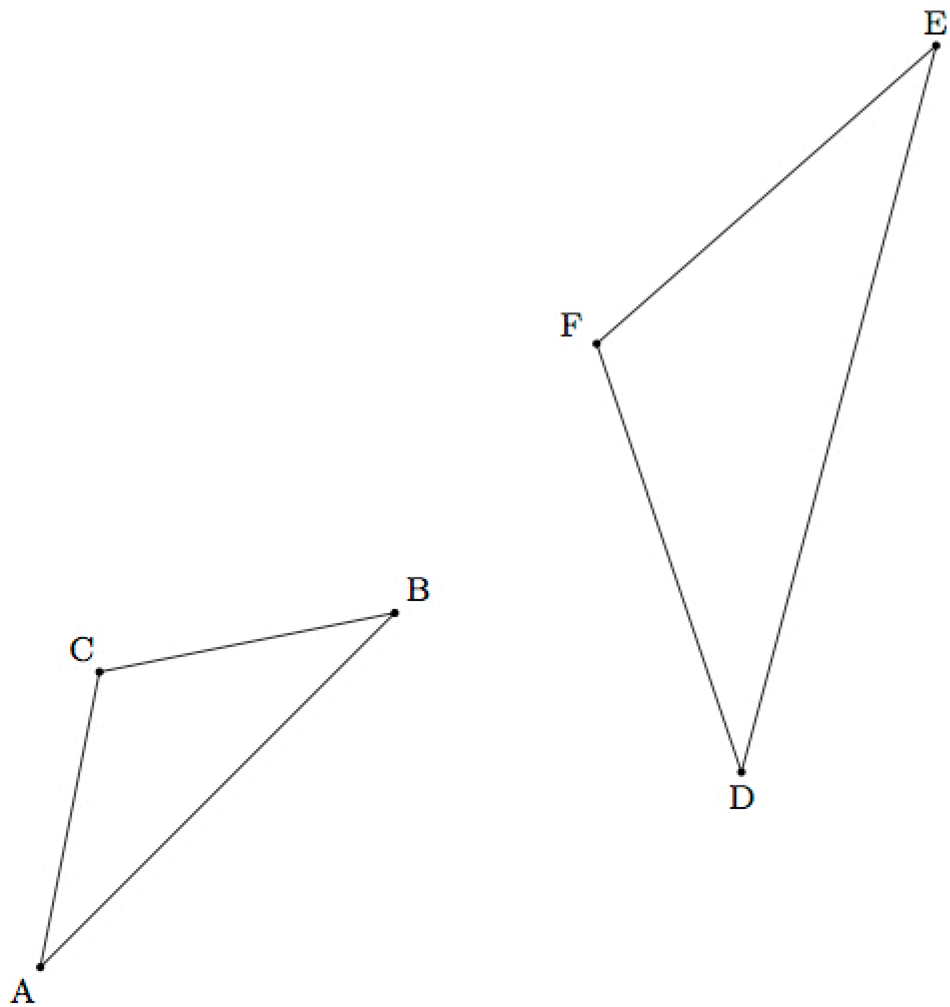
During the Task

- As with all problems where a sequence of similarity transformations is requested, there are many possibilities. Keeping to a translation followed by a rotation and dilation, as in the solution provided, we could instead translate vertex *B* to vertex *E* (or vertex *C* to vertex *F*). A rotation will then align angles *ABC* and *DEF* (or *BCA* and *EFD*) and then a dilation will finish showing the similarity.
- For students who are just beginning to experiment with these transformations, dynamic geometry software or application would be invaluable to build an intuition for the different steps in the construction. It would also provide students an opportunity to exhibit the similarity in different ways, for example using a sequence of reflections and then a dilation.

After the Task
The teacher can have students compare their solutions as there are many possible series of transformations that demonstrate the similarity.

Student Instructional Task

In the two triangles pictured below, $m\angle A = m\angle D$ and $m\angle B = m\angle E$.

