

# Proposed Updates to the K-12 Louisiana Student Standards for Mathematics

This draft of the Louisiana Student Standards for Mathematics includes the recommendations of the Math content work group. The Proposed Standard column includes any additional language added as underlined text, and the language of the original standard that was removed is identified with a strikethrough.

## Draft Standards for K-5

### Kindergarten

#### Grade Level Expected Foundational Skills

By the end of Kindergarten, mathematically proficient students can reliably use the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade K, but consists of the foundational skills required by the standards.

1. Count to 100 by tens and ones.
2. Read and write numbers 0-20.
3. Count out 0-20 objects and count groups of objects up to 20.
4. Count to answer “How many?”
5. Subitize (i.e., instantly recognize without needing to count individually) groups of images or objects to recognize the quantity.
6. Compare two numbers between 1 and 20 to identify which is greater or less.
7. Fluently add and subtract within five.

## Numeracy and Operational Fluency

Current Standard	Proposed Standard
<b>K.CC.A. K.NOF.A: Know number names and the count sequence.</b>	
<p>K.CC.A.1: Count to 100 by ones and by tens.</p> <p>K.CC.A.2: Count forward beginning from a given number within the known sequence (instead of having to begin at 1).</p>	<p><del>K.CC.A.1, K.CC.A.2</del> <b>K.NOF.A.1:</b> Count to 100 by ones and tens (i.e., multiples of 10s - 30, 40, 50, etc), beginning with any given number.</p> <p><del>Count forward beginning from a given number within the known sequence (instead of having to begin at 1).</del></p> <p><u>a. Count forward to 10 and backward from 10 by ones.</u></p> <p><u>b. Count forward to 20 and backward from 20 by ones.</u></p>
<p>K.CC.A.3: Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).</p>	<p><del>K.CC.A.3</del> <b>K.NOF.A.2:</b> <u>Read, write, and represent whole numbers from 0 to 20.</u> Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects), objects, or pictures.</p>
<b>K.CC.B K.NOF.B.: Count to tell the number of objects.</b>	
<p>K.CC.B.4: Understand the relationship between numbers and quantities; connect counting to cardinality.</p> <p>a. When counting objects in standard order, say the number names as they relate to each object in the group, demonstrating one-to-one correspondence.</p> <p>b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.</p> <p>c. Understand that each successive number name refers to a quantity that is one larger.</p>	<p><b>K.CC.B.4 K.NOF.B.3:</b> Understand the relationship between numbers and quantities; connect counting to cardinality, <u>with cardinality referring to the number that connects the final count number to its quantity of an entire set.</u></p> <p>a. When counting objects in standard order, say the number names as they relate to each object in the group, demonstrating one-to-one correspondence.</p> <p>b. Understand that the last number name said tells the number of objects counted (<u>cardinality</u>). The number of objects is the same regardless of their arrangement or the order in which they were counted.</p> <p>c. Understand that each successive number name refers to a quantity that is one larger, <u>which may include the use of objects or</u></p>

Current Standard	Proposed Standard
	<u>visual representations.</u>
<p>K.CC.B.5: Count to answer “How many?” questions.</p> <p>a. Count objects up to 20, arranged in a line, a rectangular array, or a circle.</p> <p>b. Count objects up to 10 in a scattered configuration.</p> <p>c. When given a number from 1-20, count out that many objects.</p>	<p><del>K.CC.B.5</del> <del>K.NOF.B.4: Count to a</del> Answer “How many?” questions <u>by subitizing (perceptually and conceptually)* and counting.</u></p> <p>a. <del>Count objects up to 20, arranged in a line, a rectangular array, or a circle.</del> <u>Subitize within 5.</u></p> <p>b. <del>Count objects up to 10 in a scattered configuration</del> <u>Subitize or count objects up to 20, arranged in a line, a rectangular array, or a circle.</u></p> <p>c. <del>When given a number from 1-20, count out that many objects</del> <u>Subitize or count objects up to 10 in a scattered configuration.</u></p> <p>d. <del>When given a number from 1-20, count out that many objects.</del></p>
<p><b>K.CC.C K.NOF.C: Compare numbers.</b></p>	
<p>K.CC.C.6: Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.</p> <p>*1</p> <p>*1 Include groups with up to ten objects.</p>	<p><del>K.CC.C.6</del> <del>K.NOF.C.5: Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, Compare sets of objects up to at least 20 in each set using comparative language,</del> e.g., by using matching and counting strategies.</p> <p>#1</p> <p>#1 Include groups with up to ten objects.</p>
<p>K.CC.C.7: Compare two numbers between 1 and 10 presented as written numerals.</p>	<p><del>K.CC.C.7</del> <del>K.NOF.C.6: Compare two numbers between 1 and 10 Use comparative language to describe numbers up to 20 presented as written numerals.</del></p>
<p><b>K.OA.A K.NOF.D: Add and subtract within 5.</b></p>	

Current Standard	Proposed Standard
K.OA.A.5: Fluently add and subtract within 5.	<p><b>K.OA.A.5 K.NOF.D.7:</b> Fluently add and subtract within 5, <u>varying placement of the equal sign while also ensuring accurate reading of the equation from left to right</u>.</p> <ul style="list-style-type: none"> <li>• <u><math>5 = 2 + 3</math> (five equals two plus three)</u></li> <li>• <u><math>2 + 2 = 4</math> (two plus two is equal to four)</u></li> <li>• <u><math>3 = 3 - 0</math> (three is equal to three minus zero)</u></li> <li>• <u><math>4 - 1 = 3</math> (four minus one equals three)</u></li> </ul>
<b>K.NBT.A K.NOF.E:</b> Work with numbers 11-19 to gain foundations for place value.	
<p>K.NBT.A.1: Gain an understanding of place value.</p> <p>a. Understand that the numbers 11–19 are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</p> <p>b. Compose and decompose numbers 11 to 19 using place value (e.g., by using objects or drawings).</p> <p>c. Record each composition or decomposition using a drawing or equation (e.g., 18 is one ten and eight ones, <math>18 = 1</math> ten + 8 ones, <math>18 = 10 + 8</math>).</p>	<p><b>K.NBT.A.1 K.NOF.E.8:</b> Gain an understanding of place value.</p> <p>a. Understand that the numbers 11–19 are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</p> <p>b. Compose and decompose numbers 11 to 19 using place value <u>understanding</u> (e.g., by using objects, <u>or</u> drawings, <u>or</u> verbal responses identifying tens and ones).</p> <p>c. Record each composition or decomposition using a drawing or equation.</p> <ul style="list-style-type: none"> <li>• <u>For example: (e.g., 18 is one ten and eight ones, <math>18 = 1</math> ten + 8 ones, <math>18 = 10 + 8</math>)</u></li> </ul>

## Algebraic Reasoning

Current Standard	Proposed Standard
<b>K.OA.A K.AR.A:</b> Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	
K.OA.A.1: Represent addition and subtraction with objects, fingers, mental images, drawings *2, sounds (e.g., claps), acting	K.OA.A.1 K.AR.A.1: Represent addition and subtraction <u>of two whole numbers from 0 to 10</u> with objects, fingers, mental images, drawings <sup>1</sup> *2,

Current Standard	Proposed Standard
<p>out situations, verbal explanations, expressions, or equations.</p> <p>*2 Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)</p>	<p>sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.</p> <p>*2 <sup>1</sup>Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the standards.)</p>
<p>K.OA.A.2: Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.</p>	<p><u>K.OA.A.2 K.AR.A.2: Solve addition and subtraction word problems, and Add and subtract within 10, e.g. by using objects or drawings to represent the problem.</u></p> <p>a. <u>Solve addition and subtraction word problems real-world mathematical tasks and explain the strategies used with spoken words, models, and/or equations.</u></p>
<p>K.OA.A.3: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., <math>5 = 2 + 3</math> and <math>5 = 4 + 1</math>).</p>	<p><u>K.OA.A.3 K.AR.A.3: Decompose numbers less than or equal to 10 (identified as the whole) into pairs- parts in more than one way to demonstrate the part-whole relationship, e.g., by using objects or drawings, and</u></p> <p>a. <u>Record each decomposition by with a drawing or equation.</u></p> <ul style="list-style-type: none"> <li>• <u>For example: e.g., <math>5 = 2 + 3</math> and <math>5 = 2 + 1 + 2</math>. <del>4 + 1</del></u></li> </ul>
<p>K.OA.A.4: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation</p>	<p><u>K.OA.A.4 K.AR.A.4: For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using spoken words, objects or drawings, and record the answer with a drawing or equation ; models, and/or equations.</u></p>

## Geometric Reasoning and Logic

Current Standard	Proposed Standard
<b>K.G.A <u>K.GL.A</u>: Identify and describe shapes.</b>	
K.G.A.1: Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as <i>above, below, beside, in front of, behind, and next to</i> .	<b>K.G.A.1 <u>K.GL.A.1</u></b> : Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as <i>above, below, beside, in front of, behind, and next to</i> .
K.G.A.2: Correctly name shapes regardless of their orientations or overall size.	<b>K.G.A.2 <u>K.GL.A.2</u></b> : Correctly name shapes regardless of their orientations or overall size. <ul style="list-style-type: none"><li>• <u>Identify examples and nonexamples of those shapes</u></li></ul>
K.G.A.3: Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).	<b>K.G.A.3 <u>K.GL.A.3</u></b> : Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”). <ul style="list-style-type: none"><li>• <u>Identify faces of three-dimensional shapes as two-dimensional geometric figures.</u></li></ul>
<b>K.G.B <u>K.GL.B</u>: Analyze, compare, create, and compose shapes.</b>	
K.G.B.4: Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).	<b>K.G.B.4 <u>K.GL.B.4</u></b> : Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).
K.G.B.5: Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	<b>K.G.B.5 <u>K.GL.B.5</u></b> : Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
K.G.B.6: Compose simple shapes to form larger shapes. <i>For example, "Can you join these two triangles with full sides touching to make a rectangle?"</i>	<b>K.G.B.6 <u>K.GL.B.6</u></b> : Compose simple shapes to form larger shapes. <ul style="list-style-type: none"><li>• For example, "Can you join these two triangles with full sides touching to make a rectangle?"</li></ul>

## Data Analysis & Measurement

Current Standard	Proposed Standard
<b>K.MD.A K.DM: Describe and compare measurable attributes.</b>	
K.MD.A.1: Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	K.MD.A.1 <del>K.DM.A.1: Describe measurable attributes of a set of objects, such as length or weight. Describe several measurable attributes of a single object. Given a single object, describe several measurable attributes.</del>
K.MD.A.2: Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. <i>For example, directly compare the heights of two children and describe one child as taller/shorter.</i>	K.MD.A.2 <del>K.DM.A.2: Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference (more of, less of, etc.).</del> <ul style="list-style-type: none"><li>For example, directly compare the heights of two children and describe one child as taller/shorter.</li></ul>
<b>K.MD.B K.DM.B: Classify objects and count the number of objects in categories.</b>	
K.MD.B.3: Classify objects into given categories based on their attributes; count the numbers of objects in each category and sort the categories by count. *3  *3 Limit category counts to be less than or equal to 10.	K.MD.B.3 <del>K.DM.B.3: Classify objects into two given categories based on their attributes (limit category counts to less than or equal to 20):</del> <ul style="list-style-type: none"><li>count and sort the numbers of objects in each category and sort the categories by count. *3</li></ul> <del>*3 Limit category counts to be less than or equal to 10.)</del>
<b>K.MD.C K.DM.C: Work with money.</b>	
K.MD.C.4: Recognize pennies, nickels, dimes, and quarters by name and value (e.g., This is a nickel and it is worth 5 cents.).	K.MD.C.4 <del>K.DM.C.4: Identify Recognize pennies, nickels, dimes, and quarters by name and value:</del> <ul style="list-style-type: none"><li>(e.g., This is a nickel and it is worth 25 cents.).</li><li><u>Identify fair-trade values within a dime:</u><ul style="list-style-type: none"><li>10 pennies = 1 dime</li><li>2 nickels = 1 dime</li></ul></li></ul>

Current Standard	Proposed Standard
	<ul style="list-style-type: none"> <li>○ <u>5 pennies = 1 nickel</u></li> </ul>

## Grade 1

### Grade Level Expected Foundational Skills

By the end of grade 1, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 1, but is comprised of the foundational skills required by the standards.

1. Fluently add and subtract within 10.
2. Identify equivalent sums or differences of one-digit numbers. Ex.  $6 - 2 = 2 + 2$
3. Determine the missing number in an equation with numbers less than or equal to 10. Ex.  $8 + ? = 10$ ;  $5 = \square - 3$ .
4. Given a two-digit number, mentally find 10 more or 10 less.
5. Count to 120 by 5's.
6. Read and write numbers through 120.
7. Compare numbers up to 100, identifying whether one number is greater than, less than, or equal to another.

### Numeracy & Operational Fluency

Current Standard	Proposed Standard
<b>1.OA.C.1.NOF.A: Add and subtract within 20.</b>	
1.OA.C.5: Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	<b>1.OA.C.5 1.NOF.A.1:</b> Relate counting to addition and subtraction. <ul style="list-style-type: none"> <li>● <u>For example: (e.g., by counting on 2 to add 2).</u></li> </ul>

Current Standard	Proposed Standard
<p>1.OA.C.6: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use mental strategies such as counting on; making ten (e.g., <math>8 + 6 = 8 + 2 + 4 = 10 + 4 = 14</math>); decomposing a number leading to a ten (e.g., <math>13 - 4 = 13 - 3 - 1 = 10 - 1 = 9</math>); using the relationship between addition and subtraction (e.g., knowing that <math>8 + 4 = 12</math>, one knows <math>12 - 8 = 4</math>); and creating equivalent but easier or known sums (e.g., adding <math>6 + 7</math> by creating the known equivalent <math>6 + 6 + 1 = 12 + 1 = 13</math>).</p>	<p><del>1.OA.C.6</del> <u>1.NOF.A.2</u>: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.</p> <p><u>a.</u> Use mental strategies such as:</p> <ul style="list-style-type: none"> <li>• counting on;</li> <li>• making ten (e.g., <math>8 + 6 = 8 + 2 + 4 = 10 + 4 = 14</math>);</li> <li>• decomposing a number leading to a ten (e.g., <math>13 - 4 = 13 - 3 - 1 = 10 - 1 = 9</math>);</li> <li>• using the <u>part-whole</u> relationship between addition and subtraction (e.g., knowing that <math>8 + 4 = 12</math>, one knows <math>12 - 8 = 4</math>); and</li> <li>• creating equivalent but easier or known sums (e.g., adding <math>6 + 7</math> by creating the known equivalent <math>6 + 6 + 1 = 12 + 1 = 13</math>).</li> </ul>
<p>1.NBT.C.5: Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</p>	<p><del>1.NBT.C.5</del> <u>1.NOF.A.3</u>: Given a two-digit number, mentally find <u>1 more</u>, <u>10 more</u>, <u>one less</u>, or <u>10 less</u> than the number, without having to count;</p> <p><u>a.</u> <u>explain</u> <u>justify</u> the reasoning used <u>when computing mentally with</u> <u>an oral and/or written explanation</u>.</p>
<p><b><u>1.NBT.A-1.NOF.B:</u></b> Extending the counting sequence.</p>	
<p>1.NBT.A.1: Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.</p>	<p><del>1.NBT.A.1</del> <u>1.NOF.B.4</u>: Count to 120, starting at any number less than 120.</p> <p><u>a.</u> <u>Count forward and backward to 120 by ones, fives, and tens.</u></p> <p><u>b.</u> <u>In this range, read, and write, and identify numerals and number names written in words and represent a number of objects with a written numeral.</u></p>
<p><b><u>1.NBT.B 1.NOF.C:</u></b> Understand place value.</p>	

Current Standard	Proposed Standard
<p>1.NBT.B.2: Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:</p> <p>a. 10 can be thought of as a bundle of ten ones — called a “ten.”</p> <p>b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.</p> <p>c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</p>	<p>1.NBT.B.2-1.NOF.C.5: Understand that the two digits of a two-digit number represent amounts of tens and ones.</p> <p>a. <u>Read, write, and represent two-digit numbers up to 99 using base-ten numerals/standard form, word form, and unit form (i.e., 42 and 4 tens 2 ones).</u> Understand the following as special cases:</p> <ul style="list-style-type: none"> <li>• 10 can be thought of as a bundle of ten ones — called a “ten.”</li> <li>• The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.</li> <li>• The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</li> </ul>

## Algebraic Reasoning

Current Standard	Proposed Standard
<p><b>1.OA.A 1.AR.A: Represent and solve problems involving addition and subtraction.</b></p> <p>1.OA.A.1: Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem). *2</p> <p>*2 See Glossary, Table 1.  <a href="https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn=86bb8a1f_0">https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn=86bb8a1f_0</a></p> <p>Page 60 - Common addition and subtraction situations</p>	<p><b>1.OA.A.1 1.AR.A.1: Use addition and subtraction within 20 to solve real-world mathematical tasks word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.</b></p> <p>a. <u>Represent these situations using models and equations, elevating the concrete-representational- abstract instructional framework.</u><sup>2</sup> (e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem). *2</p> <p>*2 See <a href="#">Glossary, Table 1.</a></p> <p>Page 60 - Common addition and subtraction situations</p>

Current Standard	Proposed Standard
<p>1.OA.A.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.</p>	<p>1.OA.A.2 1.AR.A.2: Solve <u>real-world mathematical tasks</u> <del>word</del> problems that call for addition of three whole numbers whose sum is less than or equal to 20.</p> <p>a. <u>Represent these situations using models and equations, elevating the concrete-representational- abstract instructional framework</u>, e.g., by using objects, drawings, and equations with a symbol for the unknown to represent the problem.</p>
<p><b>1.OA.B-1.AR.B: Understand and apply properties of operations and the relationship between addition and subtraction.</b></p>	
<p>1.OA.B.3: Apply properties of operations to add and subtract. *3  <i>Examples: If <math>8 + 3 = 11</math> is known, then <math>3 + 8 = 11</math> is also known.</i>  <i>(Commutative property of addition.) To add <math>2 + 6 + 4</math>, the second two numbers can be added to make a ten, so <math>2 + 6 + 4 = 2 + 10 = 12</math>. (Associative property of addition.)</i></p> <p>*3 Students need not use formal terms for these properties.</p>	<p>1.OA.B.3 1.AR.B.3: Apply properties of operations to add and subtract. *3</p> <ul style="list-style-type: none"> <li>• <u>For example - commutative property of addition:</u> Examples: If <math>8 + 3 = 11</math> is known, then <math>3 + 8 = 11</math> is also known. (Commutative property of addition)</li> <li>• <u>For example - associate property of addition:</u> To add <math>2 + 6 + 4</math>, the second two numbers can be added to make a ten, so <math>2 + 6 + 4 = 2 + 10 = 12</math>. (Associative property of addition)</li> </ul> <p>*3 Students need not use formal terms for these properties. <u>Teachers need to understand and use precise language.</u></p>
<p>1.OA.B.4: Understand subtraction as an unknown-addend problem. <i>For example, subtract <math>10 - 8</math> by finding the number that makes 10 when added to 8.</i></p>	<p>1.OA.B.4 1.AR.B.4: Understand subtraction as an unknown-addend problem.</p> <ul style="list-style-type: none"> <li>• <u>For example, subtract <math>10 - 8</math> by finding the number that makes 10 when added to 8.</u></li> </ul>
<p><b>1.OA.D 1.AR.C: Work with addition and subtraction equations.</b></p>	

Current Standard	Proposed Standard
<p>1.OA.D.7: Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. <i>For example, which of the following equations are true and which are false? <math>6 = 6</math>, <math>7 = 8 - 1</math>, <math>5 + 2 = 2 + 5</math>, <math>4 + 1 = 5 + 2</math></i></p>	<p>1.OA.D.7 <del>1.ARC.5: Understand Interpret</del> the meaning of the equal sign. a. <u>Understand that the equal sign represents a relationship where expressions on each side of the equal sign represent the same value(s).</u> b. Determine if equations involving addition and subtraction are true or false. <del>For example, which of the following equations are true and which are false? <math>6 = 6</math>, <math>7 = 8 - 1</math>; <math>5 + 2 = 2 + 5</math>; <math>4 + 1 = 5 + 2</math></del></p>
<p>1.OA.D.8: Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations <math>8 + ? = 11</math>, <math>5 = \square - 3</math>, <math>6 + 6 = \square</math>.</p>	<p>1.OA.D.8 <del>1.ARC.6: Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.</del> <del>For example, determine the unknown number that makes the equation true in each of the equations <math>8 + ? = 11</math>, <math>5 = \square - 3</math>, <math>6 + 6 = \square</math>.</del></p>
<p><b><u>1.AR.D: Compare numbers.</u></b></p>	
<p>1.NBT.B.3: Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, and <math>&lt;</math>.</p>	<p>1.NBT.B.3 <del>1.AR.D.7: Compare two, two-digit numbers based on values meanings of the tens and ones digits, recording the results of comparisons with comparative language or the symbols <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> and the words greater than, equal to, and less than.</del></p>
<p><b><u>1.NBT.C-1.AR.E: Use place value understanding and properties of operations to add and subtract.</u></b></p>	

Current Standard	Proposed Standard
<p>1.NBT.C.4: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10.</p> <p>a. Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a number sentence; justify the reasoning used with a written explanation.</p> <p>b. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.</p>	<p>1.NBT.C.4 1.AR.E.8: Add <del>within 100 up to 99</del>, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10.</p> <p>a. Use concrete models or drawings and strategies based on place value, properties of operations, and/or the <u>part-whole</u> relationship between addition and subtraction; relate the strategy to a number sentence; <del>justify the reasoning used with a written explanation</del>.</p> <p>b. Understand that in adding two-digit numbers, <del>one adds</del> tens and tens, ones and ones; and sometimes it is necessary to compose a ten.</p> <p>c. <u>Justify the reasoning of methods for addition used with an oral and/or written explanation.</u></p>
<p>1.NBT.C.6: Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>	<p>1.NBT.C.6 1.AR.E.9: Subtract <del>two-digit numbers limited to multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences)</del>, using concrete models or drawings and strategies based on place value, properties of operations, and/or the <u>part-whole</u> relationship between addition and subtraction; <del>relate the strategy to a written method and explain the reasoning used</del>.</p> <p>a. <u>Justify the reasoning used when choosing the model or strategy to compute with an oral and/or written explanation.</u></p>

## Geometric Reasoning and Logic

Current Standard	Proposed Standard
<p><b>1.G.A 1.GL.A:</b> Reason with shapes and their attributes.</p>	
<p>1.G.A.1: Distinguish between defining attributes (e.g., triangles</p>	<p>1.G.A.1 1.GL.A.1: Distinguish between defining attributes (e.g., triangles are</p>

Current Standard	Proposed Standard
<p>are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes that possess defining attributes.</p>	<p>closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size);</p> <ul style="list-style-type: none"> <li>• Build and draw shapes that possess defining attributes; <u>and</u></li> <li>• <u>Verbally describe why a shape belongs to a given category.</u></li> </ul>
<p>1.G.A.2: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) and three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. *4</p> <p>*4 Students do not need to learn formal names such as “right rectangular prism.”</p>	<p>1.G.A.2 <u>1.GL.A.2</u>: Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) and three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape, <u>building understanding of the part-whole relationships.</u><sup>4</sup> *4</p> <p><del>*4 Students do not need to learn formal names such as “right rectangular prism.”</del></p>
<p>1.G.A.3: Partition circles and rectangles into two and four equal shares, describe the shares using the words <i>halves</i>, <i>fourths</i>, and <i>quarters</i>, and use the phrases <i>half of</i>, <i>fourth of</i>, and <i>quarter of</i>. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.</p>	<p>1.G.A.3 <u>1.GL.A.3</u>: Partition circles and rectangles into two and four equal shares, describe the shares using the words <i>halves</i>, <i>fourths</i>, and <i>quarters</i>, and use the phrases <i>half of</i>, <i>fourth of</i>, and <i>quarter of</i>. Describe the whole as two of, or four of the shares.</p> <ol style="list-style-type: none"> <li>Understand for these examples that decomposing into more equal shares creates smaller shares.</li> <li><u>Identify examples and non-examples of halves and fourths.</u></li> </ol>

## Data Analysis & Measurement

Current Standard	Proposed Standard
<b><u>1.MD.A 1.DM.A: Measure lengths indirectly and by iterating length units.</u></b>	
1.MD.A.1: Order three objects by length; compare the lengths of two objects indirectly by using a third object.	<p><b><u>1.MD.A.1 1.DM.A.1: Apply knowledge of “longer than” and “shorter than.”</u></b></p> <p>a. Order three objects by length;</p> <p>b. Compare the lengths of two objects indirectly by using a third object.</p>
1.MD.A.2: Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. <i>Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.</i>	<p><b><u>1.MD.A.2 1.DM.A.2: Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end.</u></b></p> <p>a. Understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.</p>
<b><u>1.MD.B 1.DM.B: Tell and write time.</u></b>	
1.MD.B.3: Tell and write time in hours and half-hours using analog and digital clocks.	<b><u>1.MD.B.3 1.DM.B.3: Tell, and write, and represent time in hours and half-hours using analog and digital clocks.</u></b>
<b><u>1.MD.C 1.DM.C: Represent and interpret data.</u></b>	
1.MD.C.4: Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	<p><b><u>1.MD.C.4 1.DM.C.4: Use tally marks and t-charts/tables with up to three categories to:</u></b></p> <ul style="list-style-type: none"> <li>organize, Create bar graphs and/or picture graphs to represent data precisely with accurate scaling within a scale of 1 or 2; and</li> <li>interpret data by asking and answering questions about: with up to three categories;</li> </ul> <p>a. Ask and answer questions about: <ul style="list-style-type: none"> <li>the total number of data points;</li> <li>how many in each category; and</li> </ul> </p>

Current Standard	Proposed Standard
	<ul style="list-style-type: none"> <li>○ <u>how many more or less are in one category than in another.</u></li> </ul>
<b>1.MD.D 1.DM.D: Work with money.</b>	
1.MD.D.5: Determine the value of a collection of coins up to 50 cents. (Pennies, nickels, dimes, and quarters in isolation; not to include a combination of different coins.)	<b>1.MD.D.5 1.DM.D.5:</b> Determine the value of a collection of coins up to 50 cents. (Pennies, nickels, dimes, and quarters in isolation; not to include a combination of different coins.)

