Louisiana Believes

Common Core State Standards for Mathematics

TEACHER SELF-LEARNING SERIES

Module 3

Rigor: The Third CCSSM Shift

DEPARTMENT of EDUCATION
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Time Frame: Approximately 2 hours as the time for watching all videos is about 1.5 hours. It is not necessary that the module be completed in one session.

Audience: Teachers, principals, and additional school faculty of all grade levels whose work requires understanding of the Common Core State Standards for Mathematics (CCSSM)

Module Description: This module examines Rigor, the third of the three shifts required for implementation of the CCSSM. This module is based on the premise that the information from Modules 1 and 2 is well understood.

Course Objectives: By the end of the module, the learner will be able to:

a. identify the components of rigor and give examples of each.
b. describe the relationship among the components.
c. determine if a standard is written to align with one of the three components of rigor.
d. state fluencies required by the CCSSM and those suggested in high school by PARCC.

Materials Needed to Complete Module: copy of the Common Core State Standards for Mathematics (www.corestandards.org/the-standards) for those who may need to determine the meaning of referenced standards codes, Internet access

Pre-Assessment: Those who can answer the questions below with confidence may want to skip this module.

1. State the three components of rigor and give an example to show what each means.
2. List key words found in the CCSSM that are indicators of rigor.
3. Why are high school fluencies not provided in the CCSSM?
4. Where can suggested fluencies be found for high school courses taught in Louisiana?
5. What are three strategies that might be used to promote fluency? Describe how each would promote development of fluency in students?

Understanding Rigor

Implementation of the Common Core State Standards for Mathematics (CCSSM) requires three instructional shifts. They are Focus, Coherence, and Rigor. A brief summary of each is provided below:

1. **Focus :** Ensuring that instruction focuses strongly where the Standards focus
2. **Coherence: Think** across grades and **link** to major topics within grades
3. **Rigor**: In major topics, pursue **conceptual understanding**, procedural skill and **fluency**, and **application** with equal intensity

This module examines the shift of Rigor which refers to a balance of the components of conceptual understanding, procedural skill and fluency, and application. Here Rigor does not mean giving students really hard problems. This shift is about the depth of what is expected in the standards and what one should expect to see happening in the classroom and in curricular materials. Watch the one minute video, *Shifts in Math Practice: The Balance Between Skills and Understanding*, posted at http://www.youtube.com/watch?v=5dUQtIXoptY in which William McCallum gives a summary of these components.

Examples of Rigor in the Language of the Standards

There are key words to look for in the standards to know when a standard should be assessed for conceptual understanding, procedural skill and fluency, or through an application type problem. Key words have been bolded in the examples below.

Conceptual Understanding

3.NF.A.1 **Understand** a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b*.

HSG-SRT.C.7 **Explain** and use the relationship between the sine and cosine of complementary angles.

Other words that indicate need for conceptual understanding are **interpret, recognize, describe**, etc.

Procedural Skill and Fluency

5.NBT.B.5 **Fluently** multiply multi-digit whole numbers using the standard algorithm.

Application

7.NS.A.3 **Solve real-world and mathematical problems** involving *the* four operations with rational numbers.

HSA-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options **in a modeling context.** *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*

Conceptual Understanding

To address conceptual understanding, teachers must teach more than "how to get the answer" and instead support students' ability to access concepts from a number of perspectives so that students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of core math concepts by applying them to new situations as well as writing and speaking about their understanding.

Fluency

Bill McCallum and Jason Zimba, lead writers for the CCSSM, provide important information fluency in the video (2 minutes), *Mathematics Fluency: A Balanced Approach,* posted at http://www.youtube.com/watch?v=ZFUAV00bTwA. Please watch this video.

The use of the term *fluent* in a particular standard means "fast and accurate." It might be helpful to think of fluency as meaning the same thing as when someone is fluent in a foreign language. When one is fluent in a language, he/she has a good flow when speaking, reading and listening. One who is fluent does not halt, stumble, or reverse oneself. The word "fluency" is used judiciously in the standards to mark the endpoint of learning progressions that begin with solid underpinnings and with the understanding that students have passed through stages of growing maturity. Some fluency expectations are meant to be mental and others completed with pencil and paper. However, in each instance there should be no hesitation in getting the answer with accuracy.

It is important for teachers to structure class time and/or homework time for students to become more fluent, through repetition and practice, in core functions so that they are more able to understand and manipulate more complex concepts.

Grade	Required Fluency
K	Add and subtract within 5
1	Add and subtract within 10
2	Add/subtract within 20 Add/subtract within 100 (pencil and paper)
3	Multiply/divide within 100 Add/subtract within 1,000
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division Multi-digit decimal operations
7	Solve px+q=r, p(x+q)=r
8	Solve simple 2x2 systems by inspection

While the high school standards for mathematics do not list high school fluencies, the PARCC Model Content Frameworks for Mathematics Grades 3-11 suggests fluencies for Algebra I, Geometry and Algebra II as noted below:

Algebra I

Analytic Geometry Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

HSA-APR.A.1 Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

HSA-SSE.A.1b Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.