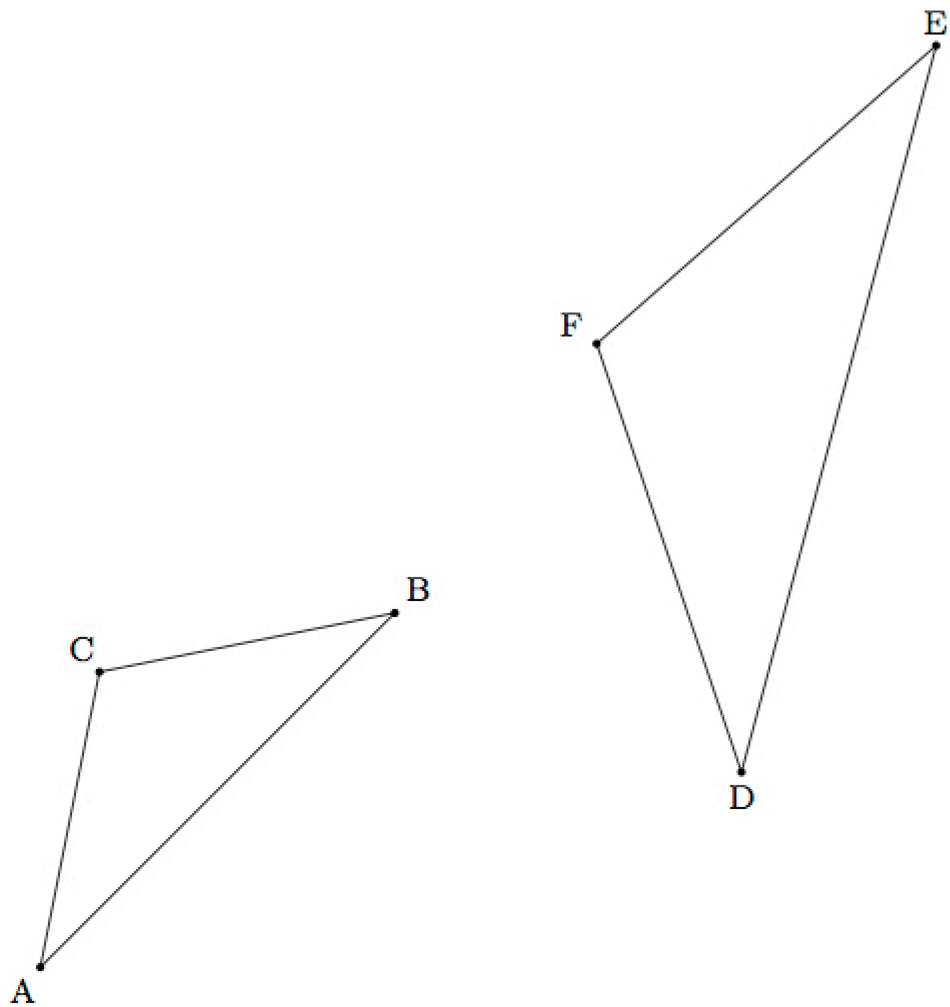


Using a sequence of translations, rotations, reflections, and/or dilations, show that ΔABC is similar to ΔDEF.

Task adapted from ***http://www.illustrativemathematics.org/illustrations/1422.***

Instructional Task Exemplar Response

In the two triangles pictured below, $m\angle A = m\angle D$ and $m\angle B = m\angle E$.



Using a sequence of translations, rotations, reflections, and/or dilations, show that ΔABC is similar to ΔDEF.

Task adapted from http://www.illustrativemathematics.org/illustrations/1422.

Sample Response:

There are many ways to complete this Instructional Task. This sample will use the following:

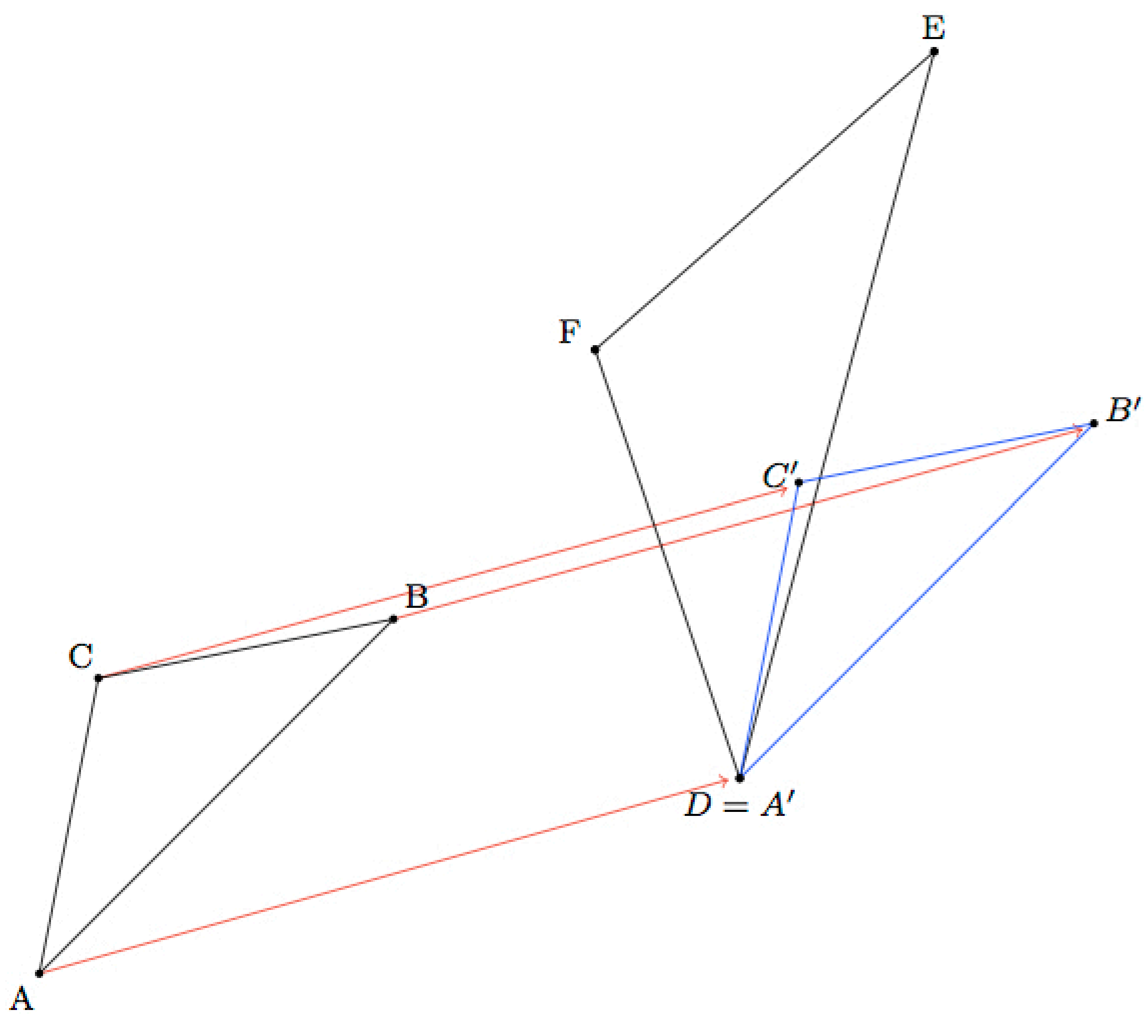
Step 1: a translation to map A to D

Step 2: a rotation to align two of the sides of the two triangles, and

Step 3: a dilation that completes the similarity transformation.

Step 1: Translation

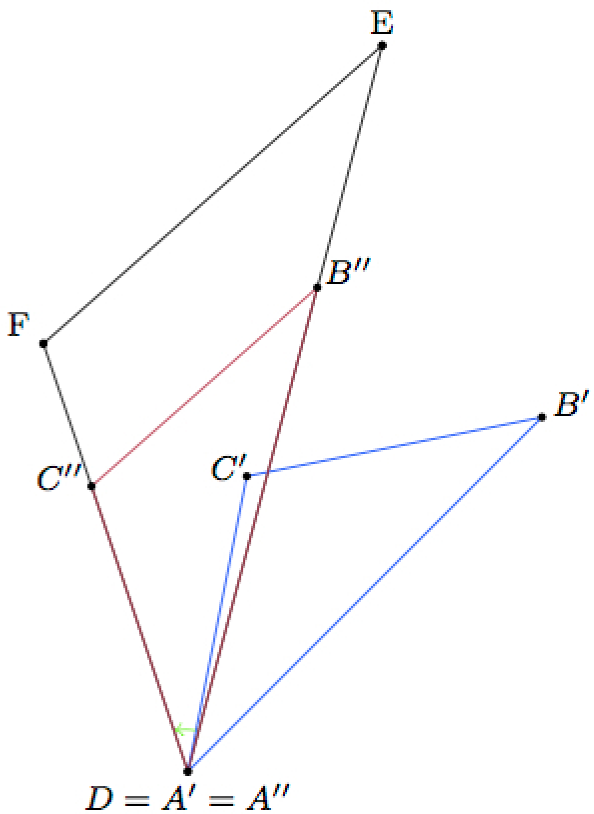
The translation taking *A* to *D* is pictured below, with the image of $\triangle ABC$ being denoted by $A'B'C'$.



Note that *A* = *D* because the translation is chosen precisely so as to map *A* to *D*.

Step 2: Rotation

Rotate △$A'B'C'$ by angle $C'A'F'$ as pictured below where the angle of rotation is marked in green:



Also pictured is the image $A''B''C''$ of △$A'B'C'$ under this rotation. Note that $\overrightarrow{A''C''} = \overrightarrow{DF}$. This is true by the choice of my angle of rotation. Note too that $\overrightarrow{A''B''} = \overrightarrow{DE}$. This is true because $m(\angle B''A''C'')=m(\angle BAC)$, since rigid motions preserve angle measures, and $m(\angle BAC) = m(\angle EDF)$ by hypothesis.

Step 3: Dilation

I have already moved A'' to D and so I chose D as the center of our dilation. I would like to move B'' to E and the dilation factor that will accomplish this is $\frac{|DE|}{|A''B''|}$. To check that C'' maps to F note that $m(\angle DEF) = m(\angle A''B''C'')$. Angles are preserved by dilations and so this means that $\overline{B''C''}$ must map to $\overline{EF}$ (if C'' mapped to a point different than F, then there would be two rays from E to points on $\overleftrightarrow{DF}$ making the same angle with $\overrightarrow{ED}$, which is not possible).