## Grade 2

### Grade Level Expected Foundational Skills

By the end of grade 2, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 2, but is comprised of the foundational skills required by the standards.

1. Know all sums of two one-digit numbers by memory.
2. Mentally add and subtract within 20.
3. Fluently add and subtract within 100 using strategies.
4. Count within 1000 by 2s, 5s, and 10s.
5. Read and write numbers within 1000.
6. Given a three-digit number, mentally find 100 more or 100 less.
7. Compare numbers up to 1000, identifying whether one number is greater than, less than, or equal to another.
8. Measure to determine how much longer one object is than another using standard length units.

## Numeracy & Operational Fluency

Current Standard	Proposed Standard
<p><b>2.OA.B 2.NOF.A: Add and subtract within 20.</b></p> <p>2.OA.B.2: Fluently add and subtract within 20 using mental strategies. *2 By the end of Grade 2, know from memory all sums of two one-digit numbers. *2 See standard 1.OA.6 for a list of mental strategies.</p>	<p><b>2.OA.B.2 2.NOF.A.1:</b> Fluently add and subtract within 20 using mental strategies.<sup>5</sup> *2 By the end of Grade 2, know from memory all sums of two one-digit numbers. *2 <sup>5</sup>See standard 1.OA.6 1.NOF.A.2 for a list of mental strategies.</p>
<p><b>2.NBT.A 2.NOF.B: Understand place value.</b></p> <p>2.NBT.A.1: Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a “hundred.” b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</p>	<p><b>2.NBT.A.1 2.NOF.B.2:</b> Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. • For example: e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: ○ 100 can be thought of as a bundle of ten tens — called a “hundred.” ○ The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</p>
<p>2.NBT.A.2: Count within 1000; skip-count by 5s, 10s, and 100s.</p>	<p><b>2.NBT.A.2 2.NOF.B.3:</b> Count <u>within</u> <u>forward and backward up to 1000</u>, <u>starting at any number less than 1000</u>. a. Skip-count by 2s, 5s, 10s, and 100s.</p>
<p>2.NBT.A.3: Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</p>	<p><b>2.NBT.A.3 2.NOF.B.4:</b> Read and write numbers <u>up to 1000</u> using base-ten numerals/<u>standard form</u>, number names/<u>written form</u>, <u>unit form</u>, and expanded form.</p>
<p>2.NBT.A.4: Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p>	<p><b>2.NBT.A.4 2.NOF.B.5:</b> Compare two <u>three-digit numbers</u> based on <u>meanings-values</u> of the hundreds, tens, and ones digits, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p>

Current Standard	Proposed Standard
<b>2.NBT.B 2.NOF.C: Use place value understanding and properties of operations to add and subtract.</b>	
2.NBT.B.5: Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	2.NBT.B.5 <u>2.NOF.C.6</u> : Fluently add and subtract <del>within</del> <u>up to</u> 100 using strategies based on place value, properties of operations, and/or the <u>part-whole</u> relationship between addition and subtraction.
2.NBT.B.6: Add up to four two-digit numbers using strategies based on place value and properties of operations.	2.NBT.B.6 <u>2.NOF.C.7</u> : Add up to four two-digit numbers using strategies based on place value and properties of operations.
2.NBT.B.8: Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100– 900.	2.NBT.B.8 <u>2.NOF.C.8</u> : <u>For a given number 100-900:</u> a. <u>Mentally add 10 or 100, to a given number 100–900, and</u> b. <u>Mentally subtract 10 or 100 from a given number 100–900.</u>
2.NBT.B.9: Explain why addition and subtraction strategies work, using place value and the properties of operations. *3  *3 Explanations may be supported by drawings or objects.	2.NBT.B.9 <u>2.NOF.C.9</u> : <u>Explain</u> <u>Construct a written explanation or drawing for</u> why addition and subtraction strategies work, using place value <u>understanding</u> and the properties of operations. *3  <del>*3 Explanations may be supported by drawings or objects.</del>

## Algebraic Reasoning

Current Standard	Proposed Standard
<p><b>2.OA.A 2.AR.A:</b> Represent and solve problems involving addition and subtraction.</p>	
<p>2.OA.A.1: Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *1</p> <p>*1 See Glossary, Table 1. <a href="https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn=86bb8a1f_0">https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn=86bb8a1f_0</a></p> <p>Page 60 - Common addition and subtraction situations</p>	<p>2.OA.A.1 <b>2.AR.A.1:</b> Use addition and subtraction within 100 to solve one- and two-step <u>real-world mathematical tasks</u> <del>word problems</del> involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions <u>for one or two operations</u>. a. <u>Represent these situations using models and equations, elevating the concrete-representational- abstract instructional framework</u><sup>5</sup> e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *1</p> <p>*1 <sup>6</sup>See <u>Glossary, Table 1</u>. Page 60 – Common addition and subtraction situations</p>
2.NBT.B.7: Add and subtract within 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; justify the reasoning used with a written explanation. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	<p>2.NBT.B.7 <b>2.AR.A.2:</b> Add and subtract <u>within up to</u> 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the <u>part: whole</u> relationship between addition and subtraction; <u>justify the reasoning used with a written explanation</u>. a. <u>Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</u> b. <u>Justify the reasoning used when choosing the model or strategy to compute with a written explanation.</u></p>
<p><b>2.OA.C 2.AR.B:</b> Work with equal groups of objects to gain foundations for multiplication.</p>	

Current Standard	Proposed Standard
2.OA.C.3: Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	2.OA.C.3 <u>2.AR.B.3</u> : Determine whether a group of objects (up to 20) has an odd or even number of members. a. <u>Represent these situations</u> , e.g., by pairing objects or counting them by 2s; b. <u>w</u> Write an equation to express an even number as a sum of <u>two</u> equal addends.
2.OA.C.4: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	2.OA.C.4 <u>2.AR.B.4</u> : Use addition to find the total number of objects arranged in <u>equal groups and</u> rectangular arrays with up to 5 rows and up to 5 columns. a. <u>w</u> Write an equation to express the total as a sum of equal addends (a repeated addition equation).

## Geometric Reasoning & Logic

Current Standard	Proposed Standard
<b><u>2.G.A 2.GL.A</u></b> : Reason with shapes and their attributes.	
2.G.A.1: Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. *5 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.  *5 Sizes are compared directly or visually, not compared by measuring.	2.G.A.1 <u>2.GL.A.1</u> : Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces, with <u>sizes being compared directly or visually</u> .*5 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.  *5 <u>Sizes are compared directly or visually, not compared by measuring</u> .

Current Standard	Proposed Standard
2.G.A.2: Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	2.G.A.2 <u>2.GL.A.2: Apply spatial structuring to P</u> partition a rectangle into rows and columns of same-size <u>squares</u> <u>units</u> , <u>interpreting both as a collection of units and as single units</u> , and count to find the total number of <u>them</u> <u>units</u> . <u>The rectangle should not be divided up into more than 5 columns and 5 rows to correlate with 2.AR.B.4.</u>
2.G.A.3: Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words <i>halves</i> , <i>thirds</i> , <i>half of</i> , <i>a third of</i> , etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	2.G.A.3 <u>2.GL.A.3: Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words <i>halves</i>, <i>thirds</i>, <i>half of</i>, <i>a third of</i>, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</u> a. Identify examples and non-examples of halves, thirds, and fourths.

## Data Analysis & Measurement

Current Standard	Proposed Standard
<b>2.MD.A 2.DM.A: Measure and estimate lengths in standard units.</b>	
2.MD.A.1: Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	2.MD.A.1 <u>2.DM.A.1: Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.</u>
2.MD.A.2: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	2.MD.A.2 <u>2.DM.A.2: Measure the length of an object twice, using <u>two different units of length</u> <u>units</u> <u>(inches, feet, centimeters, or meters)</u> <u>of different lengths</u> <u>for</u> <u>or</u> <u>the two different measurements</u>;</u> • <u>Describe how the two measurements relate to the size of the unit chosen with a written response.</u>

Current Standard	Proposed Standard
2.MD.A.3: Estimate lengths using units of inches, feet, centimeters, and meters.	2.MD.A.3 <del>2.DM.A.3</del> : Estimate lengths using units of inches, feet, centimeters, and meters <u>to the nearest whole unit</u> .
2.MD.A.4: Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	2.MD.A.4 <del>2.DM.A.4</del> : Measure to determine how much longer one object is than another, expressing the length difference in terms of <del>a</del> <u>using the same</u> standard length unit.
<b><u>2.MD.B 2.DM.B: Relate addition and subtraction to length.</u></b>	
2.MD.B.5: Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	2.MD.B.5 <del>2.DM.B.5</del> : Use addition and subtraction <u>up to</u> <del>within</del> 100 to solve <u>one- and two-step real-world mathematical tasks</u> <del>word problems</del> involving lengths that are given in the same units by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
2.MD.B.6: Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.	<p><del>2.MD.B.6 2.DM.B.6: Represent whole numbers as lengths from 0 on</del>  <u>Create</u> a number line diagram with equally spaced points corresponding to the numbers <u>(0, 1, 2, etc.)</u>, using <u>0 or another number as a starting point</u>.</p> <ul style="list-style-type: none"> <li>• <u>Recognize that each mark on a number line represents one whole number and that each position corresponds to a single value.</u></li> <li>• <u>Represent whole numbers as lengths from 0.</u></li> <li>• <u>Represent whole-number sums and differences <del>within</del> <u>up to</u> 100 on a number line diagram.</u></li> </ul>
<b><u>2.MD.C 2.DM.C: Work with time and money.</u></b>	
2.MD.C.7: Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	<p><del>2.MD.C.7 2.DM.C.7: Tell, and write, and represent</del> time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.</p> <p>a. <u>Express portions of an hour using the fractional terms half an hour, half past, quarter of an hour, quarter after, and quarter til.</u></p>

Current Standard	Proposed Standard
<p>2.MD.C.8: Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. <i>Example: If you have 2 dimes and 3 pennies, how many cents do you have?</i></p>	<p><del>2.MD.C.8</del> <del>2.DM.C.8</del>: Solve <del>real-world mathematical tasks</del> <del>word problems</del> involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately.</p>
<p><b>2.MD.D 2.DM.D: Represent and interpret data.</b></p>	
<p>2.MD.D.9: Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</p>	<p><del>2.MD.D.9</del> <del>2.DM.D.9</del>: Generate measurement data by <del>either</del> measuring the lengths of several objects to the nearest whole unit, or by <del>making</del> <del>taking</del> repeated measurements of the same object.</p> <p>a. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</p>
<p>2.MD.D.10: Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems *4 using information presented in a bar graph.</p>	<p><del>2.MD.D.10</del> <del>2.DM.D.10</del>: Draw a picture graph and a bar graph (with <del>single-unit</del> <del>a scale of 1, 2, 5, or 10</del>) to represent a data set with up to four categories.</p> <p>a. Solve simple put-together, take-apart, and compare <del>real-world mathematical tasks</del> <del>problems</del> *4 using information presented in a <del>picture graph or bar graph</del>.</p>
<p>*4 See Glossary, Table 1.  <a href="https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn=86bb8a1f_0">https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn=86bb8a1f_0</a>  Page 60 - Common addition and subtraction situations</p>	<p>*4 See Glossary, Table 1.</p>

# Grade 3

## Grade Level Expected Foundational Skills

By the end of grade 3, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 3, but is comprised of the foundational skills required by the standards.

1. Know all products of two one-digit numbers from memory.
2. Know all quotients of numbers within 100 by memory.
3. Fluently add and subtract within 1,000.
4. Compare numbers up to 100,000, identifying greater than, less than, or equal to.
5. Estimate and round whole numbers to the nearest 10 and 100.
6. Identify equivalent fractions with limited denominators.

## Numeracy & Operational Fluency

Current Standard	Proposed Standard
<p><b><u>3.NF.A.3.NOF.A.: Develop understanding of fractions as numbers.</u></b></p> <p>3.NF.A.1: Understand a fraction <math>1/b</math>, with denominators 2, 3, 4, 6, and 8, as the quantity formed by 1 part when a whole is partitioned into <math>b</math> equal parts; understand a fraction <math>a/b</math> as the quantity formed by <math>a</math> parts of size <math>1/b</math>.</p>	<p><b><u>3.NF.A.1 3.NOF.A.1: Understand and interpret a fraction <math>1/b</math>, with denominators 2, 3, 4, 6, and 8;</u></b></p> <p>a. <u>Understand a unit fraction <math>\frac{1}{b}</math> as the quantity formed by 1 part when a whole <u>or a set</u> is partitioned into <math>b</math> equal parts <u>where <math>b</math> is a non-zero whole number</u>;</u></p> <p>b. <u>Understand a fraction <math>\frac{a}{b}</math> as the quantity formed by <math>a</math> parts of size <math>\frac{1}{b}</math>.</u></p> <p>c. <u>Represent fractions greater than zero and less than or equal to one using concrete objects, number lines, or pictorial models.</u></p>

Current Standard	Proposed Standard
<p>3.NF.A.2: Understand a fraction with denominators 2, 3, 4, 6, and 8 as a number on a number line diagram.</p> <p>a. Represent a fraction <math>1/b</math> on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into <math>b</math> equal parts. Recognize that each part has size <math>1/b</math> and that the endpoint of the part based at 0 locates the number <math>1/b</math> on the number line.</p> <p>b. Represent a fraction <math>a/b</math> on a number line diagram by marking off <math>a</math> lengths <math>1/b</math> from 0. Recognize that the resulting interval has size <math>a/b</math> and that its endpoint locates the number <math>a/b</math> on the number line.</p>	<p><u>d. Read and write fractions in standard form and written unit form</u></p> <p><u>f. Solve real-world mathematical tasks involving partitioning an object or set of objects, identifying a fraction as parts of a whole.</u></p>
<p>3.NF.A.3: Explain equivalence of fractions with denominators 2, 3, 4, 6, and 8 in special cases, and compare fractions by reasoning about their size.</p> <p>a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.</p> <p>b. Recognize and generate simple equivalent fractions, e.g., <math>1/2 = 2/4</math>, <math>4/6 = 2/3</math>. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form <math>3 = 3/1</math>; recognize that <math>6/1 = 6</math>; locate <math>4/4</math> and 1 at the same point of a number line diagram.</p> <p>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that</p>	<p><u>3.NF.A.2</u> <u>3.NOF.A.2</u>: Understand a fraction with denominators 2, 3, 4, 6, and 8 as a number on a number line diagram.</p> <p>a. Represent a fraction <math>\frac{1}{b}</math> on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into <math>b</math> equal parts. Recognize that each part has size <math>\frac{1}{b}</math> and that the endpoint of the part, based at 0, locates the number <math>\frac{1}{b}</math> on the number line.</p> <p>b. Represent a fraction <math>\frac{a}{b}</math> on a number line diagram by marking off <math>a</math> lengths <math>\frac{1}{b}</math> from 0. Recognize that the resulting interval has size <math>\frac{a}{b}</math> and that its endpoint locates the number <math>\frac{a}{b}</math> on the number line.</p> <p><u>3.NF.A.3</u> <u>3.NOF.A.3</u>: Explain equivalence of fractions with denominators 2, 3, 4, 6, and 8 as fractions that have different numerators and denominators but are equal to the same value in special cases, and compare fractions by reasoning about their size.</p> <p>a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.</p> <p>b. Recognize and generate simple equivalent fractions, e.g., <math>\frac{1}{2} = \frac{2}{4}</math>, <math>\frac{4}{6} = \frac{2}{3}</math>. Explain why the fractions are equivalent, e.g., by using a visual fraction model or number line.</p> <p>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.</p> <p>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that</p>

Current Standard	Proposed Standard
<p>comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model</p>	<p>comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model <u>or</u> number line.</p>
<p><b>3.OA.C.3.NOF.B: Multiply and divide within 100.</b></p>	
<p>3.OA.C.7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that <math>8 \times 5 = 40</math>, one knows <math>40 \div 5 = 8</math>) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.</p>	<p><b>3.OA.C.7 3.NOF.B.4:</b> Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that <math>8 \times 5 = 40</math>, one knows <math>40 \div 5 = 8</math>) or properties of operations. By the end of Grade 3, know <u>from memory</u> all products of two one-digit numbers <u>from memory</u>.</p>
<p><b>3.NBT.A 3.NOF.C.: Use place value understanding and properties of operations to perform multi-digit arithmetic.</b></p>	
<p>3.NBT.A.1: Use place value understanding to round whole numbers to the nearest 10 or 100.</p>	<p><b>3.NBT.A.1 3.NOF.C.5:</b> Use place value understanding to:</p> <ol style="list-style-type: none"> <li><u>Round whole numbers to the nearest 10 or 100.</u></li> <li><u>Use compatible numbers to estimate solutions to real-world mathematical tasks.</u></li> </ol>
<p>3.NBT.A.2: Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<p><b>3.NBT.A.2 3.NOF.C.6:</b> Fluently add and subtract <u>within up to</u> 1000 using strategies and algorithms based on place value <u>understanding</u>, properties of operations, and/or the <u>part-whole</u> relationship between addition and subtraction.</p>
<p>3.NBT.A.3: Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., <math>9 \times 80, 5 \times 60</math>) using strategies based on place value and properties of operations.</p>	<p><b>3.NBT.A.3 3.NOF.C.7:</b> Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., <math>9 \times 80, 5 \times 60</math>) using strategies based on place value <u>understanding</u> and properties of operations.</p>

## Algebraic Reasoning

Current Standard	Proposed Standard
<p><b><u>3.OA.A 3.AR.A:</u></b> Represent and solve problems involving multiplication and division.</p> <p>3.OA.A.1: Interpret products of whole numbers, e.g., interpret <math>5 \times 7</math> as the total number of objects in 5 groups of 7 objects each. <i>For example, describe a context in which a total number of objects can be expressed as <math>5 \times 7</math>.</i></p>	<p><b><u>3.OA.A.1 3.AR.A.1:</u></b> Interpret products of whole numbers/factors, e.g., interpret <math>5 \times 7</math> as the total number of objects in 5 groups of 7 objects each. <i>For example, describe a context in which a total number of objects can be expressed as <math>5 \times 7</math>.</i></p>
<p>3.OA.A.2: Interpret whole-number quotients of whole numbers, e.g., interpret <math>56 \div 8</math> as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. <i>For example, describe a context in which a number of shares or a number of groups can be expressed as <math>56 \div 8</math>.</i></p>	<p><b><u>3.OA.A.2 3.AR.A.2:</u></b> Interpret whole-number quotients of whole numbers (i.e., dividends and divisors), e.g., interpret <math>56 \div 8</math> as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. <i>For example, describe a context in which a number of shares or a number of groups can be expressed as <math>56 \div 8</math>.</i></p>
<p>3.OA.A.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. *1</p> <p>*1 See Glossary, Table 2. <a href="https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn">https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn</a></p>	<p><b><u>3.OA.A.3 3.AR.A.3:</u></b> Use multiplication and division within 100 to solve word problems real-world mathematical tasks in situations involving equal groups, arrays, and measurement quantities. a, e.g., Represent these situations by using models, drawings, and equations with a symbol for the unknown number to represent the problem. *1</p> <p>*1 See Glossary, Table 2.</p>

Current Standard	Proposed Standard
<p><a href="#"><u>=86bb8a1f_0</u></a>  Page 61 - Common multiplication and division situations</p>	
<p>3.OA.A.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. <i>For example, determine the unknown number that makes the equation true in each of the equations <math>8 \times ? = 48</math>, <math>5 = \square \div 3</math>, <math>6 \times 6 = ?</math>.</i></p>	<p>3.OA.A.4 <del>3.AR.A.4</del>: Determine the unknown whole number in a multiplication or division equation relating three whole numbers <u>by representing the situation with a model or drawing and solving for the unknown. For example, determine the unknown number that makes the equation true in each of the equations <math>8 \times ? = 48</math>, <math>5 = \square \div 3</math>, <math>6 \times 6 = ?</math></u></p>
<p><b><u>3.OA.B 3.AR.B: Understand properties of multiplication and the relationship between multiplication and division.</u></b></p>	
<p>3.OA.B.5: Apply properties of operations as strategies to multiply and divide. <i>*2 Examples: If <math>6 \times 4 = 24</math> is known, then <math>4 \times 6 = 24</math> is also known. (Commutative property of multiplication.) <math>3 \times 5 \times 2</math> can be found by <math>3 \times 5 = 15</math>, then <math>15 \times 2 = 30</math>, or by <math>5 \times 2 = 10</math>, then <math>3 \times 10 = 30</math>. (Associative property of multiplication.) Knowing that <math>8 \times 5 = 40</math> and <math>8 \times 2 = 16</math>, one can find <math>8 \times 7</math> as <math>8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56</math>. (Distributive property.)</i></p> <p>*2 Students need not use formal terms for these properties.</p>	<p>3.OA.B.5 <del>3.AR.B.5</del>: Apply properties of operations as strategies to multiply and divide. <i>*2 Examples: If <math>6 \times 4 = 24</math> is known, then <math>4 \times 6 = 24</math> is also known. (Commutative property of multiplication.) <math>3 \times 5 \times 2</math> can be found by <math>3 \times 5 = 15</math>, then <math>15 \times 2 = 30</math>, or by <math>5 \times 2 = 10</math>, then <math>3 \times 10 = 30</math>. (Associative property of multiplication.) Knowing that <math>8 \times 5 = 40</math> and <math>8 \times 2 = 16</math>, one can find <math>8 \times 7</math> as <math>8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56</math>. (Distributive property.)</i></p> <p><del>*2 Students need not use formal terms for these properties.</del></p>
<p>3.OA.B.6: Understand division as an unknown-factor problem. <i>For example, find <math>32 \div 8</math> by finding the number that makes <math>32</math> when multiplied by <math>8</math>.</i></p>	<p>3.OA.B.6 <del>3.AR.B.6</del>: Understand division as an unknown-factor problem, providing an explanation that leverages the relationship between and the properties of multiplication and division. <i>For example, find <math>32 \div 8</math> by finding the number that makes <math>32</math> when multiplied by <math>8</math>.</i></p>
<p><b><u>3.OA.D 3.AR.C.: Solve problems real-world mathematical tasks involving the four operations and identify and explain patterns in</u></b></p>	

Current Standard	Proposed Standard
<p><b>arithmetic.</b></p>	
<p>3.OA.D.8: Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. *3</p> <p>*3 This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</p>	<p><b>3.OA.D.8 3.AR.C.7:</b> Solve <u>two-step word problems</u> <u>real-world mathematical tasks involving at least two steps and using at least two of</u> the four operations.</p> <p><b>a.</b> Represent these problems using equations with a letter standing for the unknown quantity.</p> <p><b>b.</b> Assess the reasonableness of answers using mental computation and estimation strategies, including rounding. *3</p> <p><del>*3 This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</del></p>
<p>3.OA.D.9: Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. <i>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</i></p>	<p><b>3.OA.D.9 3.AR.C.8:</b> Identify, <u>create, and extend</u> arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations <u>without the formal language of the properties.</u></p> <ul style="list-style-type: none"> <li>For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</li> </ul>
<p><b>3.AR.D: Compare whole numbers.</b></p> <p>n/a</p>	<p><b>3.AR.D.9: Using understanding of the base-ten system:</b></p> <p><b>a.</b> Compare whole numbers up to 100,000, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols <u>to record the results of comparisons.</u></p> <p><b>b.</b> Order a set of whole numbers up to 100,000.</p>

## Geometric Reasoning & Logic

Current Standard	Proposed Standard
<b><u>3.G.A 3.GL.A:</u> Reason with shapes and their attributes.</b>	
3.G.A.1: Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	<u>3.G.A.1 3.GL.A.1:</u> Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). <u>a.</u> Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
3.G.A.2: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. <i>For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.</i>	<u>3.G.A.2 3.GL.A.2:</u> Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. <i>For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.</i>

## Data Analysis & Measurement

Current Standard	Proposed Standard
<b><u>3.MD.A 3.DM.A:</u> Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</b>	
3.MD.A.1: Understand time to the nearest minute. a. Tell and write time to the nearest minute and measure time intervals in minutes, within 60 minutes, on an analog and digital clock.	<u>3.MD.A.1: Understand time to the nearest minute.</u> <u>a.</u> <u>Tell and write time to the nearest minute and measure time intervals in minutes, within 60 minutes, on an analog and digital clock.</u> <u>b.</u> <u>Calculate elapsed time greater than 60 minutes to the nearest quarter hour.</u>