Geometry

HSG-SRT.B.5 Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks.

HSG-GPE.B.4, 5, 7 Fluency with the use of coordinates to establish geometric results, calculate length and angle, and use geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.

HSG-CO.D.12 Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.

Algebra II

HSA-APR.D.6 This standard sets an expectation that students will divide polynomials with remainder by inspection in simple cases. For example, one can view the rational expression

$$\frac{x+4}{x+3} \text{ as } \frac{x+4}{x+3}=\frac{(x+3)+1}{x+3}=1+\frac{1}{x+3}.$$

HSA-SSE.A.2 The ability to see structure in expressions and to use this structure to rewrite expressions is a key skill in everything from advanced factoring (e.g., grouping) to summing series to the rewriting of rational expressions to examine the end behavior of the corresponding rational function.

HSF-IF.A.3 Fluency in translating between recursive definitions and closed forms is helpful when dealing with many problems involving sequences and series, with applications ranging from fitting functions to tables to problems in finance.

Observing Conceptual Understanding, Fluency, and Application in Classroom Instruction

Part I – Deep understanding, focus, and fluency

Watch the video, *Sorting and Classifying Equations Overview* (10 minutes), posted at http://tinyurl.com/aevmuma. You may also want to view the follow-up video lesson, *Sorting and Classifying Equations Class Discussion* (8 minutes), posted at http://tinyurl.com/anqqmyv. Note: A link to the discussion video can also be found by scrolling down the page after viewing the first video.

Questions to Consider

- Why is it helpful to achieve team consensus before class consensus?
- What role do equity sticks and index cards play in facilitating a productive discussion?
- How does discussing multiple strategies affect student understanding?
- Does this lesson focus on major work of Grade 8? Explain.
- How does this lesson help maintain fluencies expected of students in Grade 7?

Part II – Establishing Fluency in Daily Practice

Students need opportunities to practice speed and accuracy and to reinforce their use in conceptual development. Counting exercises, choral and whiteboard responses, and sprints are some strategies than can be used to address fluency.

Sprints are a variation of timed tests which have a focus on individual improvement and are designed to establish and enhance fluency by developing students' number sense through a focus on patterns. They were developed by Dr. Yoram Sagher, professor of mathematics at Florida Atlantic University and an expert in the field of teaching elementary mathematics. While the focus of sprints has been on elementary mathematics, middle and high school teachers can find them useful in establishing fluency in their students as well.

Watch the six minute video, *Rigor Breakdown-Sprints-Fluency in Action* (http://tinyurl.com/bhz3q7p), in which LSU professor, Dr. Scott Baldridge, leads a session on sprints for educators in New York as they engage in professional development on implementation of the CCSSM. The video includes footage of the use of a sprint in a classroom as well as discussion among teachers describing experiences in using sprints.

Bill Davidson, an author of sprints, has posted a list of things to consider should teachers want to develop their own sprints at http://tinyurl.com/byf9dce.

Other videos that demonstrate the use of fluency strategies are provided below. While the grades in which the strategies are used are primarily for elementary grades, most can be adapted for middle and high school classrooms.

EngageNY

- Counting Exercises for K-1 (7.5 minutes) - http://tinyurl.com/adex55s
- Skip Counting by Fractions (3 minutes) - http://tinyurl.com/bjzno57
- Whiteboard Exchange (4 minutes) - http://tinyurl.com/aeae8xj

The Teaching Channel

- Making Math Fun with Place Value Games (Grade 2) (6.5 minutes) - http://tinyurl.com/bbvz2ke

Part II – Application

Two videos using a real-life problem as the focus of instruction are provided listed below.

Inside Mathematics - Grade 7 – Percents – **Clip 2**
http://www.insidemathematics.org/index.php/7th-grade-whats-the-savings
(20 minutes). Clip 1 (pre-lesson discussion) and clip 3 (post-lesson debrief) may be of interest as well.

The Teaching Channel – High School - Statistical Analysis to Rank Baseball Players
(16.5 minutes) - https://www.teachingchannel.org/videos/statistical-analysis-lesson?fd=1

Problem Types Addressing Fluency, Conceptual Understanding, and Application

Student Achievement Partners has collated a set of problems based on the Item Development Invitation to Negotiate (ITN) released by PARCC. They are not intended as literal previews of future PARCC assessments. There is a list of questions for readers to consider on page 1 of the document. The problem set is posted at http://tinyurl.com/akq2vf3.

PARCC item and task prototypes are posted at http://tinyurl.com/b36xmpg. Grade 3 information will be shown when the screen opens. Scroll to the bottom of the page to get to the sample tasks for that grade, or click on links on the left side bar to access items for other grades. Items are designed to be reflective of the CCSSM shifts.

Conclusion

This module presented information on Rigor, the third of three instructional shifts required for implementation of the Common Core State Standards in Mathematics. Rigor is the idea of maintaining balance among developing deep conceptual understanding, procedural skill and fluency, and application of mathematical concepts in real-life situations.

You should now be able to answer the Pre-Assessment questions with ease.