K-12 Louisiana Student Standards for Mathematics

Table of Contents

Standards for Math Practice

The Standards for Math Practice describe how students interact with math content. These standards apply to students in grades K-12. The practice standards bring together content in a way that supports students' logical thinking and problem-solving.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and utilize structure.
8. Look for and express regularity in repeated reasoning.

Kindergarten

Grade Level Expected Foundational Skills

By the end of Kindergarten, mathematically proficient students can reliably use the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade K, but consists of the foundational skills required by the standards.

1. Count to 100 by tens and ones.
2. Read and write numbers 0-20.
3. Count out 0-20 objects and count groups of objects up to 20.
4. Count to answer "How many?"
5. Subitize (i.e., instantly recognize without needing to count individually) groups of images or objects to recognize the quantity.
6. Compare two numbers between 1 and 20 to identify which is greater or less.
7. Fluently add and subtract within five.

Numeracy & Operational Fluency

A. Know number names and the count sequence. (K.NOF.A)

1. Count forward to 100 by ones and tens (i.e., multiples of 10s-30,40,50, etc), beginning with any given number.
    a. Count forward to 10 and backward from 10 by ones.
    b. Count forward to 20 and backward from 20 by ones.
2. Read, write, and represent whole numbers from 0 to 20 with a written numeral, objects, or pictures.

B. Count to tell the number of objects. (K.NOF.B)

3. Understand the relationship between numbers and quantities; connect counting to cardinality, with cardinality referring to the number that connects the final count number to its quantity of an entire set.
    a. When counting objects in standard order, say the number names as they relate to each object in the group, demonstrating one-to-one correspondence.
    b. Understand that the last number name said tells the number of objects counted (cardinality). The number of objects is the same regardless of their arrangement or the order in which they were counted.
    c. Understand that each successive number name refers to a quantity that is one larger, which may include the use of objects or visual representations.
4. Answer "How many?" questions by subitizing (perceptually and conceptually) and counting.
    a. Subitize within 5.
    b. Subitize or count objects up to 20, arranged in a line, a rectangular array, or a circle.
    c. Subitize or count objects up to 10 in a scattered configuration.
    d. When given a number from 1-20, count out that many objects.

C. Compare numbers. (K.NOF.C)

5. Compare sets of objects up to and including 20 in each set using comparative language, e.g., by using matching and counting strategies.
6. Use comparative language to describe numbers up to 20 presented as written numerals.

D. Add and subtract within 5. (K.NOF.D)

7. Fluently add and subtract within 5, varying placement of the equal sign, while also ensuring accurate reading of the equation from left to right. For example
    - 5 = 2 + 3 (five equals two plus three)
    - 2 + 2 = 4 (two plus two is equal to four)
    - 3 = 3 - 0 (three is equal to three minus zero)
    - 4 - 1 = 3 (four minus one equals three)

E. Work with numbers 11-19 to gain foundations for place value. (K.NOF.E)

8. Gain an understanding of place value.
    a. Understand that the numbers 11–19 are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
    b. Compose and decompose numbers 11 to 19 using place value understanding (e.g., by using objects, drawings, or verbal responses identifying tens and ones).
    c. Record each composition or decomposition using a drawing or equation. For example: 18 is one ten and eight ones, 18 = 1 ten + 8 ones, 18 = 10 + 8.

Algebraic Reasoning

A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. (K.AR.A)

1. Represent addition and subtraction of two whole numbers from 0 to 10 with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Add and subtract within 10, e.g., by using objects or drawings to represent the problem.
    a. Solve addition and subtraction real-world mathematical tasks and explain the strategies used with spoken words, models, and/or equations.
3. Decompose numbers less than or equal to 10 (identified as the whole) into parts in more than one way to demonstrate the part: whole relationship.
    a. Record each decomposition with a drawing or equation.
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using spoken words, objects, models, and/or equations.

Geometric Reasoning & Logic

A. Identify and describe shapes (Squares, Circles, Triangles, Rectangles, Hexagons, Cubes, Cones, Cylinders, And Spheres. (K.GL.A)

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind, and next to.*
2. Correctly name shapes regardless of their orientations or overall size.
    - Identify examples and nonexamples of those shapes.
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).
    - Identify faces of three-dimensional shapes as two-dimensional geometric figures.

B. Analyze, compare, create, and compose shapes. (K.GL.B)

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

Data Analysis & Measurement

A. Describe and compare measurable attributes. (K.DM.A)

1. Describe measurable attributes of a set of objects, such as length or weight.
    - Given a single object, describe several measurable attributes.
2. Directly compare two objects with a measurable attribute in common, to describe the difference (more of, less of, etc.).

B. Classify objects and count the number of objects in categories. (K.DM.B)

3. Classify objects into two given categories based on their attributes (limit category counts to less than or equal to 20).
    - Countnumbers of objects in each category.
    - Order the categories by count.

C. Work with money. (K.DM.C)

4. Identify pennies, nickels, dimes, and quarters by name and value.
    - Identify fair-trade values within a dime:
        - 10 pennies = 1 dime
        - 2 nickels = 1 dime
        - 5 pennies = 1 nickel

Grade 1

Grade Level Expected Foundational Skills

By the end of grade 1, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 1, but is comprised of the foundational skills required by the standards.

1. Fluently add and subtract within 10.
2. Identify equivalent sums or differences of one-digit numbers. Ex. 6 - 2 = 2 + 2
3. Determine the missing number in an equation with numbers less than or equal to 10. Ex. 8 + ? = 10; 5 = □ - 3.
4. Given a two-digit number, mentally find 10 more or 10 less.
5. Count to 120 by 5's.
6. Read and write numbers through 120.
7. Compare numbers up to 100, identifying whether one number is *greater than, less than,* or *equal to* another.

Numeracy and Operational Fluency

A. Add and subtract. (1.NOF.A)

1. Relate counting to addition and subtraction. For example, counting on 2 to add 2.
2. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.
    a. Use mental strategies such as:
        - counting on or counting back;
        - using a number line or path;
        - making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
        - decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9);
        - using the part-whole relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and
        - creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
3. Given a two-digit number, mentally find one more, 10 more, one less, or 10 less than a given number, without having to count.
    a. Justify the reasoning used when computing mentally with an oral and/or written explanation.

B. Extending the counting sequence. (1.NOF.B)

4. Count to 120, starting at any number less than 120.
    a. Count forward and backward to 120 by ones, fives, and tens.
    b. In this range, read, write, and identify numerals and number names written in words.

C. Understand place value. (1.NOF.C)

5. Understand that the two digits of a two-digit number represent amounts of tens and ones.
    a. Read, write, and represent two-digit numbers up to 99 using base-ten numerals/standard form, word form, and unit form (i.e., 42 and 4 tens 2 ones). Understand the following as special cases:
        - 10 can be thought of as a bundle of ten ones — called a "ten."
        - The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
        - The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens and 0 ones.

Algebraic Reasoning

A. Represent and solve problems involving addition and subtraction. (1.AR.A)

1. Use addition and subtraction within 20 to solve real-world mathematical tasks involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.
    a. Represent these situations using models and equations, elevating the concrete-representational-abstract instructional framework.
2. Solve real-world mathematical tasks that call for addition of three whole numbers whose sum is less than or equal to 20.
    a. Represent these situations using models and equations, elevating the concrete-representational-abstract instructional framework.

B. Understand and apply properties of operations and the relationship between addition and subtraction. (1.AR.B)

3. Apply properties of operations to add and subtract.
4. Understand subtraction as an unknown-addend problem.

C. Work with addition and subtraction equations. (1.AR.C)

5. Interpret the meaning of the equal sign.
    a. Understand that the equal sign represents a relationship where expressions on each side of the equal sign represent the same value(s).
    b. Determine if equations involving addition and subtraction are true or false.
6. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

D. Compare numbers. (1.AR.D)

7. Compare two, two-digit numbers based on values of the tens and ones digits, recording the results of comparisons with comparative language or the symbols >, =, < and the words *greater than, equal to* and *less than.*

E. Use place value understanding and properties of operations to add and subtract. (1.AR.E)

8. Add up to 99, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10.
    a. Use concrete models or drawings and strategies based on place value, properties of operations, and/or the part:whole relationship between addition and subtraction; relate the strategy to a number sentence.
    b. Understand that in adding two-digit numbers, add tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
    c. Justify the reasoning of methods for addition used with an oral and/or written explanation.

9. Subtract two digit numbers, limited to multiples of 10 using concrete models or drawings and strategies based on place value, properties of operations, and/or the part:whole relationship between addition and subtraction.
    a. Justify the reasoning used when choosing the model or strategy to compute with an oral and/or written explanation.

Geometric Reasoning and Logic

A. Reason with shapes and their attributes. (1.GL.A)

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size).
    a. Build and draw shapes that possess defining attributes.
    b. Verbally describe why a shape belongs to a given category.
2. Compose two-dimensional shapes and three-dimensional shapes to create a composite shape, and compose new shapes from the composite shape, building understanding of the part:whole relationships.
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths, and quarters*, and use the phrases *half of, fourth of, and quarter of*. Describe the whole as two of, or four of the shares.
    a. Understand for these examples that decomposing into more equal shares creates smaller shares.
    b. Identify examples and non-examples of halves and fourths.

Data Analysis and Measurement

A. Measure lengths indirectly and by iterating length units. (1.DM.A)

1. Apply knowledge of "longer than" and "shorter than."
    a. Order three objects by length.
    b. Compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end.
    a. Understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

B. Tell and write time. (1.DM.B)

3. Tell, write, and represent time in hours and half-hours using analog and digital clocks.

C. Represent and interpret data. (1.DM.C)

4. Use tally marks and t-charts/tables with up to three categories to:
    a. Create bar graphs and/or picture graphs to represent data precisely with accurate scaling within a scale of 1 or 2; and
    b. Interpret data represented in a chart or graph by asking and answering questions about:
        - the total number of data points;
        - how many in each category; and
        - how many more or less are in one category than in another?