Current Standard	Proposed Standard
<p>b. Calculate elapsed time greater than 60 minutes to the nearest quarter and half hour on a number line diagram.</p> <p>c. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p>	<p>and half hour on a number line diagram.</p> <p>e. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p>
<p>3.MD.A.2: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). *5 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. *6</p> <p>*5 Excludes compound units such as <math>cm^3</math> and finding the geometric volume of a container.</p> <p>*6 Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).</p>	<p>3.MD.A.2 3.DM.A.1: Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). *5  <u>a.</u> Add, subtract, multiply, or divide to solve one-step <u>real-world mathematical tasks</u> word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. *6</p> <p>*5 Excludes compound units such as <math>cm^3</math> and finding the geometric volume of a container.</p> <p>*6 Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).</p>
<p><b>3.MD.B 3.DM.B: Represent and interpret data.</b></p>	
<p>3.MD.B.3: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. <i>For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</i></p>	<p>3.MD.B.3 3.DM.B.2: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.</p> <p><u>a.</u> Solve one- and two-step “how many more” and “how many less” problems <u>real-world mathematical tasks</u> using information presented in scaled bar graphs. <i>For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</i></p>
<p>3.MD.B.4: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is</p>	<p>3.MD.B.4 3.DM.B.3: Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch.</p> <p><u>a.</u> Show the data by making a line plot, where the horizontal scale is</p>

Current Standard	Proposed Standard
marked off in appropriate units — whole numbers, halves, or quarters.	marked off in appropriate units — whole numbers, halves, or quarters.
<b><u>3.MD.C 3.DM.C: Geometric measurement: Understand concepts of area and relate area to multiplication and to addition.</u></b>	
<p>3.MD.C.5: Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p>a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</p> <p>b. A plane figure that can be covered without gaps or overlaps by <math>n</math> unit squares is said to have an area of <math>n</math> square units.</p>	<p><del>3.MD.C.5</del> <del>3.DM.C.4</del>: Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p>a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</p> <p>b. <del>A plane figure that can be covered without gaps or overlaps by <math>n</math> unit squares is said to have an area of <math>n</math> square units. A plane figure has an area of <math>n</math> square units if it can be completely covered — without any gaps or overlaps — by <math>n</math> unit squares.</del></p>
3.MD.C.6: Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	3.MD.C.6 <del>3.DM.C.5</del> : Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
<p>3.MD.C.7: Relate area to the operations of multiplication and addition.</p> <p>a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.</p> <p>b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.</p> <p>c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths <math>a</math> and <math>b + c</math> is the sum of <math>a \times b</math> and <math>a \times c</math>. Use area models to represent the distributive property in mathematical reasoning.</p>	<p><del>3.MD.C.7</del> <del>3.DM.C.6</del>: Relate area to the operations of multiplication and equal groups of addition.</p> <p>a. Find the area of a rectangle with whole-number side lengths by tiling it, and <del>show connect</del> that the area is the same as <del>would be found by</del> multiplying the side lengths.</p> <p>b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving <del>real-world and mathematical problems</del> <del>real-world mathematical tasks</del>, and represent whole-number products as rectangular areas in mathematical reasoning.</p> <p>c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths <math>a</math> and <math>b + c</math> is the sum of <math>a \times b</math> and <math>a \times c</math>.</p> <p>c. Use area models to represent the distributive property in mathematical reasoning.</p>

Current Standard	Proposed Standard
<p><b><u>3.MD.D</u></b> <b><u>3.MD.D</u></b>: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.</p>	<p><b><u>3.MD.D.8</u></b> <b><u>3.DM.D.7</u></b>: Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.</p>
<p><b><u>3.MD.E</u></b> <b><u>3.DM.E</u></b>: Work with time and money.</p>	<p><b><u>3.MD.D.8</u></b> <b><u>3.DM.D.7</u></b>: Solve real-world and mathematical <u>tasks</u> problems involving perimeters of polygons, including:</p> <ul style="list-style-type: none"> <li>finding the perimeter given the side lengths,</li> <li>finding an unknown side length, and</li> <li>exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.</li> </ul>
<p><b><u>3.MD.A.1</u></b>: Understand time to the nearest minute.</p> <p>a. Tell and write time to the nearest minute and measure time intervals in minutes, within 60 minutes, on an analog and digital clock.</p> <p>b. Calculate elapsed time greater than 60 minutes to the nearest quarter and half hour on a number line diagram.</p> <p>c. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p>	<p><b><u>3.MD.A.1</u></b> <b><u>3.DM.A.1</u></b>: Understand time to the nearest minute.</p> <p>a. Tell, and write, and represent time to the nearest minute and measure time intervals in minutes, within 60 minutes, on an analog and digital clock.</p> <p>b. Calculate elapsed time greater than 60 minutes to the nearest quarter and half hour on a number line diagram.</p> <p>c. Solve <u>real-world mathematical tasks</u> <u>word problems</u> involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p>
<p><b><u>3.MD.E.9</u></b>: Solve word problems involving pennies, nickels, dimes, quarters, and bills greater than one dollar, using the dollar and cent symbols appropriately.</p>	<p><b><u>3.MD.E.9</u></b> <b><u>3.DM.E.9</u></b>: Solve <u>real-world mathematical tasks</u> <u>word problems</u> involving <u>at least two steps</u> and <u>using the four operations</u> with pennies, nickels, dimes, quarters, and bills greater than one dollar, using the <u>dollar \$</u> and <u>cent ¢</u> symbols appropriately.</p>

# Grade 4

## Grade Level Expected Foundational Skills

By the end of grade 4, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 4, but is comprised of the foundational skills required by the standards.

1. Fluently add and subtract up to 1,000,000.
2. Identify factors and multiples within 100.
3. Compare and order numbers up to 1,000,000, identifying whether one number is greater than, less than, or equal to another.
4. Multiply and divide with multi-digit whole numbers.
5. Estimate and round whole numbers within 1,000,000 to any place value.
6. Add and subtract fractions with like denominators.
7. Multiply a fraction by a whole number.

## Numeracy & Operational Fluency

Current Standard	Proposed Standard
<b>4.NF.A 4.NOF.A: Extend understanding of fraction equivalence and ordering.</b>	
4.NF.A.1: Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)	4.NF.A.1 4.NOF.A.1: Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models or number line diagrams, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
4.NF.A.2: Compare two fractions with different numerators and	4.NF.A.2 4.NOF.A.2: Compare two fractions with different numerators

Current Standard	Proposed Standard
<p>different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as <math>\frac{1}{2}</math>. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)</p>	<p>and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as <math>\frac{1}{2}</math>.</p> <ul style="list-style-type: none"> <li>• Recognize that comparisons are valid only when the two fractions refer to the same whole.</li> <li>• Record the results of comparisons with symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model <u>or a number line diagram</u>. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)</li> </ul>
<p><b>4.NF.B.4.NOF.B: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</b></p>	
<p>4.NF.B.3: Understand a fraction <math>a/b</math> with <math>a &gt; 1</math> as a sum of fractions <math>1/b</math>. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)</p> <p>a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. Example: <math>3/4 = 1/4 + 1/4 + 1/4</math>.</p> <p>b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: <math>3/8 = 1/8 + 1/8 + 1/8</math>; <math>3/8 = 1/8 + 2/8</math>; <math>2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8</math>.</p> <p>c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</p> <p>d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</p>	<p><del>4.NF.B.3</del> 4.NOF.B.3: Understand a fraction <math>\frac{a}{b}</math> with <math>a &gt; 1</math> as a sum of fractions <math>\frac{1}{b}</math>. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)</p> <p>a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. <del>Example: <math>3/4 = 1/4 + 1/4 + 1/4</math></del>.</p> <p>b. Decompose a fraction, <u>including mixed numbers and fractions greater than 1</u>, into a sum of fractions, <u>including unit fractions and non-unit fractions</u>, with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model <u>or number line</u>. Examples: <math>3/8 = 1/8 + 1/8 + 1/8</math>; <math>3/8 = 1/8 + 2/8</math>; <math>2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8</math>.</p> <p>c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</p> <p><del>d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</del></p> <p><u>c. Evaluate the reasonableness of sums and differences of fractions</u></p>

Current Standard	Proposed Standard
	<u>using benchmark fractions, 0, <math>\frac{1}{4}</math>, <math>\frac{1}{2}</math>, <math>\frac{3}{4}</math>, 1, referring to the same whole.</u>
<p>c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</p> <p>d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</p>	<p><b>4.NOF.B.4: Add and subtract fractions with like denominators.</b></p> <p>a. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction <u>greater than 1</u> and/or by using properties of operations and the relationship between addition and subtraction.</p> <p>b. Solve <u>real-world mathematical tasks word problems</u> involving addition and subtraction of fractions, <u>including mixed numbers and fractions greater than 1</u>, referring to the same whole and having like denominators, e.g., by using visual fraction models, <u>number lines</u>, or <u>equations</u> to represent the problem.</p>
<p><b>4.NF.B.4:</b> Multiply a fraction by a whole number. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)</p> <p>a. Understand a fraction <math>a/b</math> as a multiple of <math>1/b</math>. For example, use a visual fraction model to represent <math>5/4</math> as the product <math>5 \times (1/4)</math>, recording the conclusion by the equation <math>5/4 = 5 \times (1/4)</math>.</p> <p>b. Understand a multiple of <math>a/b</math> as a multiple of <math>1/b</math>, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express <math>3 \times (2/5)</math> as <math>6 \times (1/5)</math>, recognizing this product as <math>6/5</math>. (In general, <math>n \times (a/b) = (n \times a)/b</math>.)</p> <p>c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat <math>3/8</math> of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i></p>	<p><b>4.NF.B.4 4.NOF.B.5:</b> Multiply a fraction by a whole number. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)</p> <p>a. Understand a fraction <math>\frac{a}{b}</math> as a multiple of <math>\frac{1}{b}</math>. <del>For example, use a visual fraction model to represent <math>\frac{5}{4}</math> as the product <math>5 \times (\frac{1}{4})</math>, recording the conclusion by the equation <math>\frac{5}{4} = 5 \times (\frac{1}{4})</math>.</del></p> <p>b. Understand a multiple of <math>\frac{a}{b}</math> as a multiple of <math>\frac{1}{b}</math>, and use this understanding to multiply a fraction by a whole number. <del>For example, use a visual fraction model to express <math>3 \times (\frac{2}{5})</math> as <math>6 \times (\frac{1}{5})</math>, recognizing this product as <math>\frac{6}{5}</math>. (In general, <math>n \times (\frac{a}{b}) = \frac{(n \times a)}{b}</math>.)</del></p> <p>c. Solve <u>word problems real-world mathematical tasks</u> involving multiplication of a fraction by a whole number, e.g., by using visual fraction models, <u>number line diagrams</u>, and equations to represent the problem. <del>For example, if each person at a party will eat <math>3/8</math> of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers</del></p>

Current Standard	Proposed Standard
	<i>does your answer lie?</i>
<b>4.NF.C 4.NOF.C: Understand decimal notation for fractions, and compare decimal fractions.</b>	
<p>4.NF.C.5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.*2  <i>For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.</i></p>	<p><b>4.NF.C.5 4.NOF.C.6:</b> Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.*2  <i>For example, express <math>\frac{3}{10}</math> as <math>\frac{30}{100}</math>, and add <math>\frac{3}{10} + \frac{4}{100} = \frac{34}{100}</math>.</i></p>
<p>*2 Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.</p>	<p>*2 Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.</p>
<p>4.NF.C.5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.*2  <i>For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.</i></p>	<p><b>4.NF.C.5 4.NOF.C.6:</b> Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.*2  <i>For example, express <math>\frac{3}{10}</math> as <math>\frac{30}{100}</math>, and add <math>\frac{3}{10} + \frac{4}{100} = \frac{34}{100}</math>.</i></p>
<p>*2 Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.</p>	<p>*2 Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.</p>
<p>4.NF.C.6: Use decimal notation for fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram; represent 62/100 of a dollar as \$0.62.</i></p>	<p><b>4.NF.C.6 4.NOF.C.7:</b> Use decimal notation and precise language for fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as <math>\frac{62}{100}</math>, say sixty-two hundredths; describe a length as 0.62 meters; locate 0.62 on</i></p>

Current Standard	Proposed Standard
	<del>a number line diagram; represent <math>\frac{62}{100}</math> of a dollar as \$0.62.</del>
4.NF.C.7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$ , $=$ , or $<$ , and justify the conclusions, e.g., by using a visual model.	<p><del>4.NF.C.7</del> <del>4.NOF.C.8</del>: Compare two decimals to hundredths by reasoning about their size.</p> <ul style="list-style-type: none"> <li>Recognize that comparisons are valid only when the two decimals refer to the same whole.</li> <li>Record the results of comparisons with the symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual model <u>or number line</u>.</li> </ul>
<b>4.OA.B 4.NOF.D: Gain familiarity with factors and multiples.</b>	
<p>4.OA.B.4: Using whole numbers in the range 1–100,</p> <ol style="list-style-type: none"> <li>Find all factor pairs for a given whole number.</li> <li>Recognize that a given whole number is a multiple of each of its factors.</li> <li>Determine whether a given whole number is a multiple of a given one-digit number.</li> <li>Determine whether a given whole number is prime or composite.</li> </ol>	<p><del>4.OA.B.4</del> <del>4.NOF.D.9</del>: Using whole numbers in the range 1–100,</p> <ol style="list-style-type: none"> <li>Find all factor pairs for a given whole number.</li> <li>Recognize that a given whole number is a multiple of each of its factors.</li> <li>Determine whether a given whole number is a multiple of a given one-digit number.</li> <li>Determine whether a given whole number is prime or composite.</li> </ol>
<b>4.NBT.A-4.NOF.E: Generalize place value understanding for multi-digit whole numbers.</b>	
4.NBT.A.1: Recognize that in a multi-digit whole number less than or equal to 1,000,000, a digit in one place represents ten times what it represents in the place to its right. <i>For example, (1) recognize that <math>700 \div 70 = 10</math>; (2) in the number 7,246, the 2 represents 200, but in the number 7,426 the 2 represents 20, recognizing that 200 is ten times as large as 20, by applying concepts of place value and division.</i>	<p><del>4.NBT.A.1</del> <del>4.NOF.E.10</del>: Recognize that in a multi-digit whole number less than or equal to 1,000,000, a digit in one place represents ten times what it represents in the place to its right. <i>For example, (1) recognize that <math>700 \div 70 = 10</math>; (2) in the number 7,246, the 2 represents 200, but in the number 7,426 the 2 represents 20, recognizing that 200 is ten times as large as 20, by applying concepts of place value and division.</i></p>

Current Standard	Proposed Standard
4.NBT.A.2: Read and write multi-digit whole numbers less than or equal to 1,000,000 using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$ , $=$ , and $<$ symbols to record the results of comparisons.	4.NBT.A.2 <del>4.NOF.E.11</del> : Read and write multi-digit whole numbers <del>less than or equal up to</del> 1,000,000 using base-ten numerals/ <u>standard form</u> , <u>written form</u> (number names), <u>unit form</u> , and expanded form. <del>Compare two multi-digit numbers based on meanings of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</del>
4.NBT.A.3: Use place value understanding to round multi-digit whole numbers, less than or equal to 1,000,000, to any place.	4.NBT.A.3 <del>4.NOF.E.12</del> : Use place value understanding to a. <del>Round multi-digit whole numbers, less than or equal up to</del> 1,000,000, to any place. b. <u>Use compatible numbers to estimate solutions to real-world mathematical tasks.</u>
<b>4.NBT.B: Use place value understanding and properties of operations to perform multi-digit arithmetic.</b> <b>4.NOF.F: Add and subtract multi-digit numbers.</b>	
4.NBT.B.4: Fluently add and subtract multi-digit whole numbers with sums less than or equal to 1,000,000, using the standard algorithm.	4.NBT.B.4 <del>4.NOF.F.13</del> : Fluently add and subtract multi-digit whole numbers with <u>sums or differences</u> less than or equal to 1,000,000, using <u>the standard algorithm</u> .

## Algebraic Reasoning

Current Standard	Proposed Standard
<b>4.OA.A <del>4.AR.A</del>: Use the four operations with whole numbers to solve problems real-world mathematical tasks.</b>	
4.OA.A.1: Interpret a multiplication equation as a comparison and represent verbal statements of multiplicative comparisons	4.OA.A.1 <del>4.AR.A.1</del> : Interpret a multiplication equation as a comparison and represent verbal statements of multiplicative comparisons as

Current Standard	Proposed Standard
<p>as multiplication equations, e.g., interpret <math>35 = 5 \times 7</math> as a statement that 35 is 5 times as many as 7, and 7 times as many as 5.</p>	<p>multiplication equations. e.g., interpret <math>35 = 5 \times 7</math> as a statement that 35 is 5 times as many as 7, and 7 times as many as 5.</p>
<p>4.OA.A.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and/or equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison (Example: 6 times as many vs. 6 more than). *1</p>	<p><del>4.OA.A.2 4.AR.A.2: Multiply or divide to solve word problems real-world mathematical tasks involving multiplicative comparison.</del></p> <ul style="list-style-type: none"> <li>• <del>Represent these tasks, e.g., by using drawings and/or equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</del> <ul style="list-style-type: none"> <li>○ <del>(Example: 6 times as many (multiplicative comparison) vs. 6 more than (additive comparison)). *1</del></li> </ul> </li> </ul>
<p>*1 See Glossary, Table 2.  <a href="https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn=86bb8a1f_0">https://doe.louisiana.gov/docs/default-source/teacher-toolbox-resources/louisiana-student-standards-for-k-12-math.pdf?sfvrsn=86bb8a1f_0</a>  Page 61 - Common multiplication and division situations.</p>	<p><del>*1 See Glossary, Table 2.</del></p>
<p>4.OA.A.3: Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. <i>Example: Twenty-five people are going to the movies. Four people fit in each car. How many cars are needed to get all 25 people to the theater at the same time?</i></p>	<p><del>4.OA.A.3-4.AR.A.3: Solve multi-step word problems real-world mathematical tasks involving more than one operation posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.</del></p> <p><del><i>Example: Twenty-five people are going to the movies. Four people fit in each car. How many cars are needed to get all 25 people to the theater at the same time?</i></del></p> <ul style="list-style-type: none"> <li>a. <u>Represent these problems using equations with a letter standing for the unknown quantity.</u></li> <li>b. <u>Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</u></li> </ul>
<p><b>4.OA.C 4.AR.B: Generate and analyze patterns.</b></p>	
<p>4.OA.C.5: Generate a number or shape pattern that follows a</p>	<p><del>4.OA.C.5 4.AR.B.4: Generate and extend a number or shape pattern that</del></p>

Current Standard	Proposed Standard
<p>given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i></p>	<p>follows a given rule.</p> <p>a. Identify apparent features of the pattern that were not explicit in the rule itself.</p> <ul style="list-style-type: none"> <li>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</li> </ul>
<p><b>4.AR.C: Compare whole numbers.</b></p>	
<p>4.NBT.A.2: Read and write multi-digit whole numbers less than or equal to 1,000,000 using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p>	<p><del>4.NBT.A.2 4.AR.C.5: Read and write multi-digit whole numbers less than or equal to 1,000,000 using base-ten numerals, number names, and expanded form. Using understanding of the base-ten system:</del></p> <p>a. Compare two multi-digit <u>whole numbers up to 1,000,000</u> using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p> <p>b. Order a set of whole numbers up to 1,000,000.</p>
<p><b>4.NBT.B 4.AR.D: Use place value understanding and properties of operations to perform multi-digit arithmetic.</b></p>	
<p>4.NBT.B.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p><del>4.NBT.B.5 4.AR.D.6: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value <u>understanding</u> and the properties of operations.</del></p> <p>a. Illustrate and explain the calculation by using equations, rectangular arrays, <u>number line diagrams</u> and/or area models.</p>
<p>4.NBT.B.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations,</p>	<p><del>4.NBT.B.6 4.AR.D.7: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value <u>understanding</u>, the properties of operations, and/or the relationship between multiplication and division.</del></p> <p>a. <u>Represent</u> Illustrate and explain the calculation by using equations,</p>

Current Standard	Proposed Standard
rectangular arrays, and/or area models.	rectangular arrays, and/or area models.

## Geometric Reasoning & Logic

Current Standard	Proposed Standard
<b>4.G.A. <u>4.GL.A:</u> Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</b>	
4.G.A.1: Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	4.G.A.1 <u>4.GL.A.1:</u> Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
4.G.A.2: Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	4.G.A.2 <u>4.GL.A.2:</u> Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. a. Recognize right triangles as a category, and identify right triangles.
4.G.A.3: Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	4.G.A.3 <u>4.GL.A.3:</u> Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

## Data Analysis & Measurement

Current Standard	Proposed Standard
<b>4.MD.A <u>4.DM.A:</u> Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</b>	

Current Standard	Proposed Standard
<p>4.MD.A.1: Know relative sizes of measurement units within one system of units including ft, in; km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. (Conversions are limited to one-step conversions.) <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i></p>	<p><b>4.MD.A.1 4.DM.A.1:</b> Know relative sizes of measurement units within one system of units, including ft, in; km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec.</p> <p>a. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit.</p> <p>b. Record measurement equivalents in a two-column table. (Conversions are limited to one-step conversions.) <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i></p>
<p>4.MD.A.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving whole numbers and/or simple fractions (addition and subtraction of fractions with like denominators and multiplying a fraction times a fraction *3 or a whole number), and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p> <p>*3 Students in Grade 4 will be assessed on multiplying a fraction and a whole number as indicated in the NF domain. Some students may be able to multiply a fraction by a fraction as a result of generating equivalent fractions; however, mastery of multiplying two fractions occurs in Grade 5.</p>	<p><b>4.MD.A.2 4.DM.A.2:</b> Use the four operations to solve <u>multi-step real-world mathematical tasks</u> word problems involving:</p> <ul style="list-style-type: none"> <li>distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving whole numbers and/or simple fractions (addition and subtraction of fractions with like denominators and multiplying a fraction times a fraction *3 or a whole number), and</li> <li>problems that require expressing measurements given in a larger unit in terms of a smaller unit. <i>Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</i></li> </ul> <p>*3 Students in Grade 4 will be assessed on multiplying a fraction and a whole number as indicated in the NF domain. Some students may be able to multiply a fraction by a fraction as a result of generating equivalent fractions; however, mastery of multiplying two fractions occurs in Grade 5.</p>
<p>4.MD.A.3: Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the</i></p>	<p><b>4.MD.A.3 4.DM.A.3:</b> Apply the area and perimeter formulas for rectangles in <u>real world and mathematical problems</u> <u>real-world mathematical tasks</u>. <i>For example, find the width of a rectangular room</i></p>

Current Standard	Proposed Standard
<p>length, by viewing the area formula as a multiplication equation with an unknown factor.</p>	<p><del>given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</del></p>
<p><b>4.MD.B 4.DM.B:</b> Use place value understanding and properties of operations to perform multi-digit arithmetic.</p>	
<p>4.MD.B.4: Make a line plot to display a data set of measurements in fractions of a unit (<math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{1}{8}</math>). Solve problems involving addition and subtraction of fractions by using information presented in line plots. <i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p>	<p><b>4.MD.B.4 4.DM.B.4:</b> Make a line plot to display a data set of measurements in fractions of a unit (<math>\frac{1}{2}</math>, <math>\frac{1}{4}</math>, <math>\frac{1}{8}</math>).</p> <p>a. Solve problems <u>real-world mathematical tasks</u> involving addition and subtraction of fractions <u>with like denominators</u> by using information presented in line plots.</p> <p><i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p>
<p><b>4.MD.C 4.DM.C:</b> Geometric measurement: understand concepts of angle and measure angles.</p>	
<p>4.MD.C.5: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</p> <p>a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where two rays intersect the circle.</p> <p>b. An angle that turns through <math>\frac{1}{360}</math> of a circle is called a "one-degree angle," and can be used to measure angles.</p> <p>c. An angle that turns through <math>n</math> one-degree angles is said to have an angle measure of <math>n</math> degrees.</p>	<p><b>4.MD.C.5 4.DM.C.5:</b> Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</p> <p>a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where two rays intersect the circle.</p> <p>b. An angle that turns through <math>\frac{1}{360}</math> of a circle is called a "one-degree angle," and can be used to measure angles.</p> <p>c. An angle that turns through <math>n</math> one-degree angles is said to have an angle measure of <math>n</math> degrees.</p>
<p>4.MD.C.6: Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p>	<p><b>4.MD.C.6 4.DM.C.6:</b> Measure angles in whole-number degrees using a <u>standard <math>180^\circ</math> protractor</u>. Sketch angles of specified measure.</p>
<p>4.MD.C.7: Recognize angle measure as additive. When an angle</p>	<p><b>4.MD.C.7 4.DM.C.7:</b> Recognize angle measure as additive. When an angle</p>

Current Standard	Proposed Standard
<p>is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a letter for the unknown angle measure.</p>	<p>is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.</p> <p>a. Solve addition and subtraction <del>problems</del> <u>real-world mathematical tasks</u> to find unknown angles on a diagram <del>in real-world and mathematical problems</del>, e.g., by using an equation with a letter for the unknown angle measure.</p>
<p><b>4.MD.D 4.DM.D: Relate area to operations of multiplication and addition.</b></p>	
<p>4.MD.D.8: Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.</p>	<p><b>4.MD.D.8 4.DM.D.7:</b> Recognize area as additive. <del>Find areas of rectilinear figures by decomposing two dimensional composite figures whose sides meet at right angles into non-overlapping rectangles and squares, adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. Apply area formulas to find the area of each part, and use addition or subtraction to determine the total area of the composite figure.</del></p>

# Grade 5

## Grade Level Expected Foundational Skills

By the end of grade 5, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 5, but is comprised of the foundational skills required by the standards.

1. Evaluate and compare simple expressions.
2. Fluently multiply multi-digit whole numbers using a standard algorithm.
3. Divide multi-digit whole numbers with two-digit divisors.
4. Multiply a fraction by a whole number or a fraction.
5. Read and write decimals to the thousandths.
6. Compare and order whole numbers and decimals, identifying greater than, less than, or equal to.
7. Estimate and round multi-digit numbers with decimals to any place.
8. Add and subtract fractions with unlike denominators.

