Algebra II Guide to Rigor in Mathematics 2.0

In order to provide a quality mathematical education for students, instruction must be rigorous, focused, and coherent. This document provides explanations and a standards-based alignment to assist teachers in providing the first of those: a rigorous education. While this document will help teachers identify the explicit component(s) of rigor called for by each of the Louisiana Student Standards for Mathematics (LSSM), it is up to the teacher to ensure his/her instruction aligns to the expectations of the standards, allowing for the proper development of rigor in the classroom.

This rigor document is considered a "living" document as we believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to classroomsupporttoolbox@la.gov so that we may use your input when updating this guide.

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Table of Contents

Introduction

Definitions of the Components of Rigor 2
A Special Note on Procedural Skill and Fluency 2
Recognizing the Components of Rigor 3
Focus in the standards 3

Algebra II LSSM Rigor Alignments 4

Definitions of the Components of Rigor

Rigorous teaching in mathematics does not simply mean increasing the difficulty or complexity of practice problems. Incorporating rigor into classroom instruction and student learning means exploring at a greater depth, the standards and ideas with which students are grappling. There are **three** components of rigor that will be expanded upon in this document, and each is equally important to student mastery: **Conceptual Understanding, Procedural Skill and Fluency,** and **Application**.

- **Conceptual Understanding** refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.
- **Procedural Skill and Fluency** is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.
- **Application** provides valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

A Special Note on Procedural Skill and Fluency

While speed is definitely a component of fluency, it is not necessarily speed in producing an answer; rather, fluency can be observed by watching the speed with which a student engages with a particular problem. Furthermore, fluency does not require the most efficient strategy. The standards specify grade-level appropriate strategies or types of strategies with which students should demonstrate fluency (e.g., 1.OA.C.6 allows for students to use counting on, making ten, creating equivalent but easier or known sums, etc.). It should also be noted that teachers should expect some procedures to take longer than others (e.g., fluency with the standard algorithm for division, 6.NS.B.2, as compared to fluently adding and subtracting within 10, 1.OA.C.6).

Standards identified as targeting procedural skill and fluency do not all have an expectation of automaticity and/or rote recall. Only two standards, 2.OA.B.2 and 3.OA.C.7, have explicit expectations of students knowing facts from memory. Other standards targeting procedural skill and fluency do not require students to reach automaticity. For example, in 4.G.A.2, students do not need to reach automaticity in classifying two-dimensional figures.

Recognizing the Components of Rigor

In the LSSM each standard is aligned to one or more components of rigor, meaning that each standard aims to promote student growth in conceptual understanding, procedural skill and fluency, and/or application. Key words and phrases in the standards indicate which component(s) of rigor the standard is targeting: conceptual understanding standards often use terms like *understand*, *recognize*, or *interpret*; procedural skill and fluency standards tend to use words like *fluently*, *find*, or *solve*; and application standards typically use phrases like *word problems* or *real-world problems*. Key words and phrases are underlined in each standard to help clarify the identified component(s) of rigor for each standard.

Focus in the Standards

Not all content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Louisiana Standards for Mathematical Practice. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. Students should spend the large majority of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

Algebra II

LSSM – Algebra II		Explicit Component(s) of Rigor		
Code	Standard	Conceptual Understanding	Procedural Skill and Fluency	Application
A2: N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5*.	✓		
A2: N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	✓	✓	
A2: N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling. *	✓		
A2: N-CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	✓		
A2: N-CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.		✓	
A2: N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.		✓	
A2: A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	✓	✓	
A2: A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *	✓	✓	
A2: A-SSE.B.3c	Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* *		✓	
A2: A-SSE.B.4	Apply the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* *		✓	✓
A2: A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	✓	✓	
A2: A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	✓	✓	
A2: A-APR.C.4	Use polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	✓	✓	

A2: A-APR.D.6	Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + ^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	✓	✓	
A2: A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* *	✓	✓	✓
A2: A-REI.A.1	Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	✓		
A2: A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	✓	✓	
A2: A-REI.B.4	Solve quadratic equations in one variable.		✓	
A2: A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	✓	✓	
A2: A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), limited to systems of at most three equations and three variables. With graphic solutions, systems are limited to two variables.		✓	
A2: A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*		✓	
A2: A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. *	✓	✓	
A2: F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* *	✓		
A2: F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *	✓	✓	
A2: F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *	✓	✓	

A2: F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. *		✓	
A2: F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. *	✓	✓	
A2: F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. *	✓	✓	
A2: F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	✓	✓	
A2: F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.*	✓		
A2: F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, determine which has the larger maximum*	✓	✓	
A2: F-BF.A.1	Write a function that describes a relationship between two quantities. *	✓	✓	
A2: F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context. *	✓	✓	
A2: F-BF.A.1b	Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* *	✓	✓	✓
A2: F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. *	✓	✓	✓
A2: F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	✓	✓	
A2: F-BF.B.4	Find inverse functions.		✓	
A2: F-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.		✓	
A2: F-LE.A.2	Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences to solve multistep problems. *	✓	✓	✓
A2: F-LE.A.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. *	✓	✓	
A2: F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context. *	✓		

A2: F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	✓		
A2: F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	✓		
A2: F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. *	✓	✓	
A2: F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	✓	✓	
A2: S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. *	✓	✓	✓
A2: S-ID.B.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. *	✓	✓	
A2: S-ID.B.6a	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize exponential models. *	✓	✓	✓
A2: S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. *	✓		
A2: S-IC.A.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*? *	✓	✓	✓
A2: S-IC.B.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. *	✓		
A2: S-IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. *		✓	✓
A2: S-IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. *	✓	✓	✓
A2: S-IC.B.6	Evaluate reports based on data. *	✓		

*Modeling standard