D. Work with money. (1.DM.D)

5. Determine the value of a collection of coins up to 50 cents. (Pennies, nickels, dimes, and quarters in isolation; not to include a combination of different coins.)

Grade 2

Grade Level Foundational Skills

By the end of grade 2, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 2, but is comprised of the foundational skills required by the standards.

1. Know all sums of two one-digit numbers by memory.
2. Mentally add and subtract within 20.
3. Fluently add and subtract within 100 using strategies.
4. Count within 1000 by 2s, 5s, and 10s.
5. Read and write numbers within 1000.
6. Given a three-digit number, mentally find 100 more or 100 less.
7. Compare numbers up to 1000, identifying whether one number is greater than, less than, or equal to another.
8. Measure to determine how much longer one object is than another using standard length units.

Numeracy & Operational Fluency

A. Add and subtract within 20. (2.NOF.A)

1. Fluently add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers. See standard 1.NOF.A.2 for a list of mental strategies.

B. Understand place value. (2.NOF.B)

2. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. Understand the following as special cases:
    - 100 can be thought of as a bundle of ten tens — called a “hundred.”
    - The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
3. Count forward and backward up to 1000, starting at any number less than 1000.
    a. Skip-count by 2s, 5s, 10s and 100s.
4. Read and write numbers up to 1000 using base-ten numerals in standard form, number names in written form, unit form, and expanded form.
5. Compare two three-digit numbers based on values of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

C. Use place value understanding and properties of operations to add and subtract. (2.NOF.C)

6. Fluently add and subtract up to 100 using strategies based on place value, properties of operations, and/or the part-whole relationship between addition and subtraction.
7. Add up to four two-digit numbers using strategies based on place value and properties of operations.
8. For a given number 100-900:
    a. Mentally add 10 or 100; and
    b. Mentally subtract 10 or 100.
9. Construct a written explanation or drawing for why addition and subtraction strategies work, using place value understanding and the properties of operations.

Algebraic Reasoning

A. Represent and solve problems involving addition and subtraction. (2.AR.A)

1. Use addition and subtraction within 100 to solve one- and two-step real-world mathematical tasks involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions for one or two operations.
    a. Represent these situations using models and equations, elevating the concrete-representational-abstract instructional framework.
2. Add and subtract up to 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the part: whole relationship between addition and subtraction.
    a. Understand that in adding or subtracting three-digit numbers, add or subtract hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
    b. Justify the reasoning used when choosing the model or strategy to compute with a written explanation.

B. Work with equal groups of objects to gain foundations for multiplication. (2.AR.B)

3. Determine whether a group of objects (up to 20) has an odd or even number of members.
    a. Represent these situations by pairing objects or counting them by 2s.
    b. Write an equation to express an even number as a sum of equal addends.
4. Use addition to find the total number of objects arranged in equal groups and rectangular arrays with up to 5 rows and up to 5 columns.
    a. Write an equation to express the total as a sum of equal addends (a repeated addition equation).

Geometric Reasoning & Logic

A. Reason with shapes and their attributes. (2.GL.A)

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces, with sizes being compared directly or visually. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Apply spatial structuring to partition a rectangle into rows and columns of same-size units, interpreting both as a collection of units and as single units, and count to find the total number of units. The rectangle should not be divided up into more than 5 columns and 5 rows to correlate with 2.AR.B.4.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths.
    a. Identify examples and non-examples of halves, thirds, and fourths.

Data Analysis and Measurement

A. Measure and estimate lengths in standard units. (2.DM.A)

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using two different units of length (inches, feet, centimeters, or meters) or two different measurements.
    a. Describe how the two measurements relate to the size of the unit chosen with a written response.
3. Estimate lengths using units of inches, feet, centimeters, and meters to the nearest whole unit.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of the same standard length unit.

B. Relate addition and subtraction to length. (2.DM.B)

5. Use addition and subtraction up to 100 to solve one- and two-step real-world mathematical tasks involving lengths that are given in the same units by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Create a number line diagram with equally spaced points corresponding to whole numbers (0, 1, 2, etc.), using 0 or another whole number as a starting point.
    a. Recognize that each mark on a number line represents one whole number and that each position corresponds to a single value.
    b. Represent whole numbers as lengths from 0.
    c. Represent whole-number sums and differences up to 100 on a number line diagram.

C. Work with time and money. (2.DM.C)

7. Tell, write, and represent time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
    a. Express portions of an hour using the fractional terms half an hour, half past, quarter of an hour, quarter after, and quarter til.
8. Solve real-world mathematical tasks involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.

D. Represent and interpret data. (2.DM.D)

9. Generate measurement data by either measuring the lengths of several objects to the nearest whole unit or by taking repeated measurements of the same object.
    a. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph with a scale of 1, 2, 5, or 10 to represent a data set with up to four categories.
    a. Solve simple put-together, take-apart, and compare real-world mathematical tasks using information presented in a picture graph or bar graph.

Grade 3

Grade Level Expected Foundational Skills

By the end of grade 3, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 3, but is comprised of the foundational skills required by the standards.

1. Know all products of two one-digit numbers from memory.
2. Know all quotients of numbers within 100 by memory.
3. Fluently add and subtract within 1,000.
4. Compare numbers up to 100,000, identifying greater than, less than, or equal to.
5. Estimate and round whole numbers to the nearest 10 and 100.
6. Identify equivalent fractions with limited denominators.

Numeracy & Operational Fluency

A. Develop an understanding of fractions as numbers. (3.NOF.A)

1. Understand and interpret a fraction with denominators 2, 3, 4, 6, and 8.
    a. Understand a unit fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole or a set is partitioned into *b* equal parts where *b* is a non-zero whole number.
    b. Understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$.
    c. Represent fractions greater than zero and less than or equal to one using concrete objects, number lines, or pictorial models.
    d. Read and write fractions in standard form and written unit form.
    e. Solve real-world mathematical tasks involving partitioning an object or set of objects, identifying a fraction as parts of a whole.
2. Understand a fraction with denominators 2, 3, 4, 6, and 8 as a number on a number line diagram.
    a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part, based at 0, locates the number $\frac{1}{b}$ on the number line.
    b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off *a* lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.
3. Explain equivalence of fractions with denominators 2, 3, 4, 6, and 8 as fractions that have different numerators and denominators but are equal to the same value, and compare fractions by reasoning about their size.
    a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
    b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model or number line.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model or number line.

B. Multiply and divide within 100. (3.NOF.B)

4. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know all products of two one-digit numbers from memory.

C. Use place value understanding and properties of operations to perform multi-digit arithmetic. (3.NOF.C)

5. Use place value understanding to:
    a. Round whole numbers to the nearest 10 or 100.
    b. Use compatible numbers to estimate solutions to real-world mathematical tasks.
6. Fluently add and subtract up to 1000 using strategies and algorithms based on place value understanding, properties of operations, and/or the part-whole relationship between addition and subtraction.
7. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 80, 5 60) using strategies based on place value understanding and properties of operations.

Algebraic Reasoning

A. Represent and solve problems involving multiplication and division. (3.AR.A)

1. Interpret products of whole numbers/factors.
2. Interpret whole-number quotients of whole numbers (i.e., dividends and divisors).
3. Use multiplication and division within 100 to solve real-world mathematical tasks in situations involving equal groups, arrays, and measurement quantities.
    a. Represent these situations by using models, drawings, and equations with a symbol for the unknown number to represent the problem.
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers by representing the situation with a model or drawing and solving for the unknown.

B. Understand properties of multiplication and the relationship between multiplication and division. (3.AR.B)

5. Apply properties of operations as strategies to multiply and divide.
6. Understand division as an unknown-factor problem, providing an explanation that leverages the relationship between and the properties of multiplication and division.

C. Solve real-world mathematical tasks involving the four operations and identify and explain patterns in arithmetic. (3.AR.C)

7. Solve real-world mathematical tasks involving at least two steps and using at least two of the four operations.
    a. Represent these problems using equations with a letter standing for the unknown quantity.
    b. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.
8. Identify, create, and extend arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations without the formal language of the properties.

D. Compare whole numbers. (3.AR.D)

9. Using understanding of the base-ten system:
    a. Compare whole numbers up to 100,000, using >, =, and < symbols to record the results of comparisons.
    b. Order a set of whole numbers up to 100,000.

Geometric Reasoning and Logic

A. Reason with shapes and their attributes. (3.GL.A)

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
    a. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

Data Analysis & Measurement

A. Solve problems involving measurement and estimation liquid volumes and masses of objects. (3.DM.A)

1. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).
    a. Add, subtract, multiply, or divide to solve one-step real-world mathematical tasks involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

B. Represent and interpret data. (3.DM.B)

2. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.
    a. Solve one- and two-step "how many more" and "how many less" real-world mathematical tasks using information presented in scaled bar graphs.
3. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch.
    a. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units — whole numbers, halves, or quarters.

C. Understand concepts of area and relate area to multiplication and to addition. (3.DM.C)

4. Recognize area as an attribute of plane figures and understand concepts of area measurement.
    a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
    b. A plane figure has an area of *n* square units if it can be covered entirely, without any gaps or overlaps, by n unit squares.
5. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
6. Relate area to the operations of multiplication and equal groups of addition.
    a. Find the area of a rectangle with whole-number side lengths by tiling it, and connect that the area is the same as multiplying the side lengths.
    b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world mathematical tasks, and represent whole-number products as rectangular areas in mathematical reasoning.
    c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side

lengths *a* and *b + c* is the sum of *a x b* and *a x c*. Use area models to represent the distributive property in mathematical reasoning.

D. Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. (3.DM.D)

7. Solve real-world mathematical tasks involving perimeters of polygons, including:
    - finding the perimeter given the side lengths,
    - finding an unknown side length, and
    - exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

E. Work with time and money. (3.DM.E)

8. Understand time to the nearest minute.
    a. Tell, write, and represent time to the nearest minute and measure time intervals in minutes, within 60 minutes, on an analog and digital clock.
    b. Calculate elapsed time greater than 60 minutes to the nearest quarter and half hour on a number line diagram.
    c. Solve real-world mathematical tasks involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
9. Solve real-world mathematical tasks involving at least two steps and using the four operations with pennies, nickels, dimes, quarters, and bills greater than one dollar, using the $ and ¢ symbols appropriately.

Grade 4

Grade Level Expected Foundational Skills

By the end of grade 4, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 4, but is comprised of the foundational skills required by the standards.

1. Fluently add and subtract up to 1,000,000.
2. Identify factors and multiples within 100.
3. Compare and order numbers up to 1,000,000, identifying whether one number is greater than, less than, or equal to another.
4. Multiply and divide with multi-digit whole numbers.
5. Estimate and round whole numbers within 1,000,000 to any place value.
6. Add and subtract fractions with like denominators.
7. Multiply a fraction by a whole number.

Numeracy and Operational Fluency

A. Extend understanding of fraction equivalence and ordering. (4.NOF.A)

1. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$ by using visual fraction models or number line diagrams, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, and by comparing to a benchmark fraction such as $\frac{1}{2}$.
    - Recognize that comparisons are valid only when the two fractions refer to the same whole.
    - Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model or a number line diagram. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (4.NOF.B)

3. Understand a fraction $\frac{a}{b}$ with a > 1 as a sum of fractions 1/b. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
    a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
    b. Decompose a fraction, including mixed numbers and fractions greater than 1, into a sum of fractions, including unit fractions and non-unit fractions, with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model or number line diagram.
    c. Evaluate the reasonableness of sums and differences of fractions using benchmark fractions, 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, referring to the same whole.

4. Add and subtract fractions with like denominators.
    a. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction greater than 1 and/or by using properties of operations and the relationship between addition and subtraction.
    b. Solve real-world mathematical tasks involving addition and subtraction of fractions, including mixed numbers and fractions greater than 1, referring to the same whole and having like denominators, e.g., by using visual fraction models, number lines, or equations to represent the problem.
5. Multiply a fraction by a whole number. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)
    a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$.
    b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.
    c. Solve real-world mathematical tasks involving multiplication of a fraction by a whole number, e.g., by using visual fraction models, number line diagrams, and equations to represent the problem.

C. Understand decimal notation for fractions, and compare decimal fractions. (4.NOF.C)

6. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.
7. Use decimal notation and precise language for fractions with denominators 10 or 100.
8. Compare two decimals to hundredths by reasoning about their size.
    - Recognize that comparisons are valid only when the two decimals refer to the same whole.
    - Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model or number line diagram.

D. Gain familiarity with factors and multiples. (4.NOF.D)

9. Using whole numbers in the range 1–100,
    a. Find all factor pairs for a given whole number.
    b. Recognize that a given whole number is a multiple of each of its factors.
    c. Determine whether a given whole number is a multiple of a given one-digit number.
    d. Determine whether a given whole number is prime or composite.

E. Generalize place value understanding for multi-digit whole numbers. (4.NOF.E)

10. Recognize that in a multi-digit whole number less than or equal to 1,000,000, a digit in one place represents ten times what it represents in the place to its right
11. Read and write multi-digit whole numbers up to 1,000,000 using base-ten numerals/standard form, written form (number names), unit form, and expanded form.
12. Use place value understanding to
    a. Round multi-digit whole numbers up to 1,000,000, to any place.
    b. Use compatible numbers to estimate solutions to real-world mathematical tasks.

F. Add and subtract multi-digit numbers. (4.NOF.F)

13. Fluently add and subtract multi-digit whole numbers with sums or differences less than or equal to 1,000,000, using a standard algorithm.

Algebraic Reasoning

A. Use the four operations with whole numbers to solve real-world mathematical tasks. (4.AR.A)

1. Interpret a multiplication equation as a comparison and represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve real-world mathematical tasks involving multiplicative comparison.
    a. Represent these tasks by using drawings and/or equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
3. Solve multi-step real-world mathematical tasks involving more than one operation posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.
    a. Represent these problems using equations with a letter standing for the unknown quantity.
    b. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.

B. Generate and analyze patterns. (4.AR.B)

4. Generate and extend a number or shape pattern that follows a given rule.
    a. Identify apparent features of the pattern that were not explicit in the rule itself.

C. Compare whole numbers. (4.AR.C)

5. Using understanding of the base-ten system:
    a. Compare multi-digit whole numbers up to 1,000,000 using >, =, and < symbols to record the results of comparisons.
    b. Order a set of whole numbers up to 1,000,000.

D. Use place value understanding and properties of operations to perform multi-digit arithmetic. (4.AR.D)

6. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value understanding and the properties of operations.
    a. Illustrate and explain the calculation by using equations, rectangular arrays, number line diagrams, and/or area models.
7. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value understanding, the properties of operations, and/or the relationship between multiplication and division.
    a. Represent and explain the calculation by using equations, rectangular arrays, and/or area models.

Geometric Reasoning and Logic

A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles. (4.GL.A)

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size.
    a. Recognize right triangles as a category, and identify right triangles.
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Data Analysis and Measurement

A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (4.DM.A)

1. Know relative sizes of measurement units within one system of units, including ft, in; km, m, cm; kg, g; lb, oz; l, ml; hr, min, sec.
    a. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit.
    b. Record measurement equivalents in a two-column table. (Conversions are limited to one-step conversions.)
2. Use the four operations to solve multi-step real-world mathematical tasks involving:
    - distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving whole numbers and/or simple fractions, and
    - problems that require expressing measurements given in a larger unit in terms of a smaller unit.
3. Apply the area and perimeter formulas for rectangles in real-world mathematical tasks.

B. Represent and interpret data (4.DM.B)

4. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$).
    a. Solve real-world mathematical tasks involving the addition and subtraction of fractions with like denominators by using information presented in line plots.

C. Geometric measurement: understand concepts of angle and measure angles. (4.DM.C)

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.
    a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where two rays intersect the circle.
    b. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles.
    c. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
6. Measure angles in whole-number degrees using a standard 180 ° protractor.
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts.
    a. Solve addition and subtraction real-world mathematical tasks to find unknown angles on a diagram, e.g., by using an equation with a letter for the unknown angle measure.

D. Relate area to operations of multiplication and addition.

8. Recognize area as additive.
    a. Decompose two-dimensional composite figures whose sides meet at right angles into non-overlapping rectangles and squares. Apply area formulas to find the area of each part, and use addition or subtraction to determine the total area of the composite figure.

Grade 5

Grade Level Expected Foundational Skills

By the end of grade 5, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 5, but is comprised of the foundational skills required by the standards.

1. Evaluate and compare simple expressions.
2. Fluently multiply multi-digit whole numbers using a standard algorithm.
3. Divide multi-digit whole numbers with two-digit divisors.
4. Multiply a fraction by a whole number or a fraction.
5. Read and write decimals to the thousandths.
6. Compare and order whole numbers and decimals, identifying greater than, less than, or equal to.
7. Estimate and round multi-digit numbers with decimals to any place.
8. Add and subtract fractions with unlike denominators.

Numeracy and Operational Fluency

A. Use equivalent fractions as a strategy to add and subtract fractions. (5.NOF.A)

1. Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions to produce an equivalent equation with fractions that have like denominators.
2. Solve real-world mathematical tasks involving addition and subtraction of fractions.
    a. Add and subtract fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models, number line diagrams, or equations to represent the problem.
    b. Use benchmark fractions and number sense of fractions to estimate mentally and justify the reasonableness of answers.

B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (5.NOF.B)

3. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b}$ = a ÷ b). Solve real-world mathematical tasks involving division of whole numbers in the form of fractions to include fractions greater than one or mixed numbers, e.g., by using visual fraction models, number line diagrams, or equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
    a. Interpret the product ($\frac{m}{n}$) x q as *m* parts of a partition of *q* into *n* equal parts; equivalently, as the result of a sequence of operations, m x q ÷ n.
    b. Construct or critique a precise model to develop understanding of the concept of multiplying two fractions and create a story context for the equation.
    In general, ($\frac{m}{n}$) x ($\frac{c}{d}$) = $\frac{(mc)}{(nd)}$.

5. Interpret multiplication as scaling (resizing) by:
    a. Comparing the size of a product to the size of one factor based on the size of the other factor, without performing the indicated multiplication.
    b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case).
    c. Explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.
    d. Relating the principle of fraction equivalence.
6. Represent and solve real-world mathematical tasks involving multiplication of fractions, including fractions greater than 1 and mixed numbers, e.g., by using visual fraction models, number line diagrams, or equations.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions in the context of real-world mathematical tasks, e.g., by using visual fraction models, number line diagrams, or equations to represent the problem.
    a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.
    b. Interpret division of a whole number by a unit fraction, and compute such quotients.

C. Understand the place value system. (5.NOF.C)

8. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.
9. Use place value understanding to
    a. Round multi-digit numbers with decimals to any place; and
    b. Use compatible numbers to estimate solutions to real-world mathematical tasks.

D. Multiply multi-digit numbers. (5.NOF.D)

10. Fluently multiply multi-digit whole numbers using a standard algorithm.

Algebraic Reasoning

A. Write and interpret numerical expressions. (5.AR.A)

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols, attending to the order of operations and the properties of operations.
    a. Compare two simple expressions using >, =, or < to record the comparison of expressions limited to three operations and one grouping symbol.
2. Write simple expressions that record calculations with whole numbers, fractions, and decimals, and interpret numerical expressions without evaluating them.

B. Use place value understanding to apply patterns beyond whole numbers. (5.AR.B)

3. Generate and extend two numerical patterns using two given rules. Identify apparent relationships between corresponding terms.
    a. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
    b. Explain these relationships informally.