Current Standard	Proposed Standard
<p><b>5.NF.A 5.NOF.A: Use equivalent fractions as a strategy to add and subtract fractions.</b></p> <p>5.NF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, <math>2/3 + 5/4 = 8/12 + 15/12 = 23/12</math>. (In general, <math>a/b + c/d = (ad + bc)/bd</math>.)</p>	<p><b>5.NF.A.1 5.NOF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent equation with sum or difference of fractions that have with like denominators.</b></p> <ul style="list-style-type: none"><li>• For example, <math>\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}</math>. (In general, <math>\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}</math>.)</li></ul>

Current Standard	Proposed Standard
<p>5.NF.A.2: Solve word problems involving addition and subtraction of fractions.</p> <p>a. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.</p> <p>b. Use benchmark fractions and number sense of fractions to estimate mentally and justify the reasonableness of answers. <i>For example, recognize an incorrect result <math>2/5 + 1/2 = 3/7</math>, by observing that <math>3/7 &lt; 1/2</math>.</i></p>	<p><del>5.NF.A.2</del> <del>5.NOF.A.2: Solve word problems real-world mathematical tasks involving addition and subtraction of fractions.</del></p> <p>a. <del>Solve word problems involving addition and subtraction</del> <u>Add and subtract</u> <del>of</del> fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models, <u>number line diagrams</u>, or equations to represent the problem.</p> <p>b. Use benchmark fractions and number sense of fractions to estimate mentally and justify the reasonableness of answers. <i>For example, recognize an incorrect result <math>\frac{2}{5} + \frac{1}{2} = \frac{3}{7}</math>, by observing that <math>\frac{3}{7} &lt; \frac{1}{2}</math>.</i></p>
<p><b>5.NF.B 5.NOF.B: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.</b></p>	
<p>5.NF.B.3: Interpret a fraction as division of the numerator by the denominator (<math>a/b = a \div b</math>). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <i>For example, interpret <math>3/4</math> as the result of dividing 3 by 4, noting that <math>3/4</math> multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size <math>3/4</math>. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</i></p>	<p><del>5.NF.B.3</del> <del>5.NOF.B.3: Interpret a fraction as division of the numerator by the denominator (<math>\frac{a}{b} = a \div b</math>). Solve word problems real-world mathematical tasks involving division of whole numbers leading to answers in the form of fractions to include fractions greater than one or mixed numbers, e.g., by using visual fraction models, number line diagrams, or equations to represent the problem. For example, interpret <math>\frac{3}{4}</math> as the result of dividing 3 by 4, noting that <math>\frac{3}{4}</math> multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size <math>\frac{3}{4}</math>. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</del></p>

Current Standard	Proposed Standard
<p>5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p> <p>a. Interpret the product <math>(m/n) \times q</math> as <math>m</math> parts of a partition of <math>q</math> into <math>n</math> equal parts; equivalently, as the result of a sequence of operations, <math>m \times q \div n</math>. <i>For example, use a visual fraction model to show understanding, and create a story context for <math>(m/n) \times q</math>.</i></p> <p>b. Construct a model to develop understanding of the concept of multiplying two fractions and create a story context for the equation. [In general, <math>(m/n) \times (c/d) = (mc)/(nd)</math>.]</p> <p>c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.</p> <p>d. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</p>	<p><b>5.NF.B.4 5.NOF.B.4:</b> Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p> <p>a. Interpret the product <math>(\frac{m}{n}) \times q</math> as <math>m</math> parts of a partition of <math>q</math> into <math>n</math> equal parts; equivalently, as the result of a sequence of operations, <math>m \times q \div n</math>. <i>For example, use a visual fraction model or number line to show understanding, and create a story context for <math>(\frac{m}{n}) \times q</math>.</i></p> <p>b. Construct <u>or critique a precise</u> a-model to develop <u>an</u> understanding of the concept of multiplying two fractions and create a story context for the equation. [In general, <math>(\frac{m}{n}) \times (\frac{c}{d}) = \frac{(mc)}{(nd)}</math>.]</p> <p>c. <del>Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.</del></p> <p>d. <del>Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</del></p>
<p>5.NF.B.5: Interpret multiplication as scaling (resizing), by:</p> <p>a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.</p> <p>b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case).</p> <p>c. Explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.</p> <p>d. Relating the principle of fraction equivalence <math>a/b = (n \times a)/(n \times b)</math> to the effect of multiplying <math>a/b</math> by 1.</p>	<p><b>5.NF.B.5 5.NOF.B.5:</b> Interpret multiplication as scaling (resizing), by:</p> <p>a. Comparing the size of a product to the size of one factor <u>on the basis-based on</u> of the size of the other factor, without performing the indicated multiplication.</p> <p>b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number <u>(recognizing multiplication by whole numbers greater than 1 as a familiar case)</u>.</p> <p>c. Explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.</p> <p>d. Relating to the principle of fraction equivalence.</p>

Current Standard	Proposed Standard
5.NF.B.6: Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	5.NF.B.6 5.NOF.B.6: <u>Represent and solve real-world problems mathematical tasks</u> involving multiplication of fractions, <u>including fractions greater than 1</u> and mixed numbers, e.g., by using visual fraction models, <u>number line diagrams</u> , or equations to represent the problem.

Current Standard	Proposed Standard
<p>5.NF.B.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.*1</p> <p>a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. <i>For example, create a story context for <math>(1/3) \div 4</math>, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that <math>(1/3) \div 4 = 1/12</math> because <math>(1/12) \times 4 = 1/3</math>.</i></p> <p>b. Interpret division of a whole number by a unit fraction, and compute such quotients. <i>For example, create a story context for <math>4 \div (1/5)</math>, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that <math>4 \div (1/5) = 20</math> because <math>20 \times (1/5) = 4</math>.</i></p> <p>c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, how much chocolate will each person get if 3 people share <math>1/2</math> lb of chocolate equally? How many <math>1/3</math>-cup servings are in 2 cups of raisins?</i></p> <p>*1 Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.</p>	<p><b>5.NF.B.7 5.NOF.B.7:</b> Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions <u>in the context of real-world mathematical tasks, e.g., by using visual fraction models, number line diagrams, or equations to represent the problem.*1</u></p> <p>a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. <i>For example, create a story context for <math>(1/3) \div 4</math>, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that <math>(1/3) \div 4 = \frac{1}{12}</math> because <math>(\frac{1}{12}) \times 4 = \frac{1}{3}</math>.</i></p> <p>b. Interpret division of a whole number by a unit fraction, and compute such quotients. <i>For example, create a story context for <math>4 \div (1/5)</math>, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that <math>4 \div (1/5) = 20</math> because <math>20 \times (1/5) = 4</math>.</i></p> <p>c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, how much chocolate will each person get if 3 people share <math>1/2</math> lb of chocolate equally? How many <math>1/3</math>-cup servings are in 2 cups of raisins?</i></p> <p>*1 Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.</p>
<p><b>5.NBT.A 5.NOF.C: Understand the place value system.</b></p>	

Current Standard	Proposed Standard
5.NBT.A.1: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.	5.NBT.A.1-5.NOF.C.8: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.
5.NBT.A.4: Use place value understanding to round decimals to any place.	5.NBT.A.4 5.NOF.C.9: Use place value understanding to: <ul style="list-style-type: none"> <li>a. <u>Round multi-digit numbers with decimals to any place;</u> and</li> <li>b. <u>Use compatible numbers to estimate solutions to real-world mathematical tasks.</u></li> </ul>
<b>5.NBT.B: Perform operations with multi-digit whole numbers with decimals to hundredths.</b> <b>5.NOF.D: Multiply multi-digit numbers.</b>	
5.NBT.B.5: Fluently multiply multi-digit whole numbers using the standard algorithm.	5.NBT.B.5 5.NOF.D.10: Fluently multiply multi-digit whole numbers using <u>the</u> <u>a</u> standard algorithm.

## Numeracy & Operational FluencyAlgebraic Reasoning

Current Standard	Proposed Standard
<b>5.OA.A 5.AR.A: Write and interpret numerical expressions.</b>	
5.OA.A.1: Use parentheses or brackets in numerical expressions, and evaluate expressions with these symbols.	5.OA.A.1 5.AR.A.1: Use parentheses, brackets <u>or braces</u> in numerical expressions, and evaluate expressions with these symbols <u>attending to the order of operations and the properties of operations</u> . <ul style="list-style-type: none"> <li>a. <u>Compare two simple expressions using <math>&gt;</math>, <math>=</math>, or <math>&lt;</math> to record the comparison of expressions limited to three operations and one grouping symbol.</u></li> </ul>

Current Standard	Proposed Standard
<p>5.OA.A.2: Write simple expressions that record calculations with whole numbers, fractions, and decimals, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as <math>2 \times (8 + 7)</math>. Recognize that <math>3 \times (18,932 + 9.21)</math> is three times as large as <math>18,932 + 9.21</math>, without having to calculate the indicated sum or product.</i></p>	<p><b>5.OA.A.2 5.AR.A.2:</b> Write simple expressions that record calculations with whole numbers, fractions, and decimals, and interpret numerical expressions without evaluating them. <i>For example, express the calculation “add 8 and 7, then multiply by 2” as <math>2 \times (8 + 7)</math>. Recognize that <math>3 \times (18,932 + 9.21)</math> is three times as large as <math>18,932 + 9.21</math>, without having to calculate the indicated sum or product.</i></p>
<p><b>5.OA.B 5.AR.B: Analyze patterns and relationships: Compare whole numbers.</b></p>	
<p>5.OA.B.3: Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. <i>For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</i></p>	<p><b>5.OA.B.3 5.AR.B.3:</b> Generate and extend two numerical patterns using two given rules. Identify apparent relationships between corresponding terms.</p> <p>a. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. <i>For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences and observe that the terms in one sequence are twice the corresponding terms in the other sequence.</i></p> <p>b. Explain these relationships informally.</p>
<p><b>5.NBT.A: Understand the place value system.</b>  <b>5.AR.C: Use place value understanding to apply patterns beyond whole numbers.</b></p>	

Current Standard	Proposed Standard
<p>5.NBT.A.2: Explain and apply patterns in the number of zeros of the product when multiplying a number by powers of 10. Explain and apply patterns in the values of the digits in the product or the quotient, when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. <i>For example, <math>10^0 = 1</math>, <math>10^1 = 10</math> ... and <math>2.1 \times 10^2 = 210</math>.</i></p>	<p>5.NBT.A.2 <u>5.AR.C.4: Explain</u> <u>Construct a written explanation</u> and apply patterns in the number of zeros of the product when multiplying a number by powers of 10.</p> <p>a. Explain and apply patterns in the values of the digits in the product or the quotient; when a decimal is multiplied or divided by a power of 10.</p> <p>b. Use whole-number exponents to denote powers of 10. <i>For example, <math>10^0 = 1</math>, <math>10^1 = 10</math> ... and <math>2.1 \times 10^2 = 210</math>.</i></p>
<p>5.NBT.A.3: Read, write, and compare decimals to thousandths.</p> <p>a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., <math>347.392 = 3100 + 410 + 71 + 3\frac{1}{10} + 9\frac{1}{100} + 2\frac{1}{1000}</math>.</p> <p>b. Compare two decimals to thousandths based on the meanings of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p>	<p>5.NBT.A.3 <u>5.AR.C.5: Read, write, and compare decimals to thousandths.</u></p> <p><u>Using understanding of the base-ten system:</u></p> <p>a. Read and write decimals to thousandths using base-ten numerals/<u>standard form</u>, <u>written form (number names)</u>, <u>unit form</u> and expanded form, e.g., <math>347.392 = 3100 + 410 + 71 + 3\frac{1}{10} + 9\frac{1}{100} + 2\frac{1}{1000}</math>.</p> <p>b. Compare <u>and order two multi-digit whole numbers and decimals to thousandths</u> based on the <u>meanings values</u> of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p>
<p><b>5.NBT.B 5.AR.D: Perform operations with multi-digit whole numbers with decimals to hundredths.</b></p>	
<p>5.NBT.B.6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, subtracting multiples of the divisor, and/or the relationship between multiplication and division. Illustrate and/or explain the calculation by using equations, rectangular arrays, area models, or other strategies based on place value.</p>	<p>5.NBT.B.6 <u>5.AR.D.6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value <u>understanding</u>, the properties of operations, subtracting multiples of the divisor, and/or the relationship between multiplication and division.</u></p> <p>a. Illustrate and/or explain the calculation by using equations, rectangular arrays, area models, or other strategies based on place value <u>understanding</u>.</p>

Current Standard	Proposed Standard
<p>5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; justify the reasoning used with a written explanation.</p>	<p><b>5.NBT.B.7 5.AR.D.7:</b> Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value <u>understanding</u>, properties of operations, and/or the <u>part-whole</u> relationship between addition and subtraction <u>or</u> multiplication and division;  <u>a. Justify the reasoning of methods used for calculation with a written explanation.</u></p>

## Geometric Reasoning & Logic

Current Standard	Proposed Standard
<p><b>5.G.A.5.GL.A:</b> <del>Graph points on the coordinate plane to solve real-world mathematical problems. Generate and analyze patterns.</del></p> <p>5.G.A.1: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number in the ordered pair indicates how far to travel from the origin in the direction of one axis, and the second number in the ordered pair indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</p>	<p><b>5.G.A.1 5.GL.A.1:</b> <del>Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.</del></p> <ul style="list-style-type: none"> <li><u>Understand that in an ordered pair, the first number shows how far to move along the x-axis, and the second number shows how far to move along the y-axis from the origin.</u></li> </ul> <p><del>Understand that the first number in the ordered pair indicates how far to travel from the origin in the direction of one axis, and the second number in the ordered pair indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</del></p>

Current Standard	Proposed Standard
5.G.A.2: Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	5.G.A.2 <del>5.GL.A.2</del> : Represent real-world <del>and mathematical tasks</del> problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
<b><u>5.G.B. 5.GL.B:</u></b> Classify two-dimensional figures into categories based on their properties.	
5.G.B.3: Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <i>For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</i>	5.G.B.3 <del>5.GL.B.3</del> : <u>Analyze and relate</u> <del>Understand that</del> attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <i>For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</i>
5.G.B.4: Classify quadrilaterals in a hierarchy based on properties. (Students will define a trapezoid as a quadrilateral with at least one pair of parallel sides.)	5.G.B.4 <del>5.GL.B.4</del> : Classify quadrilaterals in a hierarchy based on properties. a. <u>Justify the reasoning for classification with a written response.</u> ( <i>Students will define a trapezoid as a quadrilateral with at least one pair of parallel sides.</i> )

## Data Analysis & Measurement

Current Standard	Proposed Standard
<b><u>5.MD.A 5.DM.A:</u></b> Convert like measurement units within a given measurement system.	

Current Standard	Proposed Standard
5.MD.A.1: Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world problems (e.g., convert 5 cm to 0.05 m; 9 ft to 108 in).	5.MD.A.1-5.DM.A.1: Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world <u>mathematical tasks</u> problems (e.g., convert 5 cm to 0.05 m; 9 ft to 108 in involving: <u>distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving whole numbers, decimals, and fractions.</u>
<b>5.MD.B 5.DM.B: Represent and interpret data. Compare whole numbers.</b>	
5.MD.B.2: Make a line plot to display a data set of measurements in fractions of a unit ( $1/2$ , $1/4$ , $1/8$ ). Use operations on fractions for this grade to solve problems involving information presented in line plots. <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i>	5.MD.B.2 5.DM.B.2: Make a line plot to display a data set of measurements in fractions of a unit ( $1/2$ , $1/4$ , $1/8$ , $\frac{1}{2}$ , $\frac{1}{4}$ , $\frac{1}{8}$ ). a. Use operations on fractions, <u>excluding dividing fractions by fractions, for this grade</u> to solve problems / <u>real-world mathematical tasks</u> involving information presented in line plots. <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i>
<b>5.MD.C 5.DM.C: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</b>	
5.MD.C.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure that can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.	5.MD.C.3 5.DM.C.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. b. A solid figure that can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.
5.MD.C.4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	5.MD.C.4 5.DM.C.4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

Current Standard	Proposed Standard
<p>5.MD.C.5: Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.</p> <p>a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</p> <p>b. Apply the formulas <math>V = l \times w \times h</math> and <math>V = B \times h</math> for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.</p> <p>c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.</p>	<p><b>5.MD.C.5 5.DM.C.5:</b> Relate volume to the operations of multiplication and addition and solve real-world and mathematical <u>tasks</u> problems involving volume.</p> <p>a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and <del>show</del> <u>connect</u> that the volume is the same as <del>would be found by</del> multiplying the edge lengths, <u>equivalently also</u> by multiplying the height by the area of the base (i.e., <u>compute the layers of area</u>).</p> <ul style="list-style-type: none"> <li>• Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</li> </ul> <p>b. Apply the formulas <math>V = l \times w \times h</math> and <math>V = B \times h</math> for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical <u>tasks</u> problems.</p> <p>c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world <u>mathematical problemstasks</u>.</p>
<p><b>5.DM.D: Geometric measurement: extend previous understandings of area and multiplication to multiply fractions.</b></p>	
<p>5.NF.B.4: c. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.</p> <p>d. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</p>	<p><b>5.NF.B.4 5.DM.D.6:</b> Apply and extend previous understandings of area and multiplication to multiply a fraction or whole number by a fraction.</p> <p>a. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.</p> <p>b. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</p> <p>c. Compose and decompose rectangular regions to calculate area.</p>

# Grade 6

## Grade Level Expected Foundational Skills

By the end of grade 6, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 6, but is comprised of the foundational skills required by the standards.

1. Fluently divide multi-digit numbers using standard algorithms.
2. Divide fractions by fractions.
3. Add, subtract, multiply, and divide multi-digit decimals using standard algorithms.
4. Determine the unknown number in an addition or subtraction equation.
5. Determine the unknown number in a multiplication and division equation.
6. Evaluate expressions with positive, whole-number exponents.

## Numeracy & Operational Fluency

Current Standard	Proposed Standard
<b>6.NS.A <u>6.NOF.A:</u> Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</b>	

Current Standard	Proposed Standard
<p>6.NS.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.</p> <p>For example, create a story context for <math>(2/3) \div (3/4)</math> and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that <math>(2/3) \div (3/4) = 8/9</math> because <math>3/4</math> of <math>8/9</math> is <math>2/3</math>. (In general, <math>(a/b) \div (c/d) = ad/bc</math>.) How much chocolate will each person get if 3 people share <math>1/2</math> lb of chocolate equally? How many <math>3/4</math>- cup servings are in <math>2/3</math> of a cup of yogurt? How wide is a rectangular strip of land with length <math>3/4</math> mi and area <math>1/2</math> square mi?</p>	<p>6.NS.A.1 6.NOF.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.</p> <ul style="list-style-type: none"> <li>For example, <ul style="list-style-type: none"> <li>create a story context for <math>(2/3) \div (3/4)</math> and <u>use illustrate the quotient by using a visual fraction model to show the quotient</u>;</li> <li>use the relationship between multiplication and division to explain that <math>(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}</math> because <math>\frac{3}{4}</math> of <math>\frac{8}{9}</math> is <math>\frac{2}{3}</math>.</li> <li>In general, <math>(\frac{a}{b}) \div (\frac{c}{d}) = (\frac{ad}{bc})</math>.</li> </ul> </li> </ul> <ol style="list-style-type: none"> <li>How much chocolate will each person get if 3 people share <math>1/2</math> lb of chocolate equally, how much will each person get?</li> <li>How many <math>3/4</math>- cup servings are in <math>2/3</math> of a cup of yogurt?</li> <li>What is the width of a rectangular strip of land with a length of <math>3/4</math> mile and an area of <math>1/2</math> square mile?</li> <li>—</li> </ol>
<p><b>6.NS.B.6.NOF.B:</b> Compute fluently with multi-digit numbers and find common factors and multiples.</p>	
<p>6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.</p>	<p>6.NS.B.2 6.NOF.B.2: Fluently divide <u>positive</u> multi-digit <u>whole</u> numbers using <u>the</u> standard algorithm.</p>
<p>6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</p>	<p>6.NS.B.3 6.NOF.B.3: Fluently add, subtract, multiply, and divide <u>positive</u> multi-digit decimals <u>to the thousandths</u> using <u>the</u> standard algorithm for each operation.</p>

Current Standard	Proposed Standard
<p>6.NS.B.4: Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express <math>36 + 8</math> as <math>4(9 + 2)</math>.</p>	<p><b>6.NS.B.4-6.NOF.B.4:</b> Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.</p> <ul style="list-style-type: none"> <li>For example, express <math>36 + 8</math> as <math>4(9 + 2)</math> because the greatest common factor of 36 and 8 is 4.</li> </ul>
<p><b>6.NS.C 6.NOF.C: Apply and extend previous understandings of the system of rational numbers.</b></p>	
<p>6.NS.C.5: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p>	<p><b>6.NS.C.5-6.NOF.C.5:</b> Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p>
<p>6.NS.C.6: Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., <math>-( -3 ) = 3</math>, and that 0 is its own opposite.</p> <p>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that</p>	<p><b>6.NS.C.6-6.NOF.C.6:</b> Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., <math>-( -3 ) = 3</math>, and that 0 is its own opposite.</p> <p>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered</p>

Current Standard	Proposed Standard
<p>when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p>	<p>pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p>
<p><b>6.NS.C.7:</b> Understand ordering and absolute value of rational numbers.</p> <p>a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret <math>-3 &gt; -7</math> as a statement that <math>-3</math> is located to the right of <math>-7</math> on a number line oriented from left to right.</p> <p>b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write <math>-3^{\circ}\text{C} &gt; -7^{\circ}\text{C}</math> to express the fact that <math>-3^{\circ}\text{C}</math> is warmer than <math>-7^{\circ}\text{C}</math>.</p> <p>c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of <math>-30</math> dollars, write <math> -30  = 30</math> to describe the size of the debt in dollars.</p> <p>d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than <math>-30</math> dollars represents a debt greater than <math>30</math> dollars.</p>	<p><b>6.NS.C.7 6.NOF.C.7:</b> Understand ordering and absolute value of rational numbers.</p> <p>a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <del>For example, interpret <math>-3 &gt; -7</math> as a statement that <math>-3</math> is located to the right of <math>-7</math> on a number line oriented from left to right.</del></p> <p>b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. <del>For example, write <math>-3^{\circ}\text{C} &gt; -7^{\circ}\text{C}</math> to express the fact that <math>-3^{\circ}\text{C}</math> is warmer than <math>-7^{\circ}\text{C}</math>.</del></p> <p>c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <del>For example, for an account balance of <math>-30</math> dollars, write <math> -30  = 30</math> to describe the size of the debt in dollars.</del></p> <p>d. Distinguish comparisons of absolute value from statements about order. <del>For example, recognize that an account balance less than <math>-30</math> dollars represents a debt greater than <math>30</math> dollars.</del></p>

Current Standard	Proposed Standard
<p>6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>	<p><b>6.NS.C.8</b> <b>6.NOF.C.8:</b> Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>

## Algebraic Reasoning

Current Standard	Proposed Standard
<p><b>6.EE.A</b> <b>6.AR.A:</b> Apply and extend previous understandings of arithmetic to algebraic expressions.</p>	
<p>6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.</p>	<p><b>6.EE.A.1</b> <b>6.AR.A.1:</b> Understand, write, and evaluate numerical expressions involving whole-number exponents.</p> <ol style="list-style-type: none"> <li data-bbox="1051 878 2014 943">Identify parts of exponential notation using mathematical terms (base and exponent).</li> <li data-bbox="1051 943 2014 1046">Represent and evaluate powers using a whole number or fraction as the base and a whole number as an exponent, and evaluate numerical expressions.</li> </ol>

Current Standard	Proposed Standard
<p>6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers.</p> <p>a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract <math>y</math> from 5” as <math>5 - y</math>.</p> <p>b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. For example, describe the expression <math>2(8+7)</math> as a product of two factors; view <math>(8+7)</math> as both a single entity and a sum of two terms.</p> <p>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas <math>V=s^3</math> and <math>A=6 s^2</math> to find the volume and surface area of a cube with sides of length <math>s=1/2</math>.</p>	<p>6.EE.A.2 6.AR.A.2: Write, read, and evaluate expressions in which letters stand for numbers.</p> <p>a. Write expressions that record operations with numbers and with letters standing for numbers.</p> <ul style="list-style-type: none"> <li>For example, express the calculation “Subtract <math>y</math> from 5” as <math>5 - y</math>.</li> </ul> <p>b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity.</p> <ul style="list-style-type: none"> <li>For example, describe the expression <math>2(8+7)</math> as a product of two factors; view <math>(8+7)</math> as both a single entity and a sum of two terms.</li> </ul> <p>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).</p> <ul style="list-style-type: none"> <li>For example, use the formulas <math>V=s^3</math> and <math>A=6 s^2</math> to find the volume and surface area of a cube with sides of length <math>s=\frac{1}{2}</math>.</li> </ul>

Current Standard	Proposed Standard
<p>6.EE.A.3: Apply the properties of operations to generate equivalent expressions.</p> <p>For example, apply the distributive property to the expression <math>3(2 + x)</math> to produce the equivalent expression <math>6 + 3x</math>; apply the distributive property to the expression <math>24x + 18y</math> to produce the equivalent expression <math>6(4x + 3y)</math>; apply properties of operations to <math>y + y + y</math> to produce the equivalent expression <math>3y</math>.</p>	<p><b>6.EE.A.3 and 6.EE.A.4</b> 6.AR.A.3: Apply the properties of operations to generate equivalent expressions, identify when two expressions are equivalent, and explain why two expressions are equivalent.</p> <ul style="list-style-type: none"> <li>For example, we can apply the distributive property to the expression <math>3(2 + x)</math> to produce the equivalent expression <math>6 + 3x</math>; apply the distributive property to the expression <math>24x + 18y</math> to produce the equivalent expression <math>6(4x + 3y)</math>; and apply properties of operations to <math>y + y + y</math> to produce the equivalent expression <math>3y</math>.</li> </ul>
<p><b>6.EE.B 6.AR.B: Reason about and solve one-variable equations and inequalities.</b></p>	
<p>6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</p>	<p><b>6.EE.B.5 6.AR.B4:</b> Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</p>
<p>6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p>	<p><b>6.EE.B.6 6.AR.B.5:</b> Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p>
<p>6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations and inequalities of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math> and <math>x</math> are all nonnegative rational numbers. Inequalities will include <math>&gt;</math>, <math>&lt;</math>, <math>\leq</math>, and <math>\geq</math>.</p>	<p><b>6.EE.B.7 6.AR.B.6:</b> Solve real-world and mathematical problems by writing and solving equations and inequalities of the form <math>x + p = q</math> and <math>px = q</math> for cases in which <math>p</math>, <math>q</math> and <math>x</math> are all nonnegative rational numbers. Inequalities will include <math>&gt;</math>, <math>&lt;</math>, <math>\leq</math>, and <math>\geq</math>.</p>

Current Standard	Proposed Standard
<p>6.EE.B.8: Write an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form <math>x &gt; c</math> or <math>x &lt; c</math> have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</p>	<p><del>6.EE.B.8 6.AR.B.7: Write an inequality of the form <math>x &gt; c</math> or <math>x &lt; c</math> to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form <math>x &gt; c</math> or <math>x &lt; c</math> have infinitely many solutions; represent solutions of such inequalities on number line diagrams. Translate a real-world written description into an algebraic inequality in the form of <math>x &gt; c</math> or <math>x &lt; c</math>. Describe and interpret the infinitely many solutions for <math>x &gt; c</math> or <math>x &lt; c</math> and graph the solutions on a number line.</del></p>
<p>6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.</p> <p>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation <math>d = 65t</math> to represent the relationship between distance and time.</p>	<p><del>6.EE.C.9 6.AR.C.8: Use variables to represent two quantities in a real-world problem that change in relationship to one another.</del></p> <ul style="list-style-type: none"> <li>• For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation <math>d = 65t</math> to represent the relationship between distance and time.</li> <li>a. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.</li> <li>• Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.</li> </ul>

## Geometric Reasoning & Logic

Current Standard	Proposed Standard
<b><u>6.G.A 6.GL.A:</u></b> Solve real-world and mathematical problems involving area, surface area, and volume.	
6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	6.G.A.1 <u>6.GL.A.1:</u> Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
6.G.A.2: Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volume of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	6.G.A.2 <u>6.GL.A.2:</u> Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ and $V = lwh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
6.G.A.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	6.G.A.3 <u>6.GL.A.3:</u> Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	6.G.A.4 <u>6.GL.A.4:</u> Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

## Data Analysis

Current Standard	Proposed Standard
<b>6.SP.A.6.DA.A:</b> Develop understanding of statistical variability.	
6.SP.A.1: Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.	6.SP.A.1 <b>6.DA.A.1</b> Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. <ul style="list-style-type: none"><li>For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.</li></ul>
6.SP.A.2: Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	6.SP.A.2 <b>6.DA.A.2:</b> Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6.SP.A.3: Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	6.SP.A.3 <b>6.DA.A.3:</b> Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
<b>6.SP.B.6.DA.B:</b> Summarize and describe distributions.	
6.SP.B.4: Display numerical data in plots on a number line, including dot plots, histograms, and box plots	6.SP.B.4 <b>6.DA.B.4:</b> Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Current Standard	Proposed Standard
<p>6.SP.B.5: Summarize numerical data sets in relation to their context, such as by:</p> <ul style="list-style-type: none"> <li>a. Reporting the number of observations.</li> <li>b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.</li> <li>c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</li> <li>d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data</li> </ul>	<p><b>6.SP.B.5-6.DA.B.5:</b> Summarize numerical data sets in relation to their context, such as by:</p> <ul style="list-style-type: none"> <li>a. Reporting the number of observations.</li> <li>b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.</li> <li>c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</li> <li>d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data</li> </ul>

## Proportionality and Functions

Current Standard	Proposed Standard
<p><b>6.RP.A <u>6.PF.A:</u> Understand ratio concepts and use ratio reasoning to solve problems.</b></p>	
<p>6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</p> <p>For example, “The ratio of wings to beaks in the birdhouse at the zoo was 2:1 because, for every 2 wings, there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</p>	<p><b>6.RP.A.1 <u>6.PF.A.1:</u> Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities; <u>use appropriate notation <math>a:b</math>, <math>a</math> to <math>b</math>, where <math>b \neq 0</math>.</u></b></p> <ul style="list-style-type: none"><li>For example,<ul style="list-style-type: none"><li>The ratio of wings to beaks in the birdhouse at the zoo was 2:1 (<u>2 to 1</u>) because, for every 2 wings, there was 1 beak.</li><li>For every vote candidate A received, candidate C received nearly three votes (<u>A:C or A to C</u>).</li></ul></li></ul>
<p>6.RP.A.2: Understand the concept of a unit rate <math>a/b</math> associated with a ratio <math>a:b</math> with <math>b \neq 0</math>, and use rate language in the context of a ratio relationship.</p> <p>For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is <math>\frac{3}{4}</math> cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.” *</p> <p>*Expectations for unit rates in this grade are limited to non-complex fractions.</p>	<p><b>6.RP.A.2 <u>6.PF.A.2:</u> Understand the concept of a unit rate <math>a/b</math> associated with a ratio <math>a:b</math> <u>with where</u> <math>b \neq 0</math>, and use rate language in the context of a ratio relationship. <u>Expectations for unit rates in this grade are limited to non-complex fractions.</u></b></p> <ul style="list-style-type: none"><li>For example,<ul style="list-style-type: none"><li>This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is <math>\frac{3}{4}</math> cup of flour for each cup of sugar.</li><li>We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger. <math>\ddagger</math></li></ul></li></ul> <p><b><del>*Expectations for unit rates in this grade are limited to non-complex fractions.</del></b></p>

Current Standard	Proposed Standard
<p>6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p> <p>b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what unit rate were lawns being mowed?</p> <p>c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means <math>30/100</math> times the quantity); solve problems involving finding the whole, given a part and the percent.</p> <p>d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p>	<p><b>6.RP.A.3 6.PF.A.3:</b> Use <u>and apply</u> ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p> <p>b. Solve unit rate problems, including those involving unit pricing and constant speed.</p> <ul style="list-style-type: none"> <li>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what unit rate were lawns being mowed?</li> </ul> <p>c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means <math>30/100</math> times the quantity); solve problems involving finding the whole, given a part, and the percent.</p> <p>d. Use ratio reasoning to convert measurement units <u>within and between the U.S. customary and metric systems</u>; manipulate and transform units appropriately when multiplying or dividing quantities.</p>

# Grade 7

## Grade Level Expected Foundational Skills

By the end of grade 7, mathematically proficient students can reliably use all prior foundational skills and the skills foundational to grade 7 to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 7, but consists of the foundational skills required by the standards.

1. Fluently add, subtract, multiply, and divide positive and negative rational numbers in the form of whole numbers, fractions, and decimals.
2. Accurately convert a rational number to a decimal, recognizing the decimal form of rational numbers.
3. Use properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple sets of grouping symbols.
4. Fluently solve one-variable equations.

## Numeracy & Operational Fluency

Current Standard	Proposed Standard
<b>7.NS.A 7.NOF.A:</b> Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	

Current Standard	Proposed Standard
<p>7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.</p> <p>b. Understand <math>p + q</math> as the number located a distance <math> q </math> from <math>p</math>, in the positive or negative direction depending on whether <math>q</math> is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.</p> <p>c. Understand subtraction of rational numbers as adding the additive inverse, <math>p - q = p + (-q)</math>. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.</p> <p>d. Apply properties of operations as strategies to add and subtract rational numbers.</p>	<p>7.NS.A.1-7.NOF.A.1: Apply and extend previous understandings of addition and subtraction <u>and of fractions</u> to add and subtract rational numbers <u>flexibly and accurately</u>; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>a. Describe situations in which opposite quantities combine to make 0.</p> <ul style="list-style-type: none"> <li>For example, a hydrogen atom has a <u>net charge of 0 charge</u> because its two <u>constituents are oppositely charged components carry opposite charges</u>.</li> </ul> <p>b. Understand <math>p + q</math> as the number located a distance <math> q </math> from <math>p</math>, in the positive or negative direction depending on whether <math>q</math> is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.</p> <p>c. Understand subtraction of rational numbers as adding the additive inverse, <math>p - q = p + (-q)</math>. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.</p> <p>d. Apply properties of operations as strategies to add and subtract rational numbers.</p>

Current Standard	Proposed Standard
<p>7.NS.A.2: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.</p> <p>a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as <math>(-1)(-1) = 1</math> and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.</p> <p>b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If <math>p</math> and <math>q</math> are integers, then <math>-(p/q) = (-p)/q = p/(-q)</math>. Interpret quotients of rational numbers by describing real-world contexts.</p> <p>c. Apply properties of operations as strategies to multiply and divide rational numbers.</p> <p>d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</p>	<p><b>7.NS.A.2-7.NOF.A.2:</b> Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers <u>flexibly and accurately</u>.</p> <p>a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as <math>(-1)(-1) = 1</math> and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.</p> <p>b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If <math>p</math> and <math>q</math> are integers, then <math>-(p/q) = (-p)/q = p/(-q)</math>. Interpret quotients of rational numbers by describing real-world contexts.</p> <p>c. Apply properties of operations as strategies to multiply and divide rational numbers.</p> <p>d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</p>
<p>7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers.*</p> <p>*Computations with rational numbers extend the rules for manipulating fractions to complex fractions.</p>	<p><b>7.NS.A.3 7.NOF.A.3:</b> Solve real-world and mathematical problems involving the four operations with rational numbers.<sup>7</sup></p> <p><sup>7</sup>Computations with rational numbers extend the rules for manipulating fractions to complex fractions.</p>

## Algebraic Reasoning

Current Standard	Proposed Standard
<b><u>7.EE.A-7.AR.A: Use properties of operations to generate equivalent expressions.</u></b>	
7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple grouping symbols (e.g., parentheses, brackets, and braces).	7.EE.A.1 <u>7.AR.A.1</u> : Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with <u>positive and negative</u> rational coefficients to include multiple grouping symbols (e.g., parentheses, brackets, and braces).
7.EE.A.2: Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.  For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”	7.EE.A.2 <u>7.AR.A.2</u> : Understand that rewriting an expression in different forms in a problem context <u>can help reveal relationships between</u> <u>shed light on the problem and how the quantities in it are related</u> <u>the context of a problem</u> .  • For example, $a + 0.05a$ <u>is equivalent</u> to $1.05a$ , <u>which means to</u> “increase by 5%” is the same as “multiply by 1.05.”
<b><u>7.EE.B-7.AR.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</u></b>	

Current Standard	Proposed Standard
<p>7.EE.B.4: Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.</p> <p>Example: If a woman making \$25 an hour gets a 10 percent raise, she will make an additional <math>\frac{1}{10}</math> of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar <math>9\frac{3}{4}</math> inches long in the center of a door that is <math>27\frac{1}{2}</math> inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</p>	<p><b>7.EE.B.4 7.AR.B.3:</b> Solve <u>multi-step real-life</u> and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. <u>For example:</u></p> <ul style="list-style-type: none"> <li>• If a woman making \$25 an hour gets a 10% <u>percent</u>-raise, she will make an additional <math>1\frac{1}{10} \frac{1}{10}</math> of her salary an hour, or \$2.50, for a new salary of \$27.50.</li> <li>• If you want to place a towel bar <math>9\frac{3}{4}</math> <u><math>3\frac{3}{4}</math></u> inches long in the center of a door that is <math>27\frac{1}{2}</math> <u><math>1\frac{1}{2}</math></u> inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</li> </ul>

Current Standard	Proposed Standard
<p>7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p>a. Solve word problems leading to equations of the form <math>px+q=r</math> and <math>p(x+q)=r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.</p> <p>For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</p> <p>b. Solve word problems leading to inequalities of the form <math>px+q&gt;r</math>, <math>px + q \geq r</math>, <math>px + q &lt; r</math>, or <math>px + q \leq r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.</p> <p>For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.</p>	<p><del>7.EE.B.4</del> 7.AR.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p>a. Solve word problems leading to equations of the form <math>px+q=r</math> and <math>p(x+q)=r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.</p> <ul style="list-style-type: none"> <li>For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</li> </ul> <p>b. Solve word problems leading to inequalities of the form <math>px+q&gt;r</math>, <math>px + q \geq r</math>, <math>px + q &lt; r</math>, or <math>px + q \leq r</math>, where <math>p</math>, <math>q</math>, and <math>r</math> are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.</p> <ul style="list-style-type: none"> <li>For example, as a salesperson, you are paid \$50 per week plus \$3 per sale. This week, you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.</li> </ul>

## Geometric Reasoning & Logic

Current Standard	Proposed Standard
<p><b><u>7.G.A-7.GL.A:</u></b> Draw, construct, and describe geometrical figures and describe the relationships between them.</p>	
7.G.A.1: Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	7.G.A.1 <u>7.GL.A.1:</u> Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
7.G.A.2: Draw (freehand, with ruler and protractor, or with technology) geometric shapes with given conditions. (Focus is on triangles from three measures of angles or sides, noticing when the conditions determine one and only one triangle, more than one triangle, or no triangle.)	<del>7.G.A.2 <u>7.GL.A.2:</u> Draw (freehand, with ruler and protractor, or with technology) geometric shapes with given conditions. (Focus is on triangles from three measures of angles or sides, noticing when the conditions <u>Construct triangles with given conditions. Understand the possible side lengths and angle measures that determine one and only one triangle, more than one triangle, or no triangle.</u>)</del>
7.G.A.3: Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	<del>7.G.A.3 <u>7.GL.A.3:</u> Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</del>
<p><b><u>7.G.B-7.GL.B:</u></b> Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</p>	
7.G.B.4: Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	7.G.B.4 <u>7.GL.B.4:</u> Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
7.G.B.5: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	7.G.B.5 <u>7.GL.B.5:</u> Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Current Standard	Proposed Standard
7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (Pyramids limited to surface area only).	7.G.B.6 <del>7.GL.B.6</del> : Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (Pyramids limited to surface area only).

## Data Analysis

Current Standard	Proposed Standard
<p><b>7.SP.A.7.DA.A:</b> Use <del>r</del>andom sampling to draw inferences about a population.</p>	
<p>7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</p> <p>7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</p>	<p>7.SP.A.1 <del>7.DA.A.1</del>: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</p> <p>7.SP.A.2 <del>7.DA.A.2</del>: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.</p> <ul style="list-style-type: none"> <li>For example: <ul style="list-style-type: none"> <li>Estimate the mean word length in a book by randomly sampling words from the book.</li> <li>Predict the winner of a school election based on randomly sampled survey data.</li> </ul> </li> </ul> <p><del>Gauge how far off the estimate or prediction might be.</del></p>

Current Standard	Proposed Standard
<p><b><u>7.SP.B.7. DA.B: Draw informal comparative inferences about two populations.</u></b></p>	
<p>7.SP.B.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities using quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</p>	<p><b><u>7.SP.B.3-7. DA.B.3:</u></b> Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities using quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</p>
<p>7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.</p>	<p><b><u>7.SP.B.4 7.DA.B.4:</u></b> Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.</p> <ul style="list-style-type: none"> <li>For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.</li> </ul>
<p><b><u>7.SP.C 7.DA.C: Investigate chance processes and develop, use, and evaluate probability models with and without technology.</u></b></p>	
<p>7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around <math>\frac{1}{2}</math> indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</p>	<p><b><u>7.SP.C.5 7.DA.C.5:</u></b> Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.</p> <ul style="list-style-type: none"> <li>For example: <ul style="list-style-type: none"> <li>A probability near 0 indicates an unlikely event,</li> <li>a probability around <math>\frac{1}{2}</math> indicates an event that is neither unlikely nor likely; and</li> <li>a probability near 1 indicates a likely event.</li> </ul> </li> </ul>

Current Standard	Proposed Standard
<p>7.SPC.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</p>	<p><b>7.SPC.6 7.DA.C.6:</b> Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.</p> <ul style="list-style-type: none"> <li>For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</li> </ul>
<p>7.SPC.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</p> <p>a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</p> <p>b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</p>	<p><b>7.SPC.7 7.DA.C.7:</b> Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.</p> <ul style="list-style-type: none"> <li>Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.</li> <li>For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.</li> <li>Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.</li> <li>For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?</li> </ul>

Current Standard	Proposed Standard
<p>7.SPC.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</p> <p>a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</p> <p>b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.</p> <p>c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?</p>	<p><b>7.SPC.8 7.DA.C.8:</b> Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</p> <p>a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</p> <p>b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.</p> <p>c. Design and use a simulation to generate frequencies for compound events.</p> <ul style="list-style-type: none"> <li>For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?</li> </ul>

## Proportionality and Functions

Current Standard	Proposed Standard
<b><u>7.RP.A: 7.PF.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.</u></b>	
<p>7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. For example, if a person walks <math>\frac{1}{2}</math> mile in each <math>\frac{1}{4}</math> hour, compute the unit rate as the complex fraction <math>\frac{1/2}{1/4}</math> miles per hour, equivalently 2 miles per hour.</p>	<p><b>7.RP.A.1 7.PF.A.1:</b> Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.</p> <ul style="list-style-type: none"><li>For example, if a person walks <math>\frac{1}{2}</math> mile in each <math>\frac{1}{4}</math> hour, compute the unit rate as the complex fraction <math>\frac{1/2}{1/4}</math> miles per hour, equivalently 2 miles per hour.</li></ul>
<p>7.RP.A.2: Recognize and represent proportional relationships between quantities.</p> <p>a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</p> <p>b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>c. Represent proportional relationships by equations. For example, if total cost <math>t</math> is proportional to the number <math>n</math> of items purchased at a constant price <math>p</math>, the relationship between the total cost and the number of items can be expressed as <math>t = pn</math>.</p> <p>d. Explain what a point <math>(x,y)</math> on the graph of a proportional relationship means in terms of the situation, with special attention to the points <math>(0,0)</math> and <math>(1, r)</math> where <math>r</math> is the unit rate.</p>	<p><b>7.RP.A.2 7.PF.A.2:</b> Recognize and represent proportional relationships between quantities.</p> <ol style="list-style-type: none"><li>Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</li><li>Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</li><li>Represent proportional relationships by equations.</li></ol> <ul style="list-style-type: none"><li>For example, if total cost <math>t</math> is proportional to the number <math>n</math> of items purchased at a constant price <math>p</math>, the relationship between the total cost and the number of items can be expressed as <math>t = pn</math>.</li></ul> <p>d. Explain what a point <math>(x,y)</math> on the graph of a proportional relationship means in terms of the situation, with special attention to the points <math>(0,0)</math> and <math>(1, r)</math> where <math>r</math> is the unit rate.</p>

Current Standard	Proposed Standard
7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems of simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.	7.RP.A.3 <u>7.PF.A.3</u> : Use proportional relationships to <u>reason and</u> solve multistep ratio and percent problems of simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.

# Grade 8

## Grade Level Expected Foundational Skills

By the end of grade 8, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 8, but is comprised of the foundational skills required by the standards.

1. Identify irrational numbers, recognizing the decimal patterns indicating a decimal represents a rational number.
2. Compare the magnitude of irrational numbers using approximations.
3. Fluently apply the properties of integer exponents.
4. Fluently solve simple cube and square root equations.
5. Write and perform operations with numbers written in scientific notation.
6. Solve linear equations algebraically or through graphing.
7. Solve systems of linear equations.
8. Construct a function to model a linear relationship.
9. Use transformation to discuss similarity and congruence.
10. Apply the Pythagorean Theorem to determine distance.
11. Find the volume of cones, cylinders, and spheres.

## Numeracy & Operational Fluency

Current Standard	Proposed Standard
<b>8.NS.A 8. NOF.A:</b> Know that there are numbers that are not rational, and approximate them by rational numbers.	

Current Standard	Proposed Standard
<p>8.NS.A.1: Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually. Convert a decimal expansion which repeats eventually into a rational number by analyzing repeating patterns.</p>	<p><u>8.NS.A.1 8. NOF.A.1</u> Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually. Convert a decimal expansion which repeats eventually into a rational number by analyzing repeating patterns. <u>Understand the real number system.</u></p> <ol style="list-style-type: none"> <li><u>Distinguish between rational and irrational numbers (e.g., know that <math>\sqrt{2}</math> is irrational.)</u></li> <li>Understand informally that every number has a decimal expansion.</li> <li><del>f</del>For rational numbers, show that the decimal expansion repeats eventually.</li> <li>Convert a decimal expansion that repeats eventually into a rational number by analyzing repeating patterns.</li> </ol>
<p>8.NS.A.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., <math>\pi^2</math>). For example, by truncating the decimal expansion of <math>\sqrt{2}</math>, show that <math>\sqrt{2}</math> is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximation to the hundredths place.</p>	<p><u>8.NS.A.2-8. NOF.A.2</u> Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., <math>\pi^2</math>).</p> <ul style="list-style-type: none"> <li>For example, by truncating the decimal expansion of <math>\sqrt{2}</math>, show that <math>\sqrt{2}</math> is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximation to the hundredths place.</li> </ul>

## Algebraic Reasoning

Current Standard	Proposed Standard
<p><u>8.EE.A 8.AR.A:</u> Work with radicals and integer exponents.</p>	

Current Standard	Proposed Standard
8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .	<p><b>8.EE.A.1</b> <b>8.AR.A.1</b> Know and apply the properties of integer exponents to generate equivalent numerical expressions.</p> <ul style="list-style-type: none"> <li>For example, <math>3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27</math>.</li> </ul>
8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ , where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	<p><b>8.EE.A.2</b> <b>8.AR.A.2</b> Use square root and cube root symbols to represent solutions to equations, <del>of</del> <u>in</u> the form <math>x^2 = p</math> and <math>x^3 = p</math>, <u>using the square root and cube root symbols, and determine if the solution is rational or irrational</u> <del>the form <math>x^2 = p</math> and <math>x^3 = p</math>, where <math>p</math> is a positive rational number</del>. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. <u>Know that <math>\sqrt{2}</math> is irrational</u>.</p>
8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$ , and determine that the world population is more than 20 times larger.	<p><b>8.EE.A.3</b> <b>8.AR.A.3</b> Use numbers expressed in <del>the form of a single digit times an integer power of 10</del> <u>scientific notation</u> to estimate very large or very small quantities and to express how many times larger <del>as much one is than the other or smaller one number is compared to another</del>.</p> <ul style="list-style-type: none"> <li>For example, estimate the population of the United States as <math>3 \times 10^8</math> and the population of the world as <math>7 \times 10^9</math>, and determine that the world population is more than 20 times larger.</li> </ul>
8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	<p><b>8.EE.A.4</b> <b>8.AR.A.4</b>: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</p>
<b>8.EE.B</b> <b>8.AR.B</b> : Understand the connections between proportional relationships, lines, and linear equations.	

Current Standard	Proposed Standard
<p>8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.</p> <p>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</p>	<p><b>8.EE.B.5 8.AR.B.5:</b> Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.</p> <ul style="list-style-type: none"> <li>For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</li> </ul>
<p>8.EE.B.6: Use similar triangles to explain why the slope <math>m</math> is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation <math>y = mx</math> for a line through the origin and the equation <math>y = mx + b</math> for a line intercepting the vertical axis at <math>b</math>.</p>	<p><b>8.EE.B.6 8.AR.B.6:</b> Use similar triangles to explain why the slope <math>m</math> is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation <math>y = mx</math> for a line through the origin and the equation <math>y = mx + b</math> for a line intercepting the vertical axis at <math>b</math>.</p>
<p><b>8.EE.C 8.AR.C: Analyze and solve linear equations and pairs of simultaneous linear equations.</b></p>	
<p>8.EE.C.7: Solve linear equations in one variable.</p> <p>a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form <math>x = a</math>, <math>a = a</math>, or <math>a = b</math> results (where <math>a</math> and <math>b</math> are different numbers).</p> <p>b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</p>	<p><b>8.EE.C.7 8.AR.C.7:</b> Solve linear equations in one variable.</p> <ol style="list-style-type: none"> <li>Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form <math>x = a</math>, <math>a = a</math>, or <math>a = b</math> results (where <math>a</math> and <math>b</math> are different numbers).</li> <li>Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</li> </ol>

## Geometric Reasoning & Logic

Current Standard	Proposed Standard
<b>8.G.A.8.GL.A:</b> Understand congruence and similarity using physical models, transparencies, or geometry software.	
8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations:  a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	8.G.A.1 8.GL.A.1: Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
8.G.A.2: Explain that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Rotations are only about the origin and reflections are only over the y-axis and x-axis in grade 8.)	8.G.A.2 8.GL.A.2: Explain that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Rotations are only about the origin, and reflections are only over the y-axis and x-axis in grade 8.)
8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (Rotations are only about the origin, dilations only use the origin as the center of dilation, and reflections are only over the y-axis and x-axis in grade 8).	8.G.A.3 8.GL.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (Rotations are only about the origin, dilations only use the origin as the center of dilation, and reflections are only over the y-axis and x-axis in grade 8).

Current Standard	Proposed Standard
<p>8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</p>	<p><b>8.G.A.5 8.GL.A.5:</b> Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.</p> <ul style="list-style-type: none"> <li>For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</li> </ul>
<p><b>8.G.B-8.GL.B: Understand and apply the Pythagorean Theorem.</b></p>	
<p>8.G.B.6: Explain a proof of the Pythagorean Theorem and its converse using the areas of squares.</p>	<p><b>8.G.B.6 8.GL.B.6:</b> Explain a proof of the Pythagorean Theorem and its converse using the areas of squares</p>
<p>8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</p>	<p><b>8.G.B.7 8.GL.B.7:</b> Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</p>
<p>8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p>	<p><b>8.G.B.8 8.GL.B.8:</b> Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p>
<p><b>8.G.C 8.GL.C: Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</b></p>	

Current Standard	Proposed Standard
8.G.C.9: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	<b>8.G.C.9</b> <b>8.GL.C.9:</b> Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

## Data Analysis

Current Standard	Proposed Standard
<b>8.SPA-8.DA.A:</b> Investigate patterns of association in bivariate data.	
8.SP.A.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	<b>8.SP.A.1</b> <b>8.DA.A.1:</b> Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
8.SP.A.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	<b>8.SP.A.2</b> <b>8.DA.A.2:</b> Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
8.SP.A.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.	<b>8.SP.A.3</b> <b>8.DA.A.3:</b> Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. <ul style="list-style-type: none"> <li>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</li> </ul>

Current Standard	Proposed Standard
<p>8.SP.A.4: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</p>	<p><u>8.SP.A.4 8.DA.A.4</u>: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.</p> <ul style="list-style-type: none"> <li>a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.</li> <li>b. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.</li> <li>● For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</li> </ul>

## Proportionality and Functions

Current Standard	Proposed Standard
<p><b><u>8.F.A 8.PF.A</u>: Define, evaluate, and compare functions.</b></p>	
<p>8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in this grade level.)</p>	<p><u>8.F.A.1 8.PF.A.1</u>: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in this grade level.)</p>

Current Standard	Proposed Standard
<p>8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</p>	<p>8.F.A.2 <u>8.PF.A.2</u>: Compare properties of two functions <u>where each fraction is</u> represented in a different way (algebraically, graphically, numerically <u>in tables</u>, or <u>through</u> verbal descriptions).</p> <ul style="list-style-type: none"> <li>For example, given a linear function represented by a table of values <u>(numerically)</u> and a linear function represented by an algebraic expression <u>(algebraically)</u>, determine which function has the greater rate of change.</li> </ul>
<p>8.F.A.3: Interpret the equation <math>y = mx + b</math> as defining a linear function, whose graph is a straight line; categorize functions as linear or nonlinear when given equations, graphs, or tables. For example, the function <math>A = s^2</math> giving the area of a square as a function of its side length is not linear because its graph contains the points <math>(1,1)</math>, <math>(2,4)</math> and <math>(3,9)</math>, which are not on a straight line.</p>	<p>8.F.A.3 <u>8.PF.A.3</u>: Interpret the equation <math>y = mx + b</math> as defining a linear function, whose graph is a straight line; categorize functions as linear or nonlinear when given equations, graphs, or tables.</p> <ul style="list-style-type: none"> <li>For example, the function <math>A = \underline{s^2}</math> giving the area of a square as a function of its side length is not linear because its graph contains the points <math>(1,1)</math>, <math>(2,4)</math> and <math>(3,9)</math>, which are not on a straight line.</li> </ul>
<p><b><u>8.F.B 8.PF.B: Use functions to model relationships between quantities.</u></b></p>	
<p>8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two <math>(x, y)</math> values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.</p>	<p>8.F.B.4 <u>8.PF.B.4</u>: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two <math>(x, y)</math> values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.</p>

Current Standard	Proposed Standard
8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	8.F.B.5 <u>8. PF.B.5</u> : Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

# Algebra I

## Number and Quantity

Current Standard	Proposed Standard
<b>Number and Quantity: The Real Number System (N-RN)</b> B. Use properties of rational and irrational numbers.	
A1: N-RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	No change
<b>Number and Quantity: Quantities★</b> A. Reason quantitatively and use units to solve problems.	
A1: N-Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	A1: N-Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems. a. eChoose and interpret units consistently in formulas. b. eChoose and interpret the scale and the origin in graphs and data displays.
A1:N-Q.A.2: Define appropriate quantities for the purpose of descriptive modeling	No change
A1: N-Q.A.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	No change

## Algebra

Current Standard	Proposed Standard
<b>Algebra: Seeing Structure in Expressions</b> A. Interpret the structure of expressions.	
A1: A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i>	A1: A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i>
A1: A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>, or see <math>2x^2 + 8x</math> as <math>(2x)(x) + 2x(4)</math>, thus recognizing it as a polynomial whose terms are products of monomials and the polynomial can be factored as <math>2x(x+4)</math>.</i>	A1: A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it: <i>for a specific purpose. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>, or see <math>2x^2 + 8x</math> as <math>(2x)(x) + 2x(4)</math>, thus recognizing it as a polynomial whose terms are products of monomials and the polynomial can be factored as <math>2x(x+4)</math>.</i>
<b>Algebra: Seeing Structure in Expressions</b> B. Write expressions in equivalent forms to solve problems.	
A1: A-APR.B.3: Identify zeros of quadratic functions, and use the zeros to sketch a graph of the function defined by the polynomial.	No change
<b>Algebra: Creating Equations★</b> A. Create equations that describe numbers or relationships.	
A1: A-CED.A.1: Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear, quadratic, and exponential situation functions.</i>	A1: A-CED.A.1: Create equations and inequalities <del>in one variable</del> and use them to solve problems. <i>Include equations arising from linear, quadratic, and exponential situation functions.</i>

Current Standard	Proposed Standard
<p>A1: A-CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p>	<p>A1: A-CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p>
<p>A1: A-CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i></p>	<p>No change</p>
<p><b>Algebra: Reasoning with Equations and Inequalities★</b> A. Understand solving equations as a process of reasoning and explain the reasoning.</p>	
<p>A1: A-REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p>	<p>A1: A-REI.A.1: <i>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</i> <i>Use properties of equality to justify and explain each step obtained from the previous step when solving an equation, assuming the original equation has a solution.</i> a. Construct a viable argument to justify the solution method.</p>
<p><b>Algebra: Reasoning with Equations and Inequalities★</b> B. Solve equations and inequalities in one variable.</p>	
<p>A1: A-REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</p>	<p>No change</p>

Current Standard	Proposed Standard
<p>A1: A-REI.B.4: Solve quadratic equations in one variable.</p> <p>a. Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form.</p> <p>b. Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.</p> <p>c. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."</p>	<p>A1: A-REI.B.4: Solve quadratic equations in one variable.</p> <p>a. Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. <del>Derive the quadratic formula from this form.</del></p> <p>b. Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.</p> <p>c. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."</p>
<p><b>Algebra: Reasoning with Equations and Inequalities★</b></p> <p>C. Solve systems of equations.</p>	
<p>A1: A-REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p>	<p><del>A1: A-REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</del></p>
<p>A1: A-REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p>	<p><del>A1: A-REI.C.65: Write and <del>s</del>solve systems of linear equations in two variables exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</del></p> <p>a. <u>Use methods such as substitution, elimination, and graphing to solve.</u></p> <p>b. <u>Justify a method for solving such systems.</u></p>
<p><b>Algebra: Reasoning with Equations and Inequalities★</b></p> <p>D. Represent and solve equations and inequalities graphically.</p>	
<p>A1: A-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</p>	<p>No change</p>

Current Standard	Proposed Standard
A1: A-REI.D.11: Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, piecewise linear (to include absolute value), and exponential functions.★	No change
A1: A-REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	No change

## Functions

Current Standard	Proposed Standard
<b>Functions: Interpreting Functions</b> A. Understand the concept of a function and use function notation.	
A1: F-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .	No change

Current Standard	Proposed Standard
A1: F-IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	No change
A1: F-IF.A.3: Recognize that sequences are functions whose domain is a subset of the integers. Relate arithmetic sequences to linear functions and geometric sequences to exponential functions.	Remove this standard. <del>A1: F-IF.A.3: Recognize that sequences are functions whose domain is a subset of the integers. Relate arithmetic sequences to linear functions and geometric sequences to exponential functions.</del>
<b>Functions: Interpreting Functions</b> B. Interpret functions that arise in applications in terms of the context.	
A1: F-IF.B.4: For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.</i> ★	A1: F-IF.B.4: For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities, <ol style="list-style-type: none"> <li><u>interpret key features of graphs and tables in terms of the quantities, and</u></li> <li><u>sketch graphs showing key features given a verbal description of the relationship.</u></li> <li><u>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries and end behavior.</u> ★</li> </ol>
A1: F-IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For ex, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i> ★	No change

Current Standard	Proposed Standard
A1: F-IF.B.6: Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★	No change
<b>Functions: Interpreting Functions</b> C. Analyze functions using different representations.	
A1: F-IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph piecewise linear (to include absolute value) and exponential functions.	No change
A1: F-IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	No change
A1: F-IF.C.9: Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.	No change

Current Standard	Proposed Standard
<b>Functions: Building Functions</b> A. Build a function that models a relationship between two quantities.	
A1: F-BF.A.1: Write a linear, quadratic, or exponential function that describes a relationship between two quantities. ★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	A1: F-BF.A.1: Write a linear, quadratic, or exponential function that describes a relationship between two quantities. ★ a. Determine an explicit expression, <del>a recursive process</del> , or steps for calculation from a context.
<b>Functions: Building Functions</b> B. Build new functions from existing functions.	
A1: F-BF.B.3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative). Without technology, find the value of $k$ given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where $f(x)$ is a linear, quadratic, piecewise linear (to include absolute value), or exponential function.	No change
<b>Functions: Linear, Quadratic, and Exponential Models</b> ★ A. Construct and compare linear, quadratic, and exponential models and solve problems.	

Current Standard	Proposed Standard
<p>A1: F-LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <p>a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</p> <p>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</p> <p>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</p>	<p>A1: F-LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <p>a. Prove that linear functions grow by equal differences over equal intervals.</p> <p>b. <u>Prove that exponential functions grow by equal factors over equal intervals.</u></p> <p>c. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</p> <p>d. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</p>
<p>A1: F-LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p>	<p>No change</p>
<p>A1: F-LE.A.3: Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p>	<p>A1: F-LE.A.3: Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly <u>or</u> quadratically, <u>or</u> (more generally) as a polynomial function with and without technology.</p>
<p><b>Functions: Linear, Quadratic, and Exponential Models★</b></p> <p>B. Interpret expressions for functions in terms of the situation they model.</p>	
<p>A1: F-LE.B.5: Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.</p>	<p>No change</p>

## Statistics and Probability

Current Standard	Proposed Standard
<b>Statistics and Probability★: Interpreting Categorical and Quantitative Data</b> A. Summarize, represent, and interpret data on a single count or measurement variable.	
A1: S-ID.A.2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	No change
A1: S-ID.A.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	No change
<b>Statistics and Probability★: Interpreting Categorical and Quantitative Data</b> B. Summarize, represent, and interpret data on two categorical and quantitative variables.	
A1: S-ID.B.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	No change

Current Standard	Proposed Standard
<p>A1: S-ID.B.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions or choose a function suggested by the context. Emphasize linear and quadratic models.</i></p> <p>b. Informally assess the fit of a function by plotting and analyzing residuals.</p> <p>c. Fit a linear function for a scatter plot that suggests a linear association.</p>	<p>No change</p>
<p><b>Statistics and Probability★: Interpreting Categorical and Quantitative Data</b></p> <p>C. Interpret linear models.</p>	
<p>A1: S-ID.C.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p>	<p>No change</p>
<p>A1: S-ID.C.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.</p>	<p>No change</p>
<p>A1: S-ID.C.9: Distinguish between correlation and causation.</p>	<p>No change</p>

# Geometry

## Geometric Reasoning and Logic

Current Standard	Proposed Standard
<b>Geometry: Congruence</b> A. Experiment with transformations in the plane.	
GM: G-CO.A.1: Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	GM: G-CO.A.1: <u>Based on the undefined notions of point, line, distance along a line, and distance around a circular arc, know the precise definitions of</u> <ul style="list-style-type: none"><li>• <u>angle,</u></li><li>• <u>circle,</u></li><li>• <u>perpendicular line,</u></li><li>• <u>parallel line, and</u></li><li>• <u>line segment.</u></li></ul>
GM: G-CO.A.2: Represent transformations in the plane using, e.g., transparencies, tracing paper, or geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	GM: G-CO.A.2: Use a plane to <ol style="list-style-type: none"><li>a. <u>Represent transformations with and without technology,</u></li><li>b. <u>Describe transformations as functions that take points in the plane as inputs and give other points as outputs.</u></li><li>c. <u>Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).</u></li></ol>
GM: G-CO.A.3: Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	GM: G-CO.A.3: <u>Describe the rotations and reflections that carry map a preimage onto itself when given a rectangle, parallelogram, trapezoid, or regular polygon.</u>
GM: G-CO.A.4: Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	No change

Current Standard	Proposed Standard
<p>GM: G-CO.A.5: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</p>	<p>GM: G-CO.A.5: Given a geometric figure and a rotation, reflection, translation, or sequence of transformations</p> <ol style="list-style-type: none"> <li data-bbox="1051 393 2002 458">Draw the transformed figure <u>with and without technology, using, e.g., graph paper, tracing paper, or geometry software.</u></li> <li data-bbox="1051 458 1959 491"><u>Specify a sequence that will map a given figure onto another.</u></li> </ol>
<p><b>Geometry: Congruence</b></p> <p>B. Understand congruence in terms of rigid motions.</p>	
<p>GM: G-CO.B.6: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.</p>	<p>GM: G-CO.B.6: Use geometric descriptions of rigid motions to transform figures <u>and to</u></p> <ul style="list-style-type: none"> <li data-bbox="1051 691 1917 724"><u>predict</u> the effect of a given rigid motion on a given figure.</li> <li data-bbox="1051 724 2002 789"><u>Given</u> two figures, use the definition of congruence in terms of rigid motions to <u>decide</u> <u>determine</u> if they are congruent.</li> </ul>
<p>GM: G-CO.B.7: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.</p>	<p>No change</p>
<p>GM: G-CO.B.8: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.</p>	<p>No change</p>
<p><b>Geometry: Congruence</b></p> <p>C. Prove and apply geometric theorems.</p>	

Current Standard	Proposed Standard
<p>GM: G-CO.C.9: Prove and apply theorems about lines and angles. <i>Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</i></p>	<p>GM: G-CO.C.9: Prove and apply theorems about lines and angles. <i>Theorems include, <u>but are not limited to</u>:</i></p> <ul style="list-style-type: none"> <li>• vertical angles are congruent;</li> <li>• when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent;</li> <li>• points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</li> </ul>
<p>GM: G-CO.C.10: Prove and apply theorems about triangles. <i>Theorems include: measures of interior angles of a triangle sum to <math>180^\circ</math>; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</i></p>	<p>GM: G-CO.C.10: Prove and apply theorems about triangles. <i>Theorems include, <u>but are not limited to</u>:</i></p> <ul style="list-style-type: none"> <li>• measures of interior angles of a triangle sum to <math>180^\circ</math>;</li> <li>• base angles of isosceles triangles are congruent;</li> <li>• the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;</li> <li>• the medians of a triangle meet at a point.</li> </ul>
<p>GM: G-CO.C.11: Prove and apply theorems about parallelograms. <i>Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</i></p>	<p>GM: G-CO.C.11: Prove and apply theorems about parallelograms. <i>Theorems include, <u>but are not limited to</u>:</i></p> <ul style="list-style-type: none"> <li>• opposite sides are congruent,</li> <li>• opposite angles are congruent,</li> <li>• the diagonals of a parallelogram bisect each other, and <u>conversely the converse of this theorem</u>;</li> <li>• rectangles are parallelograms with congruent diagonals <u>and the converse of this theorem</u>.</li> </ul>
<p><b>Geometry: Congruence</b> D. Make geometric constructions.</p>	

Current Standard	Proposed Standard
<p>GM: G-CO.D.12: Make formal geometric constructions with a variety of tools and methods, e.g., compass and straightedge, string, reflective devices, paper folding, or dynamic geometric software. <i>Examples: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i></p>	<p>Remove this standard.</p> <p><del>GM: G-CO.D.12: Make formal geometric constructions with a variety of tools and methods, e.g., compass and straightedge, string, reflective devices, paper folding, or dynamic geometric software. <i>Examples: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</i></del></p>
<p>GM: G-CO.D.13: Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</p>	<p>GM: G-CO.D.12: Make formal geometric constructions</p> <ul style="list-style-type: none"> <li>with a variety of tools and methods, e.g., compass and straightedge, string, reflective devices, paper folding, or dynamic geometric software. <del>W</del>ith or without technology</li> <li><del>Construct of</del> an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</li> </ul>
<p><b>Geometry: Similarity, Right Triangles, and Trigonometry</b></p> <p>A. Understand similarity in terms of similarity transformations.</p>	
<p>GM: G-SRT.A.1: Verify experimentally the properties of dilations given by a center and a scale factor:</p> <ol style="list-style-type: none"> <li>A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</li> <li>The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</li> </ol>	<p>No change</p>

Current Standard	Proposed Standard
<p>GM: G-SRT.A.2: Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</p>	<p>GM: G-SRT.A.2: Given two figures, use the definition of similarity in terms of similarity transformations to decide <u>determine</u> if they are similar. Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</p> <p>GM: G-SRT.A.2 <u>Using similarity transformations</u></p> <ul style="list-style-type: none"> <li>• <u>Determine if two figures are similar using the definition of similarity transformations.</u></li> <li>• <u>Explain the meaning of similarity for triangles as the equality of all corresponding pairs of angles and proportionality of all corresponding sides.</u></li> </ul>
<p>GM: G-SRT.A.3: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</p>	<p>No change</p>
<p><b>Geometry: Similarity, Right Triangles, and Trigonometry</b> B. Prove and apply theorems involving similarity.</p>	
<p>GM: G-SRT.B.4: Prove and apply theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity; SAS similarity criteria; SSS similarity criteria; AA similarity criteria.</p>	<p>GM: G-SRT.B.4: Prove and apply theorems about triangles. Theorems include, <u>but are not limited to</u>:</p> <ul style="list-style-type: none"> <li>• a line parallel to one side of a triangle divides the other two proportionally, and <u>conversely the converse of this theorem</u>;</li> <li>• the Pythagorean Theorem proved using triangle similarity;</li> <li>• SAS similarity criteria;</li> <li>• SSS similarity criteria;</li> <li>• AA similarity criteria.</li> </ul>

Current Standard	Proposed Standard
GM: G-SRT.B.5: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	No change
<b>Geometry: Similarity, Right Triangles, and Trigonometry</b> C. Define trigonometric ratios and solve problems involving right triangles.	
GM: G-SRT.C.6: Understand that by similarity, side ratios in right triangles, including special right triangles (30-60-90 and 45-45-90), are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	No change
GM: G-SRT.C.7: Explain and use the relationship between the sine and cosine of complementary angles.	No change
GM: G-SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★	No change
<b>Geometry: Circles</b> A. Understand and apply theorems about circles.	
GM: G-C.A.1: Prove that all circles are similar.	GM: G-C.A.1: <u>Prove</u> <u>Understand</u> that all circles are similar.
GM: G-C.A.2: Identify and describe relationships among inscribed angles, radii, and chords, including the following: <i>the relationship that exists between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; and a radius of a circle is perpendicular to the tangent where the radius intersects the circle.</i>	GM: G-C.A.2: Identify and describe relationships among inscribed angles, radii, and chords, including the following: <ul style="list-style-type: none"> <li>the relationship that exists between central, inscribed, and circumscribed angles;</li> <li>inscribed angles on a diameter are right angles; and</li> <li>a radius of a circle is perpendicular to the tangent where the radius intersects the circle.</li> </ul>

Current Standard	Proposed Standard
GM: G-C.A.3: Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	No change
<b>Geometry: Circles</b> B. Find arc lengths and areas of sectors of circles.	
GM: G-C.B.5: Use similarity to determine that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	GM: G-C.B.5: Use similarity to determine that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. apply the formula for finding area of a sector and arc length.
<b>Geometry: Expressing Geometric Properties with Equations</b> A. Translate between the geometric description and the equation for a conic section.	
GM: G-GPE.A.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	No change
<b>Geometry: Expressing Geometric Properties with Equations</b> B. Use coordinates to prove simple geometric theorems algebraically.	
GM: G-GPE.B.4: Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$ .	GM: G-GPE.B.4: Use coordinates to prove simple geometric theorems algebraically. <ul style="list-style-type: none"> <li>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle;</li> <li>For example, prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</li> </ul>

Current Standard	Proposed Standard
GM: G-GPE.B.5: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	GM: G-GPE.B.5: <u>Prove</u> <u>Determine</u> the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
GM: G-GPE.B.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	GM: G-GPE.B.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio. <ul style="list-style-type: none"> <li>• <u>Apply ratio thinking to find the midpoint of the given line segment.</u></li> </ul>
GM: G-GPE.B.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	No change
<b>Geometry: Geometric Measurement and Dimension</b> A. Explain volume formulas and use them to solve problems.	
GM: G-GMD.A.1: Give an informal argument, e.g., dissection arguments, Cavalieri's principle, or informal limit arguments, for the formulas for the circumference of a circle; area of a circle; volume of a cylinder, pyramid, and cone.	GM: G-GMD.A.1: Give an informal argument (e.g., dissection arguments, Cavalieri's principle, or informal limit arguments) for the formulas of: <ul style="list-style-type: none"> <li>• the circumference of a circle;</li> <li>• <u>area of a regular polygon;</u></li> <li>• area of a circle;</li> <li>• volume of a cylinder, pyramid, and cone.</li> </ul>
GM: G-GMD.A.3: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	No change
<b>Geometry: Geometric Measurement and Dimension</b> B. Visualize relationships between two-dimensional and three-dimensional objects.	

Current Standard	Proposed Standard
GM: G-GMD.B.4: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	<p>GM: G-GMD.B.4: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</p> <ul style="list-style-type: none"> <li>• <u>Apply properties of two-dimensional figures identified to solve problems.</u></li> </ul>
<b>Geometry: Modeling with Geometry</b> A. Apply geometric concepts in modeling situations.	
GM: G-MG.A.1: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★	No change
GM: G-MG.A.2: Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★	No change
GM: G-MG.A.3: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★	No change
<b>Statistics and Probability★: Conditional Probability and the Rules of Probability</b> A. Understand independence and conditional probability and use them to interpret data.	
GM: S-CP.A.1: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	No change

Current Standard	Proposed Standard
GM: S-CP.A.2: Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	No change
GM: S-CP.A.3: Understand the conditional probability of $A$ given $B$ as $P(A \text{ and } B)/P(B)$ , and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$ , and the conditional probability of $B$ given $A$ is the same as the probability of $B$ .	No change
GM: S-CP.A.4: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i>	GM: S-CP.A.4: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. <i>For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</i>
GM: S-CP.A.5: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i>	GM: S-CP.A.5: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i>
<p><b>Statistics and Probability★: Conditional Probability and the Rules of Probability</b></p> <p>B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.</p>	

Current Standard	Proposed Standard
GM: S-CP.B.6: Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	No change
GM: S-CP.B.7: Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.	No change

# Algebra II

## Number & Quantity

Current Standard	Proposed Standard
<b>Number and Quantity: The Real Number System</b> A. Extend the properties of exponents to rational exponents.	
A2: N-RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.	A2: N-RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
A2: N-RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.	No change
<b>Number and Quantity: Quantities★</b> A. Reason quantitatively and use units to solve problems.	
A2: N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.	No change
<b>Number and Quantity: The Complex Number System (N-CN)</b> A. Perform arithmetic operations with complex numbers.	
A2: N-CN.A.1 Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ with $a$ and $b$ real.	No change

Current Standard	Proposed Standard
<b>A2: N-CN.A.2</b> Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Remove this standard. <del>A2: N-CN.A.2 Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</del>
<b>Number and Quantity: The Complex Number System (N-CN)</b> C. Use complex numbers in polynomial identities and equations.	
<b>A2: N-CN.C.7</b> Solve quadratic equations with real coefficients that have complex solutions.	No change

## Algebra

Current Standard	Proposed Standard
<b>Algebra: Seeing Structure in Expressions (A-SSE)</b> A. Interpret the structure of expressions.	
A2: A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i>	A2: A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it <u>for a specific purpose</u> . <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i>
<b>Algebra: Seeing Structure in Expressions (A-SSE)</b> B. Write expressions in equivalent forms to solve problems.	

Current Standard	Proposed Standard
<p>A2: A-SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★</p> <p>c. Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression <math>1.15^t</math> can be rewritten as <math>(1.15^{1/12})^{12t} \approx 1.012^{12t}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%</i></p>	<p>A2: A-SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★</p> <p>c. Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression <math>1.15^t</math> can be rewritten as <math>(1.15^{1/12})^{12t} \approx 1.012^{12t}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%</i></p>
<p>A2: A-SSE.B.4: Apply the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. <i>For example, calculate mortgage payments.</i> ★</p>	<p>No change</p>
<p><b>Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)</b></p>	
<p>B. Perform arithmetic operations on polynomials. <u>Understand the relationship between zeros and factors of polynomials.</u></p>	
<p>A2: A-APR.B.2: Know and apply the Remainder Theorem: For a polynomial <math>p(x)</math> and a number <math>a</math>, the remainder on division by <math>x - a</math> is <math>p(a)</math>, so <math>p(a) = 0</math> if and only if <math>(x - a)</math> is a factor of <math>p(x)</math>.</p>	<p>No change</p>
<p>A2: A-APR.B.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</p>	<p>No change</p>
<p><b>Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)</b></p>	
<p>C. Use polynomial identities to solve problems.</p>	
<p>A2: A-APR.C.4: Use polynomial identities to describe numerical relationships. <i>For example, the polynomial identity <math>(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2</math> can be used to generate Pythagorean triples.</i></p>	<p>A2: A-APR.C.4: <u>Describe numerical relationships using polynomial identities.</u> <u>Use polynomial identities to describe numerical relationships.</u> <i>For example, the polynomial identity <math>(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2</math> can be used to generate Pythagorean triples.</i></p>

Current Standard	Proposed Standard
<b>Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)</b> D. Rewrite rational expressions.	
A2: A-APR.D.6: Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.	A2: A-APR.D.6: Rewrite simple rational expressions in different forms: <ul style="list-style-type: none"> <li>Write <math>a(x)/b(x)</math> in the form <math>q(x) + r(x)/b(x)</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system.</li> </ul>
<b>Algebra: Creating Equations ★ (A-CED)</b> A. Create equations that describe numbers or relationships.	
A2: A-CED.A.1: Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>	A2: A-CED.A.1: Create equations and inequalities <del>in one variable</del> and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>
<b>Reasoning with Equations and Inequalities (A-REI)</b> A. Understand solving equations as a process of reasoning and explain the reasoning.	
A2: A-REI.A.1: Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	A2: A-REI.A.1: <del>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</del> <del>Use properties of equality to justify and explain each step obtained from the previous step when solving an equation, assuming the original equation has a solution. Expand context to situations, including but not limited to logarithmic, exponential, rational, and radical equations.</del> a. Construct a viable argument to justify the solution method.

Current Standard	Proposed Standard
A2: A-REI.A.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	No change
<b>Reasoning with Equations and Inequalities (A-REI)</b> B. Solve equations and inequalities in one variable, including applications in mathematical modeling.	
A2: A-REI.B.4: Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ .	A2: A-REI.B.4: Solve quadratic equations in one variable by <ul style="list-style-type: none"> <li>inspection (e.g., for <math>x^2 = 49</math>),</li> <li>taking square roots,</li> <li>completing the square,</li> <li>the quadratic formula and</li> <li>factoring, as appropriate to the initial form of the equation.</li> </ul> Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ . <u>Apply these methods to represent and solve real-world problems through mathematical modeling, interpreting the meaning of solutions in context.</u>
<b>Reasoning with Equations and Inequalities (A-REI)</b> C. Solve systems of equations	
A2: A-REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to two variables.	No change
A2: A-REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</i>	A2: A-REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <del>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</del>

Current Standard	Proposed Standard
<p><b>Reasoning with Equations and Inequalities (A-REI)</b></p> <p>D. Represent and solve equations and inequalities graphically.</p> <p>A2: A-REI.D.11: Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</p>	<p>A2: A-REI.D.11: Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>;</p> <p><u>Find approximate solutions by</u></p> <ul style="list-style-type: none"> <li>• using technology to graph the functions,</li> <li>• make tables of values, or</li> <li>• find successive approximations.</li> </ul> <p>Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</p>

## Functions

Current Standard	Proposed Standard
<p><b>Functions: Interpreting Functions (F-IF)</b></p> <p>A. Understand the concept of a function and use function notation.</p> <p>Consider adding IF.A.3 since it was removed from A1.</p>	<p><u>A2: F-IF.A.3: Recognize that sequences are functions whose domain is a subset of the integers. Relate arithmetic sequences to linear functions and geometric sequences to exponential functions.</u></p>
<p><b>Functions: Interpreting Functions (F-IF)</b></p> <p>B. Interpret functions that arise in applications in terms of the context.</p>	

Current Standard	Proposed Standard
<p>A2: F-IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★</p>	<p>A2: F-IF.B.4: For a function that models a relationship between two quantities,</p> <ul style="list-style-type: none"> <li>interpret key features of graphs and tables in terms of the quantities, and</li> <li>sketch graphs showing key features given a verbal description of the relationship.</li> </ul> <p>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★</p>
<p>A2: F-IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★</p>	<p>No change</p>
<p><b>Functions: Interpreting Functions (F-IF)</b> C. Analyze functions using different representations.</p>	
<p>A2: F-IF.C.7: Graph functions are expressed symbolically and show key features of the graph by hand in simple cases and by using technology for more complicated cases. ★</p> <p>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p>	<p>No change</p>

Current Standard	Proposed Standard
<p>A2: F-IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as <math>y = (1.02)t</math>, <math>y = (0.97)t</math>, <math>y = (1.01)12t</math>, <math>y = (1.2)t/10</math>, and classify them as representing exponential growth or decay.</p>	No change
<p>A2: F-IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>	No change
<p><b>Functions: Building Functions (F-BF)</b></p> <p>A. Build a function that models a relationship between two quantities.</p>	
<p>A2: F-BF.A.1: Write a function that describes a relationship between two quantities.★</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></p>	No change
<p>A2: F-BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★</p>	No change

Current Standard	Proposed Standard
<b>Functions: Building Functions(F-BF)</b> B. Build new functions from existing functions.	
A2: F-BF.B.3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i>	A2: F-BF.B.3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); <ul style="list-style-type: none"> <li>Find the value of <math>k</math> given the graphs.</li> <li>Experiment with cases and illustrate an explanation of the effects on the graph using technology.</li> <li>Recognize even and odd functions from their graphs and algebraic expressions for them.</li> </ul>
A2: F-BF.B.4: Find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. <i>For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</i>	A2: F-BF.B.4: Find inverse functions. <ul style="list-style-type: none"> <li>Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse</li> <li>Write an expression for the inverse. <del>For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</del></li> </ul>
<b>Functions: Linear, Quadratic, and Exponential Models (F-LE)</b> A. Construct and compare linear, quadratic, and exponential models and solve problems.	
A2: F-LE.A.2: Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences to solve multi-step problems.	No change
A2: F-LE.A.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology. ★	No change
<b>Functions: Linear, Quadratic, and Exponential Models (F-LE)</b> A. Interpret expressions for functions in terms of the situation they model.	

Current Standard	Proposed Standard
A2: F-LE.B.5: Interpret the parameters in a linear, quadratic, or exponential function in terms of a context. ★	No change
<b>Functions: Trigonometric Functions (F-TF)</b> A. Extend the domain of trigonometric functions using the unit circle.	
A2: F-TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	No change
A2: F-TF.A.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	No change
<b>Functions: Trigonometric Functions (F-TF)</b> B. Model periodic phenomena with trigonometric functions.	
A2: F-TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★	No change
<b>Functions: Trigonometric Functions (F-TF)</b> <b>C. Prove and apply trigonometric identities</b>	
<b>A2: F-TF.C.8</b> Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ given $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ and the quadrant of the angle.	<b>A2: F-TF.C.8</b> Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find <u>the value of</u> $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ given $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ and the quadrant of the angle.

## Statistics & Probability

Current Standard	Proposed Standard
<b>Statistics and Probability: Interpreting Categorical and Quantitative Data «(S-ID)</b> A. Summarize, represent, and interpret data on a single count or measurement variable.	
A2: S-ID.A.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	A2: S-ID.A.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages <u>with technology</u> . a. Recognize that there are data sets for which such a procedure is not appropriate. b. Use calculators, spreadsheets, and tables to estimate areas <u>under the normal curve</u> .
<b>Statistics and Probability: Interpreting Categorical and Quantitative Data « (S-ID)</b> A. Summarize, represent, and interpret data on two categorical and quantitative variables.	
A2: S-ID.B.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. <i>Emphasize exponential models.</i>	A2: S-ID.B.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. b. Use given functions or choose a function suggested by the context. <i>Emphasize exponential models.</i>
<b>Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)</b> A. Understand and evaluate random processes underlying statistical experiments.	
A2: S-IC.A.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★	No change

Current Standard	Proposed Standard
<p>A2: S-IC.A.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <i>For example, a model says a spinning coin will fall heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? ★</i></p>	<p>A2: S-IC.A.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. <del>For example, a model says a spinning coin will fall heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? ★</del></p>
<p><b>Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)</b></p> <p>A. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</p>	
<p>A2: S-IC.B.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★</p>	<p>No change</p>
<p>A2: S-IC.B.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★</p>	<p>No change</p>
<p>A2: S-IC.B.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.★</p>	<p>No change</p>
<p>A2: S-IC.B.6: Evaluate reports based on data.★</p>	<p>A2: S-IC.B.6: Evaluate <del>reports media or academic research</del> based on data.★</p>

# Integrated Math 1

## Number and Quantity

### Proposed Standard

#### Number and Quantity: The Real Number System (N-RN)

B. Use properties of rational and irrational numbers.

IM1: N-RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

#### Number and Quantity: Quantities★

A. Reason quantitatively and use units to solve problems.

IM1: N-Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems.

- c. Choose and interpret units consistently in formulas.
- d. Choose and interpret the scale and the origin in graphs and data displays.

IM1: N-Q.A.2: Define appropriate quantities for the purpose of descriptive modeling

IM1: N-Q.A.3: Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

# Algebra

Proposed Standard
<b>Algebra: Seeing Structure in Expressions</b> A. Interpret the structure of expressions.
IM1: A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
<b>Algebra: Creating Equations★</b> A. Create equations that describe numbers or relationships.
IM1: A-CED.A.1: Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear, quadratic, and exponential situation functions.</i>
IM1: A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
IM1: A-CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
IM1: A-CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i>
<b>Algebra: Reasoning with Equations and Inequalities★</b> A. Understand solving equations as a process of reasoning and explain the reasoning.
IM1: A-REI.A.1: Use properties of equality to justify and explain each step obtained from the previous step when solving an equation, assuming the original equation has a solution. b. Construct a viable argument to justify the solution method.
<b>Algebra: Reasoning with Equations and Inequalities★</b> B. Solve equations and inequalities in one variable.

## Proposed Standard

IM1: A-REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

IM1: A-REI.B.4: Solve quadratic equations in one variable.

- a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions.
- b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.
- c. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

### **Algebra: Reasoning with Equations and Inequalities★**

C. Solve systems of equations.

IM1: A-REI.C.5: Write and solve systems of linear equations in two variables.

- a. Use methods such as substitution, elimination, and graphing to solve.
- b. Justify a method for solving such systems.

### **Algebra: Reasoning with Equations and Inequalities★**

D. Represent and solve equations and inequalities graphically.

IM1: A-REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

IM1: A-REI.D.11: Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, piecewise linear (to include absolute value), and exponential functions.★

IM1: A-REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## Functions

Proposed Standard
<b>Functions: Interpreting Functions</b> A. Understand the concept of a function and use function notation.
A1: F-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
A1: F-IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
<b>Functions: Interpreting Functions</b> B. Interpret functions that arise in applications in terms of the context.
A1: F-IF.B.4: For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities. <ol style="list-style-type: none"><li>interpret key features of graphs and tables in terms of the quantities, and</li><li>sketch graphs showing key features given a verbal description of the relationship.</li><li>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries and end behavior. ★</li></ol>
A1: F-IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. ★
A1: F-IF.B.6: Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
<b>Functions: Interpreting Functions</b> C. Analyze functions using different representations.

## Proposed Standard

A1: F-IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

- a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- b. Graph piecewise linear (to include absolute value) and exponential functions.

A1: F-IF.C.9: Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.

### Functions: Building Functions

A. Build a function that models a relationship between two quantities.

A1: F-BF.A.1: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
  - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

A1: F-BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

### Functions: Building Functions

B. Build new functions from existing functions.

A1: F-BF.B.3: Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative). Without technology, find the value of  $k$  given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where  $f(x)$  is a linear, quadratic, piecewise linear (to include absolute value), or exponential function.

### Functions: Linear, Quadratic, and Exponential Models★

A. Construct and compare linear, quadratic, and exponential models and solve problems.

### Proposed Standard

A1: F-LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

- a. Prove that linear functions grow by equal differences over equal intervals.
- b. Prove that exponential functions grow by equal factors over equal intervals.
- c. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- d. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

A1: F-LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

A1: F-LE.A.3: Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly **or** quadratically.-with and without technology.

#### **Functions: Linear, Quadratic, and Exponential Models★**

B. Interpret expressions for functions in terms of the situation they model.

A1: F-LE.B.5: Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

## Geometric Reasoning and Logic

### Proposed Standard

#### **Geometry: Congruence**

A. Experiment with transformations in the plane.

IM1: G-CO.A.1: Based on the undefined notions of point, line, distance along a line, and distance around a circular arc, know the precise definitions of

- angle,
- circle,
- perpendicular line,
- parallel line, and
- line segment.

## Proposed Standard

### IM1: G-CO.A.2: Use a plane to

- a. Represent transformations with and without technology.
- b. Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
- c. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch)

### IM1: G-CO.A.3: Describe the rotations and reflections that carry map a preimage onto itself when given a rectangle, parallelogram, trapezoid, or regular polygon.

### IM1: G-CO.A.4: Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

### IM1: G-CO.A.5: Given a geometric figure and a rotation, reflection, translation, or sequence of transformations

- a. draw the transformed figure with and without technology, using e.g., graph paper, tracing paper, or geometry software.
- b. Specify a sequence that will map a given figure onto another.

## **Geometry: Congruence**

### B. Understand congruence in terms of rigid motions.

### GM: G-CO.B.6: Use geometric descriptions of rigid motions to transform figures and to.

- Predict the effect of a given rigid motion on a given figure;
- Given two figures, use the definition of congruence in terms of rigid motions to determine if they are congruent.

### IM1: G-CO.B.7: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

### IM1: G-CO.B.8: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

## **Geometry: Congruence**

### D. Make geometric constructions.

## Proposed Standard

### IM1: G-CO.D.12: Make formal geometric constructions

- with a variety of tools and methods, with or without technology
- of an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

### **Geometry: Expressing Geometric Properties with Equations**

B. Use coordinates to prove simple geometric theorems algebraically.

### IM1: GM: G-GPE.B.4: Use coordinates to prove geometric theorems algebraically.

IM1: G-GPE.B.5: Determine the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

### IM1: G-GPE.B.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

- Apply ratio thinking to find the midpoint of the given line segment.

### IM1: G-GPE.B.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

## Statistics and Probability

## Proposed Standard

### **Statistics and Probability★: Interpreting Categorical and Quantitative Data**

A. Summarize, represent, and interpret data on a single count or measurement variable.

### IM1: S-ID.A.1: Represent and interpret data with plots on the real number line (dot plots, histograms, and box plots).

### IM1: S-ID.A.2: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

### Proposed Standard

IM1: S-ID.A.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

#### **Statistics and Probability★: Interpreting Categorical and Quantitative Data**

B. Summarize, represent, and interpret data on two categorical and quantitative variables.

IM1: S-ID.B.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

IM1: S-ID.B.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and quadratic models.*
- b. Informally assess the fit of a function by plotting and analyzing residuals.
- c. Fit a linear function for a scatter plot that suggests a linear association.

#### **Statistics and Probability★: Interpreting Categorical and Quantitative Data**

C. Interpret linear models.

IM1: S-ID.C.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

IM1: S-ID.C.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.

IM1: S-ID.C.9: Distinguish between correlation and causation.

# Integrated Math 2

## Number & Quantity

### Proposed Standard

#### Number and Quantity: The Real Number System (N-RN)

A. Extend the properties of exponents to rational exponents

IM1: N-RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

IM1: N-RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

B. Use properties of rational and irrational numbers.

IM1: N-RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

#### Number and Quantity: The Complex Number System (N-CN)

A. Perform arithmetic operations with complex numbers.

IM2: N-CN.A.1 Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.

IM2: N-CN.A.2 Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

#### Number and Quantity: The Complex Number System (N-CN)

C. Use complex numbers in polynomial identities and equations.

IM2: N-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

IM2: N-CN.C.8: Extend polynomial identities to the complex numbers. For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .

### Proposed Standard

IM2: N-CN.C.9: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## Algebra

### Proposed Standard

#### **Algebra: Seeing Structure in Expressions**

A. Interpret the structure of expressions.

IM2: A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.★

- a. Interpret parts of an expression, such as terms, factors, and coefficients.
- b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

IM2: A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it for a specific purpose.

#### **Algebra: Seeing Structure in Expressions**

A. Interpret the structure of expressions.

IM2: A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.★

- a. Interpret parts of an expression, such as terms, factors, and coefficients.
- b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A2: A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it for a specific purpose.

#### **Algebra: Seeing Structure in Expressions**

B. Write expressions in equivalent forms to solve problems.

## Proposed Standard

IM2: A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

- a. Factor a quadratic expression to reveal the zeros of the function it defines.
- b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- c. Use the properties of exponents to transform expressions for exponential functions. For example the expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%

### Arithmetic with Polynomials and Rational Expressions

A. Perform arithmetic operations on polynomials.

IM2: A-APR.A.1: Understand that polynomials form a system comparable to the integers, as they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

### Algebra: Creating Equations ★ (A-CED)

A. Create equations that describe numbers or relationships.

IM2: A-CED.A.1 Create equations and inequalities and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

IM2: A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

IM2: A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

IM2: A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *Include formulas involving quadratic terms. For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .*

### Reasoning with Equations and Inequalities

B. Solve equations and inequalities in one variable.

### Proposed Standard

#### IM2: A-REI.B.4 Solve quadratic equations in one variable.

- a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions.
- b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.
  - c. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

#### Reasoning with Equations and Inequalities

##### C. Solve systems of equations.

#### IM2: A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

## Functions

### Proposed Standard

#### Functions: Interpreting Functions (F-IF)

##### B. Interpret functions that arise in applications in terms of the context.

#### IM2: F-IF.B.4 For a function that models a relationship between two quantities,

- interpret key features of graphs and tables in terms of the quantities, and
- sketch graphs showing key features given a verbal description of the relationship.

Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★

#### IM2: F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. ★

#### IM2: F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

## Proposed Standard

### **Functions: Interpreting Functions (F-IF)**

C. Analyze linear, exponential, quadratic, absolute value, step, and piecewise-defined functions using different representations functions using different representations.

IM2: F-IF.C.7 Graph functions are expressed symbolically and show key features of the graph by hand in simple cases and by using technology for more complicated cases. ★

- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

IM2: F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- a. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)t$ ,  $y = (0.97)t$ ,  $y = (1.01)12t$ ,  $y = (1.2)t/10$ , and classify them as representing exponential growth or decay.

IM2: F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

### **Functions: Building Functions (F-BF)**

A. Build a function that models a relationship between two quantities.

IM2: F-BF.A.1 Write a function that describes a relationship between two quantities.★

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

### **Functions: Building Functions(F-BF)**

B. Build new functions from existing functions.

### Proposed Standard

IM2: F-BF.B.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative):

- Find the value of  $k$  given the graphs.
- Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- Include recognizing even and odd functions from their graphs and algebraic expressions for them.

IM2: F-BF.B.4 Find inverse functions.

- Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse.
- Write an expression for the inverse.

### **Functions: Linear, Quadratic, and Exponential Models (F-LE)**

A. Construct and compare linear, quadratic, and exponential models and solve problems.

IM2: F-LE.A.3 Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically, with and without technology.

### **Functions: Trigonometric Functions (F-TF)**

C. Prove and apply trigonometric identities

IM2: F-TF.C.8 Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find the value of  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

## Geometric Reasoning and Logic

### Proposed Standard

#### Geometry: Congruence

C. Prove and apply geometric theorems.

## Proposed Standard

GM: G-CO.C.9: Prove and apply theorems about lines and angles.

Theorems include, but are not limited to:

- vertical angles are congruent;
- when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent;
- points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

GM: G-CO.C.10: Prove and apply theorems about triangles.

Theorems include, but are not limited to:

- measures of interior angles of a triangle sum to  $180^\circ$ ;
- base angles of isosceles triangles are congruent;
- the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point.

GM: G-CO.C.11: Prove and apply theorems about parallelograms.

Theorems include, but are not limited to:

- opposite sides are congruent,
- opposite angles are congruent,
- the diagonals of a parallelogram bisect each other, and the converse of this theorem;
- rectangles are parallelograms with congruent diagonals and the converse of this theorem.

## **Geometry: Similarity, Right Triangles, and Trigonometry**

A. Understand similarity in terms of similarity transformations.

IM2: G-SRT.A.1: Verify experimentally the properties of dilations given by a center and a scale factor:

- a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

## Proposed Standard

### IM2: G-SRT.A.2

#### Using similarity transformations

- Determine if two figures are similar using the definition of similarity transformations.
- Explain the meaning of similarity for triangles as the equality of all corresponding pairs of angles and proportionality of all corresponding sides.

### IM2: G-SRT.A.3: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

### **Geometry: Similarity, Right Triangles, and Trigonometry**

#### B. Prove and apply theorems involving similarity.

### IM2: G-SRT.B.4: Prove and apply theorems about triangles.

#### Theorems include, but are not limited to:

- a line parallel to one side of a triangle divides the other two proportionally, and the converse of this theorem;
- the Pythagorean Theorem proved using triangle similarity;
- SAS similarity criteria;
- SSS similarity criteria;
- AA similarity criteria.

### IM2: G-SRT.B.5: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

### **Geometry: Similarity, Right Triangles, and Trigonometry**

#### C. Define trigonometric ratios and solve problems involving right triangles.

### IM2: G-SRT.C.6: Understand that by similarity, side ratios in right triangles, including special right triangles (30-60-90 and 45-45-90), are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

### IM2: G-SRT.C.7: Explain and use the relationship between the sine and cosine of complementary angles.

### IM2: G-SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

### **Circles (I2: G-C)**

#### A. Understand and apply theorems about circles.

## Proposed Standard

IM2: G-C.A.1 Understand that all circles are similar.

IM2: G-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords, including the following:

- the relationship that exists between central, inscribed, and circumscribed angles;
- inscribed angles on a diameter are right angles; and
- a radius of a circle is perpendicular to the tangent where the radius intersects the circ

IM2: G-C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

IM2: G-C.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

IM2: G-C.B.5: Apply the formula for finding area of a sector and arc length.

### **Expressing Geometric Properties with Equations**

A. Translate between the geometric description and the equation for a conic section.

IM2: G-GPE.A.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

IM2: G-GPE.A.2: Derive the equation of a parabola given a focus and directrix.

### **Geometry: Expressing Geometric Properties with Equations**

B. Use coordinates to prove simple geometric theorems algebraically.

IM2: G-GPE.B.4: Use coordinates to prove simple geometric theorems algebraically.

### **Geometry: Geometric Measurement and Dimension**

A. Explain volume formulas and use them to solve problems.

## Proposed Standard

IM2: G-GMD.A.1: Give an informal argument, e.g., dissection arguments, Cavalieri's principle, or informal limit arguments, for the formulas of:

- the circumference of a circle;
- area of a regular polygon;
- area of a circle;
- volume of a cylinder, pyramid, and cone.

IM2: G-GMD.A.3: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

### **Statistics and Probability★: Conditional Probability and the Rules of Probability**

A. Understand independence and conditional probability and use them to interpret data.

IM2: S-CP.A.1: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

IM2: S-CP.A.2: Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

IM2: S-CP.A.3: Understand the conditional probability of A given B as  $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

IM2: S-CP.A.4: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

IM2: S-CP.A.5: Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

### **Statistics and Probability★: Conditional Probability and the Rules of Probability**

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

IM2: S-CP.B.6: Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

### Proposed Standard

IM2: S-CP.B.7: Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

IM2: S-CP.B.8: Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.

IM2: S-CP.B.9: Understand and apply the concepts of permutations and combinations.

- Use permutations and combinations to compute probabilities of compound events and solve problems.

### **Using Probability to Make Decisions (I2: S-MD)**

B. Use probability to evaluate outcomes of decisions.

IM2: S-MD.B.6 Use probabilities to make fair decisions (drawing by lots, using a random number generator, etc).

IM2: S-MD.B.7 Analyze decisions and strategies using probability concepts.

# Integrated Math 3

## Number and Quantity

### Proposed Standard

#### Number and Quantity: The Complex Number System (N-CN)

C. Use complex numbers in polynomial identities and equations.

IM3: N-CN.C.8 Extend polynomial identities to the complex numbers. For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .

IM3: N-CN.C.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## Algebra

### Proposed Standard

#### Algebra: Seeing Structure in Expressions

A. Interpret the structure of expressions.

IM3: A-SSE.A.1: Interpret expressions that represent a quantity in terms of its context.★

- a. Interpret parts of an expression, such as terms, factors, and coefficients.
- b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

IM3: A-SSE.A.2: Use the structure of an expression to identify ways to rewrite it: for a specific purpose.

#### Algebra: Seeing Structure in Expressions

B. Write expressions in equivalent forms to solve problems.

## Proposed Standard

IM3: A-SSE.B.4 Apply the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. For example, calculate mortgage payments. ★

### **Algebra: Arithmetic with Polynomials and Rational Expressions**

A. Perform arithmetic operations on polynomials.

IM3: A-APR.A.1: Understand that polynomials form a system comparable to the integers, as they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

### **Algebra: Arithmetic with Polynomials and Rational Expressions**

B. Understand the relationship between zeros and factors of polynomial

IM3: A-APR.B.2: Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

IM3: A-APR.B.3: Identify zeros of quadratic functions, and use the zeros to sketch a graph of the function defined by the polynomial.

### **Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)**

C. Use polynomial identities to solve problems.

IM3: A-APR.C.4 Describe numerical relationships using polynomial identities.

IM3: A-APR.C.5 Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

### **Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)**

D. Rewrite rational expressions.

IM3: A-APR.D.6 Rewrite simple rational expressions in different forms:

- Write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

## Proposed Standard

IM3: A-APR.D.7 Perform operations on rational expressions, building from previous knowledge that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

### Algebra: Creating Equations ★ (A-CED)

A. Create equations that describe numbers or relationships.

IM3: A-CED.A.1 Create equations and inequalities and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

IM3: A-CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

IM3: A-CED.A.3: Represent constraints by

- a. by
  - equations or inequalities, and
  - by systems of equations and/or inequalities,
- b. and interpret solutions as viable or nonviable options in a modeling context.

IM3: A-CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .*

### Reasoning with Equations and Inequalities (A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

IM3: A-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

### Reasoning with Equations and Inequalities (A-REI)

D. Represent and solve equations and inequalities graphically.

### Proposed Standard

IM3: A-REI.D.11 Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ :

Find approximate solutions by

- using technology to graph the functions.
- make tables of values, or
- find successive approximations.

Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

## Functions

### Proposed Standard

#### Functions: Interpreting Functions

B. Interpret functions that arise in applications in terms of the context.

IM3: F-IF.B.4: For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities,

- a. interpret key features of graphs and tables in terms of the quantities, and
- b. sketch graphs showing key features given a verbal description of the relationship.
- c. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries and end behavior. ★

IM3: F-IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function. ★

IM3: F-IF.B.6: Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

## Proposed Standard

### Functions: Interpreting Functions

#### C. Analyze functions using different representations.

IM3: F-IF.C.7 Graph functions are expressed symbolically and show key features of the graph by hand in simple cases and by using technology for more complicated cases. ★

- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

IM3: F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)t$ ,  $y = (0.97)t$ ,  $y = (1.01)12t$ ,  $y = (1.2)t/10$ , and classify them as representing exponential growth or decay.

IM3: F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

### Functions: Building Functions (F-BF)

#### A. Build a function that models a relationship between two quantities.

IM3: F-BF.A.1 Write a function that describes a relationship between two quantities. ★

- b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

### Functions: Building Functions(F-BF)

#### B. Build new functions from existing functions.

## Proposed Standard

IM3: F-BF.B.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative):

- Find the value of  $k$  given the graphs.
- Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- Include recognizing even and odd functions from their graphs and algebraic expressions for them.

IM3: F-BF.B.4 Find inverse functions.

- Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse
- Write an expression for the inverse.

### **Functions: Linear, Quadratic, and Exponential Models (F-LE)**

A. Construct and compare linear, quadratic, and exponential models and solve problems.

IM3: F-LE.A.4 For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology. ★

### **Functions: Trigonometric Functions (F-TF)**

A. Extend the domain of trigonometric functions using the unit circle.

IM3: F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

IM3: F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

### **Functions: Trigonometric Functions (F-TF)**

B. Model periodic phenomena with trigonometric functions.

IM3: F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★

## Geometric Reasoning and Logic

Proposed Standard
<b><u>Similarity, Right Triangles, and Trigonometry (I3: G-SRT)</u></b> <u>D. Apply trigonometry to general triangles.</u>  <u>IM3: G-SRT.C.9: Derive the formula <math>A = 1/2 ab \sin(C)</math> for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</u>  <u>IM3: G-SRT.C.10: Prove the Laws of Sines and Cosines and use them to solve problems.</u>  <u>IM3: G-SRT.C.11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</u>
<b><u>Geometry: Geometric Measurement and Dimension</u></b> <u>B. Visualize relationships between two-dimensional and three-dimensional objects.</u>  <u>IM3: G-GMD.B.4: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</u> <ul style="list-style-type: none"><li>• <u>Apply properties of two dimensional figures identified to solve problems.</u></li></ul>
<b><u>Geometry: Modeling with Geometry</u></b> <u>A. Apply geometric concepts in modeling situations.</u>  <u>IM3: G-MG.A.1: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★</u>  <u>IM3: G-MG.A.2: Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★</u>  <u>IM3: G-MG.A.3: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★</u>

## Statistics & Probability

Proposed Standard
<b>Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID)</b> A. Summarize, represent, and interpret data on a single count or measurement variable.
IM3: S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages with technology. a. Recognize that there are data sets for which such a procedure is not appropriate.
<b>Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)</b> A. Understand and evaluate random processes underlying statistical experiments.
IM3: S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★
IM3: S-IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
<b>Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)</b> A. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
IM3: S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★
IM3: S-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★
IM3: S-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.★
IM3: S-IC.B.6 Evaluate media or academic research based on data.★
<b>Using Probability to Make Decisions (I2: S-MD)</b> B. Use probability to evaluate outcomes of decisions.

### Proposed Standard

IM3: S-MD.B.6 Use probabilities to make fair decisions (drawing by lots, using a random number generator, etc)

IM3: S-MD.B.7 Analyze decisions and strategies using probability concepts.

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# Algebra III

## Number and Quantity

Proposed Standard
<b><u>N.CN The Complex Number System</u></b>
<u>N.CN.A Perform arithmetic operations with complex numbers.</u>
<u>N.CN.1 Know there is a complex number <math>i</math> such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math> and <math>b</math> real.</u>
<u>N.CN.2 Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</u>
<u>N.CN.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</u>
<u>N.CN.B Represent complex numbers and their operations on the complex plane.</u>
<u>N.CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</u>
<u>N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, <math>(-1 + \sqrt{3}i)^3 = 8</math> because <math>(-1 + \sqrt{3}i)</math> has modulus 2 and argument <math>120^\circ</math>.</u>
<u>N.CN.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.</u>
<u>N.CN.C Use complex numbers in polynomial identities and equations.</u>
<u>N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.</u>
<u>N.CN.8 Extend polynomial identities to the complex numbers. For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</u>
<u>N.CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</u>
<b><u>N.VM Vector and Matrix Quantities</u></b>

## Proposed Standard

N.VM.A Represent and model with vector quantities.

N.VM.1 Understand vector attributes.

- Represent vector quantities as having both magnitude and direction by directed line segments.
- Use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $v$ ).

N.VM.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.B Perform operations on vectors.

N.VM.4 Add and subtract vectors.

- a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- c. Understand vector subtraction  $v - w$  as  $v + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$ , and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N.VM.5 Multiply a vector by a scalar.

- a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .
- b. Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|cv| \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ).

N.VM.C Perform operations on matrices and use matrices in applications.

N.VM.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N.VM.7 Multiply matrices by scalars to produce new matrices.

- Multiply with and without technology.
- Multiply with and without context.

N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

### Proposed Standard

N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers.

- a. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector.

## Algebra

### Proposed Standard

#### A.APR Arithmetic with Polynomials and Rational Expressions

A.APR.C Use polynomial identities to solve problems.

A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.

A.APR.5 Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

A.APR.D Rewrite rational expressions.

A.APR.6 Rewrite simple rational expressions in different forms:

- Write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.APR.7 Perform operations on rational expressions, building from previous knowledge that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

#### A.REI Reasoning with Equations and Inequalities

### Proposed Standard

#### A.REI.C Solve System of Equations

A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.

A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations

- Use technology for matrices of dimension  $3 \times 3$  or greater.

## Functions

### Proposed Standard

#### F.BF Building Functions

F.BF.A Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities

c. Compose functions. For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.

F.BF.B Build new functions from existing functions.

F.BF.4 Find inverse functions.

- Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- Verify by composition that one function is the inverse of another.
- Read values of an inverse function from a graph or a table, given that the function has an inverse.
- Produce an invertible function from a non-invertible function by restricting the domain.

F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

#### F.IF Interpreting Functions

F.IF.C Analyze functions using different representations.

### Proposed Standard

E.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

E.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

## Geometric Reasoning and Logic

### Analysis Proposed Standard

#### G.GPE Expressing Geometric Properties with Equations

G.GPE.A Translate between the geometric description and the equation for a conic section.

G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.GPE.2 Derive the equation of a parabola given a focus and directrix.

G.GPE.3 (+)Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

# Advanced Math Precalculus

## Number and Quantity

Proposed Standard
<b>N.CN The Complex Number System</b>
N.CN.A Perform arithmetic operations with complex numbers.
N.CN.1 Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ with $a$ and $b$ real.
N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N.CN.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
N.CN.B Represent complex numbers and their operations on the complex plane.
N.CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument $120^\circ$ .
N.CN.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
N.CN.C Use complex numbers in polynomial identities and equations.
N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.
N.CN.8 Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$ .
N.CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
<b>N.VM Vector and Matrix Quantities</b>
N.VM.A Represent and model with vector quantities.

## Proposed Standard

### N.VM.1 Understand vector attributes.

- Represent vector quantities as having both magnitude and direction by directed line segments.
- Use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $v$ ).

### N.VM.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

### N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

### N.VM.B Perform operations on vectors.

#### N.VM.4 Add and subtract vectors.

- a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- c. Understand vector subtraction  $v - w$  as  $v + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$ , and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

#### N.VM.5 Multiply a vector by a scalar.

- a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .
- b. Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ).

### N.VM.C Perform operations on matrices and use matrices in applications.

### N.VM.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

#### N.VM.7 Multiply matrices by scalars to produce new matrices.

- Multiply with and without technology.
- Multiply with and without context.

#### N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

### Proposed Standard

N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers.  
a. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector.

N.VM.12 Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

## Algebra

### Proposed Standard

#### A.APR Arithmetic with Polynomials and Rational Expressions

A.APR.B Understand the relationship between zeros and factors of polynomials.

A.APR.B.2 Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

A.APR.C Use polynomial identities to solve problems.

A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.

A.APR.5 Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

A.APR.D Rewrite rational expressions.

## Proposed Standard

A.APR.6 Rewrite simple rational expressions in different forms:

- Write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.APR.7 Perform operations on rational expressions, building from previous knowledge that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

## A.REI Reasoning with Equations and Inequalities

A.REI.C Solve System of Equations

A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.

A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations

- Use technology for matrices of dimension  $3 \times 3$  or greater.

A.REI.D Represent and solve equations and inequalities graphically.

A.REI.D.13 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Include rational, logarithmic, exponential, and polynomial functions.

A.REI.D.14 Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, quadratic, polynomial, rational, piecewise linear (to include absolute value), logarithmic, and exponential functions. Identify domain restrictions and real and non-real solutions algebraically. ★

# Functions

Proposed Standard
<b>F.BF Building Functions</b>
F.BF.A Build a function that models a relationship between two quantities.
F.BF.1 Write a function that describes a relationship between two quantities
c. Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
F.BF.B Build new functions from existing functions.
F.BF.4 Find inverse functions.
a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse.
b. Verify by composition that one function is the inverse of another.
c. Read values of an inverse function from a graph or a table, given that the function has an inverse.
d. Produce an invertible function from a non-invertible function by restricting the domain.
F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
<b>F.IF Interpreting Functions</b>
F.IF.C Analyze functions using different representations.
F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
<b>F.TF Trigonometric Functions</b>
F.TF.A Extend the domain of trigonometric functions using the unit circle.
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

### Proposed Standard

F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F.TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for  $x$ ,  $\pi+x$ , and  $2\pi-x$  in terms of their values for  $x$ , where  $x$  is any real number.

F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F.TF.B Model periodic phenomena with trigonometric functions.

F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

F.TF.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F.TF.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

F.TF.C Prove and apply trigonometric identities.

F.TF.8 Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

F.TF.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

## Geometric Reasoning and Logic

### Analysis Proposed Standard

#### G.GPE Expressing Geometric Properties with Equations

G.GPE.A Translate between the geometric description and the equation for a conic section.

G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

## Analysis Proposed Standard

G.GPE.2 Derive the equation of a parabola given a focus and directrix.

G.GPE.3 (+)Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

### G.SRT Similarity, Right Triangles, and Trigonometry

G.SRT.D Apply trigonometry to general triangles.

G.SRT.9 Derive the formula  $A = 1/2 ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G.SRT.10 Prove the Laws of Sines and Cosines and use them to solve problems.

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

# Advanced Math Functions and Statistics

## Number and Quantity

### Proposed Standard

#### N.CN The Complex Number System

N.CN.A Perform arithmetic operations with complex numbers.

N.CN.1 Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.

N.CN.2 Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N.CN.B Represent complex numbers and their operations on the complex plane.

N.CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example,  $(-1 + \sqrt{3}i)^3 = 8$  because  $(-1 + \sqrt{3}i)$  has modulus 2 and argument  $120^\circ$ .

N.CN.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

N.CN.C Use complex numbers in polynomial identities and equations.

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.8 Extend polynomial identities to the complex numbers. For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .

N.CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

#### N.VM Vector and Matrix Quantities

N.VM.A Represent and model with vector quantities.

## Proposed Standard

### N.VM.1 Understand vector attributes.

- Represent vector quantities as having both magnitude and direction by directed line segments.
- Use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $v$ ).

### N.VM.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

### N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

### N.VM.B Perform operations on vectors.

#### N.VM.4 Add and subtract vectors.

- Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- Understand vector subtraction  $v - w$  as  $v + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$ , and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

#### N.VM.5 Multiply a vector by a scalar.

- Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .
- Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ).

### N.VM.C Perform operations on matrices and use matrices in applications.

### N.VM.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

#### N.VM.7 Multiply matrices by scalars to produce new matrices.

- Multiply with and without technology.
- Multiply with and without context.

#### N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

### Proposed Standard

N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers.

- a. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector.

## Algebra

### Proposed Standard

#### A.APR Arithmetic with Polynomials and Rational Expressions

B. Understand the relationship between zeros and factors of polynomials.

A.APR.2 Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

A.APR.C Use polynomial identities to solve problems.

A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.

A.APR.5 Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

A.APR.D Rewrite rational expressions.

A.APR.6 Rewrite simple rational expressions in different forms:

- Write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.APR.7 Perform operations on rational expressions, building from previous knowledge that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

## Proposed Standard

### **A.REI Reasoning with Equations and Inequalities**

#### A.REI.C Solve System of Equations

A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.

A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations

- Use technology for matrices of dimension  $3 \times 3$  or greater.

#### A.REI.D Represent and solve equations and inequalities graphically.

A.REI.D.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Include rational, logarithmic, exponential, and polynomial functions.

A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, quadratic, polynomial, rational, piecewise linear (to include absolute value), logarithmic, and exponential functions. Identify domain restrictions and real and non-real solutions algebraically. ★

## Functions

## Proposed Standard

### **F.BF Building Functions**

F.BF.A Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities

c. Compose functions. For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.

F.BF.B Build new functions from existing functions.

### Proposed Standard

#### E.BF.4 Find inverse functions.

- a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse.
- b. Verify by composition that one function is the inverse of another.
- c. Read values of an inverse function from a graph or a table, given that the function has an inverse.
- d. Produce an invertible function from a non-invertible function by restricting the domain.

#### F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

### **F.IF Interpreting Functions**

#### F.IF.C Analyze functions using different representations.

#### F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

#### F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

## Statistics and Probability

### Proposed Standard

#### Data in Context

#### DC.1 Use a statistical cycle to formulate questions, describe types of data, data sources, and constraints within the context of a problem.

- a. Define the stages of the statistical cycle and how each stage relates to the others.
- b. Formulate questions and conclusions based on context.
- c. Understand the type of data relevant to the question at hand (e.g., quantitative versus categorical).
- d. Compare and contrast population and sample, and parameter and statistic.
- e. Identify and explain constraints of the statistical approach.

## Proposed Standard

DC.2 Compare and contrast data collection methods to plan and conduct an observational study.

- a. Investigate and describe sampling techniques (e.g., simple random sampling, stratified sampling, systematic sampling, cluster sampling).
- b. Determine which sampling technique is best, given a particular context.
- c. Investigate and explain biased influences inherent within sampling methods and various forms of response bias.
- d. Use the statistical cycle to plan and conduct an observational study to answer a question or address a problem.

DC.3 Utilize the principles of experimental design to plan and conduct a well-designed experiment.

- a. Describe the principles of experimental design, including:
  - i. treatment/control groups;
  - ii. blinding/placebo effects;
  - iii. experimental units/subjects; and
  - iv. blocking/matched pairs and completely randomized designs.
- b. Evaluate the principles of experimental design to address comparison, randomization, replication, and control within the context of the problem.
- c. Compare and contrast controlled experiments and observational studies and the conclusions that may be drawn from each.
- d. Use the statistical cycle to plan and conduct a well-designed experiment to answer a question or address a problem.
- e. Select a data collection method appropriate for a given context.

**All Standards from this point forward were already vetted as STAT A.**

### **S.ID Interpreting Categorical and Quantitative Data**

S.ID.A Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1 Represent and interpret data with plots on the real number line (dot plots, histograms, and box plots).

S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use technology when appropriate.

S.ID.B Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

## Proposed Standard

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- a. Fit a function to the data using technology:
  - o Use functions fitted to data to solve problems in the context.
  - o Use given functions or choose a function suggested by the context.
- b. Informally assess the fit of a function by plotting and analyzing residuals.
- c. Fit a linear function for a scatter plot that suggests a linear association.

S.ID.C Interpret linear models

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S.ID.9 Distinguish between correlation and causation.

### **S.IC Making Inferences and Justifying Conclusions**

S.IC.A Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

S.IC.B Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S.IC.6 Evaluate reports based on data.

### **S.CP Conditional Probability and the Rules of Probability**

S.CP.A Understand independence and conditional probability and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions,

## Proposed Standard

intersections, or complements of other events (“or,” “and,” “not”).

S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.3 Understand the conditional probability of A given B as  $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities..

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

S.CP.B Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S.CP.7 Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

S.CP.8 Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.

S.CP.9 Understand and apply the concepts of permutations and combinations.

- Use permutations and combinations to compute probabilities of compound events and solve problems.

## S.MD Using Probability to Make Decisions

S.MD.A Calculate expected values and use them to solve problems.

S.MD.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

S.MD.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution

S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

- For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

### Proposed Standard

S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

S.MD.5 Use probability to evaluate outcomes of decisions.

S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

- a. Find the expected payoff for a game of chance.
- b. Evaluate and compare strategies on the basis of expected values.

S.MD.6 Use probabilities to make fair decisions (drawing by lots, using a random number generator, etc).

S.MD.7 Analyze decisions and strategies using probability concepts.

# Statistics and Probability

## Statistics & Probability

### Proposed Standard

#### DC Data in Context

DC.1 Use a statistical cycle to formulate questions, describe types of data, data sources, and constraints within the context of a problem.

- a. Define the stages of the statistical cycle and how each stage relates to the others.
- b. Formulate questions and conclusions based on context.
- c. Understand the type of data relevant to the question at hand (e.g., quantitative versus categorical).
- d. Compare and contrast population and sample, and parameter and statistic.
- e. Identify and explain constraints of the statistical approach.

DC.2 Compare and contrast data collection methods to plan and conduct an observational study.

- a. Investigate and describe sampling techniques (e.g., simple random sampling, stratified sampling, systematic sampling, cluster sampling).
- b. Determine which sampling technique is best, given a particular context.
- c. Investigate and explain biased influences inherent within sampling methods and various forms of response bias.
- d. Use the statistical cycle to plan and conduct an observational study to answer a question or address a problem.

DC.3 Utilize the principles of experimental design to plan and conduct a well-designed experiment.

- a. Describe the principles of experimental design, including:
  - i. treatment/control groups;
  - ii. blinding/placebo effects;
  - iii. experimental units/subjects; and
  - iv. blocking/matched pairs and completely randomized designs.
- b. Evaluate the principles of experimental design to address comparison, randomization, replication, and control within the context of the problem.
- c. Compare and contrast controlled experiments and observational studies and the conclusions that may be drawn from each.
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S.ID.A Summarize, represent, and interpret data on a single count or measurement variable.

## Proposed Standard

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S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use technology when appropriate.

S.ID.B Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- a. Fit a function to the data using technology:
  - o Use functions fitted to data to solve problems in the context.
  - o Use given functions or choose a function suggested by the context.
- b. Informally assess the fit of a function by plotting and analyzing residuals.
- c. Fit a linear function for a scatter plot that suggests a linear association.

S.ID.C Interpret linear models

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

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S.IC.A Understand and evaluate random processes underlying statistical experiments.

S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

S.IC.B Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

## Proposed Standard

S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

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### **S.CP Conditional Probability and the Rules of Probability**

S.CP.A Understand independence and conditional probability and use them to interpret data.

S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.3 Understand the conditional probability of A given B as  $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities..

S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

S.CP.B Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

S.CP.7 Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

S.CP.8 Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.

S.CP.9 Understand and apply the concepts of permutations and combinations.

- Use permutations and combinations to compute probabilities of compound events and solve problems.

## Proposed Standard

### S.MD Using Probability to Make Decisions

S.MD.A Calculate expected values and use them to solve problems.

S.MD.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

S.MD.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

- For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

S.MD.B Use probability to evaluate outcomes of decisions.

S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

- Find the expected payoff for a game of chance.
- Evaluate and compare strategies on the basis of expected values.

S.MD.6 Use probabilities to make fair decisions (drawing by lots, using a random number generator, etc).

S.MD.7 Analyze decisions and strategies using probability concepts.

# Math Essentials

Proposed Standards
<b><u>ME.PF Applying mathematics in Personal Finance</u></b>
<b><u>ME.PF.1. Use graphical and numerical techniques to study patterns and analyze data related to personal finance.</u></b>
<ul style="list-style-type: none"><li>a. <u>Use rates and linear functions to solve problems involving personal finance and budgeting, including compensation and deductions.</u></li><li>b. <u>Solve problems involving personal taxes</u></li><li>c. <u>Analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees.</u></li></ul>
<b><u>2. Use mathematical processes with algebraic formulas, graphs, and amortization modeling with technology to solve problems involving credit.</u></b>
<ul style="list-style-type: none"><li>a. <u>Use formulas with technology to generate tables to display a series of payments for loan amortizations resulting from financed purchases;</u></li><li>b. <u>Analyze personal credit options in retail purchasing and compare relative advantages and disadvantages of each option;</u></li><li>c. <u>Use technology to create amortization models to investigate home financing and compare buying a home to renting a home.</u></li><li>d. <u>Use technology to create amortization models to investigate automobile financing and compare buying a vehicle to leasing a vehicle.</u></li></ul>
<b><u>3. Use mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning.</u></b>
<ul style="list-style-type: none"><li>a. <u>Analyze and compare insurance coverage options and rates.</u></li><li>b. <u>Investigate and compare investment options, including stocks, bonds, annuities, certificates of deposit, and retirement plans.</u></li><li>c. <u>Analyze types of savings options involving simple and compound interest and compare the relative advantages of these options.</u></li></ul>
<b><u>ME.SE Applying Mathematics in Science and Engineering</u></b>
<b><u>1. Apply mathematical processes and algebraic techniques to study patterns and analyze data related to science.</u></b>
<ul style="list-style-type: none"><li>a. <u>Use proportional and inversely proportional relationships to describe physical laws</u></li><li>b. <u>Use exponential models available through technology to model growth and decay in areas such as population, biology, ecology, and chemistry, including radioactive decay; and</u></li><li>c. <u>Use quadratic functions to model motion, such as an object dropped, bounced, thrown, or kicked.</u></li></ul>

## Proposed Standards

### 2. Apply mathematical processes using algebra and geometry, with and without technology, to study patterns and analyze data related to architecture and engineering

- a. Use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in architecture.
- b. Use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields such as engineering drawing, architecture, and construction.
- c. Use the Pythagorean Theorem and special right-triangle relationships to calculate distances.
- d. Use trigonometric ratios to calculate distances and angle measures as applied to fields such as surveying, navigation, and orienteering.

### 3. Apply mathematics of measurement to industrial design problems.

- a. Identify, compare, and use various tools, such as rulers and measuring tapes in both the imperial and metric systems.
- b. Identify, convert, and apply units of length, weight, volume, time, and temperature between imperial and metric systems given a situation or context.
- c. Apply perimeter, circumference, volume and area formulas as a way to understand problems and to guide the solution of multi-step problems.
- d. Choose and interpret units and tools consistently in formulas
- e. Justify the choice of unit and tools in a given context.

## **ME.FA Applying Mathematics in Fine Arts**

### 1. Use mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts.

- a. Use trigonometric ratios and functions available through technology to model periodic behavior in art and music;
- b. Use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography;
- c. Use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music; and
- d. Use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields such as painting, sculpture, and photography.

## **ME. SS Applying Mathematics in Social Sciences**

### 1. Determine the number of elements in a finite sample space and compute the probability of an event.

- a. Determine the number of ways an event may occur using combinations, permutations, and the Fundamental Counting Principle.
- b. Compare theoretical to empirical probability in chance events.
- c. Use experiments to determine the reasonableness of a theoretical model, such as the binomial or geometric distribution.

## Proposed Standards

2. Apply mathematical processes and mathematical models to analyze data as it applies to social sciences.

- a. Interpret information from various graphs to draw conclusions from the data and determine the strengths and weaknesses of conclusions;
- b. Analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions.
- c. Distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies;
- d. Use data from a sample to estimate the population mean or population proportion;
- e. Analyze marketing claims based on graphs and statistics from electronic and print media and justify the validity of stated or implied conclusions.
- f. Use regression methods available through technology to model situations best fit by linear and exponential functions. Use the model to interpret correlations, and make predictions.

3. Apply mathematical processes to design a study and utilize graphical, numerical, and analytical techniques to communicate the study's results effectively.

- a. Formulate a meaningful question, determine the necessary data to answer the question, gather the relevant data, analyze the data, and draw reasonable conclusions.
- b. Communicate and present methods used, analyses conducted, recommendations, limitations, and conclusions drawn for a data-analysis project.  
Presentation styles can include one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation.