Geometry Guide to Rigor in Mathematics 2.0

In order to provide a quality mathematical education for students, instruction must be rigorous, focused, and coherent. This document provides explanations and a standards-based alignment to assist teachers in providing the first of those: a rigorous education. While this document will help teachers identify the explicit component(s) of rigor called for by each of the Louisiana Student Standards for Mathematics (LSSM), it is up to the teacher to ensure his/her instruction aligns to the expectations of the standards, allowing for the proper development of rigor in the classroom.

This rigor document is considered a "living" document as we believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to classroomsupporttoolbox@la.gov so that we may use your input when updating this guide.

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Definitions of the Components of Rigor

Rigorous teaching in mathematics does not simply mean increasing the difficulty or complexity of practice problems. Incorporating rigor into classroom instruction and student learning means exploring at a greater depth, the standards and ideas with which students are grappling. There are **three** components of rigor that will be expanded upon in this document, and each is equally important to student mastery: **Conceptual Understanding, Procedural Skill and Fluency,** and **Application**.

- **Conceptual Understanding** refers to understanding mathematical concepts, operations, and relations. It is more than knowing isolated facts and methods. Students should be able to make sense of why a mathematical idea is important and the kinds of contexts in which it is useful. It also allows students to connect prior knowledge to new ideas and concepts.
- **Procedural Skill and Fluency** is the ability to apply procedures accurately, efficiently, and flexibly. It requires speed and accuracy in calculation while giving students opportunities to practice basic skills. Students' ability to solve more complex application tasks is dependent on procedural skill and fluency.
- **Application** provides valuable context for learning and the opportunity to solve problems in a relevant and a meaningful way. It is through real-world application that students learn to select an efficient method to find a solution, determine whether the solution makes sense by reasoning, and develop critical thinking skills.

A Special Note on Procedural Skill and Fluency

While speed is definitely a component of fluency, it is not necessarily speed in producing an answer; rather, fluency can be observed by watching the speed with which a student engages with a particular problem. Furthermore, fluency does not require the most efficient strategy. The standards specify grade-level appropriate strategies or types of strategies with which students should demonstrate fluency (e.g., 1.OA.C.6 allows for students to use counting on, making ten, creating equivalent but easier or known sums, etc.). It should also be noted that teachers should expect some procedures to take longer than others (e.g., fluency with the standard algorithm for division, 6.NS.B.2, as compared to fluently adding and subtracting within 10, 1.OA.C.6).

Standards identified as targeting procedural skill and fluency do not all have an expectation of automaticity and/or rote recall. Only two standards, 2.OA.B.2 and 3.OA.C.7, have explicit expectations of students knowing facts from memory. Other standards targeting procedural skill and fluency do not require students to reach automaticity. For example, in 4.G.A.2, students do not need to reach automaticity in classifying two-dimensional figures.

Recognizing the Components of Rigor

In the LSSM each standard is aligned to one or more components of rigor, meaning that each standard aims to promote student growth in conceptual understanding, procedural skill and fluency, and/or application. Key words and phrases in the standards indicate which component(s) of rigor the standard is targeting: conceptual understanding standards often use terms like *understand*, *recognize*, or *interpret*; procedural skill and fluency standards tend to use words like *fluently*, *find*, or *solve*; and application standards typically use phrases like *word problems* or *real-world problems*. Key words and phrases are underlined in each standard to help clarify the identified component(s) of rigor for each standard.

Focus in the Standards

Not all content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Louisiana Standards for Mathematical Practice. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. Students should spend the large majority of their time on the major work of the grade ( ). Supporting work ( ) and, where appropriate, additional work ( ) can engage students in the major work of the grade.

Geometry

LSSM – Geometry		Explicit Component(s) of Rigor		
Code	Standard	Conceptual Understanding	Procedural Skill and Fluency	Application
GM: G-CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	✓		
GM: G-CO.A.2	Represent transformations in the plane using, e.g., transparencies, tracing paper, or geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	✓		
GM: G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	✓	✓	
GM: G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	✓		
GM: G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	✓	✓	
GM: G-CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	✓	✓	
GM: G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	✓		
GM: G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	✓		
GM: G-CO.C.9	Prove and apply theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	✓	✓	
GM: G-CO.C.10	Prove and apply theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	✓	✓	

GM: G-CO.C.11	Prove and apply theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	✓	✓	
GM: G-CO.D.12	Make formal geometric constructions with a variety of tools and methods, e.g., compass and straightedge, string, reflective devices, paper folding, or dynamic geometric software. Examples: *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*		✓	
GM: G-CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.		✓	
GM: G-SRT.A.1	Verify experimentally the properties of dilations given by a center and a scale factor:	✓		
GM: G-SRT.A.1a	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	✓		
GM: G-SRT.A.1b	The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	✓		
GM: G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	✓	✓	
GM: G-SRT.A.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	✓		
GM: G-SRT.B.4	Prove and apply theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity; SAS similarity criteria, SSS similarity criteria, AA similarity criteria.*	✓	✓	
GM: G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	✓	✓	✓
GM: G-SRT.C.6	Understand that by similarity, side ratios in right triangles, including special right triangles (30-60-90 and 45-45-90), are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	✓		
GM: G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.	✓	✓	
GM: G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*			✓
GM: G-C.A.1	Prove that all circles are similar.	✓		
GM: G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords, including the following: *the relationship that exists between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; and a radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	✓		

GM: G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	✓	✓	
GM: G-C.B.5	Use similarity to determine that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	✓	✓	
GM: G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		✓	
GM: G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*		✓	
GM: G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	✓	✓	
GM: G-GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	✓	✓	
GM: G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*		✓	
GM: G-GMD.A.1	Give an informal argument, e.g., dissection arguments, Cavalieri's principle, and informal limit arguments, for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	✓		
GM: G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*		✓	✓
GM: G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	✓		
GM: G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	✓		
GM: G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*			✓
GM: G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*			✓
GM: S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*	✓		

GM: S-CP.A.2	Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*	✓	✓	
GM: S-CP.A.3	Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.*	✓		
GM: S-CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.**	✓	✓	✓
GM: S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.**	✓		
GM: S-CP.B.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.*	✓	✓	✓
GM: S-CP.B.7	Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model.*	✓	✓	✓

*Modeling standard