C. Use place value understanding to apply patterns beyond whole numbers. (5.AR.C)

4. Construct a written explanation and apply patterns in the number of zeros of the product when multiplying a number by powers of 10.
    a. Explain and apply patterns in the values of the digits in the product or the quotient when a decimal is multiplied or divided by a power of 10.
    b. Use whole-number exponents to denote powers of 10.
5. Using understanding of the base-ten system:
    a. Read and write decimals to thousandths using base-ten numerals/standard form, written form (number names), unit form, and expanded form.
    b. Compare and order multi-digit whole numbers and decimals to thousandths based on the values of the digits in each place, using >, =, and < symbols to record the results of comparisons.

D. Perform operations with multi-digit whole numbers with decimals to hundredths. (5.AR.D)

6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value understanding, the properties of operations, subtracting multiples of the divisor, and/or the relationship between multiplication and division.
    a. Illustrate and/or explain the calculation by using equations, rectangular arrays, area models, or other strategies based on place value understanding.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value understanding, properties of operations, and/or the part-whole relationship between addition and subtraction or multiplication and division.
    a. Justify the reasoning of methods used for calculation with a written explanation.

Geometric Reasoning and Logic

A. Generate and analyze patterns. (5.GL.A)

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.
    - Understand that in an ordered pair, the first number shows how far to move along the x-axis, and the second number shows how far to move along the y-axis from the origin.
2. Represent real-world mathematical tasks by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

B. Classify two-dimensional figures into categories based on their properties. (5.GL.B)

3. Analyze and relate attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.
4. Classify quadrilaterals in a hierarchy based on properties.
    a. Justify the reasoning for classification with a written response.

Data Analysis and Measurement

A. Convert like measurement units within a given measurement system. (5.DM.A)

1. Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step, real-world mathematical tasks involving:
    - distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving whole numbers, decimals, and fractions.

B. Represent and interpret data (5.DM.B)

2. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$).
    - Use operations on fractions, excluding dividing fractions by fractions, to solve real-world mathematical tasks involving information presented in line plots.

C. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. (5.DM.C)

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
    a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
    b. A solid figure that can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
5. Relate volume to the operations of multiplication and addition and solve real-world mathematical tasks involving volume.
    a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and connect that the volume is the same as multiplying the edge lengths, also by multiplying the height by the area of the base (i.e., compute the layers of area).
        - Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
    b. Apply the formulas *V = l x w x h* and *V = B x h* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world mathematical tasks.
    c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world mathematical tasks.

D. Geometric measurement: extend previous understandings of area and multiplication to multiply fractions. (5.DM.D)

6. Apply and extend previous understandings of area and multiplication to multiply a fraction or whole number by a fraction.
    a. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.
    b. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
    c. Compose and decompose rectangular regions to calculate area.

Grade 6

Grade Level Expected Foundational Skills

By the end of grade 6, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 6, but is comprised of the foundational skills required by the standards.

1. Fluently divide multi-digit numbers using standard algorithms.
2. Divide fractions by fractions.
3. Add, subtract, multiply, and divide multi-digit decimals using standard algorithms.
4. Determine the unknown number in an addition or subtraction equation.
5. Determine the unknown number in a multiplication and division equation.
6. Evaluate expressions with positive, whole number exponents.

Numeracy and Operational Fluency

A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions. (6.NOF.A)

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example,
    - create a story context for (2/3) ÷ (3/4) and illustrate the quotient by using a visual fraction model;
    - use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3.
    - In general, (a/b) ÷ (c/d) = ad/bc.

B. Compute fluently with multi-digit numbers and find common factors and multiples. (6.NOF.B)

2. Fluently divide positive multi-digit whole numbers using a standard algorithm.
3. Fluently add, subtract, multiply, and divide positive multi-digit decimals to the thousandths using a standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2) because the greatest common factor of 36 and 8 is 4.

C. Apply and extend previous understandings of the system of rational numbers. (6.NOF.C)

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.
b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

7. Understand ordering and absolute value of rational numbers.
   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.
   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
   d. Distinguish comparisons of absolute value from statements about order.
8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Algebraic Reasoning

A. Apply and extend previous understandings of arithmetic to algebraic expressions. (6.AR.A)

1. Understand, write, and evaluate numerical expressions involving whole-number exponents.
   a. Identify parts of exponential notation using mathematical terms (base and exponent).
   b. Represent and evaluate powers using a whole number or fraction as the base and a whole number as an exponent, and evaluate numerical expressions.
2. Write, read, and evaluate expressions in which letters stand for numbers.
   a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as 5–y.
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, and coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8+7) as a product of two factors; view (8+7) as both a single entity and a sum of two terms.
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V=s^3$ and $A=6\ s^2$ to find the volume and surface area of a cube with sides of length s=1/2.
3. Apply the properties of operations to generate equivalent expressions, identify when two expressions are equivalent, and explain why two expressions are equivalent. For example, we can apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); and apply properties of operations to y + y + y to produce the equivalent expression 3y.

B. Reason about and solve one-variable equations and inequalities. (6.AR.B)

4. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
5. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6. Solve real-world and mathematical problems by writing and solving equations and inequalities of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers. Inequalities will include >, <, ≤, and ≥.
7. Translate a real-world written description into an algebraic inequality in the form of x > c or x < c. Describe and interpret the infinitely many solutions for x>c or x<c and graph the solutions on a number line.

C. Represent and analyze quantitative relationships between dependent and independent variables. (6.AR.C)

8. Use variables to represent two quantities in a real-world problem that change in relationship to one another. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.
    a. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.
    b. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Geometric Reasoning and Logic

A. Solve real-world and mathematical problems involving area, surface area, and volume. (6.GL.A)

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = Bh$ *and* $V = lwh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Data Analysis

A. Develop understanding of statistical variability. (6.DA.A)

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

B. Summarize and describe distributions. (6.DA.B)

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
    a. Reporting the number of observations.
    b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
    c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
    d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Proportionality and Functions

A. Understand ratio concepts and use ratio reasoning to solve problems. (6.PF.A)

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities; use appropriate notation a:b, a to b, where $b \neq 0$. For example,
    - "The ratio of wings to beaks in the birdhouse at the zoo was 2:1 (2 to 1) because, for every 2 wings, there was 1 beak."
    - "For every vote candidate A received, candidate C received nearly three votes (A: C or A to C)."
2. Understand the concept of a unit rate a/b associated with a ratio a:b where b ≠ 0, and use rate language in the context of a ratio relationship. Expectations for unit rates in this grade are limited to non-complex fractions. For example,
    - "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar."
    - "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."
3. Use and apply ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
    a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
    b. Solve unit rate problems, including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what unit rate were lawns being mowed?
    c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part, and the percent.
    d. Use ratio reasoning to convert measurement units within and between the U.S. customary and metric systems; manipulate and transform units appropriately when multiplying or dividing quantities.

Grade 7

Grade Level Foundational Skills

By the end of grade 7, mathematically proficient students can reliably use all prior foundational skills and the skills foundational to grade 7 to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 7, but consists of the foundational skills required by the standards.

1. Fluently add, subtract, multiply, and divide positive and negative rational numbers in the form of whole numbers, fractions, and decimals.
2. Accurately convert a rational number to a decimal, recognizing the decimal form of rational numbers.
3. Use properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients to include multiple sets of grouping symbols.
4. Fluently solve one-variable equations.

Numeracy and Operational Fluency

A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. (7.NOF.A)

1. Apply and extend previous understandings of addition and subtraction and of fractions to add and subtract rational numbers flexibly and accurately; represent addition and subtraction on a horizontal or vertical number line diagram.
    a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has a net charge of 0 because its two components carry opposite charges.
    b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0, as they are additive inverses. Interpret sums of rational numbers by describing real-world contexts.
    c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (– q). Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.
    d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers flexibly and accurately.
    a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
    b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts.
    c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Algebraic Reasoning

A. Use properties of operations to generate equivalent expressions. (7.AR.A)

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with positive and negative rational coefficients to include multiple grouping symbols (e.g., parentheses, brackets, and braces).
2. Understand that rewriting an expression in different forms can help reveal relationships between quantities in the context of a problem. For example, a + 0.05a is equivalent to 1.05a, which means to "increase by 5%" is the same as "multiply by 1.05."

B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (7.AR.B)

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example:
    - If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50.
    - If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.
4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities
    a. Solve word problems leading to equations of the form px+q=r and p(x+q)=r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
    b. Solve word problems leading to inequalities of the form px+q>r, px + q ≥ r, px + q < r, or px + q ≤ r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week, you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

Geometric Reasoning and Logic

A. Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
2. Construct triangles with given conditions. Understand the possible side lengths and angle measures that determine one and only one triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (Pyramids limited to surface area only).

Data Analysis

A. Random sampling to draw inferences about a population. (7.DA.A)

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example:
    - Estimate the mean word length in a book by randomly sampling words from the book.
    - Predict the winner of a school election based on randomly sampled survey data.

B. Draw informal comparative inferences about two populations. (7.DA.B)

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities using quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

C. Investigate chance processes and develop, use, and evaluate probability models with and without technology. (7.DA.C)

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. For example:
    - A probability near 0 indicates an unlikely event;
    - A probability around ½ indicates an event that is neither unlikely nor likely; and
    - A probability near 1 indicates a likely event.
6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
    a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
    b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
    c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Proportionality and Functions

A. Analyze proportional relationships and use them to solve real-world and mathematical problems. (7.PF.A)

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.
2. Recognize and represent proportional relationships between quantities.
    a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
    b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
    c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
    d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and (1, r) where r is the unit rate.
3. Use proportional relationships to reason and solve multistep ratio and percent problems of simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.

Grade 8

Grade Level Foundational Skills

By the end of grade 8, mathematically proficient students can reliably apply the following skills to engage in grade-appropriate mathematical tasks. This list does not represent the full depth of learning expected in grade 8, but is comprised of the foundational skills required by the standards.

1. Identify irrational numbers, recognizing the decimal patterns indicating a decimal represents a rational number.
2. Compare the magnitude of irrational numbers using approximations.
3. Fluently apply the properties of integer exponents.
4. Fluently solve simple cube and square root equations.
5. Write and perform operations with numbers written in scientific notation.
6. Solve linear equations algebraically or through graphing.
7. Solve systems of linear equations.
8. Construct a function to model a linear relationship.
9. Use transformation to discuss similarity and congruence.
10. Apply the Pythagorean Theorem to determine distance.
11. Find the volume of cones, cylinders, and spheres.

Numeracy and Operational Fluency

A. Know that there are numbers that are not rational, and approximate them by rational numbers. (8.NOF.A)

1. Understand the real number system.
    a. Distinguish between rational and irrational numbers (e.g., know that $\sqrt{2}$ is irrational.)
    b. Understand informally that every number has a decimal expansion.
    c. For rational numbers, show that the decimal expansion repeats eventually.
    d. Convert a decimal expansion that repeats eventually into a rational number by analyzing repeating patterns.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximation to the hundredths place.

Algebraic Reasoning

A. Work with radicals and integer exponents. (8.AR.A)

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
2. Represent solutions to equations, in the form $x^2 = p$ and $x^3 = p$, using the square root and cube root symbols, and determine if the solution is rational or irrational. Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

3. Use numbers expressed in scientific notation to estimate very large or very small quantities and to express how many times larger or smaller one number is compared to another. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger and the population of the United States is $\frac{1}{20}$ of the population of the world.
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

B. Understand the connections between proportional relationships, lines, and linear equations. (8AR.B)

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

C. Analyze and solve linear equations and pairs of simultaneous linear equations. (8.AR.C)

7. Solve linear equations in one variable.
    a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).
    b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
    b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
    For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.
    c. Solve real-world and mathematical problems leading to two linear equations in two variables.
    For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Geometric Reasoning and Logic

A. Understand congruence and similarity using physical models, transparencies, or geometry software. (8.GL.A)

1. Verify experimentally the properties of rotations, reflections, and translations:
    a. Lines are taken to lines, and line segments to line segments of the same length.
    b. Angles are taken to angles of the same measure.
    c. Parallel lines are taken to parallel lines.

2. Explain that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Rotations are only about the origin, and reflections are only over the y-axis and x-axis in grade 8.)
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. (Rotations are only about the origin, dilations only use the origin as the center of dilation, and reflections are only over the y-axis and x-axis in grade 8.)
4. Explain that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Rotations are only about the origin, dilations only use the origin as the center of dilation, and reflections are only over the y-axis and x-axis in grade 8.)
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

B. Understand and apply the Pythagorean Theorem. (8.GL.B)

6. Explain a proof of the Pythagorean Theorem and its converse using the areas of squares.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. (8.GL.C)

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Data Analysis

A. Investigate patterns of association in bivariate data. (8.DA.A)

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.
    a. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.
    b. Use relative frequencies calculated for rows or columns to describe the possible association between the two variables. For example, collect data from students in your class on whether they have a curfew on school nights and whether they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Proportionality and Functions

A. Define, evaluate, and compare functions. (8.PF.A)

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in this grade level.)
2. Compare properties of two functions where each function is represented in a different way (algebraically, graphically, numerically, or through verbal descriptions). For example, given a linear function represented by a table of values (numerically) and a linear function represented by an algebraic expression (algebraically), determine which function has the greater rate of change.
3. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; categorize functions as linear or nonlinear when given equations, graphs, or tables. For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4), and (3,9), which are not on a straight line.

B. Use functions to model relationships between quantities. (8.PF.B)

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Algebra I

Number and Quantity

The Real Number System (A1:N-RN)

B. Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Number and quantity: Quantities (A1:N-Q)

A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems.
    a. Choose and interpret units consistently in formulas.
    b. Choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

Algebra: Seeing Structure in Expressions (A1: A-SSE)

A. Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. ★
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
2. Use the structure of an expression to identify ways to rewrite it for a specific purpose.

B. Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions, emphasizing integer exponents. *For example, the growth of bacteria can be modeled by either* $f(t) = 3^{(t+2)}$ *or* $g(t) = 9(3^t)$ *because the expression* $3^{(t+2)}$ *can be rewritten as* $(3^t)(3^2) = 9(3^t)$.

Algebra: Arithmetic with Polynomials and Rational Expressions (A1:A-APR)

A. Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system comparable to the integers, as they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials.

3. Identify zeros of quadratic functions, and use the zeros to sketch a graph of the function defined by the polynomial.

Algebra: Creating Equations★ (A1: A-CED)

A. Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear, quadratic, and exponential situation functions.*
2. Create equations in two variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Algebra: Reasoning with Equations and Inequalities★ (A1: A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

1. Use properties of equality to justify and explain each step obtained from the previous step when solving an equation, assuming the original equation has a solution.
    a. Construct a viable argument to justify the solution method.

B. Solve equations and inequalities in one variable.

3. Solve linear and absolute value equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.
    c. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

C. Write and solve systems of equations.

5. Write and solve systems of linear equations in two variables.
    a. Use methods such as substitution, elimination, and graphing to solve.
    b. Justify a method for solving such systems.

D. Represent and solve equations and inequalities graphically.

6. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
7. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, piecewise linear (to include absolute value), and exponential functions.★
8. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions

Functions: Interpreting Functions (A1: F-IF)

A. Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

B. Interpret functions that arise in applications in terms of the context.

4. For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities,
    a. interpret key features of graphs and tables in terms of the quantities, and
    b. sketch graphs showing key features given a verbal description of the relationship.
    c. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries and end behavior. ★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
6. Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

C. Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph piecewise linear (to include absolute value) and exponential functions.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
9. Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential), each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.

Functions: Building Functions (A1: F-BF)

A. Build a function that models a relationship between two quantities.

1. Write a linear, quadratic, or exponential function that describes a relationship between two quantities. ★
    a. Determine an explicit expression or steps for calculation from a context.

B. Build new functions from existing functions

3. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, kf(x),* and *f(x + k)* for specific values of *k* (both positive and negative). Without technology, find the value of k given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where *f(x)* is a linear, quadratic, piecewise linear (to include absolute value), or exponential function.

Functions: Linear, Quadratic, and Exponential Models★ (A1: F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
    a. Prove that linear functions grow by equal differences over equal intervals.
    b. Prove that exponential functions grow by equal factors over equal intervals.
    c. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    d. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically, with and without technology.

B. Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

Statistics and Probability

Statistics and Probability: Interpreting Categorical and Quantitative Data (A1: S-ID) ★

A. Summarize, represent, and interpret data on a single count or measurement variable.

2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

B. Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and quadratic models.*
    b. Informally assess the fit of a function by plotting and analyzing residuals.
    c. Fit a linear function for a scatter plot that suggests a linear association.

C. Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Geometry

Geometric Reasoning and Logic

Congruence (GM: G-CO)

A. Experiment with transformations in the plane.

1. Based on the undefined notions of point, line, distance along a line, and distance around a circular arc, know the precise definitions of
    - angle,
    - circle,
    - perpendicular line,
    - parallel line, and
    - line segment.
2. Use a plane to
    a. Represent transformations with and without technology,
    b. Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
    c. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Describe the rotations and reflections that map a preimage onto itself when given a rectangle, parallelogram, trapezoid, or regular polygon.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, translation, or sequence of transformations
    a. draw the transformed figure with and without technology.
    b. specify a sequence that will map a given figure onto another.

B. Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures.
    - Predict the effect of a given rigid motion on a given figure.
    - Given two figures, use the definition of congruence in terms of rigid motions to determine if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

C. Prove and apply geometric theorems.

9. Prove and apply theorems about lines and angles.
    Theorems include but are not limited to:
    - *vertical angles are congruent;*
    - *when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent;*
    - *points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

10. Prove and apply theorems about triangles.
    Theorems include but are not limited to:
    - *measures of interior angles of a triangle sum to 180°;*
    - *base angles of isosceles triangles are congruent;*
    - *the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;*
    - *the medians of a triangle meet at a point.*
11. Prove and apply theorems about parallelograms.
    Theorems include but not limited to:
    - *opposite sides are congruent,*
    - *opposite angles are congruent,*
    - *the diagonals of a parallelogram bisect each other, and the converse of this theorem;*
    - *rectangles are parallelograms with congruent diagonals and the converse of this theorem.*

D. Make geometric constructions.

12. Make formal geometric constructions
    - with a variety of tools and methods, with or without technology
    - of an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry (GM: G-SRT)

A. Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor:
    a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
    b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Using similarity transformations
    - Determine if two figures are similar using the definition of similarity transformations.
    - Explain the meaning of similarity for triangles as the equality of all corresponding pairs of angles and proportionality of all corresponding sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

B. Prove and apply theorems involving similarity.

4. Prove and apply theorems about triangles. *Theorems include but not limited to:*
    - *a line parallel to one side of a triangle divides the other two proportionally, and the converse of this theorem;*
    - *the Pythagorean Theorem proved using triangle similarity;*
    - *SAS similarity criteria;*
    - *SSS similarity criteria;*
    - *AA similarity criteria.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

C. Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles, including special right triangles (30-60-90 and 45-45-90), are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Circles (GM: G-C)

A. Understand and apply theorems about circles.

1. Understand that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords, including the following:
    - *the relationship that exists between central, inscribed, and circumscribed angles;*
    - *inscribed angles on a diameter are right angles; and*
    - *a radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

B. Find arc lengths and areas of sectors of circles.

5. Derive and apply the formula for finding area of a sector and arc length.

Expressing Geometric Properties with Equations (GM: G-GPE)

A. Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

B. Use coordinates to prove simple geometric theorems algebraically.

4. Use coordinates to prove geometric theorems algebraically.
5. Determine the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
    - Apply ratio thinking to find the midpoint of the given line segment.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Geometric Measurement and Dimension (GM: G-MD)

A. Explain volume formulas and use them to solve problems.

1. Give an informal argument, e.g., dissection arguments, Cavalieri's principle, or informal limit arguments for the formulas of:
    - circumference of a circle;
    - area of a regular polygon;
    - area of a circle; and
    - volume of a cylinder, pyramid, and cone.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

B. Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
    - Apply properties of two-dimensional figures identified to solve problems.

Modeling with Geometry (GM: G-MG)

A. Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★

3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Statistics & Probability

Statistics and Probability★: Conditional Probability and the Rules of Probability (GM-S-CP)

A. Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B.*
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7. Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B),* and interpret the answer in terms of the model.

Algebra II

Number and Quantity

The Real Number System (A2: N-RN)

A. Extend the properties of exponents to rational exponents

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Quantities★(A2: N-Q)

A. Reason quantitatively and use units to solve problems.

2. Define appropriate quantities for the purpose of descriptive modeling.

The Complex Number System (A2: N-CN)

A. Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

C. Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.

Algebra

Seeing Structure in Expressions (A2: A-SSE)

A. Interpret the structure of expressions.

2. Use the structure of an expression to identify ways to rewrite it for a specific purpose.

B. Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
    c. Use the properties of exponents to transform expressions for exponential functions.
4. Apply the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. *For example, calculate mortgage payments.* ★

Algebra: Arithmetic with Polynomials and Rational Expressions (A2: A-APR)

B. Understand the relationship between zeros and factors of polynomials.

2. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

C. Use polynomial identities to solve problems.

4. Describe numerical relationships using polynomial identities.

D. Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms:
    - Write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Algebra: Creating Equations ★ (A2: A-CED)

A. Create equations that describe numbers or relationships.

1. Create equations and inequalities and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Reasoning with Equations and Inequalities (A2: A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

1. Use properties of equality to justify and explain each step obtained from the previous step when solving an equation, assuming the original equation has a solution. Expand context to situations, including but not limited to logarithmic, exponential, rational, and radical equations.
    a. Construct a viable argument to justify the solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

B. Solve equations and inequalities in one variable, including applications in mathematical modeling.

4. Solve quadratic equations in one variable by
    - inspection (e.g., for $x^2 = 49$),
    - taking square roots,
    - completing the square,
    - the quadratic formula and
    - factoring, as appropriate to the initial form of the equation.

    Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. Apply these methods to represent and solve real-world problems through mathematical modeling, interpreting the meaning of solutions in context.

C. Solve systems of equations.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

D. Represent and solve equations and inequalities graphically.

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$;

    Find approximate solutions by
    - using technology to graph the functions,
    - make tables of values, or
    - find successive approximations.

    Include cases where $f(x)$ and/or $g(x)$ are polynomial, rational, absolute value, exponential, and logarithmic functions.

Functions

Interpreting Functions (A2: F-IF)

A. Understand the concept of a function and use function notation.

3. Recognize that sequences are functions whose domain is a subset of the integers. Relate arithmetic sequences to linear functions and geometric sequences to exponential functions.

B. Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities
   - interpret key features of graphs and tables in terms of the quantities, and
   - sketch graphs showing key features given a verbal description of the relationship.

   Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

C. Analyze functions using different representations

7. Graph functions are expressed symbolically and show key features of the graph by hand in simple cases and by using technology for more complicated cases. ★
   b. Graph square root, cube root, and piecewise-defined functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions (A2: F-BF)

A. Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

B. Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).
   - Find the value of k given the graphs.
   - Experiment with cases and illustrate an explanation of the effects on the graph using technology.
   - Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
   - Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse
   - Write an expression for the inverse.

Linear, Quadratic, and Exponential Models (A2: F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems.

2. Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences to solve multi-step problems.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ★

B. Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context. ★

Trigonometric Functions (A2: F-TF)

A. Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

B. Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★

Statistics & Probability

Interpreting Categorical and Quantitative Data★ (A2: S-ID)

A. Summarize, represent, and interpret data on a single count or measurement variable.

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages with technology.
    a. Recognize that there are data sets for which such a procedure is not appropriate.

B. Summarize, represent, and interpret data on a two categorical and quantitative variables.

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
    b. Use given functions or choose a function suggested by the context. *Emphasize exponential models.*

Statistics and Probability: Making Inferences and Justifying Conclusions ★ (A2: S-IC)

A. Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Making Inferences and Justifying Conclusions ★ (A2: S-IC)

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★

5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.★
6. Evaluate media or academic research based on data.★

Integrated Math 1

Number and Quantity

Quantities (I1:N-Q)

A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems.
    a. Choose and interpret units consistently in formulas.
    b. Choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

Seeing Structure in Expressions (I1: A-SSE)

A. Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. ★
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Creating Equations★ (I1: A-CED)

A. Create equations that describe numbers or relationships.

1. Create equations and inequalities and use them to solve problems. *Include equations arising from linear, quadratic, and exponential situation functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving linear equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Algebra: Reasoning with Equations and Inequalities★ (I1: A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

1. Use properties of equality to justify and explain each step obtained from the previous step when solving an equation, assuming the original equation has a solution.
    a. Construct a viable argument to justify the solution method.

B. Solve equations and inequalities in one variable.

3. Solve linear and absolute value equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.
    c. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

C. Solve systems of equations.

5. Solve systems of linear equations in two variables exactly and approximately.
    a. Use methods such as substitution, elimination, and graphing to solve.
    b. Justify a method for solving such systems.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to two variables.

D. Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, piecewise linear (to include absolute value), and exponential functions.★
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions

Interpreting Functions (I1: F-IF)

A. Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

B. Interpret functions that arise in applications in terms of the context.

4. For linear, piecewise linear (to include absolute value), quadratic, and exponential functions that model a relationship between two quantities,
    a. interpret key features of graphs and tables in terms of the quantities, and
    b. sketch graphs showing key features given a verbal description of the relationship.
    c. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries and end behavior. ★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
6. Calculate and interpret the average rate of change of a linear, quadratic, piecewise linear (to include absolute value), and exponential function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

C. Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
9. Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential), each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.

Functions: Building Functions (I1: F-BF)

A. Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

B. Build new functions from existing functions.

3. Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative). Without technology, find the value of k given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph, including cases where f(x) is a linear, quadratic, piecewise linear (to include absolute value), or exponential function.

Linear, Quadratic, and Exponential Models★ (I1: F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
    a. Prove that linear functions grow by equal differences over equal intervals.
    b. Prove that exponential functions grow by equal factors over equal intervals.
    c. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically, with and without technology.

B. Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.

Geometric Reasoning and Logic

Congruence (I1: G-CO)

A. Experiment with transformations in the plane.

1. Based on the undefined notions of point, line, distance along a line, and distance around a circular arc, know the precise definitions of
    - angle,
    - circle,
    - perpendicular line,
    - parallel line, and
    - line segment.
2. Use a plane to
    a. Represent transformations with and without technology,
    b. Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
    c. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Describe the rotations and reflections that map a preimage onto itself when given a rectangle, parallelogram, trapezoid, or regular polygon.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, translation, or sequence of transformations
    a. draw the transformed figure with and without technology.
    b. specify a sequence that will map a given figure onto another.

B. Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures.
    - Predict the effect of a given rigid motion on a given figure.
    - Given two figures, use the definition of congruence in terms of rigid motions to determine if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

D. Make geometric constructions.

12. Make formal geometric constructions
    - with a variety of tools and methods, with or without technology
    - of an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Expressing Geometric Properties with Equations (I1: G-GPE)

B. Use coordinates to prove simple geometric theorems algebraically.

4. Use coordinates to prove geometric theorems algebraically.
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
    - Apply ratio thinking to find the midpoint of the given line segment.

7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Statistics and Probability

Interpreting Categorical and Quantitative Data (I1: S-ID) ★

A. Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent and interpret data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

B. Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and quadratic models.*
    b. Informally assess the fit of a function by plotting and analyzing residuals.
    c. Fit a linear function for a scatter plot that suggests a linear association.

C. Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Integrated Math 2

Number & Quantity

Number and Quantity: The Real Number System (I2: N-RN)

A. Extend the properties of exponents to rational exponents

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

B. Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

The Complex Number System (I2: N-CN)

A. Perform arithmetic operations with complex numbers.

1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

C. Use complex numbers in polynomial identities and equations. *Limit to quadratics with real coefficients.*

7. Solve quadratic equations with real coefficients that have complex solutions.
8. Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i).
9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Algebra

Seeing Structure in Expressions (I2: A-SSE)

A. Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. ★
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
2. Use the structure of an expression to identify ways to rewrite it for a specific purpose.

B. Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
    a. Factor a quadratic expression to reveal the zeros of the function it defines.
    b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
    c. Use the properties of exponents to transform expressions for exponential functions, emphasizing integer exponents. *For example, the growth of bacteria can be modeled by either* $f(t) = 3^{(t+2)}$ *or* $g(t) = 9(3^t)$ *because the expression* $3^{(t+2)}$ *can be rewritten as* $(3^t)(3^2) = 9(3^t)$.

Arithmetic with Polynomials and Rational Expressions (I2:A-APR)

A. Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system comparable to the integers, as they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Creating Equations★ (I2: A-CED)

A. Create equations that describe numbers or relationships.

1. Create equations and inequalities and use them to solve problems. *Include equations arising from linear, quadratic, and exponential situation functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Include formulas involving quadratic terms. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities (I2: A-REI)

B. Solve equations and inequalities in one variable.

4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.
    c. Recognize when the quadratic formula gives complex solutions and write them as "no real solution."

C. Solve systems of equations.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Functions

Functions: Interpreting Functions (I2: F-IF)

B. Interpret quadratic functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities,
    a. interpret key features of graphs and tables in terms of the quantities, and
    b. sketch graphs showing key features given a verbal description of the relationship.
    c. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries and end behavior. ★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
6. Calculate and interpret the average rate of change of a quadratic function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

C. Analyze linear, exponential, quadratic, absolute value, step, and piecewise-defined functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph piecewise linear (to include absolute value) and exponential functions.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
9. Compare properties of two functions (linear, quadratic, piecewise linear [to include absolute value] or exponential), each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum.

Building Functions (I2: F-BF)

A. Build a function that models a relationship between two quantities.

1. Write a linear, quadratic, or exponential function that describes a relationship between two quantities. ★
    a. Determine an explicit expression or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

B. Build new functions from existing functions

3. Identify the effect on the graph of replacing *f(x) by f(x) + k, kf(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative). Without technology, find the value of k given the graphs of linear and quadratic functions. With technology, experiment with cases and illustrate an explanation of the effects on the graph that include cases where *f(x)* is a linear, quadratic, piecewise linear (to include absolute value), or exponential function.
4. Find inverse functions.
    - Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse.
    - Write an expression for the inverse.

Linear, Quadratic, and Exponential Models★ (I2: F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems.

3. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically, with and without technology.

Trigonometric Functions (I2: F-TF)

C. Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find the value of sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.

Geometric Reasoning and Logic

Congruence (I2: G-CO)

C. Prove and apply geometric theorems.

3. Prove and apply theorems about lines and angles.
   Theorems include but are not limited to:
    - *vertical angles are congruent;*
    - *when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent;*
    - *points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

4. Prove and apply theorems about triangles.
   Theorems include but are not limited to:
   - *measures of interior angles of a triangle sum to 180°;*
   - *base angles of isosceles triangles are congruent;*
   - *the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;*
   - *the medians of a triangle meet at a point.*
5. Prove and apply theorems about parallelograms.
   Theorems include but not limited to:
   - *opposite sides are congruent,*
   - *opposite angles are congruent,*
   - *the diagonals of a parallelogram bisect each other, and the converse of this theorem;*
   - *rectangles are parallelograms with congruent diagonals and the converse of this theorem.*

Similarity, Right Triangles, and Trigonometry (I2: G-SRT)

A. Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Using similarity transformations
   - Determine if two figures are similar using the definition of similarity transformations.
   - Explain the meaning of similarity for triangles as the equality of all corresponding pairs of angles and proportionality of all corresponding sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

B. Prove and apply theorems involving similarity.

4. Prove and apply theorems about triangles. *Theorems include but not limited to:*
   - *a line parallel to one side of a triangle divides the other two proportionally, and the converse of this theorem;*
   - *the Pythagorean Theorem proved using triangle similarity;*
   - *SAS similarity criteria;*
   - *SSS similarity criteria;*
   - *AA similarity criteria.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

C. Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles, including special right triangles (30-60-90 and 45-45-90), are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Circles (I2: G-C)

A. Understand and apply theorems about circles.

1. Understand that all circles are similar.

2. Identify and describe relationships among inscribed angles, radii, and chords, including the following:
    - the relationship that exists between central, inscribed, and circumscribed angles;
    - inscribed angles on a diameter are right angles; and
    - a radius of a circle is perpendicular to the tangent where the radius intersects the circle.
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

B. Find arc lengths and areas of sectors of circles.

5. Apply the formula for finding area of a sector and arc length. Radians are introduced only as a unit of measure.

Expressing Geometric Properties with Equations (I2: G-GPE)

A. Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.

B. Use coordinates to prove simple geometric theorems algebraically.

4. Use coordinates to prove geometric theorems algebraically.

Geometric Measurement and Dimension (I2: G-MD)

A. Explain volume formulas and use them to solve problems.

1. Give an informal argument, e.g., dissection arguments, Cavalieri's principle, or informal limit arguments~~,~~ for the formulas of:
    - circumference of a circle;
    - area of a circle; and
    - volume of a cylinder, pyramid, and cone.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

Conditional Probability and the Rules of Probability (I2: S-CP)

A. Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9. Understand and apply the concepts of permutations and combinations.
    - Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions (I2: S-MD)

B. Use probability to evaluate outcomes of decisions.

6. Use probabilities to make fair decisions (drawing by lots, using a random number generator, etc).
7. Analyze decisions and strategies using probability concepts.

Integrated Math 3

Number and Quantity

The Complex Number System (I3: N-CN)

C. Use complex numbers in polynomial identities and equations.

8. Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Algebra

Seeing Structure in Expressions (I3: A-SSE)

A. Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. ★
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
2. Use the structure of an expression to identify ways to rewrite it for a specific purpose.

B. Write expressions in equivalent forms to solve problems.

4. Apply the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. *For example, calculate mortgage payments.* ★

Arithmetic with Polynomials and Rational Expressions (I3:A-APR)

A. Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system comparable to the integers, as they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

C. Use polynomial identities to solve problems.

4. Describe numerical relationships using polynomial identities.
5. Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

D. Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms:
    - Write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. Perform operations on rational expressions, building from previous knowledge that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

Creating Equations★ (I3: A-CED)

A. Create equations that describe numbers or relationships.

1. Create equations and inequalities and use them to solve problems. *Include equations arising from linear, quadratic, and exponential situation functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities (I3: A-REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise

D. Represent and solve equations and inequalities graphically.

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$;

    Find approximate solutions by

    - using technology to graph the functions,
    - make tables of values, or
    - find successive approximations.

    Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Functions

Interpreting Functions (I3: F-IF)

B. Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities
    - interpret key features of graphs and tables in terms of the quantities, and
    - sketch graphs showing key features given a verbal description of the relationship.

    Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

C. Analyze functions using different representations

7. Graph functions are expressed symbolically and show key features of the graph by hand in simple cases and by using technology for more complicated cases. ★
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and

trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions (I3: F-BF)

A. Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
    b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

B. Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).
    - Find the value of k given the graphs.
    - Experiment with cases and illustrate an explanation of the effects on the graph using technology.
    - Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. Find inverse functions.
    - Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse.
    - Write an expression for the inverse.

Linear, Quadratic, and Exponential Models (I3: F-LE)

A. Construct and compare linear, quadratic, and exponential models and solve problems.

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ★

Trigonometric Functions (I3: F-TF)

A. Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

B. Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★

Geometric Reasoning and Logic

Similarity, Right Triangles, and Trigonometry (I3: G-SRT)

D. Apply trigonometry to general triangles.

9. Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. Prove the Laws of Sines and Cosines and use them to solve problems.
11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Geometric Measurement and Dimension (I3: G-MD)

B. Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
    - Apply properties of two-dimensional figures identified to solve problems.

Modeling with Geometry (I3: G-MG)

A. Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

Statistics & Probability

Interpreting Categorical and Quantitative Data★ (I3: S-ID)

A. Summarize, represent, and interpret data on a single count or measurement variable.

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages with technology.
    a. Recognize that there are data sets for which such a procedure is not appropriate.

Statistics and Probability: Making Inferences and Justifying Conclusions ★ (I3: S-IC)

A. Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

Making Inferences and Justifying Conclusions ★ (I3:S-IC)

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.★
6. Evaluate media or academic research based on data.★

Using Probability to Make Decisions (I3: S-MD)

B. Use probability to evaluate outcomes of decisions.
    6. Use probabilities to make fair decisions (drawing by lots, using a random number generator, etc).
    7. Analyze decisions and strategies using probability concepts.

Algebra III

Number & Quantity

The Complex Number System (A3: N-CN)

A. Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that i2 = –1, and every complex number has the form a + bi with a and b real.
2. Use the relation i2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

B. Represent complex numbers and their operations on the complex plane.

4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.
6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

C. Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities (A3: N-VM)

A. Represent and model with vector quantities.

1. Understand vector attributes.
    a. Represent vector quantities as having both magnitude and direction by directed line segments.
    b. Use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. Solve problems involving velocity and other quantities that can be represented by vectors.

B. Perform operations on vectors.

4. Add and subtract vectors.
    a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w, and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. Multiply a vector by a scalar.
    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
    b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

C. Perform operations on matrices and use matrices in applications.

6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. Multiply matrices by scalars to produce new matrices,
    a. Multiply with and without technology.
    b. Multiply with and without context.
8. Add, subtract, and multiply matrices of appropriate dimensions.
9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers.
    a. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector.

Algebra

Arithmetic with Polynomials and Rational Expressions (A3: A-APR)

B. Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

C. Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
5. Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

D. Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms:
    - Write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, synthetic

division, or, for the more complicated examples, a computer algebra system.

7. Perform operations on rational expressions, building from previous knowledge that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

Reasoning with Equations and Inequalities (A3: A-REI)

C. Solve System of Equations.

8. Represent a system of linear equations as a single matrix equation in a vector variable.
9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations
    - Use technology for matrices of dimension 3 × 3 or greater.

Functions

Building Functions (A3: A-BF)

A. Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
    c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

B. Build new functions from existing functions.

4. Find inverse functions.
    a. Solve an equation of the from f(x) = c for a simple function f that has an inverse and write an expression for the inverse.
    b. Verify by composition that one function is the inverse of another.
    c. Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. Produce an invertible function from a non-invertible function by restricting the domain.
5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Interpreting Functions (A3: F-IF)

C. Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Geometric Reasoning & Logic

Expressing Geometric Properties with Equations (GM: G-GPE)

A. Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

2. Derive the equation of a parabola given a focus and directrix.
3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Advanced Math Precalculus

Number & Quantity

The Complex Number System (PC: N-CN)

A. Perform arithmetic operations with complex numbers.

2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

B. Represent complex numbers and their operations on the complex plane.

4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.
6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

C. Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities (PC: N-VM)

A. Represent and model with vector quantities.

1. Understand vector attributes.
    a. Represent vector quantities as having both magnitude and direction by directed line segments.
    b. Use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, **v**).
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. Solve problems involving velocity and other quantities that can be represented by vectors.

B. Perform operations on vectors.

4. Add and subtract vectors.
    a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
    c. Understand vector subtraction **v** – **w** as **v** + (–**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w**, and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. Multiply a vector by a scalar.
    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
    b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

C. Perform operations on matrices and use matrices in applications.

6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. Multiply matrices by scalars to produce new matrices,
    a. Multiply with and without technology.
    b. Multiply with and without context.
8. Add, subtract, and multiply matrices of appropriate dimensions.
9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers.
    a. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse
11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector.
12. Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Arithmetic with Polynomials and Rational Expressions (PC: A-APR)

B. Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

C. Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
5. Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)

D. Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms:
    - Write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, synthetic division, or, for the more complicated examples, a computer algebra system.
7. Perform operations on rational expressions, building from previous knowledge that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction,

multiplication, and division by a nonzero rational expression.

Reasoning with Equations and Inequalities (PC: A-REI)

C. Solve System of Equations.

8. Represent a system of linear equations as a single matrix equation in a vector variable.
9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations
    - Use technology for matrices of dimension 3 × 3 or greater.

D. Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Include rational, logarithmic, exponential and polynomial funcitons.
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic, polynomial, rational, piecewise linear (to include absolute value), logarithmic and exponential functions. Identify domain restricions and real and non-real solutions algebraically. ★

Functions

Building Functions (PC: F-BF)

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    c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

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5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Interpreting Functions (PC: F-IF)

C. Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
    d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one

quadratic function and an algebraic expression for another, say which has the larger maximum.

Trigonometric Functions (PC: F-TF)

A. Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, π+x, and 2π–x in terms of their values for x, where x is any real number.
4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

B. Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
    a. Construct models through graphing and analyzing graphs of trigonometric functions (including finding period, amplitude, and phase shift).
6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

C. Prove and apply trigonometric identities.

8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

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3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Similarity, Right Triangles, and Trigonometry (PC: G-SRT)

D. Apply trigonometry to general triangles.

9. Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. Prove the Laws of Sines and Cosines and use them to solve problems.
11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Advanced Math Functions and Statistics

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Statistics & Probability

Data in Context (S-DC)

A. Use a statistical cycle to formulate questions, describe types of data, data sources, and constraints within the context of a problem.
   1. Define the stages of the statistical cycle and how each stage relates to the others.
   2. Formulate questions and conclusions based on context.
   3. Understand the type of data relevant to the question at hand (e.g. quantitative versus categorical).
   4. Compare and contrast population and sample, and parameter and statistic.
   5. Identify and explain constraints of the statistical approach.

B. Compare and contrast data collection methods to plan and conduct an observational study.
   6. Investigate and describe sampling techniques (e.g., simple random sampling, stratified sampling, systematic sampling, cluster sampling).
   7. Determine which sampling technique is best, given a particular context.
   8. Investigate and explain the statistical biases inherent in sampling methods and various responses to statistical bias.
   9. Use the statistical cycle to plan and conduct an observational study to answer a question or address a problem.

C. Utilize the principles of experimental design to plan and conduct a well-designed experiment.
   10. Describe the principles of experimental design, including:
       - treatment/control groups;
       - blinding/placebo effects;
       - experimental units/subjects; and
       - blocking/matched pairs and completely randomized designs.
   11. Evaluate the principles of experimental design to address comparison, randomization, replication, and control within the context of the problem.
   12. Compare and contrast controlled experiments and observational studies and the conclusions that may be drawn from each.
   13. Use the statistical cycle to plan and conduct a well-designed experiment to answer a question or address a problem.
   14. Select a data collection method appropriate for a given context.

Interpreting Categorical and Quantitative Data (S-ID)

A. Summarize, represent, and interpret data on a single count or measurement variable.
   1. Represent and interpret data with plots on the real number line (dot plots, histograms, and box plots).
   2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
   3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
   4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to

estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use technology when appropriate.

B. Summarize, represent, and interpret data on two categorical and quantitative variables.
    5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
    6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
        a. Fit a function to the data using technology.
            - Use functions fitted to data to solve problems in the context.
            - Use given functions or choose a function suggested by the context.
        b. Informally assess the fit of a function by plotting and analyzing residuals.
        c. Fit a linear function for a scatter plot that suggests a linear association.

C. Interpret linear models.
    7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
    8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
    9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions (S-IC)

A. Understand and evaluate random processes underlying statistical experiments.
    1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
    2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
    3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
    4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
    5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
    6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability (S-CP)

A. Understand independence and conditional probability and use them to interpret data.
    1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
    2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
    3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret

independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9. Understand and apply the concepts of permutations and combinations.
    - Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions (S-MD)

A. Calculate expected values and use them to solve problems.

1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
    - For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

B. Use probability to evaluate outcomes of decisions.

5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
    a. Find the expected payoff for a game of chance.
    b. Evaluate and compare strategies on the basis of expected values.
6. Use probabilities to make fair decisions (drawing by lots, using a random number generator, etc).
7. Analyze decisions and strategies using probability concepts.

Statistics and Probability

Statistics & Probability

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3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use technology when appropriate.

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9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions (S-IC)

A. Understand and evaluate random processes underlying statistical experiments.
1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Evaluate reports based on data.

Conditional Probability and the Rules of Probability (S-CP)

A. Understand independence and conditional probability and use them to interpret data.
1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

A. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9. Understand and apply the concepts of permutations and combinations.
    - Use permutaitons and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions (S-MD)

A. Calculate expected values and use them to solve problems.

1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
    - For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

B. Use probability to evaluate outcomes of decisions.

5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
    a. Find the expected payoff for a game of chance.
    b. Evaluate and compare strategies on the basis of expected values.

6. Use probabilities to make fair decisions (drawing by lots, using a random number generator, etc).
7. Analyze decisions and strategies using probability concepts.

Math Essentials

Applying mathematics in Personal Finance (ME-PF)

A. Use graphical and numerical techniques to study patterns and analyze data related to personal finance.
   1. Use rates and linear functions to solve problems involving personal finance and budgeting, including compensation and deductions.
   2. Solve problems involving personal taxes
   3. Analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees.

B. Use mathematical processes with algebraic formulas, graphs, and amortization modeling with technology to solve problems involving credit.
   4. Use formulas with technology to generate tables to display a series of payments for loan amortizations resulting from financed purchases.
   5. Analyze personal credit options in retail purchasing and compare relative advantages and disadvantages of each option. Include credit scenarios involving interest rates, terms, compound interest and continuously compounded interest using technology to compare terms, rates and interest types.
   6. Use technology to create amortization models to investigate home financing and compare buying a home to renting a home.
   7. Use technology to create amortization models to investigate automobile financing and compare buying a vehicle to leasing a vehicle.

C. Use mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning.
   8. Analyze and compare insurance coverage options and rates.
   9. Investigate and compare investment options, including stocks, bonds, annuities, certificates of deposit, and retirement plans.
   10. Analyze types of savings options involving simple and compound interest and compare the relative advantages of these options.

Applying Mathematics in Science and Engineering (ME-SE)

A. Apply mathematical processes and algebraic techniques to study patterns and analyze data related to science.
   1. Use proportional and inversely proportional relationships to describe physical laws
   2. Use exponential models available through technology to model growth and decay in areas such as population, biology, ecology, and chemistry, including radioactive decay; and
   3. Use quadratic functions to model motion, such as an object dropped, bounced, thrown, or kicked.

B. Apply mathematical processes using algebra and geometry, with and without technology, to study patterns and analyze data related to architecture and engineering
   4. Use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in architecture.
   5. Use scale factors with two-dimensional and three-dimensional objects to demonstrate

proportional and non-proportional changes in surface area and volume as applied to fields such as engineering drawing, architecture, and construction.

6. Use the Pythagorean Theorem and special right-triangle relationships to calculate distances.
7. Use trigonometric ratios to calculate distances and angle measures as applied to fields such as surveying, navigation, and orienteering.

C. Apply mathematics of measurement to industrial design problems.

8. Identify, compare, and use various tools, such as rulers and measuring tapes in both the imperial and metric systems.
9. Identify, convert, and apply units of length, weight, volume, time, and temperature between imperial and metric systems given a situation or context.
10. Apply perimeter, circumference, volume and area formulas as a way to understand problems and to guide the solution of multi-step problems.
11. Choose and interpret units and tools consistently in formulas
12. Justify the choice of unit and tools in a given context.

Applying Mathematics in Fine Arts (ME-FA)

A. Use mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts.

1. Use trigonometric ratios and functions available through technology to model periodic behavior in art and music;
2. Use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography;
3. Use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music; and
4. Use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields such as painting, sculpture, and photography.

Applying Mathematics in Social Sciences (ME-SS)

A. Determine the number of elements in a finite sample space and compute the probability of an event.

1. Determine the number of ways an event may occur using combinations, permutations, and the Fundamental Counting Principle.
2. Compare theoretical to empirical probability in chance events.

B. Apply mathematical processes and mathematical models to analyze data as it applies to social sciences.

3. Interpret information from various graphs to draw conclusions from the data and determine the strengths and weaknesses of conclusions;
4. Analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions.
5. Distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies;

6. Use data from a sample to estimate the population mean or population proportion;
7. Analyze marketing claims based on graphs and statistics from electronic and print media and justify the validity of stated or implied conclusions.
8. Use regression methods available through technology to model situations best fit by linear and exponential functions. Use the model to interpret correlations, and make predictions.

C. Apply mathematical processes to design a study and utilize graphical, numerical, and analytical techniques to communicate the study's results effectively.

9. Formulate a meaningful question, determine the necessary data to answer the question, gather the relevant data, analyze the data, and draw reasonable conclusions.
10. Communicate and present methods used, analyses conducted, recommendations, limitations, and conclusions drawn for a data-analysis project. Presentation styles can include one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation.